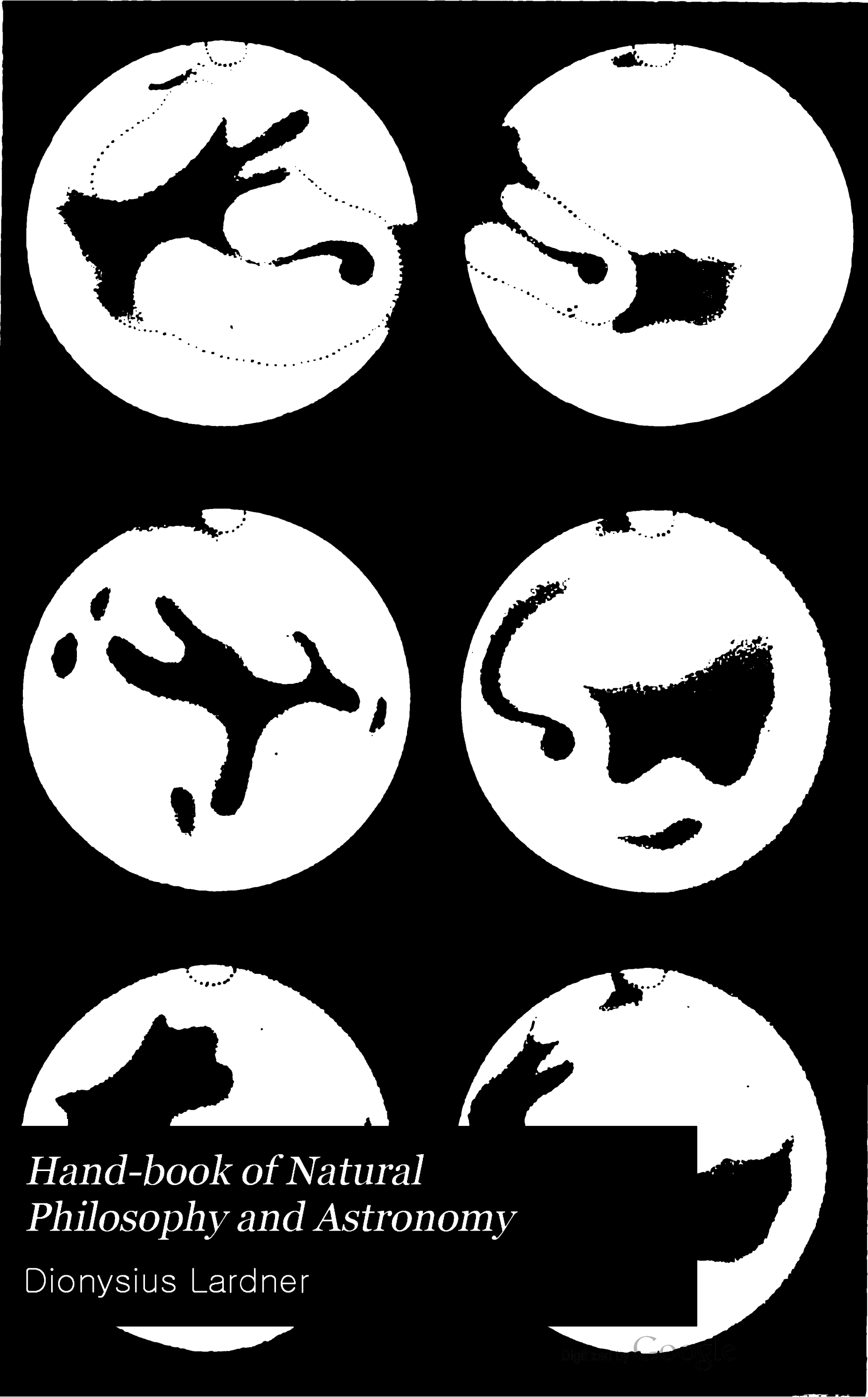

Hand-book of Natural Philosophy and Astronomy

Dionysius Lardner

HAND-BOOK

OF

NATURAL PHILOSOPHY

AND

ASTRONOMY.

BY DIONYSIUS LARDNER, D.C.L.

FORMERLY PROFESSOR OF NATURAL PHILOSOPHY AND ASTRONOMY
IN UNIVERSITY COLLEGE, LONDON.

THIRD COURSE.

METEOROLOGY — ASTRONOMY.

WITH THIRTY-SEVEN LITHOGRAPHIC PLATES, AND UPWARDS OF
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PREFACE.

To supply the means of acquiring a competent knowledge of the methods and results of the physical sciences, without any unusual acquaintance with mathematics, has been the purpose of the Author in the composition of this series of treatises. The methods of demonstration and illustration have been adopted with this view. It is, however, neither possible nor desirable invariably to exclude the use of mathematical *symbols*.

Some of these, expressing mere arithmetical operations effected upon numbers, are easily understood by all persons to whom such a work as the present is addressed, and, as they express in many cases the relations of quantities and the laws which govern them with greater brevity and clearness than ordinary language, to exclude the use of them altogether would be to deprive the reader of one of the most powerful aids to the comprehension of the laws of nature.

Nevertheless such symbols are used sparingly, and never without ample explanation of their signification. The principles of the sciences are in the main developed and demonstrated in ordinary and popular language. The series has been compiled with the view of affording that amount of information on the several subjects comprised in it which is demanded by the student in law and in medicine, by the engineer and artisan, by the superior classes in schools, and by those who having already entered on the active business of life are still desirous to sustain and extend their knowledge of the general truths of physics and of those laws by which the order and stability of the material world are maintained.

It is well known that many students who enter the Universities pass through them without acquiring even so much as a superficial knowledge of geometry and algebra. To all such persons mathematical treatises on physics and astronomy must be sealed books. They may, however, by these volumes acquire with great facility a considerable acquaintance with these sciences; and although such knowledge be not in all cases based on rigorous mathematical demonstration, it is founded on reasoning sufficiently satisfactory and conclusive.

Great pains have been taken to render the present series complete in all respects, and as nearly co-extensive with the actual state of the sciences as the objects to which it is directed admit. Each of the classes of readers for whose more especial use it is designed will doubtless find in it something which, for their purpose, is superfluous; but it must be considered that the parts which are thus superfluous for one are precisely those which are most essential for another. It is hoped that no student will find that anything important for his objects has been omitted.

The rapid succession of discoveries by which astronomy has of late years been extended has rendered elementary works in that science previously published to a certain extent obsolete, while the increasing taste for its cultivation and the multiplication of private observatories and amateur observers, have created a demand for treatises upon it which, without being less elementary in their style, shall comprise a greater amount of that vast mass of knowledge which has hitherto been shut up in the transactions of learned societies and other forms of publication equally inaccessible to the student and aspirant.

A large space has therefore been assigned to this science in the present series. The results of the researches of original inquirers and of the labour of observers have been carefully reviewed and large selections made from them are now for the first time presented to the student in an elementary form. In cases where the subject required for its better elucidation

graphic illustrations, and where such representations could be obtained from original and authentic sources, they have been unsparingly supplied.

As examples of this, we may refer among the planetary objects to the beautiful delineations of the Moon and Mars by MM. Beer and Mädler, those of Jupiter by MM. Mädler and Herschel, and those of Saturn by MM. Dawes and Schmidt; among cometary objects to the magnificent drawings of Encké's comet by Struve, and those of Halley's comet by MM. Struve, Maclear, and Smith; and among stellar objects to the splendid selection of stellar clusters and nebulæ which are reproduced from the originals of the Earl of Rosse and Sir John Herschel. In fine, among the illustrations now produced for the first time in an elementary work, the remarkable drawings of solar spots by Pastorff and Capocci ought not to be passed without notice.

To have entered into the details of the business of the observatory, beyond those explanations which are necessary and sufficient to give the reader a general notion of the processes by which the principal astronomical data are obtained, would not have been compatible with the popular character and limited dimensions of such a treatise as the present.

It has, nevertheless, been thought advisable to append to this volume a short notice of the most remarkable instruments of observation, accompanied by well executed drawings of them, the originals for some of which have been either supplied by or made under the superintendence of the eminent astronomers under whose direction the instruments are placed.

In the composition of this part of the series, it has been the good fortune of the author to detect several errors of considerable importance which have been hitherto almost universally disseminated in elementary works and under the authority of the most eminent names. Several examples of this will be noticed by the reader, among which we may refer more particularly to the Uranography of Saturn, a subject which has been hitherto completely misapprehended, phenomena being described as mani-

fested on that planet which are demonstrably impossible.* The correction of other errors less striking, though of great scientific importance, will be found in the chapter on Perturbations, and in other parts of the treatise.

This series of elementary treatises consists of three courses, which are saleable separately, and are as independent each of the others as the nature of the subject allows.

The first course consists of Mechanics — Hydrostatics — Hydraulics — Pneumatics — Sound and Optics; the second of Heat — Common Electricity — Magnetism and Voltaic Electricity; and the third of Meteorology and Astronomy.

The Index which is given in the present volume, and which will be found extremely copious and useful for the purposes of general reference, is intended to serve in common for all the three courses. The references in it are made to the numbers of the paragraphs, and not to those of the pages, and it will be found convenient to observe that the first paragraph of the second course is 1304, and that of the third 2160. The paragraphs being numbered continuously throughout the three courses, it has not been necessary in the Index to make any reference to the courses or volumes.

It will be found that this Index, combined with the analytical Table of Contents, will give to the entire series all the usefulness of a compendious Encyclopædia of Natural Philosophy and Astronomy.

* See a Memoir, by the Author, on the Uranography of Saturn, in Vol. XXII. of the Memoirs of the Royal Astronomical Society, London, Sept. 1853.

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ERRATA.

Page 21., line 13., *for* having *read* have.
27., line 14., *for* 224° *read* — 224°.
320., line 17., *for* VII. *read* VIII.
18., *for* VIII. *read* IX.
519., line 1., *for* invisible *read* visible.

HAND-BOOK

OF

NATURAL PHILOSOPHY

AND

ASTRONOMY.

THIRD COURSE.

BOOK THE FIRST.

METEOROLOGY.

CHAPTER I.

TERRESTRIAL HEAT.

2160. *Insufficiency of thermal observations.* — To ascertain the laws which regulate the distribution of heat and the periodical vicissitudes of temperature on and below the surface of the earth and in the superior strata of the atmosphere, is a problem of which the complete solution would require a collection of exact thermal observations, made not only in every part of the earth, but for a long series of years, not to say ages. Experimental research has not yet supplied such data. Observations on temperature made at periods even so recent as those within which physical science has been cultivated with more or less ardour and success, were in general scattered and unconnected, and marked neither by system nor precision. It was only since the commencement of the present century that observations on terrestrial heat were accumulated in sufficient quantity, and directed with the skill and precision indispensable

to render them the source from which the laws of temperature could be evolved. The experiments and observations of Humboldt, and the profound theoretical researches of Fourier and Laplace, supplied at once the nucleus of our present knowledge in this department of physics, and gave an impulse to enquiry, by which others have been carried forward and guided; so that if we do not yet possess all the data which sufficiently extended and long-continued observation and experiment might afford, enough has at least been done to establish with certainty some general laws which prevail in the physics of heat, and to shadow forth others which future enquirers will confirm or modify.

2161. *Local variations of temperature.* — The superficial temperature of the earth varies with the latitude, gradually decreasing in proceeding from the equator towards the poles.

It also varies with the elevation of the point of observation, decreasing in proceeding to heights above the level of the sea, and varying according to certain conditions below that level, but in all cases increasing gradually for all depths below a certain stratum, at which the temperature is invariable.

At a given latitude and a given elevation the temperature varies with the character of the surface, according as the place of observation is on sea or land; and if on land, according to the nature, productions, or condition of the soil, and the accidents of the surface, such as its inclination or aspect.

2162. *Diurnal thermometric period.* — At a given place the temperature undergoes two principal periodic variations, *diurnal* and *annual.*

The temperature falling to a minimum at a certain moment near sunrise, augments until it attains a maximum, at a certain moment after the sun has passed the meridian. The temperature then gradually falls until it returns to the minimum in the morning.

This diurnal thermometric period varies with the latitude, the elevation of the place, the character of the surface, and with a great variety of local conditions, which not only affect the hours of the maximum, minimum, and mean temperatures, but also the difference between the maximum and minimum, or the extent of the variation.

2163. *Annual thermometric period.* — The annual thermometric period also varies with the latitude, and with all the other conditions that affect the thermal phenomena.

In order to be enabled to evolve the general thermal laws from phenomena so complicated and shifting, it is above all things necessary to define and ascertain those mean conditions or states, round which the thermometric oscillations take place.

2164. *The mean diurnal temperature.*—This is a temperature so taken between the extremes, that all those temperatures which are superior to it shall exceed it by exactly as much as those which are inferior to it shall fall short of it.

To render this more clear, let us suppose that the temperature is observed every second in twenty-four hours. This would give 86,400 observed temperatures. Suppose, that of these 43,000 are above, and 43,400 below the mean temperature. If, then, the mean temperature be subtracted from each of those above it, and if each of those below it be subtracted from it, the sum of the remainders in the one case must be equal to the sum of the remainders in the other.

This is equivalent to stating that the mean temperature, multiplied by 86,400, will give the same result as would be obtained by adding together all the 86,400 observed temperatures.

But the thermometric column is not subject to such rapid changes as to show any observable difference of elevation from second to second, nor even from minute to minute. If its height be observed every hour, the mean diurnal temperature will be obtained by adding together the twelve horary temperatures, and dividing their sum by 12. But even this is not necessary, and the same result is more easily obtained, either by taking the sum of the temperatures at sunrise, at 2 P. M., and at sunset, and dividing the result by 3, or more simply still by adding together the maximum and minimum temperatures, and taking half their sum. Whichever of these methods be adopted, the same result very nearly will be obtained.

2165. *The mean temperature of the month.*—This is found by dividing the sum of the mean diurnal temperatures by the number of days.

2166. *The mean temperature of the year.*—This may be found by dividing the sum of the mean monthly temperatures by 12.

2167. *Month of mean temperature.*—It is found that in each climate there is a certain month of which the mean temperature is identical with the mean temperature of the year, or very nearly so. This circumstance, when the month is known,

supplies an easy method of observing the mean temperature of the year.

In our climate this month is October.

2168. *The temperature of the place.* — The mean annual temperature being observed in a given place for a series of years, the comparison of these means, one with another, will show whether the mean annual temperature is subject to variation, and, if so, whether the variation is periodic or progressive. All observations hitherto made and recorded tend to support the conclusion, that the variations of the mean annual temperature are, like all other cosmical phenomena, periodic, and that the oscillations are made within definite limits and definite intervals. There exists, therefore, another mean temperature superior to the annual, and which is called the *temperature of the place.* This is obtained by adding together the mean annual temperatures of all the years which constitute the thermometric period, and dividing the sum thus obtained by the number of years.

But even though the period of the variation of the mean annual temperature be not known, a near approximation to the mean temperature of the place may be obtained by adding together any attainable number of mean annual temperatures and dividing their sum by their number. The probable accuracy of the result will be greater, the less the difference between the temperatures computed.

Thus it was found by a comparison of thirty mean annual temperatures at Paris, that the mean was 51°·44, and that the difference between the greatest and least of the mean annual temperatures was only 5°·4. It may therefore be assumed that 51°·44 does not differ by so much as two-tenths of a degree from the true mean temperature of that place.

Observation, however, has been hitherto so limited, both as to extent and duration, that this thermal character has been determined for a very limited number of places. Indications, nevertheless, have been obtained sufficiently clear and satisfactory to enable Humboldt to arrive at some general conclusions, which we shall now briefly state.

2169. *Isothermal lines.*—In proceeding successively along the same meridian from the equator towards the pole, the mean temperature decreases generally, but not regularly nor uniformly. At some points it even happens that the mean

temperature augments, instead of decreasing. These irregularities are caused partly by the varying character of the surface, over which the meridian passes, and partly by the atmospheric effects produced by adjacent regions, and a multitude of other causes, local and accidental. As these causes of irregularity in the rate of decrease of the mean temperature, proceeding from the equator to the poles, are different upon different meridians, it is evident that the points of the meridians which surround the globe, at which the mean temperatures are equal, do not lie upon a parallel of latitude, as they would if the causes which affect the distribution of heat were free from all such irregularities and accidental influences.

If, then, a series of points be taken upon all the meridians surrounding the globe, having the same mean temperature, the line upon which such points are placed is called an *isothermal line.*

Each isothermal line is therefore characterized by the uniform mean temperature, which prevails upon every part of it.

2170. *Isothermal zones.*—The space included between two isothermal lines of given temperatures is called an *isothermal zone.*

The northern hemisphere has been distributed in relation to its thermal condition into six zones, limited by the six isothermal lines, characterized by the mean temperatures, 86°, 74°, 68°, 59°, 50°, 41° and 32°.

2171. *The first thermal or torrid zone.*—This zone is a space surrounding the globe, included between the equator and the isothermal line, whose temperature is 74°.

The mean temperature of the terrestrial equator is subject to very little variation, and it may therefore be considered as very nearly an isothermal line. Its mean temperature varies between the narrow limits of 81½° and 82½°.

2172. *Thermal equator.*—If, upon each meridian, the point of greatest mean temperature be taken, the series of such points will follow a certain course round the globe, which has been designated as the *thermal equator.* This line departs from the terrestrial equator, to the extent of ten or twelve degrees on the north and about eight degrees on the south side, following a sinuous and irregular course, intersecting the terrestrial equator at about 100°, and 160° east longitude.—It

attains its greatest distances north at Jamaica, and at a point in Central Africa, having a latitude of 15°, and east longitude 10° or 12°. The greatest mean temperature of the thermal equator is 86°.

The isothermal line having the temperature of 74° is not very sinuous in its course, and does not much depart from the tropics.

2173. *The second thermal zone.* — This zone, which is included between the isothermal parallels characterized by the mean temperatures of 74° and 68° is much more sinuous, and includes very various latitudes. At the points where it intersects the meridians of Europe, it is convex towards the north, and attains its greatest latitude in Algeria.

2174. *The third thermal zone.* — This zone, included between the isothermal parallels which have the mean temperatures of 68° and 59°, passes over the coasts of France upon the Mediterranean, about the latitude 43°, and from thence bends southwards, both east and west, on the east towards Nangasaki and the coasts of Japan, and on the west to Natchez on the Mississippi.

2175. *The fourth thermal zone.* — This zone is included between the parallels of mean temperatures 59° and 50°. It is convex to the north in Europe, including the chief part of France, and thence falls to the south on both sides, including Pekin on the east, and Philadelphia, New York, and Cincinnati on the west. It is evident from this arrangement of the fourth thermal zone, that the climate of Europe is warmer than that of those parts of the eastern and western continents which have the same latitude.

2176. *The fifth and sixth thermal zones.* — The sixth zone, included between the mean temperatures of 50° and 41°, is more sinuous, and includes latitudes more various even than the preceding. The thermometric observations, however, which have been hitherto made in these regions, are too limited to supply ground for any general inferences respecting it.

2177. *The polar regions.* — The circle whose area is comprised within the isothermal parallel whose mean temperature is 32°, is still less known. Nevertheless, the results of the observations made by arctic voyagers within the last twenty years, afford ground for inferring that the mean temperature of the pole itself must be somewhere from 13° to 35° below the zero

of Fahrenheit, or 45° to 67° below the temperature of melting ice.

2178. *Climate varies on the same isothermal line.*—When it is considered how different are the vegetable productions of places situate upon the same isothermal line, it will be evident that other thermal conditions besides the mean temperature must be ascertained before the climate of a place can be known. Thus London, New York, and Pekin are nearly on the same isothermal line, yet their climates and vegetable productions are extremely different.

2179. *Constant, variable, and extreme climates.* — One of the circumstances which produce the most marked difference in the climates of places having the same mean temperature is the difference between the extreme temperatures. In this respect climates are classed as *constant*, *variable*, and *extreme.*

Constant climates are those in which the maximum and minimum monthly temperatures differ but little; variable climates are those in which the difference between these extremes is more considerable, and extreme climates are those in which this difference is very great.

Constant climates are sometimes called insular, because the effect of the ocean in equalising the temperature of the air is such as to give this character to the climates of islands.

2180. *Examples of the classification of climates.* — The following examples will illustrate this classification of climates:—

Places.	Mean Temperature.	Highest mean Monthly Temperature.	Lowest mean Monthly Temperature.	Difference.
	°	°	°	°
Funchal	69	75·56	62·96	12·60
London	50·36	66·92	41·72	25·20
Paris	51·08	65·30	36·14	29·16
St. Malo	54·14	64·40	37·76	26·64
New York	53·78	80·78	25·34	55·44
Pekin	53·86	84·38	24·62	49·76

Funchal offers the example of a constant or insular climate; London, Paris, and St. Malo, of a variable; and New York and Pekin of an extreme climate.

2181. *Climatological conditions.* — A complete analysis of those conditions on which climate depends, requires also that the epochs of the extreme temperature, and, in a word, the general distribution of heat through the seasons should be stated. For this purpose we should have an exact record, not only of the extreme temperatures and the mean annual and monthly temperatures, but also the mean diurnal. The importance of such data in any climatological inquiries will be perceived when it is considered that a few degrees difference in the lowest temperature, will decide the question of the possibility of certain vegetable productions continuing to live, and the difference of a few degrees in the highest temperature will render it possible or not for certain fruits to ripen.

2182. *Table of Paris temperatures.* — The following table, published by M. Arago, shows the extremes of the temperature of the air in Paris for more than a century: —

Greatest Heat.			Greatest Cold.		
Year.	Month.	Temperature, Fahrenheit.	Year.	Month.	Temperature, Fahrenheit.
		°			°
1706	August 8.	95·5	1709	January 13.	− 9·6
1753	July 7.	96·1	1716	„ 13.	− 1·7
1754	„ 14.	95·0	1754	„ 8.	+ 6·6
1755	„ 14.	94·5	1755	„ 8.	+ 3·9
1793	„ 8.	101·1	1768	„ 8.	+ 1·2
1793	„ 16.	99·1	1776	„ 29.	− 2·4
1800	August 18.	95·9	1783	December 30.	− 2·4
1802	„ 8.	97·5	1788	„ 31.	− 8·1
1803	„ 8.	98·1	1795	January 25.	−10·3
1808	July 15.	97·2	1798	December 26.	+ 0·3
1818	„ 24.	94·1	1823	January 14.	+ 5·9

2183. *Extreme temperature in torrid zone.* — The highest temperature of the air which has been observed within the torrid zone is 130°, which was observed by MM. Lyon and Ritchie, in the Oasis of Mourzouk. This, however, is an extreme and exceptional case, the temperature, even in this zone, rarely exceeding 120°.

2184. *Extreme temperature in polar regions.*—The lowest temperatures observed by arctic voyagers in the polar regions range from 40° to 60° below zero of Fahrenheit, which is from 70° to 90° below the temperature of melting ice. Thus it appears that the air at the surface of the earth ranges between −60° and +120°, the extremes differing by 180°.

2185. *The variation of temperature depending on the elevation of the observer above the level of the sea.*—Innumerable phenomena show that the temperature of the air falls as the elevation increases. The presence of eternal snow on the elevated parts of mountain ranges, in every part of the globe, not excepting even the torrid zone, is a striking evidence of this.

Numerous observations have been made on the slopes of mountains, and by means of balloons and kites, to ascertain the law according to which the temperature falls as the height increases. Captain Parry raised a self-registering thermometer to the height of about 400 feet, by means of a kite, at Ingloolick, latitude 69° 21′. At this elevation the temperature was 55° below zero, being the same temperature as at the surface. At the equator Humboldt made an extensive series of observations, the general results of which are as follows:—

Elevation in Feet.	Mean Temperature.	Difference.
	°	°
0	81	0
3250	71	10
6500	65	6
9750	58	7
13,000	44½	13½
16,250	34¾	9¾

It appears from these observations, which were made upon the declivities of the vast mountain ranges which traverse the equatorial regions, that the decrease of temperature is neither uniform nor regular. The rate of decrease is least between the elevation of 3000 and 6000 feet. This is explained by the fact, that this stratum of the atmosphere at the Line is the habitual region of clouds. It is there that the vapours ascending from the surface, being more or less condensed, absorb a large portion of the solar heat, and it is not therefore surprising that this stratum should be cooled in a less degree than the strata consisting of air less charged with vapour.

The observations made in temperate climates give results equally irregular. Gay-Lussac found ascending in a balloon, that the thermometric column fell one degree for an elevation of about 320 feet. On the Alps the height which produces a fall of one degree is from 260 to 280 feet, and on the Pyrenees from 220 to 430 feet. It may therefore be assumed, that in the tropical regions an elevation of 300 feet, and in our latitudes

from 300 to 330 feet, corresponds to a fall of one degree of temperature on an average, subject, however, to considerable local variation.

2185. *Elevation of the limit of perpetual snow.*—It might appear that in those elevations at which the temperature falls to 32°, water cannot exist in the liquid state, and we might expect that above this limit we should find the surface invested with perpetual snow. Observation nevertheless shows such an inference to be erroneous. Humboldt in the equatorial regions, and M. Leopold de Buch in Norway and Lapland, have shown that the SNOW-LINE does not correspond with a mean temperature of 32° for the superficial atmosphere, but that on the contrary, within the tropics, it is marked by a mean temperature of about 35°, while in the northern regions, in latitudes of from 60° to 70°, the mean temperature is 26½°.

2186. *Conditions which affect it.*—It appears that the snow-line is determined not so much by the mean annual temperature of the air as by the temperature of the hottest month. The higher this temperature is, the more elevated will be the limit of perpetual snow. But the temperature of the hottest month depends on a great variety of local conditions, such as the cloudy state of the atmosphere, the nature of the soil, the inclination and aspect of the surface, the prevailing winds, &c.

2187. *Table of heights of snow-line observed.*—In the following table are collected and arranged, the results of the most important and accurate observations on the snow-line.

Observer.	Lat.	Place.	Height in Feet.	Mean Temperature.
	°			°
Humboldt - - -	0 to 10	Rucupichincha - - -	15,730	34·7
„ - - -	„	Huaupichincha - - -	„	„
„ - - -	„	Antisana - - - -	„	„
„ - - -	„	Corazon - - - -	„	„
„ - - -	„	Cotopaxi - - - -	„	„
„ - - -	„	Chimborazo - - - -	„	„
Pentland - - -	14 to 19	Eastern Cordilleras of Upper Peru	17,060	
„ - - -	„	Western ditto ditto	16,830	
Humboldt - - -	19 to 20	Oribaza - - - -	15,026	
„ - - -	„	Popocatepetl - - -	„	
„ - - -	„	Femmeblanche - - -	„	
„ - - -	„	Nevado de Toluca - - -	„	
Webb - - -	27 to 36	Himalaya (south side) - -	12,630	
„ - - -	„	„ (north side) - -	16,400	
Engelhardt and Parrot	42 to 43	Caucasus - - - -	10,550	38·3
Ramond - - -	„	Pyrenees - - - -	8950	„
„ - - -	45 to 46	Alps - - - - -	8760	39·2
Wahlenberg - -	49	Carpathians - - - -	8500	
Leopold de Buch -	61	Peak of Saletind - - -	5540	42·8
„ - - -	70	The Storvans Field - -	3480	41·0

2188. *Further results of Humboldt's and Pentland's researches.*—To these general results may be added the following observations of M. Humboldt*:—

"1. The snow-line on the Andes does not vary more than 70 to 100 feet in its elevation.

"The plains of Antisana, at an elevation of 13,800 feet, clothed with a rich vegetation of aromatic herb, are covered with a depth of three or four feet of snow for five or six weeks.

"In Quito, mean temperature 48°, snow is never seen below the elevation of 12,000 feet.

"Hail falls in the tropical regions at elevations of from 2000 to 3000 feet, but is never witnessed on the lower plateaux. It falls once in five or six years.

"No mountains have been observed in tropical Africa which rise to the snow-line.

"2. Pentland found that from 14° to 19° lat. S. the snow-line is higher than upon the Line. This might probably be explained by the nature and configuration of the surface.

"3. Between the Line and 20° lat. N. the snow-line falls only 700 feet. The variation of the height of the snow-line increases with the latitude.

"The summit of Mowna Roa (Owhyhee), Sandwich Islands, whose height exceeds 16,000 feet, is sometimes divested of snow.

"4. The elevation of the snow-line on the southern declivity of the Himalaya agrees with observations made in Mexico; but the northern declivity presents a singular anomaly, the snow line rising to 16,000 feet, a greater elevation than upon the Line.

"5. The snow-line on the Caucasus is higher by 1300 feet than on the Pyrenees, which are, nevertheless, in the same latitude.

"6. The snow-line on the chain of mountains which extend along Norway, from 58° to 70° lat., is at an elevation of 5000 feet. This great elevation in latitudes so high is probably explicable by local atmospheric phenomena, and the proximity of the sea."

2189. *Thermal phenomena below the surface.*—At a given place the surface of the ground undergoes a periodical variation of temperature, attaining a certain maximum in summer, and a

* "Notice on the Snow-line," *Ann. de Ch. et Phys.* tom. xiv. p. 1.

minimum in winter, and gradually, but not regularly or uniformly, augmenting from the minimum to the maximum, and decreasing from the maximum to the minimum.

The question then arises as to whether this periodic variation of temperature is propagated downwards through the crust of the earth, and if so, whether in its descent it undergoes any and what modifications?

To explain the phenomena which have been ascertained by observation, let us express the mean temperature by M, and let the maximum and minimum temperatures be T and *t*.

If we penetrate to depths more or less considerable, we shall find that the mean temperature M of the strata will be very nearly the same as at the surface. The extreme temperatures T and *t*, will, however, undergo a considerable change, T decreasing, and *t* increasing. Thus the extremes gradually approach each other as the depth increases, the mean M remaining nearly unaltered.

2190. *Stratum of invariable temperature.* — A certain depth will therefore be attained at length, when the maximum temperature T, by its continual decrease, and the minimum temperature *t*, by its continual increase, will become respectively equal to the mean temperature M. At this depth, therefore, the periodical variations at the surface disappear; and the mean temperature M is maintained permanently without the least change.

This mean temperature, however, though nearly is not precisely equal to the mean temperature at the surface. In descending M undergoes a slight increase, and at the depth where T and *t* become equal to M, and the variation disappears, the mean temperature is a little higher than the mean temperature of the surface.

2191. *Its depth varies with the latitude.* — The depth at which the superficial vicissitudes of temperature disappear varies with the latitude, with the nature of the surface, and other circumstances. In our climates it varies from 80 to 100 feet. It diminishes in proceeding towards the equator, and increases towards the pole. The excess of the permanent temperature at this depth above the mean temperature at the surface, increases with the latitude.

2192. *Its depth and temperature at Paris.* — The same thermometer which has been kept for sixty years in the vaults of

the Observatory at Paris, at the depth of eighty-eight feet below the surface, has shown, during that interval, the temperature of 11°·82 Cent., which is equal to $53\frac{1}{4}$° Fahr., without varying more than half a degree of Fahr., and even this variation, small as it is, has been explained by the effects of currents of air produced by the quarrying operations in the neighbourhood of the Observatory.

2193. *Its form.* — We must therefore infer, that within the surface of the earth there exists a stratum of which the temperature is invariable, and so placed that all strata superior to it are more or less affected by the thermal vicissitudes of the surface, more so the nearer they are to the surface, and that this stratum of invariable temperature has an irregular form, approaching nearer to the surface at some places, and receding further from it at others, the nature and character of the surface, mountains, valleys, and plains, seas, lakes, and rivers, the greater or less distance from the equator or poles, and a thousand other circumstances, imparting to it variations of form, which it will require observations and experiments much more long continued and extensive than have hitherto been made, to render manifest.

2194. *Thermal phenomena between the surface and the stratum of invariable temperature.* — The thermometric observations on the periodical changes which take place above the stratum of invariable temperature are not so numerous as could be desired: nevertheless, the following general conditions have been ascertained, especially in the middle latitudes of the northern hemisphere:—

1. The diurnal variations of temperature are not sensible to a greater depth than $3\frac{1}{2}$ feet.

2. The difference $T - t$ between the extreme temperatures of the strata decreases in geometrical progression for depths measured in arithmetical progression, or nearly so.

3. At the depth of 25 feet, $T - t = 2°$. At 50 feet $T - t = 0·2$; and at 60 to 80 feet, $T - t = 0°·02$.

4. Since the effects of the superficial variation must require a certain time to penetrate the strata, it is evident that the epoch at which each stratum attains its maximum and minimum temperatures will be different from those at which the other strata and the surface attain them. The lower the strata the greater will be the difference between the times of

attaining those limits, as compared with the surface. Thus, it is found, that at the depth of twenty-five feet the maximum is not attained until the surface has attained its minimum. The seasons, therefore, at this depth are reversed, the temperature of July being manifested in January, and *vice versâ.*

2195. *Thermal phenomena below the stratum of uniform temperature.* — The same uniformity of temperature which prevails in the invariable stratum is also observed at all greater depths; but the temperature increases with the depth. Thus, each successive stratum, in descending, has a characteristic temperature, which never changes. The rate at which this temperature augments with the depth below the invariable stratum is extremely different in different localities. In some there is an increase of one degree for every thirty feet, while in others the same increase corresponds to a depth of 100 feet. It may be assumed, in general, that an increase of one degree of temperature will take place for every fifty or sixty feet of depth.

2196. *Temperature of springs.* — The permanency of the temperatures of the inferior strata is rendered manifest by the uniformity of the temperature of springs, of which the water rises from any considerable depths. At all seasons of the year the water of such springs maintains the same uniform temperature.

It may be assumed that the temperature of the water proceeding from such springs is that of the strata from which they rise. In these latitudes it is found in general to be a little above the mean temperature of the air for ordinary springs, that is from those which probably rise from strata not below the invariable stratum. In higher latitudes the excess of temperature is greater, a fact which is in accordance with what has been already explained.

It has not been certainly ascertained whether the hot springs, some of which rise to a temperature little less than that of boiling water, derive their heat from the great depth of the strata from which they rise, or from local conditions affecting the strata. The uniformity of the temperature of many of them appears to favour the former hypothesis; but it must not be forgotten that other geological conditions besides mere depth may operate with the same permanency and regularity.

2197. *Thermal conditions of seas and lakes.* — The anomalous quality manifested in the dilatation of water when its

temperature falls below 38°·8 Fahr. (1395), and its consequent maximum density at that temperature, is attended with most remarkable and important consequences in the phenomena of the waters of the globe, and in the economy of the tribes of organised creatures which inhabit them. It is easy to show that, but for this provision, exceptional and anomalous as it seems, disturbances would take place, and changes ensue, which would be attended with effects of the most injurious description in the economy of nature.

If a large collection of water, such as an ocean, a sea, or a lake, be exposed to continued cold, so that its superficial stratum shall have its temperature constantly reduced, the following effects will be manifested.

The superficial stratum falling in temperature, will become heavier, volume for volume, than the strata below it, and will therefore sink, the inferior strata rising and taking its place. These in their turn being cooled will sink, and in this manner a continual system of downward and upward currents will be maintained, by means of which the temperature of the entire mass of liquid will be continually equalized and rendered uniform from the surface to the bottom. This will continue so long as the superficial stratum is rendered heavier, volume for volume, than those below it, by being lowered in temperature. But the superficial stratum, and all the inferior strata, will at length be reduced to the uniform temperature of 38°·8. After this the system of currents upwards and downwards will cease. The several strata will assume a state of repose. When the superficial stratum is reduced to a temperature lower than 38°·8 (which is that of the maximum density of water), it will become lighter, volume for volume, instead of being heavier than the inferior strata. It will therefore float upon them. The stratum immediately below it, and in contact with it, will be reduced in temperature, but in a less degree; and in like manner a succession of strata, one below the other, to a certain depth, will be lowered in temperature by the cold of those above them, but each stratum being lighter than those below, will remain at rest, and no interchange by currents will take place between stratum and stratum. If water were a good conductor of heat, the cooling effect of the surface would extend downwards to a considerable depth. But water being, on the

contrary, an extremely imperfect conductor, the effect of the superficial temperature will extend only to a very limited depth; and at and below that limit, the uniform temperature of 38°·8, that of the greatest density, will be maintained.

This state of repose will continue until the superficial stratum falls to 32° *, after which it will be congealed. When its surface is solidified, if it be still exposed to a cold lower than 32°, the temperature of the surface of the ice will continue to fall, and this reduced temperature will be propagated downward, diminishing, however, in degree, so as to reduce the temperature of the stratum on which the ice rests to 32°, and therefore to continue the process of congelation, and to thicken the ice.

If ice were a good conductor of heat, this downward process of congelation would be continued indefinitely, and it would not be impossible that the entire mass of water from the surface to the bottom, whatever be the depth, might be solidified. Ice, however, is nearly as bad a conductor of heat as water, so that the superficial temperature can be propagated only to a very inconsiderable depth, and it is found accordingly, that the crust of ice formed even on the surface of the polar seas, does not exceed the average thickness of twenty feet.

2198. *Thermal condition of a frozen sea.*— The thermal condition, therefore, of a frozen sea, is a state of molecular repose, as absolute as if the whole mass of liquid were solid. The temperature at the surface of the ice being below the freezing point, increases in descending until it rises to the freezing point, at the stratum where the ice ceases, and the liquid water commences. Below this the temperature still augments until it reaches 38°·8, the temperature of maximum density of water, and this temperature is continued uniform to the bottom.

2199. *Process of thawing.* — Let us now consider what effects will be produced, if the superficial strata be exposed to an increase of temperature. After the fusion of the ice, the temperature of the surface will gradually rise from 32° to 38°·8, the temperature of greatest density. When the superficial stratum rises above 32°, it will become heavier than the stratum under it, and an interchange by currents, and a con-

* For sea water the freezing point is 28½°.

sequent equalization of temperature, will take place, and this will continue until the superficial stratum attain the temperature of 38°·8, when the temperature of the whole mass of water from the surface to the bottom will become uniform.

After this a further elevation of the temperature of the superficial stratum will render it lighter than those below it, and no currents will be produced, the liquid remaining at rest; and this state of repose will continue so long as the temperature continues to rise.

Every fall of the superficial temperature, so long as it continues above 38°·8, will be attended with an interchange of currents between the superficial and those inferior strata whose temperature is above 38°·8, and a consequent equalization of temperature.

2200. *Depth of stratum of constant temperature in oceans and seas.*—It appears, therefore, to result as a necessary consequence from what has been explained, and this inference is fully confirmed by experiment and observations, that there exists in oceans, seas, and other large and deep collections of water, a certain stratum, which retains permanently, and without the slightest variation, the temperature of 38°·8, which characterizes the state of greatest density, and that all the inferior strata equally share this temperature. At the lower latitudes, the superior strata have a higher, at the higher latitudes a lower temperature, and at a certain mean latitude the stratum of invariable temperature coincides with the surface.

In accordance with this, it has been found by observation that in the torrid zone, where the superficial temperature of the sea is about 83°, the temperature decreases with the depth until we attain the stratum of invariable temperature, the depth of which, upon the Line, is estimated at about 7000 feet. The depth of this stratum gradually diminishes as the latitude increases, and the limit at which it coincides with the surface is somewhere between 55° and 60°. Above this the temperature of the sea increases as the depth of the stratum increases, until we sink to the stratum of invariable temperature, the depth of which at the highest latitudes (at which observations have been made) is estimated at about 4500 feet.

2201. *Effect of superficial agitation of the sea extends to only a small depth.*—It might be imagined that the tempera-

ture of the surface would be propagated downwards, and that a thermal equalization might therefore be produced by the intermixture of the superior with the inferior strata, arising from the agitation of the surface of the waters by atmospheric commotions. It is found, however, that these effects, even in the case of the most violent storms and hurricanes, extend to no great depth, and that while the surface of the ocean is furrowed by waves of the greatest height and extent, the inferior strata are in the most absolute repose.

2202. *Destructive effects which would be produced if water had not a point of maximum density above its point of congelation.*—If water followed the general law, in virtue of which all bodies become more dense as their temperature is lowered, a continued frost might congeal the ocean from its surface to the bottom, and certainly would do so in the polar regions; for in that case the system of vertical currents, passing upwards and downwards and producing an equalization of temperature, which has been shown to prevail above 38°·8, would equally prevail below that point, and consequently the same equalization of temperature would be continued, until the entire mass of water, from the surface to the bottom, would be reduced to the point of congelation, and would consequently be converted into a solid mass, all the organized tribes inhabiting the waters being destroyed.

The existence of a temperature of maximum density at a point of the thermometric scale above the point of congelation of water, combined with the very feeble conducting power of water, whether in the liquid or solid state, renders such a catastrophe impossible.

2203. *Variations of the temperature of the air at sea and on land.*—The air is subject to less extreme changes of temperature at sea than on land. Thus, in the torrid zone, while the temperature on land suffers a diurnal variation amounting to 10°, the extreme diurnal variation at sea does not exceed 3½°. In the temperate zone the diurnal variation at sea is limited generally to about 5½°, while on continents it is very various and everywhere considerable. In different parts of Europe it varies from 20° to 25°.

At sea as on land the time of lowest temperature is that of sunrise, but the time of greatest heat is about noon, while on land it is at two or three hours after noon.

On comparing the temperature of the air at sea with the superficial temperature of the water, it has been found that between the tropics the air, when at its highest temperature, is warmer than the water, but that its mean diurnal temperature is lower than that of the water.

In latitudes between 25° and 50° the temperature of the air is very rarely higher than that of the water, and in the polar regions the air is never found as warm as the surface of the water. It is, on the contrary, in general at a very much lower temperature.

2204. *Interchange of equatorial and polar waters.*— Much uncertainty prevails as to the thermal phenomena manifested in the vast collections of water which cover the greater part of the surface of the globe. It appears, however, to be admitted that the currents caused by the difference of the pressures of strata at the same level in the polar and equatorial seas, produce an interchange of waters, which contributes in a great degree to moderate the extreme thermal effects of these regions, the current from the pole reducing the temperature of the equatorial waters, and that from the line raising the temperature of the polar waters and contributing to the fusion of the ice. A superficial current directed from the line towards the poles carries to the colder regions the heated waters of the tropics, while a counter current in the inferior strata carries from the poles towards the line the colder waters. Although the prevalence of these currents may be regarded as established, they are nevertheless modified, both in their intensity and direction, by a multitude of causes connected with the depth and form of the bottom, and the local influence of winds and tides.

2205. *Polar ice.*— The stupendous mass of water in the solid state which forms an eternal crust encasing the regions of the globe immediately around the poles, presents one of the grandest and most imposing classes of natural phenomena. The observations and researches of Captain Scoresby have supplied a great mass of valuable information in this department of physical geography.

2206. *Extent and character of the ice fields.*—Upon the coasts of Spitzbergen and Greenland vast fields of ice are found, the extent of which amounts to not less than twelve to fifteen hundred square miles, the thickness varying from twenty to twenty-five feet. The surface is sometimes so even that a sledge can

run without difficulty for an hundred miles in the same direction. It is, however, in some places, on the contrary, as uneven as the surface of land, the masses of ice collecting in columns and eminences of a variety of forms, rising to heights of from twenty to thirty feet, and presenting the most striking and picturesque appearances. These prodigious crystals sometimes exhibit gorgeous tints of greenish blue, resembling the topaz, and sometimes this is varied by a thick covering of snow upon their summits, which are marked by an endless variety of form and outline.

2207. *Production of icebergs by their fracture.* — These vast ice fields are sometimes suddenly broken, by the pressure of the subjacent waters, into fragments presenting a surface of from 100 to 200 square yards. These being dispersed, are carried in various directions by currents, and sometimes by the effect of intersecting currents they are brought into collision with a fearful crash. A ship, which might chance in such a case to be found between them could no more resist their force than could a glass vessel the effect of a cannon ball. Terrible disasters occur from time to time from this cause. It is by the effects of these currents upon the floating masses of broken ice that these seas are opened to the polar navigators. It is thus that whalers are enabled to reach the parallels from 70° to 80°, which are the favourite resort of those monsters of the deep which they pursue.

2208. *Their forms, and magnitude.* — Sometimes after such collisions new icebergs arise from the fragments which are heaped one upon another, "Pelion on Ossa," more stupendous still than those which have been broken. In such cases the masses which result assume forms infinitely various, rising often to an elevation of thirty to fifty feet above the surface of the water; and since the weight of ice is about four-fifths of the weight of its own bulk of water (787), it follows that the magnitude of these masses submerged is four times as great as that which is above the surface. The total height of these floating icebergs, therefore, including the part submerged, must be from 150 to 250 feet.

2209. *Sunken icebergs.* — It happens sometimes that two such icebergs resting on the extremities of a fragment of ice 100 or 120 feet in length, keep it sunk at a certain depth below the surface of the water. A vessel in such cases may

sail between the icebergs and over the sunken ice; but such a course is attended with the greatest danger, for if any accidental cause should detach either of the icebergs which keep down the intermediate mass while the ship is passing, the latter by its buoyancy will rise above the surface, and will throw up the ship with irresistible force.

2210. *Singular effects of their superficial fusion.* — Icebergs are observed in Baffin's Bay of much greater magnitude than off the coast of Greenland. They rise there frequently to the height of 100 to 130 feet above the surface, and their total height, including the part immersed, must therefore amount to 500 or 650 feet. These masses appear generally of a beautiful blue colour, and having all the transparency of crystals. During the summer months, when the sun in these high latitudes never sets, a superficial fusion is produced, which causes immense cascades, which, descending from their summit and increasing in volume as they descend, are precipitated into the sea in parabolic curves. Sometimes, on the approach of the cold season, these liquid arches are seized and solidified by the intensity of the cold without losing their form, and seem as if caught in their flight between the brink from which they were projected and the surface, and suddenly congealed. These stupendous arches, however, do not always possess cohesion in proportion to their weight, and after augmenting in volume to a certain limit, sink under their weight, and, breaking with a terrific crash, fall into the sea.

2211. *Depth of polar seas.* — The depth of the seas off the coast of Greenland is not considerable. Whales, being harpooned, often plunge in their agony to the bottom, carrying with them the harpoon and line attached to it. When they float they bear upon their bodies evidence of having reached the bottom by the impression they retain of it, and the length of line they carry with them in such cases shows that depth does not exceed 3000 or 4000 feet. About the middle of the space between Spitzbergen and Greenland the soundings have reached 8000 feet without finding bottom.

2212. *Cold of the polar regions.* — The degree of cold of the polar regions, like the temperature of all other parts of the globe, depends on the extent and depth of the seas. If there be extensive tracts of surface not covered by water, or covered

only by a small depth, the influence of the water in moderating and equalizing the temperature is greatly diminished. Hence it is that the temperature of the south polar regions is more moderate than that of the north. After passing the latitude of the New Orcades and the New Shetlands, which form a barrier of ice, the navigator enters an open sea, which, according to all appearance, extends to the pole. Much, however, still remains to be discovered respecting the physical condition of these regions.

2213. *Solar and celestial heat.* — Whatever may be the sources of internal heat, the globe of the earth would, after a certain time, be reduced to a state of absolute cold, if it did not receive from external sources the quantity of heat necessary to repair its losses. If the globe were suspended in space, all other bodies from which heat could be supplied to it being removed, the heat which now pervades the earth and its surrounding atmosphere would be necessarily dissipated by radiation, and would thus escape into the infinite depths of space. The temperature of the atmosphere, and those of the successive strata, extending from the surface to the centre of the globe, would thus be continually and indefinitely diminished.

As no such fall of temperature takes place, and as, on the contrary, the mean temperature of the globe is maintained at an invariable standard, the variations incidental to season and climate being all periodical, and producing in their ultimate result a mutual compensation, it remains to be shown from what sources the heat is derived which maintains the mean temperature of the globe at this invariable standard, notwithstanding the large amount of heat which it loses by radiation into the surrounding space.

All the bodies of the material universe, which are distributed in countless numbers throughout the infinitude of space, are sources of heat, and centres from which that physical agent is radiated in all directions. The effect produced by the radiation of each of these diminishes in the same proportion as the square of its distance increases. The fixed stars are bodies analogous to our sun, and at distances so enormous that the effect of the radiation of any individual star is altogether insensible. When, however, it is considered that the multitude of these stars spread over the firmament is so prodigious that in some

places many thousand are crowded together within a space no greater than that occupied by the disc of the full moon, it will not be matter of surprise that the feebleness of thermal influence, due to their immense distances, is compensated to a great extent by their countless number; and that, consequently, their calorific effects in those regions of space through which the earth passes in its annual course is, as will presently appear, not only far from being insensible, but is very little inferior to the calorific power of the sun itself.

We are, then, to consider the waste of heat which the earth suffers by radiation as repaired by the heat which it receives from two sources, the sun and the stellar universe; and it remains to explain what is the actual quantity of heat thus supplied to the earth, and what proportion of it is due to each of these causes.

2214. *Quantity of heat emitted by the sun.*—An elaborate series of experiments were made by M. Pouillet, and concluded in 1838, with the view of obtaining, by means independent of all hypothesis as to the physical character of the sun, an estimate of the actual calorific power of that luminary. A detailed report of these observations and experiments, and an elaborate analysis of the results derived from them, appeared in the Transactions of the Academy of Sciences of Paris for that year.

It would be incompatible with the elementary nature and the consequent limits of this work, to enter into the details of these researches. We shall, therefore, confine ourselves here briefly to state their results.

When the firmament is quite unclouded, the atmosphere absorbs about one-fourth of the heat of those solar rays, which enter it vertically. A greater absorption takes place for rays which enter it obliquely, and the absorption is augmented in a certain ascertained proportion, with the increase of obliquity. It results from the analysis of the results obtained in the researches of M. Pouillet, that about forty per cent. of all the heat transmitted by the sun to the earth, is absorbed by the atmosphere, and that consequently only sixty per cent. of this heat reaches the surface. It must, however, be observed that a part of the radiant heat, intercepted by the atmosphere, raising the temperature of the air, is afterwards transmitted, as well by radiation as by contact, from the atmosphere to the earth.

By means of direct observation and experiment made with instruments contrived by him, called *pyrheliometers*, by means of which the heat of the solar radiation was made to affect a known weight of water at a known temperature, M. Pouillet ascertained the actual quantity of heat which the solar rays would impart per minute to a surface of a given magnitude, on which they would fall vertically. This being determined, it was easy to calculate the quantity of heat imparted by the sun in a minute to the hemisphere of the earth which is presented to it, for that quantity is the same which would be imparted to the surface of the great circle which forms the base of that hemisphere, if the solar rays were incident perpendicularly upon it.

2215. *Solar heat at the earth would melt a shell of ice* 100 *feet thick in a year.* — In this manner it was ascertained by M. Pouillet, that if the total quantity of heat which the earth receives from the sun in a year were uniformly diffused over all parts of the surface, and were completely absorbed in the fusion of a shell of ice encrusting the globe, it would be sufficient to liquefy a depth of 100 feet of such shell.

Since a cubic foot of ice weighs 54 lbs., it follows that the average annual supply of heat received from the sun per square foot of the earth's surface would be sufficient to dissolve 5400 lbs. weight of ice.

2216. *Calculation of the actual quantity of heat emitted by the sun.* — This fact being ascertained supplies the means of calculating the quantity of heat emitted from the surface of the sun, independently of any hypothesis respecting its physical constitution.

It is evident from the uniform calorific effects produced by the solar rays at the earth, while the sun revolves on its axis exposing successively every side to the earth in the course of about twenty-five days, that the calorific emanation from all parts of the solar surface is the same. Assuming this, then, it will follow, that the heat which the surface of a sphere surrounding the sun at the distance of the earth would receive would be so many times more than the heat received by the earth as the entire surface of such sphere would be greater than that part of it which the earth would occupy. The calculation of this is a simple problem of elementary geometry.

But such a spherical surface surrounding the sun and con-

centrical with it, would necessarily receive all the heat radiated by that luminary, and the result of the calculation proves that the quantity of heat emitted by the sun per minute is such as would suffice to dissolve a shell of ice enveloping the sun, and having a thickness of $38\frac{6}{10}$ feet; and that the heat emitted per day would dissolve such a shell, having a thickness of 55748 feet, or about $10\frac{1}{2}$ miles.

2217. *Heat at sun's surface seven times as intense as that of a blast furnace.* — The most powerful blast furnaces do not emit for a given extent of fire surface more than the seventh part of this quantity of heat. It must therefore be inferred that each square foot of the surface of the sun emits about seven times as much heat as is issued by a square foot of the fire surface of the fiercest blast furnace.

2218. *Temperature of the celestial spaces.*—When the surface of the earth during the night is exposed to an unclouded sky, an interchange of heat takes place by radiation. It radiates a certain part of the heat which pervades it, and it receives, on the other hand, the heat radiated from two sources, 1st, from the strata of atmosphere, extending from the surface of the earth to the summit of the atmospheric column, and 2d, from the celestial spaces, which lie outside this limit, and which receive their heat from the radiation of the countless numbers of suns which compose the stellar universe. M. Pouillet, by a series of ingeniously contrived experiments and observations, made with the aid of an apparatus contrived by him, called an *actinometer*, has been enabled to obtain an approximate estimate of the proportion of the heat received by the earth which is due to each of these two sources, and thereby to determine the actual temperature of the region of space through which the earth and planets move. The objects and limits of this work do not permit us to give the details of these researches, and we must therefore confine ourselves here to the statement of their results.

It appears from the observations, that the actual temperature of space is included between the minor limit of 315°, and the major limit of 207° below the temperature of melting ice, or between −283° and −175° Fahr. At what point between these limits the real temperature lies, is not yet satisfactorily ascertained, but M. Pouillet thinks that it cannot differ much from −224° Fahr.

2219. *Heat received by earth from celestial space would melt, in a year, eighty-five feet thick of ice.*—It is proved from these results, that the quantity of heat imparted to the earth in a year, by the radiation of the celestial space, is such as would liquefy a spherical shell of ice, covering the entire surface of the earth, the thickness of which would be eighty-five feet, and that forty per cent. of this quantity is absorbed by the atmosphere.

Thus the total quantity of heat received annually by the earth is such as would liquefy a spherical shell of ice 185 feet thick, of which 100 feet are due to the sun, and 85 feet to the heat which emanates from the stellar universe.

The fact that the celestial spaces supply very little less heat to the earth annually than the sun, may appear strange, when the very low temperature of these spaces is considered, a temperature 180° lower than the cold of the pole during the presence of the sun. It must, however, be remembered that while the space from which the solar radiation emanates, is only that part of the firmament occupied by the disc of the sun, that from which the celestial radiation proceeds is the entire celestial sphere, the area of which is about five million times greater than the solar disc. It will therefore cease to create surprise, that the collective effect of an area so extensive should be little short of that of the sun.

The calorific effect due to the solar radiation, according to the calculations and observations of M. Pouillet, exceeds that which resulted from the formulæ of Poisson. These formulæ were obtained from the consideration of the variation of the temperature of the strata of the earth at different depths below the surface. M. Pouillet thinks that the results proceeding from the two methods would be brought into accordance if the influence of the atmosphere on solar heat, which, as appears from what has been explained, is very considerable, could be introduced in a more direct manner into Poisson's formulæ.

2220. *Summary of the thermal effects.*—In fine, therefore, the researches of M. Pouillet give the following results, which must be received as mere approximations subject to correction by future observation :

1st. That the sun supplies the earth annually with as much heat as would liquefy 100 feet thick of ice covering the entire globe.

2d. That the celestial spaces supply as much as would liquefy 85 feet thick.

3d. That 40 per cent. of the one and the other supply is absorbed by the atmosphere, and 60 per cent. received by the earth.

4th. That of the heat radiated by the earth, 90 per cent. is intercepted by the atmosphere, and 10 per cent. dispersed in space.

5th. That the heat evolved on the surface of the sun in a day would liquefy a shell of ice 10½ miles thick, enveloping the sun, and the intensity of the solar fire is seven times greater than that of the fiercest blast furnace.

6th. That the temperature of space outside the atmosphere of the earth is 224° Fahr., or 256° below that of melting ice.

7th. That the solar heat alone, constitutes only two-thirds of the entire quantity of heat supplied to the earth to repair its thermal losses by terrestrial radiation ; and that without the heat supplied by stellar radiation, the temperature of the earth would fall to a point which would be incompatible with organic life.

CHAP. II.

THE AIR AND ATMOSPHERIC VAPOURS.

2221. *Periodical changes in the atmospheric pressure.* — The periodical changes to which the pressure of the atmosphere is subject, and the principal causes which produce them have been already briefly indicated (719. *et seq.*). We shall now explain more fully some of the more important of these phenomena.

It has been customary in these climates to observe and register the height of the barometric column four times a day, at 9 A.M., at noon, at 3 P.M., and at 9 P.M.

The mean monthly and mean annual heights are obtained from a comparison of the noon observations. The diurnal period is obtained from a comparison of the morning and afternoon observations.

2222. *Mean annual height of barometer.* — The mean height of the barometer at Paris obtained from observations continued

from 1816 to 1836, has been ascertained to be 29·764 inches. The mean annual height during this period did not vary so much as twelve hundredths of an inch.

2223. *Effect of winds on the barometric column.* — It has been found that the barometric column is affected by the direction and continuance of the wind, but these effects are not the same in all localities. At Paris, the height is greatest when the wind blows from the north or north-east, and least when from the south and south-west. The extreme difference of the mean heights during such winds was found to be twenty-seven hundredths of an inch. Observations made at Metz by Schuster gave a like result, but with a little less difference. At Marseilles, however, no such effect has been observed, but rather a tendency to a contrary change, the height being generally above the mean in southerly winds, and below it in north-westerly.

2224. *Diurnal variations of the barometer.* — A long series of observations on the diurnal changes in the barometer establish the existence of two periods, a period of decrease from 9 A.M. to 3 P.M., and a period of increase from 3 P.M. to 9 P.M. The mean amount of the former, taken from ten years' observation at Paris, was 0·0294 in., and of the latter 0·0146 in. The decrease from 9 A.M. to 3 P.M. is therefore less than the thirtieth of an inch, and the increase from 3 P.M. to 9 P.M. less than the sixtieth of an inch.

A comparison of these variations in different seasons of the year shows that the increase of the evening is subject to very minute and irregular changes, but that the changes of the decrease in the morning are both more considerable and more regular, the amount of the decrease being always least in November, December, and January, and greatest in February, March, and April.

During the night the barometer falls from 9 P.M. to 4 A.M., and rises from 4 A.M. to 9 A.M.

2225. *The winds.* — No meteorological phenomenon has had so many observers, and there is none of which the theory is so little understood, as the winds. The art of navigation has produced in every seaman an observer, profoundly interested in the discovery of the laws which govern a class of phenomena, upon the knowledge of which depends not only his professional success but his personal security, and the lives and property committed to his charge.

The chief part of the knowledge which has been collected

respecting the causes which produce these atmospheric currents is derived, nevertheless, much more from the comparison of the registers of observatories than from the practical experience of mariners.

2226. *Winds by compression and rarefaction.* — Winds are propagated either by *compression* or by *rarefaction*. In the former case they are developed in the same direction in which they blow; in the latter case they are developed in the contrary direction. To render this intelligible, let us imagine a column of air included in a tube. If a piston inserted in one end of the tube be driven from the mouth inwards, the air contiguous to it will be compressed, and this portion of air will compress the succeeding portion, and so on; the compression being propagated from the end at which the piston enters toward the opposite end. The remote end being open, the air will flow in a current driven before the piston in the same direction in which the compression is propagated.

If we imagine, on the other hand, a piston inserted in the tube at some distance from its mouth, to be drawn outwards toward the mouth, the air behind it will expand into the space deserted by the piston, and a momentary rarefaction will be produced. The next portion of air will in like manner follow that which is next the piston, the rarefaction which begins at the piston being propagated backwards through the tube in a direction contrary to the motion of the piston and that of the current of air which follows it.

What is here supposed to take place in the tube is exhibited on a larger scale in the atmosphere. Any physical cause which produces a compression of the atmosphere from north to south will produce a north wind; and any cause which produces a rarefaction from north to south will produce a south wind.

2227. *Effect of sudden condensation of vapour.* — Of all the causes by which winds are produced, the most frequent is the sudden condensation of vapour suspended in the atmosphere. In general the atmosphere above us consists of a mixture of air properly so called, and water, either in the state of vapour, or in a vesicular state, the nature and origin of which has not yet been clearly ascertained. In either case its sudden conversion into the liquid state, and its consequent precipitation to the earth, leaves the space it occupied in the atmosphere a vacuum, and a corresponding rarefaction of the air previously

mixed with the vapour ensues. The adjacent strata immediately rush in to re-establish the equilibrium of pneumatic pressure, and winds are consequently produced.

The propagation of winds by rarefaction manifested in directions contrary to that of the winds themselves, is common in the North of Europe. Wargentin gives various examples of this. When a west wind springs up, it is felt, he observes, at Moscow before it reaches Abo, although the latter city is four hundred leagues west of Moscow, and it does not reach Sweden until after it has passed over Finland.

2228. *Hurricanes.*—The intertropical regions are the theatre of hurricanes. It is there only that these atmospheric commotions are displayed in all their terrors. In the temperate zones tempests are not only more rare in their occurrence but much less violent in their force. In the circumpolar zone the winds seldom acquire the force which would justify the title of a storm.

The hurricanes of the warm climates spread over a considerable width, and extend through a still more considerable length. Some are recorded which have swept over a distance of four or five hundred leagues with a nearly uniform violence.

It is only by recounting the effects produced by these vast commotions of the atmospheric ocean, that any estimate can be formed of the force which air, attenuated and light as that fluid is, may acquire when a great velocity is given to it. In hurricanes such as that which took place at Guadaloupe on the 25th July, 1825, houses the most solidly constructed were overthrown. A new building erected in the most durable manner by the government was rased to the ground. Tiles carried from the roof were projected against thick doors with such force as to pass through them like a cannon ball. A plank of wood 3½ feet long, 9 inches wide, and an inch thick, was projected with such force as to cut through a branch of palm wood 18 inches in diameter. A piece of wood 15 feet long and 8 inches square in its cross section, was projected upon a hard paved road, and buried to a depth of more than three feet in it. A strong iron gate in front of the governor's house was carried away, and three twenty-four pounders erected on the fort were dismounted.

2229. *The probable causes explained.* — These effects, prodigious as they are, all arise from mechanical causes. There is no agent engaged in hurricanes more subtle than the me-

chanical force of air in motion, and since the weight and density of the air suffer no important change, the vast momentum manifested by such effects as those described above, must be ascribed altogether to the extraordinary velocity imparted to the air by the magnitude of the local vacuum produced, as already stated, by the sudden condensation of vapour. To form some approximate estimate of this it may be stated that, in the intertropical regions, a fall of rain often takes place over a vast extent of surface, sufficient in quantity to cover it with a stratum of water more than an inch in depth. If such a fall of rain were to take place over the extent of a hundred square leagues, as sometimes happens, the vapour from which such a quantity of liquid would be produced by condensation would, at the temperature of only 50°, occupy a volume 100,000 times greater than that of the liquid; and, consequently, in the atmosphere over the surface of 100 square leagues it would fill a space 9000 feet, or nearly two miles in length. The extent of the vacuum produced by its condensation would be a volume nearly equal to 200 cubic miles, or to the volume of a column whose base is a square mile and whose height is 200 miles.

2230. *Water spouts and land spouts.* — These phenomena, called water or land spouts according as they are manifested at sea or on land, consist apparently of dense masses of aqueous vapour and air, having at once a gyratory and progressive motion, and resembling in form a conical cloud, the base of which is presented upwards, and the vertex of which generally rests upon the ground, but sometimes assumes a contrary position. This phenomenon is attended with a sound like that of a waggon rolling on a rough pavement.

Violent mechanical effects sometimes attend these meteors. Large trees torn up by the roots, stripped of their leaves, and exhibiting all the appearances of having been struck by lightning, are projected to great distances. Houses are often thrown down, unroofed, and otherwise injured or destroyed, when they lie in the course of these meteors. Rain, hail, and frequently globes of fire, like the ball lightning, also accompany them.

The various appearances exhibited by water spouts are represented in *fig.* 669.

No satisfactory theory has yet connected these phenomena with the general laws of physics.

2231. *Evaporation from the surface of water.* — If the surface of a sea, lake, or other large collection of water were exposed to the atmosphere consisting of pure air without any admixture of vapour, evaporation would immediately commence, and the vapour developed at the surface of the water would ascend into and mix with the atmosphere. The pressure of the atmosphere would then be the sum of the pressures of the atmosphere, properly so called, and of the vapour suspended in it, since neither of these elastic fluids can augment or diminish the pressure of the other.

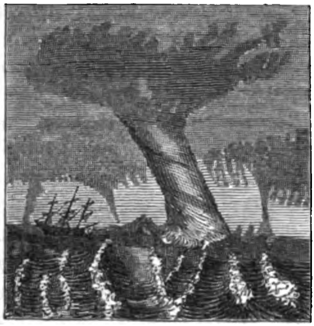


Fig. 669.

The vapour developed from the surface of the water thus mingling with the atmosphere, acquires a common temperature with it. This vapour, therefore, receiving thus from the air with which it is intermixed more or less heat, after having passed into the vaporous state, is *superheated vapour* (1496.). It has, therefore, a greater temperature than that which corresponds to its density, or, what is the same, it has a less density than that which corresponds to its temperature. Such vapour may therefore lose temperature to a certain extent without being condensed.

2232. *Air may be saturated with vapour.* — But if the same atmosphere continue to be suspended over the surface of water, the process of evaporation being continued, the quantity of vapour which rises into the air and mingles with it will be continually increased until it acquires the greatest density which is compatible with its temperature. Evaporation must then cease, and the air is said to be *saturated* with vapour.

If the temperature of the air in such case rise, evaporation will recommence and will continue until the vapour shall acquire the greatest density compatible with the increased temperature, and will then cease, the air being, as before, *saturated.*

2233. *If the temperature of saturated air fall, condensation will take place.* — But if the temperature fall, the greatest density of vapour compatible with it being less than at the higher

temperature, a part of the vapour must be condensed, and this condensation must continue until the vapour suspended in the air shall be reduced to that state of density which is the greatest compatible with the reduced temperature.

2234. *Atmosphere rarely saturated.* — A fluid so light and mobile as the atmosphere, can never remain long in a state of repose, and the column of air suspended over the surface of any collection of water however extensive, is subject to frequent change. In general, therefore, before any such portion of the atmosphere become saturated by evaporation, it is removed and replaced by another portion. It happens, consequently, that the atmosphere rarely becomes saturated by the immediate effect of evaporation.

2235. *May become so by reduced temperature or intermingling strata.*—The state of saturation is, however, often attained either by loss of temperature, or by the intermixture of strata of air of different temperatures and differently charged with vapours. Thus, if air which is below the point of saturation suffer a loss of heat, its temperature may fall to that point which is the highest compatible with the density of the vapour actually suspended in it. The air will then become saturated, not by receiving any increased quantity of vapour, but by losing that caloric by which the vapour it contained was previously *superheated*.

If two strata of air at different temperatures, and both charged with vapour to a point below saturation, be intermingled, they will take an intermediate temperature, that which had the higher temperature imparting a portion of its heat to that which had a lower temperature. The vapour with which they were previously charged will likewise be intermixed and reduced to the common temperature. Now, in this case it may happen that the common temperature to which the entire mass is reduced, after intermixture, shall be either equal to or less than the greatest temperature compatible with the density of the vapour in the mass of air thus mixed. If it be equal to that temperature, the mass of air after intermixture will be *saturated*, though the strata before intermixture were both below saturation; and if less, condensation must take place until the density of the vapour suspended in the mixture be reduced to the greatest density compatible with the temperature.

2236. *Air and vapour intermingle though of different specific gravities.* — It might be supposed that air and vapour being mixed together without combining chemically, would arrange themselves in strata, the lighter floating above the heavier as oil floats above water. This statical law, however, which prevails in liquids, is in the case of elastic fluids subject to important qualifications. The latter class of fluids have a tendency to intermingle and diffuse themselves through and among each other in opposition to their specific gravities. Thus if a stratum of hydrogen, the lightest of the gases, rest upon a stratum of carbonic acid, which is the heaviest, they will by slow degrees intermingle, a part of the hydrogen descending among the carbonic acid, and a part of the carbonic acid ascending among the hydrogen, and this will continue until the mixture becomes perfectly uniform, every part of it containing the two gases in the proportion of their entire quantities.

The same law prevails in the case of vapours mixed with gases; and thus may be explained the fact, that although the aqueous vapour suspended in the air, and having the same temperature, is always lighter bulk for bulk than the air, it does not ascend to the upper strata of the atmosphere, but is uniformly diffused through it.

2237. *The pressure of air retards, but does not diminish evaporation.* — It may be stated generally, that the effect of a column of air superposed upon the surface of water is only to retard, but not either to prevent or diminish, the evaporation. The same quantity of vapour will be developed as would be produced at the same temperature if no air were superposed on the water; but while in the latter case the entire quantity of vapour would be developed instantaneously, it is produced gradually, and completed only after a certain interval of time when the air is present. The quantity of vapour developed, and its density and pressure, are however exactly the same, whether the space through which it is diffused be a vacuum, or be filled by air, no matter what the density of the air may be. The properties of the air, therefore, neither modify nor are modified by those of the vapour which is diffused through it.

2238. *When vapour intermixes with air, it renders it specifically lighter.* — Since, at the same temperature and pres-

sure, the density of the vapour of water is less than that of air in the ratio of 5 to 8, it follows that when air becomes charged with vapour of its own temperature, the volume will be augmented, but the density diminished. If a certain volume of air weigh 8 grains, an equal volume of vapour will weigh 5 grains, the two volumes mixed together will weigh 13 grains, and, consequently, an equal volume of the mixture will weigh 6½ grains. In this case, therefore, the density of the air charged with vapour is less than the density of dry air of the same temperature in the ratio of 6½ to 8.

CHAP. III.

HYGROMETRY.

2239. *Hygrometry.*—This is the name given to that branch of meteorology which treats of the methods of measuring the elastic force and the quantity of aqueous vapour which is suspended in the atmosphere, and in which the influence of various natural bodies and physical agents upon this vapour is explained.

If the atmosphere were always charged with vapour to saturation, the pressure and density of the vapour contained in it would be immediately determined by its temperature, for there would then be the greatest pressure and density compatible with the temperature, and the pressure and density would be given by the tables (1494).

2240. *The dew point.*—But when the air, as generally happens, is not saturated, it becomes necessary to contrive means by which the temperature to which it must be reduced, in order to become saturated by the quantity of vapour actually suspended in it, can be determined.

Such temperature is called the DEW POINT, inasmuch as after reduction below that temperature, more or less condensation, and the consequent deposition of moisture or DEW, will take place.

2241. *Method of determining the pressure and density of the*

vapour suspended in the air. — When the actual temperature of the air and the dew point are known, the pressure and density of the vapour suspended in the air may be found.

Let T express the temperature of the air, t the dew point, P the pressure of the vapour which would saturate the air at the temperature T, p the pressure of the vapour which would saturate it at the temperature t, and, in fine, let P′ express the pressure of the vapour actually suspended in the air.

This pressure P′ is greater than the pressure p, which the same vapour having the same density has at the temperature t, by that increase of pressure which is due to the increase of temperature from t to T. If the increase of pressure due to one degree of augmented temperature be expressed by n, the increase due to $(T-t)$ degrees will be expressed by $(T-t)\times n$. Hence, we shall have

$$P' = p \times \{1 + (T-t) \times n\}$$

So that when p, the pressure of the saturating vapour at the dew point, is known, P′, the actual pressure, can be found.

But any means by which the temperature t at the due point can be determined, will necessarily also determine the pressure p, inasmuch as this pressure is that which corresponds to vapour having the greatest density compatible with the temperature t, and is therefore given by the tables (1494). This being found, P′ may be computed by the preceding formula.

To find the density of the vapour actually suspended in the air, or, what is the same, the weight of water in the state of vapour contained in a cubic foot of air, let this weight be expressed by W′, and let W express the weight of vapour which would saturate a cubic foot of air at the temperature T.

Since the pressure is proportional to the density when the temperature is the same, we shall have

$$P : P' :: W : W';$$

Therefore,

$$W' = W \times \frac{P'}{P} = \frac{W}{P} \times p \times \{1 + (T-t) \times n\}$$

By this formula, therefore, the weight W′ of vapour contained in a cubic foot of air can be found, provided the weight and pressure of the vapour which would saturate it at the same temperature, its dew point, and the pressure of the vapour which would saturate it at that point, are severally known.

2242. *Table of pressures and densities of saturating vapours.*—The following table, in which are given the pressure and weight of the saturating vapour in a cubic foot of air, at the several temperatures expressed in the first column, will supply all the data necessary for such calculations, provided only that means be obtained for determining by experiment the dew point.

TABLE showing the Pressure and Weight of saturating Vapour contained in a Cubic Foot of Air at Temperatures varying from −4° Fahr. to +104° Fahr.

Temperature.	Pressure: Inches, Mercury.	Weight of Vapour in a Cubic Foot of Air.	Temperature.	Pressure: Inches, Mercury.	Weight of Vapour in a Cubic Foot of Air.
°		Grains.	°		Grains.
− 4·0	·05	1	66·2	·64	7
5·0	·08	1	68·0	·68	7
14·0	·10	1	69·8	·72	8
23·0	·15	2	71·6	·76	8
32·0	·20	2	73·4	·81	9
33·8	·22	2	75·2	·86	9
35·6	·23	3	77·0	·91	10
37·4	·24	3	78·8	·96	10
39·2	·26	3	80·6	1·02	11
41·0	·28	3	82·4	1·08	12
42·8	·30	3	84·2	1·14	12
44·6	·32	4	86·0	1·21	13
46·4	·34	4	87·8	1·28	14
48·2	·36	4	89·6	1·35	14
50·0	·38	4	91·4	1·43	15
51·8	·40	5	93·2	1·51	16
53·6	·43	5	95·0	1·59	17
55·4	·45	5	96·8	1·68	18
57·2	·48	5	98·6	1·77	18
59·0	·51	6	100·4	1·87	19
60·8	·54	6	102·2	1·97	20
62·6	·58	6	104·0	2·09	21
64·4	·61	7			

2243. *Example of such a calculation.*—As an example of the application of the preceding formulæ, let us suppose that the temperature of the air is 77°, and that the dew point is ascertained to be 54½°.

By the preceding table then we obtain the following data:—

$$\text{T}=77^{\circ}, \qquad \text{P}=0{\cdot}91, \qquad t=54\tfrac{1}{2}, \qquad p=0{\cdot}44.$$

But it appears from what has been already explained (1495), that $n=0{\cdot}002037=\frac{1}{490}$.

Hence we find

$$\text{P}'=0{\cdot}44\times\{1+22{\cdot}5+0{\cdot}002037\}=0{\cdot}46.$$

It follows, therefore, that the actual pressure of the vapour suspended in the air is 46 per cent. of the pressure of the vapour which would saturate it.

We have also

$$P = 0{\cdot}96, \qquad W = 10:$$

And therefore

$$W' = 5{\cdot}05.$$

2244. *Method of ascertaining the dew point.*—To determine the dew point let a thin glass or decanter be filled with water, and, immersing a thermometer in it, let it be exposed in the open air. Let ice cold water be poured into it by small quantities and mixed with it, so as to reduce its temperature by slow degrees below that of the surrounding air. A temperature will at length be attained at which a cloudy deposition of moisture will be manifested on the external surface of the glass. The temperature at which this effect first begins to be manifested is the DEW POINT.

To explain this it must be considered that the shell of air in immediate contact with the glass is reduced to the temperature of the glass, and when that temperature has been reduced so low that the vapour suspended in the air saturates it, any further diminution of temperature is attended with condensation, which is in effect manifested by the dew which then immediately begins to collect upon the surface of the glass.

2245. *Daniel's Hygrometers.*—Hygrometers have been constructed in different forms, on this principle, to indicate the dew point. That of Daniel has been most generally adopted. This instrument consists of a glass tube, having a thin bulb blown on each end of it, and being bent into the rectangular form represented in *fig.* 670. The bulb *a* is filled to two-thirds of its capacity with ether, which being boiled produces vapour which fills the tube *t* and the bulb *b*, and escapes through a small opening in the bottom of *b*. In this manner the air is expelled from the ether, the tube, and the bulbs. The opening in *b* is then closed with the blowpipe, and the heat being removed from the bulb *a*, the vapour in the tube and bulb *b* is condensed, so that the space within the instrument above the surface of the ether contains only the vapour of ether, which corresponds to the temperature of the fluid in the bulb *a*. A thermometer is

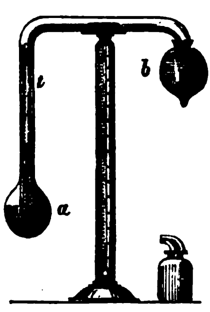


Fig. 670.

previously inserted in the tube *t*, the bulb of which is plunged in the ether, and the bulb *b* is surrounded by a linen or muslin cloth, which, being saturated with ether by means of a small phial provided with a fine rectangular spout, evaporation takes place, by which the bulb *b* is cooled. The vapour of the ether which fills the bulb *b* is thus condensed in it, and more vapour flows in to fill its place from the tube *t*. The surface of the ether in *a* being thus continually released from the pressure of the vapour condensed, further evaporation and a consequent depression of the temperature of the fluid in the bulb *a* ensues, and this continues until the temperature of the bulb *a* is reduced to the dew point, when a cloudy deposition will be manifested on the glass of the bulb *a*.

2246. *August's Psychrometer.*—Professor August of Berlin has constructed an hygrometer, the indications of which depend on the depression of temperature produced by evaporation in an atmosphere which is below the point of saturation. Two thermometers, exactly alike in all respects, are mounted on a support in immediate juxtaposition, the bulb of one being enveloped in a cloth, which is kept constantly wetted with distilled water. If the atmosphere were already saturated no evaporation would ensue; but if it be not saturated evaporation will take place from the wet cloth surrounding the bulb, and a depression of temperature will be indicated, which will bear a certain relation to the rate of this evaporation. The thermometer therefore, enveloped in the wet cloth, will fall below the other, which gives the true temperature of the air, and the difference between the two thermometers thus becomes a measure of the rate of evaporation from the cloth, and thereby of the degree of dryness of the air. The greater the quantity of vapour with which the air is charged, the less will be the difference of the temperatures indicated by the two thermometers.

When the air is extremely dry, the difference between the two thermometers sometimes amounts to from 14° to 18°.

Professor August has constructed tables by which the pressure of the vapour suspended in the air, which corresponds to the various indications of the two thermometers, can be immediately found.

2247. *Saussure's Hygrometer.*—Hygrometric substances are those porous bodies whose affinity for moisture is so strong, that when they are exposed to an atmosphere in which more or

less vapour is suspended, they will attract this vapour and condense it in their pores, so that they will become wet. The quantity of moisture which they imbibe in this manner is more or less, according to the quantity of vapour with which the atmosphere is charged.

The varying absorption of vapour causes in some bodies a corresponding variation of dimensions. The hygrometer of Saussure is founded on this property. A hair well prepared and deprived of all greasy matter is attached to a point of suspension, and being carried round a small wheel is kept extended by suspending to its extremity a small weight. Being hygrometric, it absorbs moisture from the atmosphere, by which it is made to contract and shorten its length. This causes the wheel round which it is coiled to turn through a corresponding space, which is shown by an index fixed upon the centre of the wheel, which plays upon a graduated arch.

As the vapour suspended in the air increases or diminishes, the contraction of the hair varies in corresponding manner, and the index shows the changes, indicating extreme dryness at one extremity of the scale, and extreme humidity at the other.

Tables have been constructed by which the indications of this instrument give the pressure of the vapour suspended in the air.

2248. *Dew.*—The evaporation produced during the day by the action of solar heat on the surface of water, and on all bodies charged with moisture, causes the atmosphere at the time of sunset to be more or less charged with vapour, especially in the warm season. On hot days, and in the absence of winds, the atmosphere at sunset is generally at or near the point of saturation.

Immediately after sunset the temperature of the air falls. If it were previously in a state of saturation condensation must ensue, which will be considerable if the heat of the day and the consequent change of temperature after sunset be great. In such case, the vapour condensed often assumes the appearance of a fine rain or mist taking the liquid form before its actual deposition on the surface.

The deposition of dew, however, also takes place even where the atmosphere is not reduced to its point of saturation. When the firmament is unclouded after sunset, all objects which are good radiators of heat, among which the foliage and flowers of

vegetables are the foremost, lose by radiation the heat which they had received before sunset without receiving any heat from the firmament sufficient to replace it. The temperature of such objects, therefore, falls much below that of the air, on which they produce an effect precisely similar to that which a glass of very cold water produces when exposed to a warm atmosphere charged with vapour. The air contiguous to their surface being reduced to the dew point by contact with them, a part of the vapour which it holds in suspension is condensed, and collects upon them in the form of dew.

It follows from this reasoning, that the dew produced by the fall of temperature of the air below the point of saturation will be deposited equally and indifferently on the surfaces of all objects exposed in the open air, but that which is produced by the loss of temperature of objects which radiate freely, will only be deposited on those surfaces which are good radiators. Foreign writers on physics accordingly class these depositions as different phenomena, the former being called by French meteorologists *serein*, and the latter *rosée* or dew. We are not aware that there is in English any term corresponding to *serein*.

Dew will fail to be deposited even on objects which are good radiators, when the firmament is clouded. For although heat be radiated as abundantly from objects on the surface of the earth as when the sky is unclouded, yet the clouds being also good radiators, transmit heat, which being absorbed by the bodies on the earth, compensates for the heat they lose by radiation, and prevents their temperature from falling so much below that of the air as to produce the condensation of vapour in contact with them.

Wind also prevents the deposition of dew by carrying off the air from contact with the surface of the cold object before condensation has time to take place. Meanwhile, by the contact of succeeding portions of air, the radiator recovers its temperature.

In general, therefore, the conditions necessary to insure the deposition of dew is, 1st, a warm day to charge the air with vapour; 2d, an unclouded night; 3d, a calm atmosphere; and, 4th, objects exposed to it which are good radiators of heat.

In the close and sheltered streets of cities the deposition of dew is rarely observed, because there the objects are necessarily exposed to each other's influence, and an interchange of heat by

radiation takes place so as to maintain their temperature; besides which, the objects found there are not as strong radiators as the foliage and flowers of vegetables.

2249. *Hoar frost.*—When the cold which follows the condensation of vapour falls below 32°, what would otherwise be DEW becomes HOAR FROST. For the same reason that dew is deposited when the temperature of the air is above the point of saturation, hoar frost may be manifested when the temperature of the air is many degrees above the point of congelation; for in this case, as in that of dew, the objects on which the hoar frost collects lose so much heat by their strong radiation, that while the atmosphere may be above 40° they will fall below 32°. In such cases, a dew is first deposited upon them which soon congeals, and forms the needles and crystals with which every observer is familiar.

The hoar frost is sparingly or not at all formed upon the naked earth, or on stones or wood, while it is profusely collected on leaves and flowers. The latter are strong, the former feeble radiators.

Glass is a good radiator. The panes of a window fall during the night to a temperature below 32°, although the air of the room be at a much higher temperature. Condensation and a profuse deposition of moisture takes place on their inner surfaces, which soon congeals and exhibits the crystallized coating so often witnessed.

The frosts of spring and autumn, which so frequently are attended with injury to the crops of the farmer and gardener, proceed generally not from the congelation of moisture deposited from the atmosphere, but from the congelation of their own proper moisture by the radiation of their temperature caused by the nocturnal radiation, which in other cases produces dew or hoar frost. The young buds of leaves and flowers in spring, and the grain and fruit in autumn, being reduced by radiation below 32°, while the atmosphere is many degrees above that temperature, the water which forms part of their composition is frozen, and blight ensues.

These principles, which serve to explain the cause of the evil, also suggest its remedy. It is only necessary to shelter the object from exposure to the unclouded sky, which may be done by matting, gauze, and various other expedients.

2250. *Fabrication of ice in hot climates.* — In tropical cli-

mates the principle of nocturnal radiation has supplied the means of the artificial production of ice. This process, which is conducted on a considerable scale in Bengal, where some establishments for the purpose employ several hundred men, consists in placing water in shallow pans of unglazed pottery in a situation which is exposed to the clear sky and sheltered from currents of air. Evaporation is promoted by the porous quality of the pans which become soaked with water, and radiation takes place at the same time both from the water and the pans. Both these causes combine in lowering the temperature of the water in the pans, which congeals when it falls below 32°.

2251. *Fogs and clouds.* — When the steam issuing from the surface of warm water ascends into air which is at a lower temperature, it is condensed, but the particles of water formed by such condensation are so minute, that they float in the air as would the minute particles of an extremely fine dust. These particles lose their transparency by reason of their minuteness, according to a general law of physical optics. The vapour of water is transparent and colourless. It is only when it loses the character and qualities of true vapour, that it acquires the cloudy and semi-opaque appearance just mentioned.

Fogs are nothing more than such condensed vapour produced from the surface of seas, lakes, or rivers, when the water has a higher temperature than the stratum of air which rests upon it. These fogs are more thick and frequent when the air, besides having a lower temperature than the water, is already saturated with vapour, because in that case all the vapour developed must be immediately condensed, whereas, if the air be not saturated, it will absorb more or less of the vapour which rises from the water.

Clouds are nothing but fogs suspended in the more elevated strata of the atmosphere. Clouds are most frequently produced by the intermixture of two strata of air, having different temperatures and differently charged with vapour, the mixture being supersaturated, and therefore being attended with partial condensation as already explained (2235).

2252. *Rain.* — When condensation of vapour takes place in the upper strata of the atmosphere, a fog or mist is first produced, after which the aqueous particles coalescing form themselves in virtue of the attraction of cohesion into spherules, and fall by their gravity to the earth, producing the phenomenon of rain.

2253. *Rain gauge.*—An instrument by which the quantity of rain which falls upon an area of given magnitude, at a given place, within a given time, is called a RAIN GAUGE or UDOMETER.

These instruments, which vary in form, in magnitude, and in the provisions by which the quantity falling is measured and registered, consist, in general, of a cylindrical reservoir of known diameter, the bottom of which being funnel-shaped, terminates in a discharge pipe, through which the contents pass into a close vessel. The quantity received from time to time by this vessel is measured and indicated by a great variety of expedients.

2254. *Quantity of rain falling in various places.*—The quantity of rain which falls in a given time at a given place, is expressed by stating the depth which it would have if it were received upon a plane and level surface, into which no part of it would penetrate.

At Paris, the average annual quantity of rain which falls, obtained from observations continued for thirty years at the Observatory, is 23·6 inches. There is, however, considerable variation in the quantities from which this average is deduced; the smallest quantity observed being 16·9 inches, and the greatest 27·9 inches.

The greatest annual fall of rain is that observed at Maranham, lat. $2\frac{1}{2}°$ S., which is stated by Humboldt to amount to 277 inches, more than double the annual quantity hitherto observed elsewhere. The following are the annual quantities at the under-named places:—

	In.		*In.*
St. Domingo	120	Petersburg	18·2
Cayenne	116	Rome	31·2
Island Granada	112	Rotterdam	22·4
Havannah	91	Stockholm	18·7
Calcutta	76 to 118	Vienna	17·0
Bombay	83 to 96	Alais (35 years)	39·0
Martinique	87	Algiers (10 years)	36·0
Sierra Leone	86	Bourdeaux (7 years)	33·5
Berlin	20·9	Chalons (43 years)	23·5
Brussels	19·0	Dijon (34 years)	27·6
Florence	41·3	Dieppe (8 years)	32·5
Lyons	39·5	Metz (22 years)	26·0
Maestricht	36·1	Nantes (7 years)	64·5
Marseilles	18·4	Orange (30 years)	29·5
Padua	36·6	Jerwas (6 years)	50·0

	In.		In.
Rouen (3 years) - -	38·5	Liverpool - - -	34·12
St. Lo (3 years) - -	31·5	Lancaster - - -	39·71
Toulouse (8 years) - -	25·0	Glasgow - - -	21·33
Basin of the Rhone (4 yrs.)	35·0	London (Dalton) - -	20·69
Kendal - - - -	53·94	„ (Howard) -	24·90
Dumfries - - -	36·92	York - - - -	25·70
Manchester - - -	36·14	Edinburgh - - -	25·00

Among the exceptional pluvial phenomena, the following may be mentioned:—

At Bombay, six inches of rain fell in a single day.

At Cayenne, ten inches fell in ten hours.

At Genoa, on the 25th of Oct. 1822, thirty inches of rain fell on the occurrence of a water spout. This is the greatest fall of rain on record.

2255. *Snow.*—The physical conditions which determine the production of snow are not ascertained. It is not known whether the flakes as they fall are immediately produced by the congelation of condensed vapour in the cloud whence they first proceed, or whether being at first minute particles of frozen vapour, they coalesce with other frozen particles in falling through the successive strata of the air, and thus finally attain the magnitude which they have on reaching the ground.

The only exact observations which have been made on snow refer to the forms of the crystals composing it, which Captain Scoresby has observed with great accuracy in his Polar Voyages, and of which he has given drawings. The flakes appear to consist of fine needles, grouped with singular symmetry. A few of the most remarkable forms are represented in *fig.* 671.

2256. *Hail.*—The physical causes which produce this formidable scourge of the agriculturist are uncertain. Hypotheses have been advanced to explain it which are more or less plausible, but which do not fulfil the conditions that would entitle them to the place of physical causes. Volta proposed a theory, which has obtained some celebrity, and which is characterized by the ingenuity that marked every physical investigation of that great philosopher. Two strata of clouds, each charged with vapour, and with opposite electricities, are supposed to be carried by different atmospheric currents at different elevations to such a position, that one is vertical above the other, and separated from it by a stratum of the atmosphere of

a certain thickness. Assuming that condensation and congelation is produced in the superior cloud, and that hailstones of

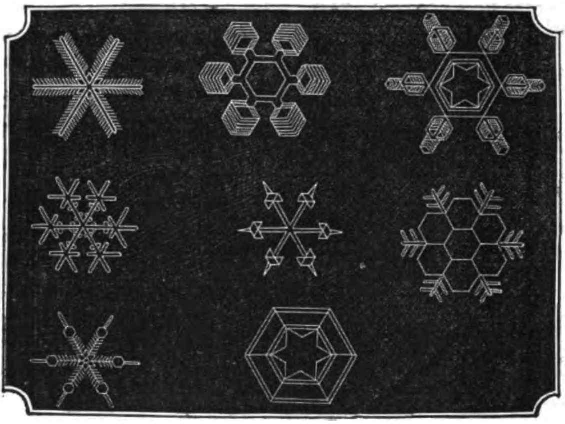


Fig. 671.

small magnitude result directly from the congelation of particles of water, these fall in a shower upon the inferior cloud, where their electricity is first neutralized by an equal charge of the contrary fluid, and they are then charged with that fluid, when they are repelled upwards, and rise again to the superior cloud, where like effects ensue, and they fall again to the inferior cloud, and so continue to rise and fall between the two clouds upon the same principle as the pith-balls move in the experiment described in (1794). The gradual increase of magnitude of the hailstones during this reverberation between the two clouds is thus explained by Volta: — When they fall from the superior upon the inferior cloud they penetrate it to a certain depth, and because of their low temperature, the vapour condenses and congeals upon their surface, thus increasing their volume. The same effect is produced when they rise again to the superior cloud, and is repeated each time that they pass to and fro from cloud to cloud, until the weight of the stones become so great that it resists the electric attraction, and they then fall to the earth.

Volta also explained how two clouds might thus be charged with contrary electricities, by the effect of solar heat in pro-

ducing evaporation, and by the assumption that vaporization develops positive and condensation negative electricity. This explanation is inadmissible, inasmuch as it is now established that evaporation and condensation are only attended with the development of electricity when they cause decomposition. However, as it is well ascertained that clouds are frequently charged with opposite electricities, this part of the hypothesis of Volta might be received without objection as a possibility. But even admitting this, the hypothesis cannot be regarded as more than an ingenious conjecture.

2257. *The phenomena attending hailstorms.*—In the absence of any satisfactory explanation of the phenomenon, it is important to ascertain with precision and certainty the circumstances which attend it, and the conditions under which it is produced.

It may then, in the first place, be considered as certain that the formation of hail is an effect of sudden electrical changes in clouds charged with vapour; for there is no instance known of hail which is not either preceded or accompanied by thunder and lightning.

Before the fall of hail, during an interval more or less, but sometimes of several minutes' duration, a rattling noise is generally heard in the air, which has been compared to that produced by shaking violently bags of nuts.

Hail falls much more frequently by day than by night. Hail clouds have generally great extent and thickness, as is indicated by the obscuration they produce. They are observed also to have a peculiar colour, a grey having sometimes a reddish tint. Their form is also peculiar, their inferior surfaces having enormous protuberances, and their edges being indented and ragged.

These clouds are often at very low elevations. Observers on mountains very frequently see a hail cloud below them.

It appears, from an examination of the structure of hailstones, that at their centre there is generally an opaque nucleus, resembling the spongy snow that forms sleet. Round this is formed a congealed mass, which is semi-transparent. Sometimes this mass consists of a succession of layers or strata. These layers are sometimes all transparent, but in different degrees. Sometimes they are alternately opaque and semi-transparent.

2258. *Extraordinary examples of hailstones.*—Extraordinary reports of the magnitude of hailstones, which have fallen during storms so memorable as to find a place in general history, have come down from periods of antiquity more or less remote. According to the Chronicles, a hailstorm occurred in the reign of Charlemagne, in which hailstones fell which measured fifteen feet in length by six feet in breadth, and eleven feet in thickness; and under the reign of Tippoo Saib, hailstones equal in magnitude to elephants are said to have fallen. Setting aside these and like recitals, as partaking rather of the character of fable than of history, we shall find sufficient to create astonishment in well authenticated observations on this subject.

In a hailstorm which took place in Flintshire on the 9th April, 1697, Halley saw hailstones which weighed five ounces.

On 4th May, 1697, Robert Taylor saw fall hailstones measuring fourteen inches in circumference.

In the storm which ravaged Como on 20th August, 1787, Volta saw hailstones which weighed nine ounces.

On 22d May, 1822, Dr. Noggerath saw fall at Bonn hailstones which weighed from twelve to thirteen ounces.

It appears, therefore, certain that in different countries hailstorms have occurred in which stones weighing from half to three quarters of a pound have fallen.

CHAP. IV.

ATMOSPHERIC ELECTRICITY.

2259. *The air generally charged with positive electricity.*—The terrestrial globe which we inhabit is invested with an ocean of air the depth of which is about the 200th part of its diameter. It may therefore be conceived by imagining a coating of air, the tenth of an inch thick, investing a twenty inch globe. This aerial ocean, relatively shallow as it is, at the bottom of which the tribes of organized nature have their

dwelling, is nevertheless the theatre of stupendous electrical phenomena.

It may be stated as a general fact, that the atmosphere which thus covers the globe is charged with positive electricity, which, acting by induction on the superficial stratum of the globe on which it rests, decomposes the natural electricity, attracting the negative fluid to the surface and repelling the positive fluid to the inferior strata. The globe and its atmosphere may therefore be not inaptly compared to a Leyden phial, the outer coating of which being placed in connexion with the prime conductor of a machine, is charged with positive electricity, and the inner coating being in connexion with the ground, is charged by induction with negative electricity. The outer coating represents the atmosphere, and the inner the superficial stratum of the globe.

2260. *This state subject to variations and exceptions.* — This normal state of the general atmospheric ocean is subject to variations and exceptions, variations of intensity and exceptions in quality or name. The variations are periodical and accidental. The exceptions local, patches of the general atmosphere in which clouds float being occasionally charged with negative electricity.

2261. *Diurnal variations of electrical intensity.* — The intensity of the electricity with which the atmosphere is charged varies, in the course of twenty-four hours, alternately increasing and decreasing. It begins to decrease at a few minutes after sunrise, and continues to decrease, until two or three o'clock in the afternoon, when it attains a minimum. It then increases and continues to increase until some minutes after sunset, when it attains a maximum. After that it again decreases, attaining a minimum at a certain time in the night, which varies in different places and different seasons, after which it again increases and attains a maximum at a few minutes after sunrise.

In general, in winter, the electricity of the air is more intense than in summer.

2262. *Observations of Quetelet.* — These were the general results of the extensive series of observations on atmospheric electricity made by Saussure. More recently they have been confirmed by the observations of M. Quetelet, which have been continued without interruption daily at the Observatory of

Brussels for the last ten years. M. Quetelet found that the first maximum was manifested about 8 A. M., and the second about 9 P. M. The minimum in the day was at 3 P. M. He found also that the mean intensity was greatest in January and least in June.

Such are the normal changes which the electrical condition of the air undergoes when the atmosphere is clear and unclouded. When, however, the firmament is covered with clouds, the electricity is subject during the day to frequent and irregular changes not only in intensity but in name; the electricity being often negative, owing to the pressure of clouds over the place of observation charged some with positive and some with negative electricity.

2263. *Irregular and local variations and exceptions.* — The intensity of the electricity of the air is also affected by the season of the year, and by the prevalent character and direction of the winds; it varies also with the elevation of the strata, being in general greater in the higher than in the lower regions of the atmosphere. The intensity is generally greater in winter, and especially in frosty weather, than in summer, and when the air is calm than when winds prevail.

Atmospheric deposits, such as rain, hail, snow, &c., are sometimes positive and sometimes negative, varying with the direction of the wind. North winds give positive, and south winds negative deposits.

2264. *Methods of observing atmospheric electricity.* — The electricity of the atmosphere is observed by erecting in it, to any desired elevation, pointed metallic conductors, from the lower extremities of which wires are carried to electroscopes of various forms, according to the intensity of the electricity to be observed. All the usual effects of artificial electricity may be reproduced by such means; sparks may be taken, light bodies attracted and repelled, electrical bells, such as those described in (1792), affected; and, in fine, all the usual effects of the fluid produced. So immediate is the increase of electrical tension in rising through the strata of the air, that a gold leaf electroscope properly adapted to the purpose, and reduced to its natural state when placed horizontally on the ground, will show a sensible divergence when raised to the level of the eyes.

2265. *Methods of ascertaining the electrical condition of the higher strata.* — To ascertain the electrical condition of strata

too elevated to be reached by a fixed conductor, the extremity of a flexible wire, to which a metallic point is attached, is connected with a heavy ball, which is projected into the air by a gun or pistol, or to an arrow projected by a bow. The projectile, when it attains the limit of its flight, detaches the wire from the electroscope, which then indicates the electrical state of the air at the highest point attained by the projectile.

The expedient of a kite, used with so much success by Franklin, Romas, and others, to draw electricity from the clouds, may also be adopted with advantage, more especially in cases where the atmospheric strata to be examined are at a considerable elevation.

2266. *Remarkable experiments of Romas*, 1757. — The vast quantities of electricity with which the clouds are sometimes charged were rendered manifest in a striking manner by the well-known experiments made by means of kites by Romas in 1757. The kite, carrying a metallic point, was elevated to the strata in which the electric cloud floated. A wire was connected with the cord, and carried from the pointed conductor borne by the kite to a part of the cord at some distance from the lower extremity, where it was turned aside and brought into connexion with an electroscope, or other experimental means of testing the quantity and quality of the electricity with which it was charged. Romas drew from the extremity of this conducting wire not only strong electric sparks, but blades of fire nine or ten feet in length and an inch in thickness, the discharge of which was attended with a report as loud as that of a pistol. In less time than an hour, not less than thirty flashes of this magnitude and intensity were often drawn from the conductor, besides many of six or seven feet and of less length.

2267. *Electrical charge of clouds varies.* — It has been shown by means of kites thus applied, that the clouds are charged some with positive and some with negative electricity, while some are observed to be in their natural state. These circumstances serve to explain some phenomena observed in the motions of the clouds which are manifested in stormy weather. Clouds which are similarly electrified repel, and those which are oppositely electrified attract each other. Hence arise motions among such clouds of the most opposite and complicated kind. While they are thus reciprocally attracted and repelled in virtue of the electricity with which they are charged,

they are also transported in various directions by the currents which prevail in the atmospheric strata in which they float, these currents often having themselves different directions.

2268. *Thunder and lightning.*—Such appearances are the sure prognostics of a thunderstorm. Clouds charged with contrary electricities affect each other by induction, and mutually attract, whether they float in the same stratum or in strata at different elevations. When they come within *striking distance*, that is to say, such a distance that the force of the fluids with which they are charged surpasses the resistance of the intervening air, the contrary fluids rush to each other, and an electrical discharge takes place, upon the same principle as the same phenomenon on a smaller scale is produced when the charges of the internal and external coatings of a Leyden jar, overcoming the resistance of the uncoated part, rush together and a spontaneous discharge is made.

The sound and the flash, the thunder and the lightning, are only the reproduction on a more vast scale of the explosion and spark of the jar.

The clouds, however, unlike the metallic coatings of the jar, are very imperfect conductors, and consequently, when discharged at one part of their vast extent, they preserve elsewhere their electricity in its original intensity. Thus, the first discharge, instead of establishing equilibrium, rather disturbs it, for the part of the cloud which is still charged is alone attracted by the part of the other cloud in which the fluid has not yet been neutralized. Hence arise various and complicated motions and variations of form of the clouds, and a succession of discharges between the same clouds must take place before the electrical equilibrium is established. This is necessarily attended by a corresponding succession of flashes of lightning and claps of thunder.

2269. *Form and extent of the flash of lightning.*—The form of the flash in the case of lightning, like that of the spark taken from an electrified conductor, is zigzag. The doublings or acute angles formed at the successive points when the flash changes its direction vary in number and proximity. The cause of this zigzag course, whether of the electric spark or of lightning, has not been explained in any clear or satisfactory manner.

The length of the flashes of lightning also varies; in some

cases they have been ascertained to extend to from two and a half to three miles. It is probable, if not certain, that the line of light exhibited by flashes of forked lightning are not in reality one continued line simultaneously luminous, but that on the contrary the light is developed successively as the electricity proceeds in its course, the appearance of a continuous line of light being an optical effect analogous to the continuous line of light exhibited when a lighted stick is moved rapidly in a circle, the same explanation being applicable to the case of lightning (1143).

2270. *Causes of the rolling of thunder.*—As the sound of thunder is produced by the passage of the electric fluid through the air which it suddenly compresses, it is evolved progressively along the entire space along which the lightning moves. But since sound moves only at the rate of 1100 feet per second, while the transmission of light is so rapid that in this case it may be considered as practically instantaneous, the sound will not reach the ear for an interval greater or less after the perception of the light, just as the flash of a gun is seen before the report is heard (831).

By noting the interval, therefore, which elapses between the perception of the flash and that of the sound, the distance of the point where the discharge takes place can be computed approximately by allowing 1100 feet for every second in the interval.

But since a separate sound is produced at every point through which the flash passes, and as these points are at distances from the observer which vary according to the position, length, direction, and form of the flash, it will follow necessarily that the sounds produced by the same flash, though practically simultaneous, because of the great velocity with which the electricity moves, arrive at the ear in comparatively slow succession. Thus, if the flash be transmitted in the exact direction in which the observer is placed, and its length be 11,000 feet, the distances of the points where the first and last sounds are produced will differ by ten times the space through which sound moves in one second. The first sound will, therefore, be heard ten seconds before the last, and the intermediate sounds will be heard during the interval.

The varying loudness of the successive sounds heard in the rolling of thunder proceeds in part from the same causes as the varying intensity of the light of the flash. But it may, perhaps,

be more satisfactorily explained by the combination of the successive discharges of the same cloud rapidly succeeding each other, and combining their effects with those arising from the varying distances of different parts of the same flash.

2271. *Affected by the zigzag form of lightning.* — It appears to us that the varying intensity of the rolling of thunder may also be very clearly and satisfactorily explained by the zigzag form of the flash, combined with the effect of the varying distance; and it seems extraordinary that an explanation so obvious has not been suggested. Let A, B, C, D, *fig.* 672., be a

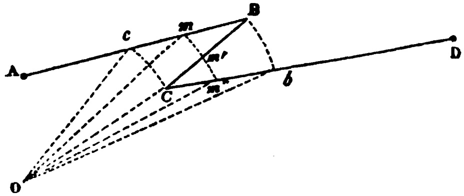


Fig. 672.

part of a zigzag flash seen by an observer at O. Taking O as a centre, suppose arcs C *c* and B *b* of circles to be drawn, with O C and O B as radii. It is clear that the points C and *c*, and B and *b*, being respectively equally distant from the observer, the sounds produced there will be heard simultaneously, and, supposing them equal, will produce the perception of a sound twice as loud as either heard alone would do. All the points on the zigzag *c* B C *b* are so placed that three of them are equidistant from O. Thus, if with O as centre, and O *m* as radius, a circular arc be described, it will intersect the path of the lightning at the three points *m*, *m'*, and *m''*, and these three points being, therefore, at the same distance from O, the sounds produced at them will reach the observer at the same moment, and if they be equally intense will produce on the ear the same effect as a single sound three times as loud. The same will be true for all the points of the zigzag between *c* and *b*. Thus, in this case, supposing the intensity of the lightning to be uniform from A to D, there will be three degrees of loudness in the sound produced, the least between A and *c* and between *b* and D, the greatest between *c* and *b* along the zigzag, and the intermediate at the points C *c* and B *b*.

It is evident, that from the infinite variety of form and position with relation to the observer, of which the course of the lightning is susceptible, the variations of intensity of the rolling of thunder which may be explained in this way have no limit.

2272. *Affected by the varying distance of different parts of the flash.*— Since the loudness of a sound diminishes as the square of the distance of the observer is increased (844), it is clear that this affords another means of explaining the varying loudness of the rolling of thunder.

2273. *Affected by echo and by interference.*—As the rolling of thunder is much more varied and of longer continuance in mountainous regions than in open plane countries, it is, no doubt, also affected by reverberation from every surface which it encounters while capable of reflecting sound. A part therefore of the rolling must be in such cases the effect of echo.

It has been also conjectured that the acoustic effects are modified by the effects of interference (836).

2274. *Inductive action of clouds on the earth.*—A cloud charged with electricity, whatever be the quality of the fluid or the state of the atmosphere around it, exercises by induction an action on all bodies upon the earth's surface immediately under it. It has a tendency to decompose their natural electricity, repelling the fluid of the same name, and attracting to the highest points the fluid of a contrary name. The effects thus actually produced upon objects exposed to such induction will depend on the intensity and quality of the electricity with which the cloud is charged, its distance, the conductibility of the materials of which the bodies affected consist, their magnitude, position, and, above all, their form.

Water being a much better conductor than earth in any state of aggregation, thunder clouds act with great energy on the sea, lakes, and other large collections of water. The flash has a tendency to pass between the cloud and the water, just as the spark passes between the conductor of an electric machine and the hand presented to it. If the water were covered with a thin sheet of glass, the lightning would still pass, breaking through the glass, because, although the glass be a non-conductor, it does not intercept the inductive action of the cloud, any more than a thin glove of varnished silk on the hand would intercept the spark from the conductor.

2275. *Formation of fulgurites explained.*—This explains the fact that lightning sometimes penetrates strata of the solid ground under which subterranean reservoirs of water are found. The water of such reservoirs is affected by the inductive action of an electrified cloud, and in its turn reacts upon the cloud as one coating of a Leyden jar reacts upon the other. When this mutual action is sufficiently strong to overcome the resistance of the subjacent atmosphere, and the strata of soil under which the subterranean reservoir lies, a discharge takes place, and the lightning penetrates the strata, fusing the materials of which it is composed, and leaving a tubular hole with a hard vitrefied coating.

Tubes thus formed have been called *fulgurites*, or *thunder tubes.*

2276. *Accidents of the surface which attract lightning.*—The properties of points, edges, and other projecting parts of conductors, which have been already stated (1776), will render easily intelligible the influence of mountains, peaked hills, projecting rocks, trees, lofty edifices, and other objects, natural and artificial, which project upwards from the general surface of the ground. Lightning never strikes the bottom of deep and close valleys. In Switzerland, on the slopes of the Alps and Pyrenees, and in other mountainous countries, multitudes of cultivated valleys are found, the inhabitants of which know by secular tradition that they have nothing to fear from thunderstorms. If, however, the width of the valleys were so great as twenty or thirty times their depth, clouds would occasionally descend upon them in masses sufficiently considerable, and lightning would strike.

Solitary hills, or elevated buildings rising in the centre of an extensive plain, are peculiarly exposed to lightning, since there are no other projecting objects near them to divert its course.

Trees, especially if they stand singly apart from others, are likely to be struck. Being from their nature more or less impregnated with sap, which is a conductor of electricity, they attract the fluid, and are struck.

The effects of such objects are, however, sometimes modified by the agency of unseen causes below the surface. The condition of the soil, subsoil, and even the inferior strata, the depth of the roots and their dimensions, also exercise considerable influence on the phenomena, so that in the places where there

is the greatest apparent safety there is often the greatest danger. It is, nevertheless, a good general maxim not to take a position in a thunderstorm either under a tree or close to an elevated building, but to keep as much as possible in the open plain.

2277. *Lightning follows conductors by preference. — Its effects on buildings.*—Lightning falling upon buildings chooses by preference the points which are the best conductors. It sometimes strikes and destroys objects which are non-conductors, but this happens generally when such bodies lie in its direct course towards conductors. Thus lightning has been found to penetrate a wall attracted by a mass of metal placed within it.

Metallic roofs, beams, braces, and other parts in buildings, are liable thus to attract lightning. The heated and rarefied air in chimneys acquires conductibility. Hence it happens often that lightning descends chimneys, and thus passes into rooms. It follows bell-wires, metallic mouldings of walls and furniture, and fuses gilding.

2278. *Conductors or paratonnerres for the protection of buildings.*—The purpose of paratonnerres or conductors, erected for the protection of buildings, is not to repel, but rather to attract lightning, and divert it into a course in which it will be innoxious.

A paratonnerre is a pointed metallic rod, the length of which varies with the building on which it is placed, but which is generally from thirty to forty feet. It is erected vertically over the object it is intended to protect. From its base an unbroken series of metallic bars, soldered or welded together end to end, are continued to the ground, where they are buried in moist soil, or, better still, immersed in water, so as to facilitate the escape of the fluid which descends upon them. If water, or moist soil, cannot be conveniently found, it should be connected with a sheet of metal of considerable superficial magnitude, buried in a pit filled with pounded charcoal, or, better still, with *braise*.

The parts of a well-constructed paratonnerre are represented in *fig.* 673. The rod, which is of iron, is round at its base, then square, and decreases gradually in thickness to the summit. It is composed commonly of three pieces closely jointed together, and secured by pins passed transversely through them. In the figure are represented only the two extremities

of the lowest, and those of the intermediate piece, to avoid giving inconvenient magnitude to the diagram. The superior piece, *g*, is represented complete. It is a rod of brass or copper, about two feet in length, terminating in a platinum point, about three inches long, attached to the rod by silver solder, which is further secured by a brass ferule, which gives the projecting appearance in the diagram below the point.

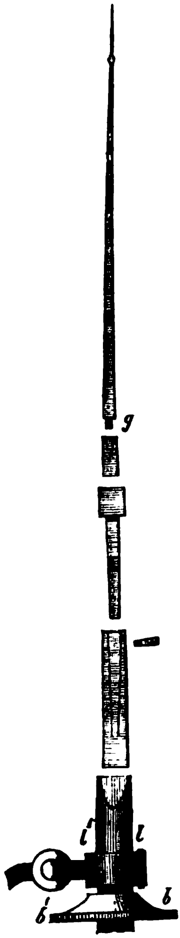


Fig. 673.

Three of the methods, reputed the most efficient for attaching the paratonnerre to the roof, are represented in *fig.* 674., at *p*, *l*, and *f*. At *p* the rod is supported against a vertical piece, to which it is attached by stirrups; at *l* it is bolted upon a diagonal brace; and at *f* it is simply secured by bolts to a horizontal beam through which it passes. The last is evidently the least solid method of fixing it.

The conductor is continued downwards along the wall of the edifice, or in any other convenient course, to the ground, either by bars of iron, round or square, or by a cable of iron or copper wires, such as is sometimes used for the lighter sort of suspension bridges. This is attached, at its upper extremity, to the base of the paratonnerre by a joint, which is hermetically closed, so as to prevent oxydation, which would produce a dangerous solution of continuity.

To comprehend the protective influence of this apparatus, it must be considered that the inductive action of a thunder-cloud decomposes the natural electricity of the rod more energetically than that of surrounding objects, both on account of the material and the form of the rod (1776). The point becoming surcharged with the fluid of a contrary name from that of the cloud suspended over it, discharges this fluid in a jet towards the cloud, where it combines with and neutralizes an equal quantity of the electricity with which the cloud is charged, and,

by the continuance of this process, ultimately reduces the cloud to its natural state.

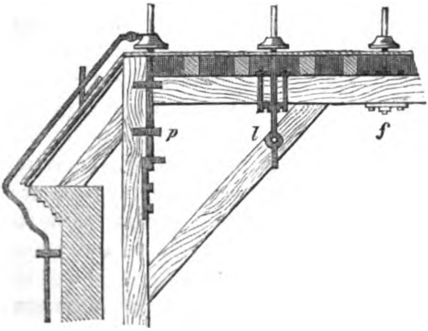


Fig. 674.

It is therefore more correct to say that the paratonnerre draws electricity from the ground and projects it to the cloud, than that it draws it from the cloud and transmits it to the earth.

It is evidently desirable that all conducting bodies to be protected by the paratonnerre should be placed in metallic connexion with it, since in that case their electricity, decomposed by the inductive action of the clouds, will necessarily escape by the conductor either to the earth or to the cloud by the point.

It is considered generally that the range of protection of a paratonnerre is a circle round its base, whose radius is two or three times its length.

2279. *Effects of lightning on bodies which it strikes.* — The effects of lightning, like those of electricity evolved by artificial means, are threefold, physiological, physical, and mechanical.

When lightning kills, the parts where it has struck bear the marks of severe burning; the bones are often broken and crushed as if they had been subjected to violent mechanical pressure. When it acts on the system by induction only, which is called the secondary or indirect shock, it does not immediately kill, but inflicts nervous shocks so severe as sometimes to leave effects which are incurable.

The physical effects of lightning produced upon conductors is to raise their temperature. This elevation is sometimes so

great that they are rendered incandescent, fused, and even burned. This happens occasionally with bell-wires, especially in exposed and unprotected positions, as in courts or gardens. The drops of molten metal produced in such cases set fire to any combustible matter on which they may chance to fall. Wood, straw, and such non-conducting bodies are ignited generally by the lightning drawn through them by the attraction of other bodies near them which are good conductors.

The mechanical effects of lightning, the physical cause of which has not been satisfactorily explained, are very extraordinary. Enormous masses of metal are torn from their supports, vast blocks of stone are broken, and massive buildings are razed to the ground.

2280. *The Aurora Borealis — the phenomenon unexplained.* — No theory or hypothesis which has commanded general acceptation, has yet been suggested for the explanation of this meteor. All the appearances which attend the phenomenon are, however, electrical; and its forms, directions, and positions, though ever varying, always bear a remarkable relation to the magnetic meridians and poles. Whatever, therefore, be its physical cause, it is evident that the theatre of its action is the atmosphere; that the agent to which the development is due is electricity, influenced in some unascertained manner by terrestrial magnetism. In the absence of any satisfactory theory for the explanation of the phenomenon, we shall confine ourselves here to a short description of it, derived from the most extensive and exact series of observations which have been made in those regions where the meteor has been seen with the most marked characters and in the greatest splendour.

2281. *General character of the meteor.* — The aurora borealis is a luminous phenomenon, which appears in the heavens, and is seen in high latitudes in both hemispheres. The term aurora borealis, or northern lights, has been applied to it because the opportunities of witnessing it are, from the geographical character of the globe, much more frequent in the northern than in the southern hemisphere. The term *aurora polaris* would be a more proper designation.

This phenomenon consists of luminous rays of various colors, issuing from every direction, but converging to the same point, which appear after sunset generally toward the north, occasionally toward the west, and sometimes, but rarely, toward the

south. It frequently appears near the horizon, as a vague and diffuse light, something like the faint streaks which harbinger the rising sun and form the dawn. Hence the phenomenon has derived its name, the northern morning. Sometimes, however, it is presented under the form of a sombre cloud, from which luminous jets issue, which are often variously coloured, and illuminate the entire atmosphere.

The more conspicuous auroras commence to be formed soon after the close of twilight. At first a dark mist or foggy cloud is perceived in the north, and a little more brightness towards the west than in the other parts of the heavens. The mist gradually takes the form of a circular segment, resting at each corner on the horizon. The visible part of the arc soon becomes surrounded with a pale light, which is followed by the formation of one or several luminous arcs. Then come jets and rays of light variously coloured, which issue from the dark part of the segment, the continuity of which is broken by bright emanations, indicating a movement of the mass, which seems agitated by internal shocks, during the formation of these luminous radiations, that issue from it as flames do from a conflagration. When this species of fire has ceased, and the aurora has become extended, a crown is formed at the zenith, to which these rays converge. From this time the phenomenon diminishes in its intensity, exhibiting, nevertheless, from time to time, sometimes on one side of the heavens and sometimes on another, jets of light, a crown and colours more or less vivid. Finally the motion ceases, the light approaches gradually to the horizon; the cloud quitting the other parts of the firmament settles in the north. The dark part of the segment becomes luminous, its brightness being greatest near the horizon, and becoming more feeble as the altitude augments until it loses its light altogether.

The aurora is sometimes composed of two luminous segments, which are concentric, and separated from each other by one dark space, and from the earth by another. Sometimes, though rarely, there is only one dark segment, which is symmetrically pierced round its border by openings, through which light or fire is seen.

2282. *Description of auroras seen in the polar regions by M. Lottin.* — One of the most recent and exact descriptions of this meteor is the following, supplied by M. Lottin, an officer

of the French navy, and a member of the Scientific Commission, sent some years ago to the North Seas. Between September 1838 and April 1839, this savant observed nearly 150 meteors of this class. They were most frequent from the 17th November to the 25th January, being the interval during which the sun remained constantly below the horizon. During this period there were sixty-four auroras visible, besides many which a clouded sky concealed from the eye, but the presence of which was indicated by the disturbances they produced upon the magnetic needle.

The succession of appearances and changes presented by these meteors are thus described by M. Lottin:—

Between four and eight o'clock, P.M., a light fog, rising to the altitude of six degrees, became coloured on its upper edge, being fringed with the light of the meteor rising behind it. This border becoming gradually more regular took the form of an arc, of a pale yellow colour, the edges of which were diffuse, the extremities resting on the horizon. This bow swelled slowly upwards, its vertex being constantly on the magnetic meridian. Blackish streaks divided regularly the luminous arc, and resolved it into a system of rays; these rays were alternately extended and contracted; sometimes slowly, sometimes instantaneously; sometimes they would dart out, increasing and diminishing suddenly in splendour. The inferior parts, or the feet of the rays, presented always the most vivid light, and formed an arc more or less regular. The length of these rays was very various, but they all converged to that point of the heavens indicated by the direction of the southern pole of the dipping needle. Sometimes they were prolonged to the point where their directions intersected, and formed the summit of an enormous dome of light.

The bow then would continue to ascend toward the zenith: it would suffer an undulatory motion in its light — that is to say, that from one extremity to the other the brightness of the rays would increase successively in intensity. This luminous current would appear several times in quick succession, and it would pass much more frequently from west to east than in the opposite direction. Sometimes, but rarely, a retrograde motion would take place immediately afterward; and as soon as this wave of light had run successively over all the rays of the aurora from west to east, it would return, in the contrary di-

rection, to the point of its departure, producing such an effect that it was impossible to say whether the rays themselves were actually affected by a motion of translation in a direction nearly horizontal, or if this more vivid light was transferred from ray to ray, the system of rays themselves suffering no change of position. The bow, thus presenting the appearance of an alternate motion in a direction nearly horizontal, had usually the appearance of the undulations or folds of a ribbon or flag agitated by the wind. Sometimes one and sometimes both of its extremities would desert the horizon, and then its folds would become more numerous and marked, the bow would change its character, and assume the form of a long sheet of rays returning into itself, and consisting of several parts forming graceful curves. The brightness of the rays would vary suddenly, sometimes surpassing in splendour stars of the first magnitude; these rays would rapidly dart out, and curves would be formed and developed like the folds of a serpent; then the rays would affect various colours, the base would be red, the middle green, and the remainder would preserve its clear yellow hue. Such was the arrangement which the colours always preserved; they were of admirable transparency, the base exhibiting blood-red, and the green of the middle being that of the pale emerald; the brightness would diminish, the colors disappear, and all be extinguished, sometimes suddenly, and sometimes by slow degrees. After this disappearance, fragments of the bow would be reproduced, would continue their upward movement, and approach the zenith; the rays, by the effect of perspective, would be gradually shortened; the thickness of the arc, which presented then the appearance of a large zone of parallel rays, would be estimated; then the vertex of the bow would reach the magnetic zenith, or the point to which the south pole of the dipping needle is directed. At that moment the rays would be seen in the direction of their feet. If they were coloured, they would appear as a large red band, through which the green tints of their superior parts could be distinguished; and if the wave of light above mentioned passed along them, their feet would form a long sinuous undulating zone; while, throughout all these changes, the rays would never suffer any oscillation in the direction of their axis, and would constantly preserve their mutual parallelisms.

While these appearances are manifested, new bows are

formed, either commencing in the same diffuse manner, or with vivid and ready-formed rays: they succeed each other, passing through nearly the same phases, and arrange themselves at certain distances from each other. As many as nine have been counted, forming as many bows, having their ends supported on the earth, and, in their arrangement, resembling the short curtains suspended one behind the other over the scene of a theatre, and intended to represent the sky. Sometimes the intervals between these bows diminish, and two or more of them close upon each other, forming one large zone, traversing the heavens, and disappearing toward the south, becoming rapidly feeble after passing the zenith. But sometimes, also, when this zone extends over the summit of the firmament from east to west, the mass of rays which have already passed beyond the magnetic zenith appear suddenly to come from the south, and to form with those from the north the real boreal corona, all the rays of which converge to the zenith. This appearance of a crown, therefore, is doubtless the mere effect of perspective; and an observer, placed at the same instant at a certain distance to the north or to the south, would perceive only an arc.

The total zone, measuring less in the direction north and south than in the direction east and west, since it often leans upon the earth, the corona would be expected to have an elliptical form; but that does not always happen: it has been seen circular, the unequal rays not extending to a greater distance than from eight to twelve degrees from the zenith, while at other times they reach the horizon.

Let it, then, be imagined, that all these vivid rays of light issue forth with splendour, subject to continual and sudden variations in their length and brightness; that these beautiful red and green tints colour them at intervals; that waves of light undulate over them; that currents of light succeed each other; and, in fine, that the vast firmament presents one immense and magnificent dome of light, reposing on the snow-covered base supplied by the ground — which itself serves as a dazzling frame for a sea, calm and black as a pitchy lake — and some idea, though an imperfect one, may be obtained of the splendid spectacle which presents itself to him who witnesses the aurora from the bay of Alten.

The corona, when it is formed, only lasts for some minutes:

it sometimes forms suddenly, without any previous bow. There are rarely more than two on the same night; and many of the auroras are attended with no crown at all.

The corona becomes gradually faint, the whole phenomenon being to the south of the zenith, forming bows gradually paler, and generally disappearing before they reach the southern horizon. All this most commonly takes place in the first half of the night, after which the aurora appears to have lost its intensity: the pencils of rays, the bands and the fragments of bows, appear and disappear at intervals; then the rays become more and more diffused, and ultimately merge into the vague and feeble light which is spread over the heavens grouped like little clouds, and designated by the name of *auroral plates* (*plaques aurorales*). Their milky light frequently undergoes striking changes in its brightness, like motions of dilatation and contraction, which are propagated reciprocally between the centre and the circumference, like those which are observed in marine animals called Medusæ. The phenomena become gradually more faint, and generally disappear altogether on the appearance of twilight. Sometimes, however, the aurora continues after the commencement of daybreak, when the light is so strong that a printed book may be read. It then disappears, sometimes suddenly; but it often happens that, as the daylight augments, the aurora becomes gradually vague and undefined, takes a whitish colour, and is ultimately so mingled with the cirrho-stratus clouds that it is impossible to distinguish it from them.

Some of the appearances here described are represented in *figs.* 675, 676, 677, 678., copied from the memoir of M. Lottin.

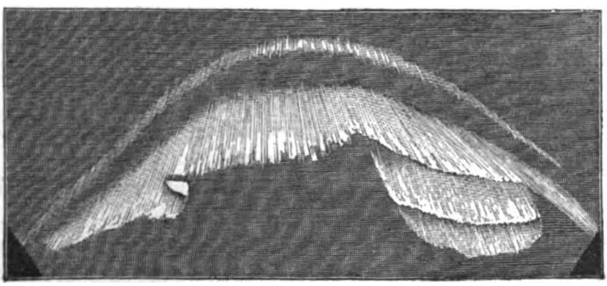


Fig. 675.

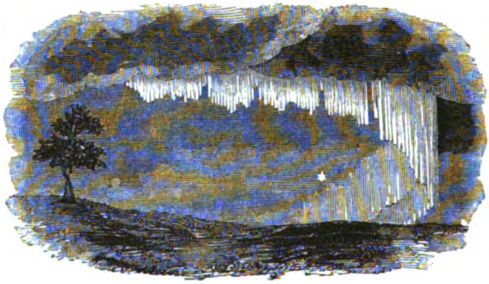


Fig. 676

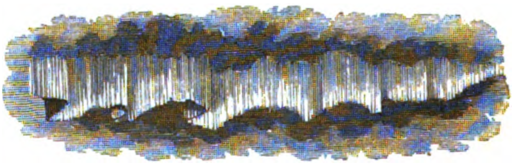


Fig. 677.

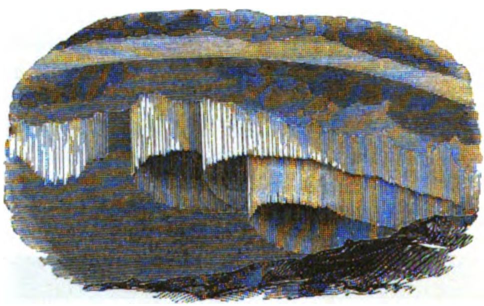
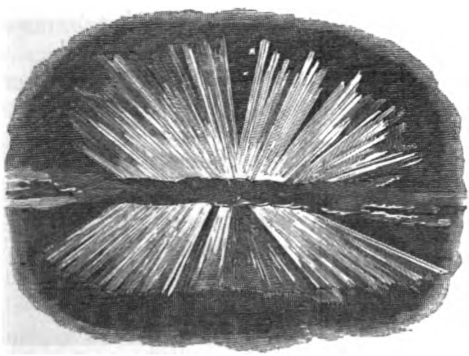


Fig. 678.

There is great difficulty in determining the exact height of the aurora borealis above the earth, and accordingly the opinions given on this subject by different observers are widely discordant. Mairan supposed the mean height to be 175 French leagues; Bergman says 460, and Euler several thousand miles. From the comparison of a number of observations of an aurora that appeared in March, 1826, made at different

places in the north of England and south of Scotland, Dr. Dalton, in a paper presented to the Royal Society, computed its height to be about 100 miles. But a calculation of this sort, in which it is of necessity supposed that the meteor is seen in exactly the same place by the different observers, is subject to very great uncertainty. The observations of Dr. Richardson, Franklin, Hood, Parry, and others, seem to prove that the place of the aurora is far within the limits of the atmosphere, and scarcely above the region of the clouds; in fact, as the diurnal rotation of the earth produces no change in its apparent position, it must necessarily partake of that motion, and consequently be regarded as an atmospherical phenomenon.



BOOK THE SECOND.

ASTRONOMY.

CHAPTER I.

METHODS OF INVESTIGATION AND MEANS OF OBSERVATION.

2283. *The solar system.* — The earth, which in the economy of the universe has become the habitation of the races of men, is a globular mass of matter, and one of an assemblage of bodies of like form and analogous magnitude which revolve in paths nearly circular round a common centre, in which the sun, a globe having dimensions vastly greater than all the others, is established, maintaining physical order among them by his predominant attraction, and ministering to the well-being of the tribes which inhabit them by a fit and regulated supply of light and heat.

This group of bodies is the SOLAR SYSTEM.

2284. *The stellar universe.* — In the vast regions of space which surround this system other bodies similar to the sun are placed, countless in number, and most of them, according to all probability, superior in magnitude and splendour to that luminary. With these bodies, which seem to be scattered throughout the depths of immensity without any discoverable limit, we acquire some acquaintance by the mere powers of natural vision, aided by those of the understanding; but this knowledge has received, especially in modern times, prodigious extension from the augmented range given to the eye by the telescope, and by the great advances which have been made in mathematical science, which may be considered as conferring upon the mind the same sort of enlarged power as the telescope has conferred upon the eye.

2285. *Subject of astronomy — origin of the name.* — The investigation of the magnitudes, distances, motions, local arrangements, and, so far as it can be ascertained, the physical condition of these great bodies composing the UNIVERSE, constitutes the subject of that branch of science called ASTRONOMY, a term derived from the Greek words αστηρ (aster), a star (under which all the heavenly bodies were included), and νομος (nomos), A LAW — the science which expounds the LAWS which govern the motions of the STARS.

2286. *It treats of inaccessible objects.* — It is evident, therefore, that astronomy is distinguished from all other divisions of natural science by this peculiarity, that the bodies which are the subjects of observation and enquiry are all of them INACCESSIBLE. Even the earth itself, which the astronomer regards as a celestial object — an αστηρ, — is to him, in a certain sense, even more inaccessible than the others; for the very fact of his place of observation being confined strictly to its surface, an insignificant part of which alone can be observed by him at any one moment, renders it impossible for him to examine, by direct observation, the earth AS A WHOLE — the only way in which he desires to consider it, — and obliges him to resort to a variety of indirect expedients to acquire that knowledge of its dimensions, form, and motions which, with regard to other and more distant objects, results from direct and immediate observation.

2287. *Hence arise peculiar methods of investigation and peculiar instruments of observation.* — This circumstance of having to deal exclusively with inaccessible objects has obliged the astronomer to invent peculiar modes of reasoning and peculiar instruments of observation, adapted to the solution of such problems, and to the discovery of the necessary data. Much needless repetition will then be saved by explaining once for all, with as much brevity as is compatible with clearness, the most important classes of those problems which determine the circumstances of each particular celestial object, and by describing the principal instruments of observation by which the necessary data are obtained.

2288. *Direction and bearing of visible objects.* — The eye estimates only the direction or relative bearings of objects within the range of vision, but supplies no direct means of determining their distances from each other, or from the eye itself (1168, *et seq.*).

The absolute direction of a visible object is that of a straight line drawn from the eye to the object.

The relative direction or bearing of an object is determined by the angle formed by the absolute direction with some other fixed or known direction, such as that of a line drawn to the north, south, east, or west.

2289. *They supply the means of ascertaining the distances and positions of inaccessible objects.* — By comparing the relative bearings of inaccessible objects, taken from two or more accessible points whose distance from each other is known, or can be ascertained by actual measurement, the distances of such inaccessible objects from the accessible objects, from the observer, and from each other, may be determined by computation. Such distances being once known, become the data by which the mutual distances of other inaccessible objects from the former, and from each other, may be in like manner computed; so that, by starting in this manner from two objects whose mutual distance can be actually measured, we may proceed, by a chain of computations, to determine the relative distances and positions of all other objects, however inaccessible, that fall within the range of vision.

2290. *Angular magnitude — its importance.* — It will be apparent, therefore, that ANGULAR MAGNITUDE plays a most prominent part in astronomical investigations, and it is, before all, necessary that the student should be rendered familiar with it.

2291. *Division of the circle — its nomenclature.* — A circle is divided into four equal arcs, called quadrants, by two diameters AA′ and BB′ intersecting at right angles at the centre C, *fig.* 679.

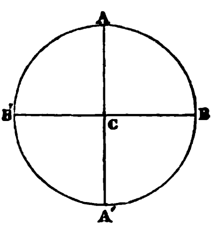


Fig. 679.

The circumference being supposed to be divided into 360 equal parts, each of which is called a DEGREE, a quadrant will consist of 90 degrees.

Angles are subdivided in the same manner as the arcs which measure them, and accordingly a right angle, such as ACB, being divided into 90 equal angles, each of these is a DEGREE.

If an angle or arc of one degree be divided into 60 equal parts, each of these is called a MINUTE.

If an angle or arc of one minute be divided into 60 equal parts, each such part is called a SECOND.

Angles less than a second are usually expressed in decimal parts of a second.

Degrees, minutes, and seconds are usually expressed by the signs, $^\circ$, $'$, $''$; thus, $25^\circ\ 30'\ 40''\cdot9$ means an angle or arc which measures 25 degrees, 30 minutes, 40 seconds, and 9-10ths of a second.

The letters *m* and *s* have sometimes been used to express minutes and seconds; but since it is frequently necessary to express TIME as well as SPACE, it will be more convenient to reserve these letters for that purpose. Thus, $25^h\ 30^m\ 40^s\cdot9$ expresses an interval of time consisting of 25 hours, 30 minutes, 40 seconds, and 9-10ths of a second.

2292. *Relative magnitudes of arcs of* 1°, $1'$, *and* $1''$, *and the radius.*—It is proved in geometry that the length of the entire circumference of a circle whose radius is expressed by 1·000 exceeds 6·283 by less than the 5000th part of the radius. As the exact length of the circumference does not admit of any numerical expression, it will therefore be sufficient for all practical purposes to take 6·283 to express it.

If *d*, *m*, and *s* express respectively the actual lengths or *linear values* of a degree, a minute, and a second of a circle the length of whose radius is expressed by *r*, we shall therefore have the following numerical relations between these several lengths:—

$$60 \times s = m, \qquad 60 \times m = d, \qquad 3600 \times s = d,$$
$$360\, d = 6{\cdot}283\, r, \qquad 360 \times 60 \times m = 21600 \times m = 6{\cdot}283 \times r,$$
$$21600 \times 60 \times s = 1296000 \times s = 6{\cdot}283\, r;$$

and from these may be deduced the following:

$$r = 57{\cdot}3 \times d = 3437{\cdot}8 \times m = 206265 \times s.$$

By these formulæ respectively the length of the radius may be computed when the linear value of an arc of 1°, $1'$, or $1''$ is known.

In like manner, if the length of the radius *r* be given, the linear value of an arc of 1°, $1'$, or $1''$ may be computed by the formulæ

$$d = \frac{1}{57{\cdot}3} \times r, \qquad m = \frac{1}{3437{\cdot}8} \times r, \qquad s = \frac{1}{206265} \times r.$$

2293. *The linear and angular magnitude of an arc.*—By the linear magnitude of an arc is to be understood its actual length if extended in a straight line, or the number expressing its length in units of some known modulus of length, such as an *inch*, a *foot*, or a *mile*. By its angular magnitude is to be understood the angle formed by two lines or radii drawn to its extremities from the centre of the circle of which it forms a part, or the number expressing the magnitude of this angle in angular units of known value, as degrees, minutes, and seconds.

2294. *Of the three following quantities,—the linear value of an arc, its angular value, and the length of the radius,—any two being given, the third may be computed.* — Let α express the angular and a the linear value of the arc, and r the radius.

1°. Let α and a be given to compute r. By dividing a by α we shall find the linear value of 1°, 1′, or 1″, according as α is expressed in degrees, minutes or seconds, and r may then be computed by (2293). Thus, according to the angular units in which α is expressed, we shall have

$$r=\frac{a}{\alpha^\circ}\times 57{\cdot}3=\frac{a}{\alpha'}\times 3437{\cdot}8=\frac{a}{\alpha''}\times 206265.$$

2°. Let a and r be given to compute α. By (2293) the linear values of 1°, 1′, or 1″ may be computed, since r is given, and by dividing a by one or other of these values α will be found: thus we shall have

$$\alpha^\circ=\frac{a}{\frac{1}{57\cdot3}\times r},\qquad \alpha'=\frac{a}{\frac{1}{3437\cdot8}\times r},\qquad \alpha''=\frac{a}{\frac{1}{206265}\times r}.$$

3°. Let α and r be given to compute a. By (2293), as before, the linear values of 1°, 1′, or 1″ may be found, and by multiplying one or other of these by α the value of a will be obtained. Thus we shall have

$$a=\frac{1}{57\cdot3}\times r\times\alpha^\circ=\frac{1}{3437\cdot8}\times r\times\alpha'=\frac{1}{206265}\times r\times\alpha''.$$

2295. *The arc, the cord, and the sine may be considered as equal when the angle is small.*—If A C B, *fig.* 680., be the angle, A B the arc, and A C=A B the radius, a line A D drawn from one extremity of the arc perpendicular to the radius A B, through

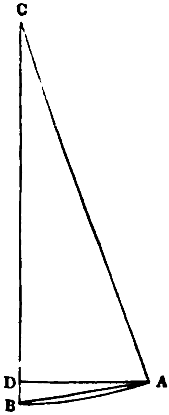


Fig. 680.

the other extremity, is called its *sine;* and the straight line A B joining the extremities of the arc is called its *cord.*

It will be evident by merely drawing the diagram with a gradually decreasing angle, that the three lengths, the sine A D, the cord A B, and the arc A B, will approach to equality as the arc diminishes. Even where the arc is so large as 30°, it does not exceed the length of the cord by more than three-tenths of a degree; and therefore, for all angles less than this, the cord and arc may be considered as equal where the most extreme precision is not required.

In like manner, if the angle A C B be 15°, the sine A D will be less than the arc by only two-tenths of a degree, that is, by the 75th part of the entire length of the arc. In all cases, therefore, where greater precision than this is not required, the sine A D, the cord, and the arc may be considered for such angles as interchangeable.

When the angles are so small as a degree or two, these quantities may for all practical purposes be considered to be equal.

2296. *To ascertain the distances of an inaccessible object from two accessible stations.*—This, which is a problem of the highest importance, being in fact the basis of all the knowledge we possess of the distances, dimensions, and motions of the great bodies of the universe, admits of easy solution.

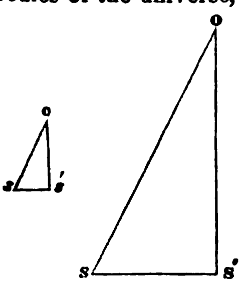


Fig. 681.

Let S and S′, *fig.* 681., be the two stations, and O the object of observation; and let the visual angles subtended by O and S′ at S, and by O and S at S′, be observed, and the distance S S′ measured.

Take a line *s s′*, consisting of as many inches as there are miles in S S′, and draw two lines *s o* and *s′o* from *s* and *s′*, making with *s s′* the same angles as S O and S′ O are ascertained by observation to make with S S′. In that case the triangle *s o s′* will be in all respects similar to the triangle S O S′, only drawn on a smaller scale, an inch in any of its sides corresponding

to a mile in one of the sides of the great triangle s o s′. If the sides *s o* and *s′ o* be therefore measured, they will consist of as many inches as there are miles in the corresponding sides s o and s′ o of the great triangle. Now, since the small triangle is always accessible to direct measurement, and as the relation of its scale to that of the great triangle is known, the magnitude of the sides of the great triangle may be ascertained.

Without being identical in its actual details with the process by which this problem is solved, the preceding reasoning is the same in principle and spirit. Trigonometrical tables supply much more accurate means of determining the proportion of the sides of the triangle, but such tables are nothing more than the arithmetical representation of such diagrams as *fig.* 681.

2297. *Case in which the distance of the object is great relatively to the distance between the stations.*—If in this case the stations be so selected with reference to the object that the directions of the object as seen from them shall form angles with the line joining the stations which shall be equal or nearly so, this latter line may be considered as the cord of the arc described, with the object as a centre and the distance as a radius; and if the direction of the object from either station be at right angles to the line joining the stations, this latter line may be considered as the sine of the arc. In either case, the distance of the object bearing a high ratio to the distance between the stations, the angle formed by the two directions of the object, as seen from the stations, will be so small, that the cord or sine may be considered as equal to the arc, and the solution of the problem will be simplified by the principles established in (2295).

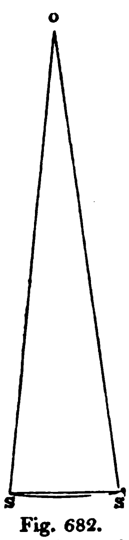


Fig. 682.

Let s and s′, *fig.* 682., be the stations, and o the object; under the conditions supposed the angle o will necessarily be small. Its magnitude may be ascertained by measuring the visual angles o s s′ and o s′ s, which may be done, since both stations s and s′ are accessible, and each of them is visible from the other. By a well-known principle established in elementary geometry, the three angles of every triangle added together make 180°. If, therefore, the sum of the two observed angles at s and s′ be

subtracted from 180°, the remainder will be the angle O. Now if this angle be expressed by a in seconds, we shall have (2294)

$$SO = SS' \times \frac{206265''}{a''}.$$

2298. *Given the apparent distance between two distant objects, such distance being at right angles, or nearly so, to their visual directions, and their distance from the observer, to find the actual distance between them.*—This problem is only a particular application of the general principle explained in (2294), a being the apparent distance of the objects, a the real distance between them, and r their distance from the observer.

This method may be applied without practical error, if the apparent distance between the objects be not greater than two or three degrees, and it may be used as a rough approximation in cases where the apparent distance is even so great as 30°. When the apparent distance amounts to so much as 60°, the actual distance computed in this way will not exceed the true distance by more than a 24th part of its whole amount; for the cord of 60° is equal to the radius, and therefore to an arc of 57°·3, being less than the arc by only 2°·7, or about a 24th part of its length.

2299. *Given the apparent diameter of a spherical object and its distance from the observer, to find its real diameter.*—This is also a particular application of the general problem (2294), a being the apparent diameter and r the distance, and in all cases which occur in astronomy the apparent diameters are so small that the results of the computation may be considered as perfectly exact.

2300. *Methods of ascertaining the direction of a visible and distant object.* — It might appear an easy matter to observe the exact direction of any point placed within the range of vision, since that direction must be that of a straight line passing directly from the eye of the observer to the point to be observed. If the eye were supplied with the appendages necessary to record and measure the directions of visible objects, this would be true, and the organ of sight would be in fact a philosophical instrument. The eye is, however, adapted to other and different uses, and constructed to play a different part in the animal economy; and invention has been stimulated to supply expedients, by means of which the exact directions of visible distant

points can be ascertained, observed, and compared one with another, so as to supply the various data necessary in the classes of problems which have just been noticed, and others which we shall have occasion hereafter to advert to.

2301. *Use of sights.* — The most simple expedient by which the visual direction of a distant point can be determined is by SIGHTS, which are small holes or narrow slits made in two thin opaque plates placed at right angles, or nearly so, to the line of vision, and so arranged, that when the eye is placed behind the posterior opening the object of observation shall be visible through the anterior opening. Every one is rendered familiar with this expedient by its application to fire-arms as a method of "taking aim."

This contrivance is, however, too rude and susceptible of error within too wide limits, to be available for astronomical purposes.

2302. *Application of the telescope to indicate the visual direction of micrometric wires.* — The telescope (1212) supplies means of determining the direction of the visual ray with all the necessary precision.

If T T′, *fig.* 683., represent the tube of a telescope, T the ex-

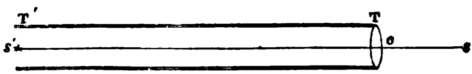


Fig. 683.

tremity in which the object-glass is fixed, and T′ the end where the images of distant objects to which the tube is directed are formed, the visual direction of any object will be that of the line *s′c* drawn from the image of such object formed in the *field of view* of the telescope to the centre *c* of the object-glass, for if this line be continued it will pass through the object *s*.

But since the field of view of the telescope is a circular space of definite extent, within which many objects in different directions may at the same time be visible, some expedient is necessary by which one or more fixed points in it may be permanently marked, or by which the entire field may be spaced out as a map is by the lines of latitude and longitude.

This is accomplished by a system of fibres, or wires (38) so thin that even when magnified they will appear like hairs. These are extended in a frame fixed within the eye-piece of the telescope, so that they appear when seen through the eye-glass

like fine lines drawn across the field of view. They are differently arranged, according to the sort of observation to which the instrument is to be applied.

2303. *Line of collimation.*—In some cases two wires intersect at right angles at the centre of the field of view, dividing it into quadrants, as represented in *fig.* 679. The wires are so adjusted that their point of intersection C coincides with the axis of the telescopic tube; and when the instrument is so adjusted that the point of observation, a star for example, is seen precisely upon the intersection C of the wires, the line of direction, or visual ray of that star, will be the line *s'c*, *fig.* 683., joining the intersection C, *fig.* 679., of the wires with the centre *c*, *fig.* 683. of the object-glass.

The line *s' c*, *fig.* 683., is technically called the *line of collimation.*

2304. *Application of the telescope to a graduated instrument.*—The telescope thus prepared is attached to a graduated instrument by which angular magnitudes can be observed and measured. Such instruments vary infinitely in form, magnitude, and mode of mounting and adjustment, according to the purposes to which they are applied, and to the degree of precision necessary in the observations to be made with them. To explain and illustrate the general principles on which they are constructed we shall take the example of one, which consists of a complete circle graduated in the usual manner, being the most common form of instrument used in astronomy for the measurement of angular distances.

Such an apparatus is represented in *fig.* 684. The circle

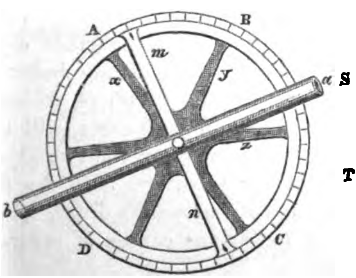


Fig. 684.

A B C D, on which the divisions of the graduation are accurately engraved, is connected with its centre by a series of spokes

x y z. At its centre is a circular hole, in which an axle is inserted so as to turn smoothly in it, and while it turns to be always concentric with the circle A B C D. To this axle the telescope *a b* is attached in such a manner that the imaginary line *s′ c*, *fig.* 683., which joins the intersection of the wires, *fig.* 679., with the centre of the object-glass, shall be parallel to the plane of the circle, and in a plane passing through its centre and at right angles to it.

At right angles to the axis of the telescope are two arms, *m n*, which form one piece with the tube, so that when the tube is turned with the axis to which it is attached, the arms *m n* shall turn also, always preserving their direction at right angles to the tube. Marks or indices are engraved upon the extremities *m* and *n* of the arms which point to the divisions upon the LIMB (as the divided arc is called).

A clamp is provided on the instrument, by which the telescope, being brought to any desired position, can be fixed immoveably in that position, while the observer examines the points upon the limb to which the indices *m* and *n* are directed.

Now let us suppose that the visual angle under the directions of two distant objects within the range of vision is required to be measured. The circle being brought into the plane of the objects, and fixed in it, the telescope is moved upon its axis until it is directed to one of the objects, so that its image shall coincide exactly with the intersection of the wires. The telescope is then clamped, and the observer examines the points of the divided limb, to which one of the indices, *m* for example, is directed. This process is called "reading off." The clamp being disengaged, the telescope is then in like manner directed to the other object, and being clamped as before, the position of the index is "read off." The difference between the numbers which indicate the position of the same index in both cases, will evidently be the visual angle under the directions of the two objects.

As a means of further accuracy, both the indices *m* and *n* may be "read off," and if the results differ, which they always will slightly, owing to various causes of error, a mean of the two may be taken.

It is evident that the same results would be obtained if, instead of making the telescope move upon the circle, it were immoveably attached to it, and that the circle itself turned upon its centre, as a wheel does upon its axle, carrying the

telescope with it. In this case the divided limb of the circle is made to move before a fixed index, and the angle under the directions of the objects will be measured by the length of the arc which passes before the index.

Such a combination is represented in section in *fig.* 685.,

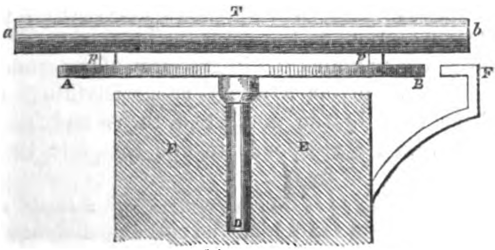


Fig. 685.

where T is the telescope, *p* the pieces by which it is attached to the circle A B seen edgewise, the axis of which D works in a solid block of metal. The fixed index F is directed to the graduated limb which moves before it.

This is the most frequent method of mounting instruments used in astronomy for angular measurement.

2305. *Expedients for measuring the fraction of a division.*— It will happen in general that the index will be directed, not to any exact division, but to some point intermediate between two divisions of the limb. In that case expedients are provided by which the fraction of a degree between the index, and the last division which it has passed, may be ascertained with an extraordinary degree of precision.

2306. *By a vernier.*—This may be accomplished by means of a supplemental scale called a VERNIER, already described (1354).

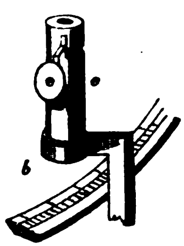


Fig. 686.

2307. *By a compound microscope, and micrometric screw.*—The same object may, however, be attained with far greater accuracy by means of a compound microscope mounted as represented in *fig.* 686., so that the observer looks at the index through it. A system of cross wires is placed in the field of view of the microscope, and the whole may be so adjusted by the action of a fine screw, that the index shall coincide precisely

with the intersection of the wires. The screw is then turned until the intersection of the cross is brought to coincide with the previous division of the limb; and the number of turns and fraction of a turn of the screw will give the fraction of a degree between the index and the previous division of the limb.

It is necessary, however, to ascertain previously the value of a complete revolution of the screw. This is easily done by turning the screw on which the intersection of the cross is moved from one division to the adjacent one. Dividing, then, one degree of the limb by the number of turns and fraction of a turn, the arc which corresponds to one complete turn will be found.

2308. *Observation and measurement of minute angles.*— When the points between which the angular distance required to be ascertained are so close together as to be seen at one and the same time within the field of view of the telescope, a method of measurement is applicable, which admits of even greater relative accuracy than do the methods of observing large angular distances. This arises from the fact that the distance between such points may be determined by various forms of micrometric instruments, in which fine wires, or lines of spider's web, are moved in a direction perpendicular to their length, so as to pass successively through the points whose distance is to be observed.

2309. *The parallel wire micrometer.*—One of the forms of micrometric apparatus used for this purpose is represented in transverse section in *fig.* 687. This, which is called the PA-

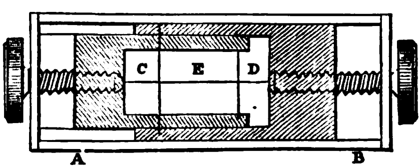


Fig. 687.

RALLEL WIRE MICROMETER, consists of two sliding frames, across which the parallel wires or threads C and D are stretched. These frames are both moved in a direction perpendicular to that of the wires by screws, constructed with very fine threads, and called from their use MICROMETER SCREWS. This frame is placed in the focus of the object-glass of the telescope, so that

the eye viewing the objects under observation sees also distinctly the parallel and moveable wires. These wires are moved by the screws until they pass through the points whose distance asunder is to be measured. This being accomplished, one of them is moved until it coincides with the other, and the number of turns and parts of a turn of the screw necessary to produce this motion gives the angular distance between the points under observation.

In this, as in the case explained in (2307), it is necessary that the angle corresponding to one complete revolution of the screw be previously ascertained, and this is done by a process precisely similar to that explained in the former case. An object of known angular magnitude, as, for example, a foot rule at the distance of a hundred yards, is observed, and the number of turns necessary to carry the wire from end to end of its image is ascertained. The angle such a rule subtends at that distance being divided by the number of turns and parts of a turn, the quotient is the angle corresponding to one complete revolution of the screw.

2310. *Measurement of the apparent diameter of an object.*— When an object is not too great to be included in the field of view of the telescope, its apparent diameter (1117) can be measured by such an apparatus. To accomplish this the screws are turned until the wires C and D, *fig.* 687., are made to touch opposite sides of the disk of the object. One of the screws is then turned until the wires coincide, and the number of turns and parts of a turn gives the apparent magnitude.

CHAP. II.

THE GENERAL ROTUNDITY AND DIMENSIONS OF THE EARTH.

2311. *The earth a station from which the universe is observed.* — The earth is, in various points of view, an interesting object of scientific investigation. The naturalist regards it as the habitation of the numerous tribes of organized beings which are the special subject of his observation and inquiry, and examines curiously those properties and qualities of soil, climate, and atmosphere, by which it is fitted for their main-

tenance and propagation, and the conditions which govern their distribution over its surface. The geologist and mineralogist regard it as the theatre of vast physical operations continued through periods of time extending infinitely beyond the records of human history, the results of which are seen in the state of its crust. The astronomer, rising above these details, regards it as a whole, examines its form, investigates its motions, measures its magnitude, and, above all, considers it as the station from which alone he can take a survey of that universe which forms the peculiar object of his study, and as the only modulus or standard by which the magnitudes of all the other bodies in the universe, and the distances which separate them from the earth and from each other, can be measured.

2312. *Necessary to ascertain its form, dimensions, and motions.* — But since the apparent magnitudes, motions, and relative arrangement of surrounding objects severally vary, not only with every change in the position of the station of the observer, but even with every change of position of the observer on that station, it is most necessary to ascertain with all attainable accuracy the dimensions of the earth, which is the station of the astronomical observer, its form, and the changes of position in relation to surrounding objects to which it is subject.

2313. *Form globular.* — The first impression produced by the aspect presented by the surface of the earth is that of a vast indefinite plane surface, broken only by the accidents of the ground on land, such as hills and mountains, and by the more mutable forms due to the agitation of the fluid mass on the sea. Even this departure from the appearance of an extensive plane surface ceases on the sea out of sight of land in a perfect calm, and on certain planes of vast extent on land, such as some of the prairies of the American continents.

This first impression is soon shown to be fallacious; and it is easily demonstrated that the immediate indications of the unaided sense of vision, such as they are, are loosely and incorrectly interpreted, and that, in fact, even that small part of the earth's surface which falls at once within the range of the eye in a fixed position *does not appear to be a plane.*

Supposing that any extensive part of the surface of the earth were really a plane, let several stakes or posts, of equal height, be erected along the same straight line, and at equal distances,

say a mile apart. Let these stakes be represented by s *s*, s′ *s′*, s″ *s″*, &c., *fig.* 688., and let a stake of equal height o *o* be erected at

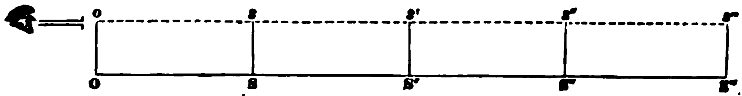


Fig. 688.

the station of the observer. Now if the surface were a plane, it is evident that the points *s*, *s′*, *s″*, &c. must appear to an eye placed at *o* in the same visual line, and would each be visible through a tube directed at *o* parallel to the surface o s. But such will not be found to be the case. When the tube is directed to s, all the succeeding points *s′*, *s″*, &c. will be *below* its direction. If it be directed to *s′*, the point *s* will be *above*, and *s″* and all the succeeding points will be *below* its direction. In like manner, if it be directed to *s″*, the preceding points *s* and *s′* will be *above*, and the succeeding points *below* its direction. In effect it will appear as though each succeeding stake were a little shorter than the preceding one. But as the stakes are all precisely equal, it must be inferred that the successive points of the surface s, s′, s″, s‴, &c. are relatively lower than the station o. Nor will the effects be explained by the supposition that the surface o s s′ s″, &c., is a descending but still a *plane* surface, because in that case the points *s*, *s′*, *s″*, &c. must still be in the same visual line directed from *o*. It therefore follows that the surface in the direction o s′ s″ s‴, &c. is not *plane* but *curved*, as represented in *fig.* 689., where the visual lines are in obvious accordance with the actual appearances as above explained.

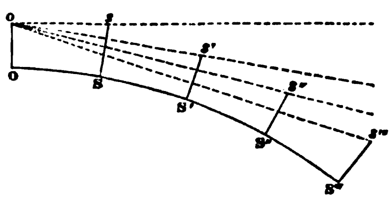


Fig. 689.

Now since these effects are found to prevail in every direction around the point of observation o, it follows that the curvature of the surface prevails all around that point; and since

the *extent of the depression* of the points s, s′, s″, &c. at equal distances from o, are equal in every direction around o, it follows that the curvature is in every direction sensibly uniform around that point.

But by shifting the centre of observation o, and making similar observations elsewhere, and on every part of the earth where such a process is practicable, not only are like effects observed, but the *degree of depression* corresponding to equal distances from the centres of observation is the same.

Hence we infer that the surface of the earth, *as observed directly by the eye*, is not a plane surface, but one everywhere curved, and that the curvature is everywhere uniform, at least that no departure from perfect uniformity in its general curvature exists sufficiently considerable to be discovered by this method.

But the only form of a solid body which has a surface of uniform curvature is a sphere or globe, and it is therefore established that such is the form of the earth.

2314. *This conclusion corroborated by circumnavigation.*— If a vessel sail, as far as it is practicable to do so, constantly in the same direction, it will at length return to the port of its departure, having circumnavigated the earth, and during its course it appears to pass over an uniform surface. This is obviously what must take place so far as regards that part of the earth which is covered with water, supposing it to be a globe.

2315. *Corroborated by lunar eclipses.*—But the most striking and conclusive corroboration of the inference just made, and indeed a phenomenon which alone would demonstrate the form of the earth, is that which is exhibited in lunar eclipses. These appearances, which are so frequently witnessed, are caused by the earth coming between the sun and the moon, so as to cast its shadow upon the latter. Now the form of that shadow is *always* precisely that which *one globe would project upon another.* The phenomenon thus at once establishes not only the globular form of the earth, but that of the moon also.

2316. *Various effects indicating the earth's rotundity.*—The rotundity of the earth being once admitted, a multitude of its consequences and effects present themselves, which supply corroborative evidence of that important proposition.

When a ship sails from the observer, the first part which should cease to be visible, if the earth was a plane, would be the

rod of the top-mast, having the smallest dimensions, and the last the hull and sails, being the greatest in magnitude;—but, in fact, the very reverse takes place. The hull first disappears, then the sails, and in fine the top-mast alone is visible by a telescope, appearing like a pole planted in the water. This becomes gradually shorter, appearing to sink in the water as the vessel recedes from the eye.

These appearances are the obvious consequences of the gradual interposition of the convexity of the part of the earth's surface over which the vessel has passed, and will be readily comprehended by the *fig.* 690.

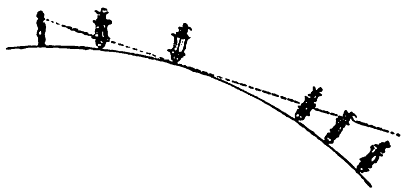


Fig. 690.

If the observer take a more elevated position, the same succession of phenomena will be presented, only greater distances will be necessary to produce the same degree of apparent sinking of the vessel.

Land is visible from the top-mast in approaching the shore, when it cannot be seen from the deck.

The top of the peak of Teneriffe can be seen from a distance when the base of the mountain is invisible.

The sun shines on the summits of the Alps long after sunset in the valleys.

An aeronaut ascending after sunset has witnessed the sun to reappear with all the effects of sunrise. On descending, he witnessed a second sunset.

2317. *Dimensions of the earth.—Method of measuring a degree.*— Having thus ascertained that the form of the earth is a globe, it now remains to discover its magnitude, or, what is the same, its diameter.

For this purpose it will be necessary first to ascertain the actual length of a degree upon its surface, that is, the distance between two points on the surface, so placed that the lines drawn from them to the centre shall make with each other an angle of one degree.

Let p and p', *fig.* 691., represent two places upon the earth's surface, distant from each other from 60 to 100 miles, and let C be the centre of the earth. Now, let us suppose that two observers at the places p and p' observe two stars s and s', which at the same time are vertically over the two places, and to which, therefore, plumb-lines suspended at the two places would be directed. The direction of these plumb-lines, if continued downwards, would intersect at C, the centre of the earth.

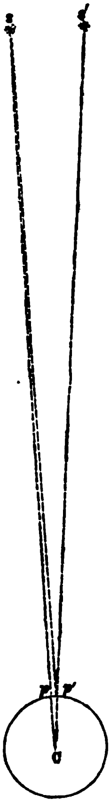


Fig. 691.

The visual angle under the directions of these stars s and s' at p' is $sp's'$, and at C is $s\text{C}s'$. But, owing to the insignificant proportion which the distances pp' and pC bear to the distances of the stars (as will be made evident hereafter), the visual angle of the stars, whether seen from p or C, will be the same. If, then, this visual angle at p' be measured, as it may be with the greatest precision, we may consider it as the magnitude of the angle $p\text{C}p'$.

Let the actual distance D, between the places p and p', be measured or ascertained by the process of surveying, and the number of seconds in the observed angle $sp's'$ be expressed by a. If d express the distance of two points on the earth which would subtend at the centre C an angle of 1°, we shall then have —

$$a : 3600 :: \text{D} : d = \text{D} \times \frac{3600}{a},$$

since the number of seconds in a degree is 3600 (2292).

2318. *Length of a degree.* — In this way it has been ascertained that the length of a degree of the earth's surface is a little less than 70 British statute miles, and may be expressed in feet (in round numbers) by 365,000.

It will therefore be easy to remember that the length of a degree is as many thousand feet as there are days in the year.

2319. *Length of a second of the earth.* — Since a second is the 3600th part of a degree, it follows also that the length of a second is an hundred feet very nearly, a measure also easily remembered.

2320. *Change of direction of the plumb-line in passing over a given distance.* — From what has just been explained it will be understood that since a plumb-line always points to the centre of the earth (when its direction is undisturbed by any local attraction), its change of direction, in passing from any one place to any other, may be always found by allowing 1″ for every hundred feet in the direct line joining the places, or still more exactly by allowing 365,000 feet for every degree, and a proportional part of this length for every fraction of a degree between the places.

2321. *To find the earth's diameter.* — Nothing can be more easy, after what has been stated, than the solution of the problem to determine the earth's diameter. By what has been explained (2294), if r express the radius or semidiameter of the earth Cp, a the arc pp' of the earth's surface between the two places, and α the angle pCp', we shall have —

$$r = a \times \frac{206265}{\alpha''}.$$

If the distance a be one degree, this will become —

$$r = \frac{365000}{3600} \times 206265 = 20{,}912979,$$

or very nearly 21 million feet, which is equal to 3960 statute miles. So that the diameter of the earth would be 7920 miles, or in round numbers (for we are not here pretending to extreme arithmetical precision) 8000 miles.

The process of observation above explained is not in its details exactly that by which the magnitude of the earth is ascertained, but it is in spirit and principle the method of observation and calculation. It would not be easy to find, for example, any two sufficiently observable stars which at one and the same moment would be vertically over the two places p and p', but any two stars nearly over them would equally answer the purpose by observing the extent of their departure from the vertical direction. Neither is it necessary that the two observations should exactly coincide as to time; but these details do not affect the principle of the method, and will be more clearly intelligible as the student advances.

2322. *Superficial inequalities of the earth relatively insignificant.* — It is by comparison alone that we can acquire any clear or definite notions of distances and magnitudes which do

not come under the immediate cognizance of the senses. If we desire to acquire a notion of a vast distance over which we cannot pass, we compare it with one with which we have immediate and actual acquaintance, such as a foot, a yard, or a mile. And since the area or superficial extent of surfaces and the volume or bulk of solids are respectively determined by the length of their linear dimensions, the same expedient suffices to acquire notions of them. In Astronomy, having to deal with magnitudes exceeding in enormous proportions those of all objects, even the most stupendous, which are so approachable as to afford means of direct sensible observation, we are incessantly obliged to have recourse to such comparisons in order to give some degree of clearness to our ideas, since without them our knowledge would become a mere assemblage of words, numbers, and geometrical diagrams.

Let us, then, consider the dimensions and form of the earth, as they have been ascertained in the preceding paragraphs.

When it is stated that the earth is a globe, the first objection which will be raised by the uninformed student is that the continents, islands, and tracts of land with which it is covered are marked by considerable inequalities of level; that mountains rise into ridges and peaks of vast height; that the seas and oceans, though level at their surface in a certain general sense, are agitated by great waves, and alternately swelled and depressed by tides, and that the solid bottom of them is known to be subject to inequalities analogous in character, and not less in depth, than those which prevail on the land. Since, then, it is the characteristic property of a globe that all points on its surface are equally distant from its centre, how, it may be demanded, can a mass of matter, so unequal in its surface as the earth is, be a globe?

It may be conceded at once, in reply to this objection, that the earth is not, in the strict geometric sense of the term, a globe. But let us consider the extent of its departure from the globular form, so far as relates to the superficial inequalities just adverted to.

The most lofty mountain peaks do not exceed five miles in height. Few, indeed, approach that limit. Most of the considerable mountainous districts are limited to less than half that height. No considerable tract of land has a general elevation even of one mile. The deepest parts of the sea have not been

sounded; but it is certain that their depth does not exceed the heights of the most lofty mountains, and the general depth is incomparably less. The superficial inequalities of the aqueous surface produced by waves and tides are comparatively insignificant.

Now, let us consider how these several superficial inequalities would be represented, observing a due proportion of scale, even on the most stupendous model.

Construct a globe 20 feet in diameter, as a model of the earth. Since 20 feet represents 8000 miles, 1-400th part of a foot, or 3-100th parts of an inch, represents a mile. The height, therefore, of the most lofty mountain peak, and the greatest depth of the ocean, would be represented by a protuberance or a hole having no greater elevation or depth than 15-100ths, or about the seventh part of an inch. The general elevation of a continent would be fairly represented by a leaf of paper pasted upon the surface, having the thickness of less than the fiftieth of an inch; and a depression of little greater amount would express the depth of the general bed of the sea.

It will therefore be apparent, that the departure of such a model from the true form of a globe would be in all, save a strictly geometrical sense, absolutely insignificant.

2323. *Relative dimensions of the atmosphere.* — The surface of the earth is covered by an ocean of air which floats upon it, as the waters of the seas rest upon their solid bed. The density of this fluid is greatest in the stratum which is in immediate contact with the surface of the land and water of the earth, and it diminishes in a very rapid ratio in ascending, so that one half of the entire atmosphere is included in the strata whose height is within 3½ miles of the surface. At an altitude of 80 miles, or the hundredth part of the earth's diameter, the rarefaction must be so extreme, that neither animal life nor combustion could be maintained.

The atmosphere, being then limited to such a height, would be represented on the model above described by a stratum two inches and a half thick.

2324. *If the earth moved, how could its motion be perceived?* — Having thus ascertained, in a rough way, the form and dimensions of the earth, let us consider the question of its rest or mobility.

Nothing is more repugnant to the first impressions received

from the aspect of the surface of the earth, and all upon it, than the idea that it is in motion. But if this universal impression be traced to its origin, and rightly interpreted, it will not be found erroneous, and will form no exception to the general maxim which induces all persons, not even excepting philosophers, to regard without disrespect notions which have obtained universal popular acceptation.

What is the stability and repose ascribed by the popular judgment to the earth? Repose certainly absolute, so far as regards all objects of vulgar or popular contemplation. It is maintained, and maintained truly, that everything upon the earth, so far as the agency of external causes is concerned, is at relative rest. Hills, mountains, and valleys, oceans, seas, and rivers, as well as all artificial structures, are in relative repose; and if our observation did not extend to objects exterior to the globe, the popular maxim would be indisputable. But the astronomer contemplates objects which either escape the attention of, or are imperfectly known to, mankind in general; and the phenomena which attend these render it manifest, that while the earth, in relation to all objects upon it and forming part of it, is at rest, it is in motion with relation to all the other bodies of the universe.

The motion of objects external to the observer is perceived by the sense of sight only, and is manifested by the relative displacement it produces among the objects affected by it, with relation to objects around them which are not in motion, and with relation to each other. Motions in which the person of the observer participates may affect the senses both of feeling and sight. The feeling is affected by the agitation to which the body of the observer is exposed. Thus, in a carriage which starts or stops, or suddenly increases or slackens its speed, the matter composing the person of the observer has a tendency to retain the motion which it had previous to the change, and is accordingly affected with a certain force, as if it were pushed or drawn from rest in one direction or the other. But once in a state of uniform motion, the sense of feeling is only affected by the agitation proceeding from the inequalities of the road. If these inequalities are totally removed, as they are in a boat drawn at a uniform rate on a canal, the sense of feeling no longer affords any evidence whatever of the motion.

A remarkable example of the absence of all consciousness of

motion, so far as mere feeling is concerned, is presented to all who have ascended in a balloon. As the aerial vehicle floats with the stratum of the air in which it is suspended, the feeling of the aeronaut is that of the most absolute repose. The balloon seems as fixed and immoveable as the solid globe itself, and nothing could produce in the voyager, blindfolded, any consciousness whatever of motion. When however his eyes, unbandaged, are turned downwards, he sees the vast diorama below moving under him. Fields and woods, villages and towns pass in succession, and the phenomena are such as to impress on the eye, and through the eye upon the mind, the conviction that the balloon is stationary, and the earth moving under it. A certain effort of the understanding, slight, it is true, but still an effort, is required to arrive at the inference that the impression thus produced on the sense of vision is an illusion, that the motion with which the landscape seems to be affected is one which in reality affects the balloon in which the spectator is suspended, and that this motion is equal in speed, and contrary in direction, to that which appears to affect the subjacent country.

Now it will be evident, that if the globe of the earth, and all upon it, were floating in space, and moving in any direction at any uniform rate, no consciousness of such motion could affect any sensitive being upon it. All objects partaking in common in such motion, no more derangement among them would ensue than among the persons and objects transported in the car of the balloon, where the aeronaut, no matter what be the speed of the motion, can fill a glass to the brim as easily as if he were upon the solid ground. Supposing, then, that the earth were affected by any motion in which all objects upon it, including the waters of the ocean, the atmosphere, and clouds, would all participate, would the existence of such a motion be perceived by a spectator placed upon the earth who would himself partake of it? It is clear that he must remain for ever unconscious of it, unless he could find within the range of his vision some objects which, not partaking of the motion, would appear to have a motion contrary to that which the observer has in common with the earth.

But such objects are only to be looked for in the regions of space beyond the limits of the atmosphere. We find them in fine in the sun, the moon, the stars, and all the objects which the firmament presents. Whatever motion the earth may have

will impart to all these distant objects the appearance of a motion in the contrary direction.

But how, it may be asked, is the apparent motion produced in distant objects by a real motion of the station in which the observer is placed, to be distinguished from the real motion of the distant objects themselves, which would give them the same apparent motion? Since the phenomena are absolutely identified, whether the apparent motion observed is produced by a real motion in the observer, or a real motion in the object observed, it is necessary to seek for evidence; either that the object observed cannot have the real motion which would produce the phenomena, or that the station of the observer has it. But before engaging in this question, it is necessary *first*, to obtain a clear and definite knowledge of what the apparent motion in question is; *secondly*, what is the real motion of the earth which could produce it; and *thirdly*, what would be the real motion, or motions, of the objects observed, which would produce the same phenomena.

2325. *Parallax.* — Since the apparent place of a distant object depends on the direction of the visual line drawn from the observer to such object, and since while the object remains stationary the direction of this visual line is changed with every change of position of the observer, such change of position produces necessarily a displacement in the apparent position of the object.

This apparent displacement of any object seen at a distance, due to the change of position of the observer, is called PARALLAX.

It follows that a distant object seen by two observers at different places on the earth is seen in different directions, so that its apparent place in the firmament will be different. It would therefore follow, that the aspect of the heavens would vary with every change of position of the observer on the earth, just as the relative position of objects on land which are stationary changes when viewed from the deck of a vessel which sails or steams along the coast. But it so happens, as will appear hereafter, that even the greatest difference of position which can exist between observers on the earth's surface is so small compared even with the nearest bodies to the earth, that the apparent displacement, or PARALLAX, thus produced is very small; while for the most numerous of celestial objects, the stars, it is

absolutely inappreciable by the most refined means of observation and measurement.

Small as it is, however, so far as relates to the nearer bodies of the universe, it is capable of definite measurement, and its amount for each of them supplies one of the data by which their distances are calculated.

2326. *Apparent and true place of an object.—Diurnal parallax.* — When an object is within such a limit of distance as would cause a sensible displacement to be produced when it is viewed from different parts of the earth's surface, it is convenient, in registering its apparent position at any given time, to adopt some fixed station from which it is supposed to be observed. The station selected by astronomers for this purpose is the centre of the earth. The direction in which an object would be seen if viewed from the centre of the earth is called its TRUE PLACE. The direction in which it is seen from any place of observation on the surface is called its APPARENT PLACE, and the apparent displacement which would be produced by the transfer of the observer from the centre to the surface or *vice versâ*, or, what is the same, the difference between the true and apparent places, is called the DIURNAL PARALLAX.

In *fig.* 692., let C represent the centre of the earth, P a place

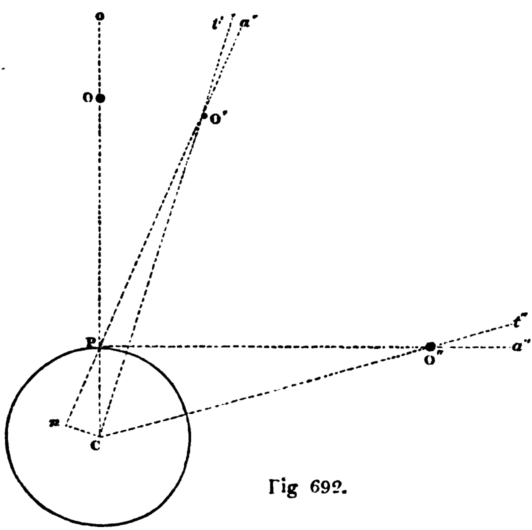


Fig 692.

of observation on its surface, O an object seen in the zenith of P, O′ the same object seen at the zenith distance O P O′, and O″ the same object seen in the horizon.

It is evident that O will appear in the same direction, whether it be viewed from P or C. Hence it follows that in the zenith there is no diurnal parallax, and that there the apparent place of an object is its true place.

But if the object be at O′, then the apparent direction is P O′, while the true direction is C O′, and the apparent place of the object will be *a′*, while its true place will be *t′*; and the diurnal parallax corresponding to the zenith distance O P O′ will be *t′ a′*, or the angle *t′* O′ *a′*, which is equal to P O′ C.

As the object is more remote from the zenith the parallax is augmented, because the semidiameter C P of the earth, which passes through the place of observation, is more and more nearly at right angles to the directions C O′ and P O′.

2327. *Horizontal parallax.* — When the object is in the horizon, as at O″, the diurnal parallax becomes greatest, and is called the HORIZONTAL PARALLAX. It is the angle P O″ C which the semidiameter of the earth subtends at the object.

If Z express the zenith distance, or the angle P C O′, a line C *n* drawn from C at right angles to P O′ *n* will be expressed by $z \times r \times \sin. z$, r being the semidiameter C P of the earth. If D express the distance of the object O′, and ϖ the parallactic angle P O′ C, which is always very small, we shall have, by the principle explained in (2294):

$$\varpi'' = 206265'' \times \sin. z \times \frac{r}{D},$$

the parallax being expressed in seconds.

If the object be in the horizon as at O″, we shall have $z = 90°$, and therefore

$$\varpi'' = 206265'' \times \frac{r}{D}.$$

2328. *Given the horizontal parallax and the earth's semidiameter, to compute the distance of the object.* — It is evident that this important problem can be solved by the preceding formula; for we have (2297)

$$D = \frac{206265}{\varpi''} \times r.$$

CHAP. III.

APPARENT FORM AND MOTION OF THE FIRMAMENT.

2329. *Aspect of the firmament.*—If we examine the heavens with attention on clear starlight nights, we shall soon be struck with the fact, that the brilliant objects scattered over them in such incalculable numbers maintain constantly the same relative position and arrangement. Every eye is familiar with certain groups of stars called constellations. These are never observed to change their relative position. A diagram representing them now would equally represent them at any future time; and if a general map be made, showing the relative arrangement of these bodies on any night, the same map will represent them with equal exactness and fidelity on any other night. There are a few,—some thirty or forty or so,—among many thousands, which are exceptions to this, with which, however, for the present we need not concern ourselves.

2330. *The celestial hemisphere.*—The impression produced upon the sight by these objects is that they are at a vast distance, but all at the same distance. They seem as though they were attached in fixed and unalterable positions upon the surface of a vast hemisphere, of which the place of the observer is the centre. Setting aside the accidental inequalities of the ground, the observer seems to stand in the centre of a vast circular plane, which is the base of this celestial hemisphere.

2331. *Horizon and zenith.*—This plane, extended indefinitely around the observer, meets the celestial hemisphere in a circle which is called the HORIZON, from the Greek word ὁρίζειν (orizein), to *terminate* or *bound*, being the boundary or limit of the visible heavens.

The centre point of the visible hemisphere—that point which is perpendicularly above the observer, and to which a plumb-line suspended at rest would be directed—is called the ZENITH.

2332. *Apparent rotation of the firmament.*—A few hours' attentive contemplation of the firmament at night will enable any common observer to perceive, that although the stars are, relatively to each other, fixed, the hemisphere, *as a whole*, is in

motion. Looking at the zenith, constellation after constellation will appear to pass across it, having risen in an oblique direction from the horizon at one side, and, after passing the zenith, descending on the other side to the horizon, in a direction similarly oblique. Still more careful and longer continued observation, and a comparison, so far as can be made by the eye, of the different directions successively assumed by the same object, creates a suspicion, which every additional observation strengthens, that the celestial vault has a motion of slow and uniform rotation round a certain diameter as an axis, carrying with it all the objects visible upon it, without in the least deranging their relative positions or disturbing their arrangement.

Such an impression, if well founded, would involve, as a necessary consequence, that a certain point in the heavens placed at the extremity of the axis of its rotation, would be fixed, and that all other points would appear to be carried around it in circles; each such point preserving therefore, constantly, the same distance from the point thus fixed.

2333. *The pole star.*—To verify this inference, we must look for a star which is not affected by the apparent rotation of the heavens, which affects more or less every other star.

Such a star is accordingly found, which is always seen in the same direction,—so far at least as the eye, unaided by more accurate means of observation, can determine.

The place of this star is called the Pole, and the star is called the Pole Star.

2334. *Rotation proved by instrumental observation.*—Mere visual observation, however, can at most only supply grounds for probable conjecture, either as to the rotation of the sphere, or the position of its pole, if such rotation take place. To verify this conjecture, to determine with certainty whether the motion of the sphere be one of rotation, and if so, to ascertain with precision the direction of the axis round which this rotation takes place, its velocity, and, in fine, whether it be uniform or variable,—are problems of the highest importance, but which are altogether beyond the powers of mere visual observation, unaided by instruments of precision.

2335. *Exact direction of the axis and position of the pole.*—Suppose a telescope of low magnifying power, supplied with micrometric wires (2302), to be directed to the pole star, so that the star may be seen exactly upon the intersection of the

middle wires. If this star were precisely at the extremity of the axis of the hemisphere, or at the pole, it would remain permanently at the intersection of the wires, notwithstanding the rotation of the firmament. Such is not, however, found to be the case. The star will appear to move; but, if the magnifying power of the telescope be low enough, it will not leave the field of view. It will appear to move in a small circle, the diameter of which is about three degrees. The telescope may be so adjusted that the star will move in a circle round the intersection of the middle wires as a centre; and in that case the point marked by the intersection of the middle wires is the true position of the POLE, round which the pole star is carried in a circle, at the distance of about $1\frac{1}{2}°$, by the rotation of the sphere.

2336. *Equatorial instrument.*—The exact direction of the axis of the celestial sphere being thus ascertained, it is possible to construct an apparatus which shall be capable of revolving upon a fixed axis, the direction of which shall coincide with that of the sphere; so that if a telescope were fixed in the direction of this axis, its line of collimation (2303) would exactly point to the celestial pole.

Upon this axis, thus directed and fixed, suppose a telescope to be so mounted that it may be placed with its line of collimation at any desired angle with the axis, and let a properly graduated arc be provided, by which the magnitude of this angle may be measured with all practicable precision.

Thus, let A A′, *fig.* 693., represent the direction of the axis on which the instrument is made to revolve. The line A A′, if continued to the firmament, would pass through the pole *p*. Let C O represent the line of collimation of a telescope, so attached to the axis at C that it may be placed at any desired angle with it; which may be accomplished by placing a joint at C on which the telescope can turn. Let N O N′ be a graduated arc, to which the telescope is attached at O, and which turns with the telescope round the axis A A′. When the telescope, being fixed at any proposed angle O C A′ with the axis, is turned round A A′, the line of collimation describes a cone of which C is the vertex and C A′ the axis, and the extremity O describes an arc O O′ of a circle at a distance from N′ measured by the angle O C A′.

If the line of collimation C O or C O′ be imagined to be continued to the heavens, it will describe, as the telescope revolves, a

circle *o o′* on the firmament corresponding to the circle O O′, and at the same angular distance *o p*, *o′ p* from the celestial pole

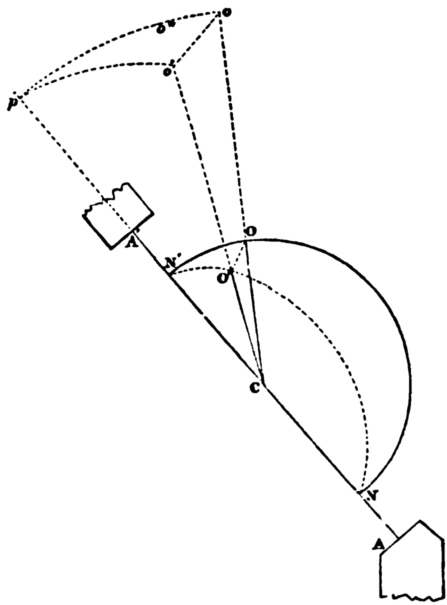


Fig. 693.

p, as the end O or O′ of the line of collimation of the telescope is from N′ or A′. In short, the angle O C N′ equally measures the two arcs, the celestial arc *o p* and the instrumental arc O N′.

The instrument thus described in its principle is one of most extensive utility in observatories, and is called an EQUATORIAL.

In its practical construction it is very variously mounted, and is sometimes acted upon by clock-work, which imparts to it a motion round the axis A A′, corresponding with the rotation of the celestial sphere.

One of the many mechanical arrangements by which this may be effected is represented in *fig.* 694., as given by the Astronomer Royal, in his lectures delivered at the Ipswich Museum.

The instrument is supported upon pivots, so that its axis A′ B′ shall coincide exactly with the direction of the celestial axis.

The telescope C D turns upon a joint at the centre, so that different directions, such as C′ D′, C″ D″, may be given to it.

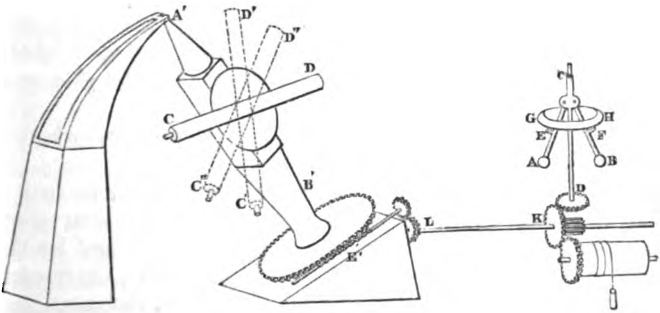


Fig. 694.

The motion upon its axis is imparted to it by wheel-work E L K, impelled by clock-work, as already mentioned.

2337. *Rotation of firmament proved by equatorial.*—Now, to establish, by means of this instrument, the fact that the firmament really has a motion of apparent rotation with a velocity rigorously uniform round the axis, let the telescope be first directed to any star, *o*, *fig.* 693., for example, so that it shall be seen upon the intersection of the middle wires. The line of collimation will then be directed to the star, and the angle O C N′ or the arc O N′ will express the apparent distance of such star from the pole *p*.

Let the instrument be then turned upon its axis from east to west (that is, in the same direction as the rotation of the firmament), through any proposed angle, say 90°, and let it be fixed in that position. The firmament will follow it, and after a certain interval the same star will be seen upon the intersection of the wires; and in the same manner, whatever be the change of position of the instrument upon its axis, provided the direction of the telescope upon the arc O N′, *fig.* 693., be not changed, the star will always arrive, after an interval more or less, according to the angle through which this instrument has been turned, upon the intersection of the wires.

It follows, therefore, from this, that the particular star here observed is carried in a circle round the heavens, always at the same distance, *o p*, from the celestial pole.

The same observations being made with a like result upon

every star to which the telescope is directed, it follows that the motion of the firmament is such that all objects upon it describe circles at right angles to its axis, each object always remaining at the same distance from the pole.

This is precisely the effect which would be produced by the rotation of the heavens round an axis directed to the pole from the place of the observer.

But it remains to ascertain the time of rotation, and whether the rotation be uniform.

If the telescope be directed as before to any star, so that it shall be seen at the intersection of the wires, let the instrument be then fixed, being detached from the clock-work, and let the exact time of the star passing the wires be noted. On the following night, at the approach of the same hour, the same star will be seen approaching to the same position, and it will at length arrive again upon the wires. The time being again exactly observed, it will be found that the interval which has elapsed between the two successive passages of the star over the wires is

$$23^{h.}\ 56^{m.}\ 4{\cdot}09^{s.}$$

Such is, therefore, the time in which the celestial sphere makes one complete revolution, and this time will be always found to be the same, whatever be the star to which the telescope is directed.

To prove that not only every complete revolution is performed in the same time, but that the rotation during the same revolution is uniform, let the instrument, after being directed to any star, be turned in the direction of the motion of the sphere through any proposed angle, 90° for example. It will be found that the interval which will elapse between the passage of the star over the wires in the two positions will, in this case, be the fourth part of $23^{h.}\ 56^{m.}\ 4{\cdot}09^{s.}$; and, in general, whatever be the angle through which the instrument may be turned, the interval between the passages of the same star over the wires in the two positions will bear the same proportion to $23^{h.}\ 56^{m.}\ 4{\cdot}09^{s.}$, as the angle bears to 360°.

It follows, therefore, that the apparent rotation of the heavens is rigorously uniform.

It will be observed that the time of one complete revolution is $3^{m.}\ 55{\cdot}91^{s.}$ less than twenty-four hours, or a common day. The cause of this difference will be explained hereafter.

2338. *Sidereal time.*—The time of one complete revolution of the firmament is called a SIDEREAL DAY. This interval is divided, like a common day, into 24 hours, each hour into 60 minutes, and each minute into 60 seconds.

Since in 24 sidereal hours the sphere turns through 360°, and since its motion is rigorously uniform, it turns through 15° in a sidereal hour, and through 1° in four sidereal minutes.

2339. *The same apparent motion observed by day.*—It may be objected that although this description of the movement of the heavens accords with the appearances during the night, there is no evidence of the continuance of the same rotation during the day, since in a cloudless firmament no object is visible except the sun, which being alone cannot manifest the same community of motion as is exhibited by the multitudinous objects which, being crowded so thickly on the firmament at night, move together without any change in their apparent relative position. To this objection it may be answered that the moon is occasionally seen in the day-time as well as the sun; and, moreover, that before sunset and sunrise the planets Jupiter and Venus are occasionally seen under favourable atmospheric circumstances. Besides, with telescopes of sufficient power properly directed, all the brighter stars can be distinctly seen when not situated very near the position of the sun. Now, in all these cases, the objects thus seen appear to be carried round by the same motion of the firmament, which is so much more conspicuously manifested in the absence of the sun and at night.

2340. *Certain fixed points and circles necessary to express the position of objects on the heavens.*—It will greatly contribute to the facility and clearness with which the celestial phenomena and their causes shall be understood if the student will impress upon his memory the names and positions of certain fixed points, lines, and circles of the celestial sphere, by reference to which the position of objects upon it are expressed. Without incumbering him with a more complex nomenclature than is indispensably necessary for this purpose, we shall therefore explain some of the principal of these landmarks of the heavens.

2341. *Vertical circles, zenith, and nadir.*—If from the place of the observer a straight line be imagined to be drawn perpendicular to the plane of the horizon, and to be continued indefi-

nitely both upwards and downwards, it will meet the visible hemisphere at its vertex, the ZENITH, and the invisible hemisphere, which is under the plane of the horizon, at a corresponding point called the NADIR.

If a plane be supposed to pass through the place of the observer and the zenith, it will meet the celestial surface in a series of points, forming a circle at right angles to the horizon. Such a circle is called a VERTICAL CIRCLE, or, shortly, a VERTICAL.

If this plane be supposed to be turned round the line passing upwards to the zenith, it will assume successively every direction round the observer, and will meet the heavens in every possible vertical circle.

The vertical circles, therefore, all intersecting at the zenith as a common point, divide the horizon as the divisions of the hours and minutes divide the dial-plate of a clock.

2342. *The celestial meridian and prime vertical.* — That vertical which passes though the celestial pole is called the MERIDIAN.

The meridian is, therefore, the only circle of the heavens which passes at once through the two principal fixed points, the pole and the zenith.

It divides the visible hemisphere into two regions on the right and left of the observer; as he looks to the north, that which is on his right being called the EASTERN, and that which is on his left the WESTERN.

Another vertical at right angles to the meridian is called the PRIME VERTICAL. This is comparatively little used for reference.

2343. *Cardinal points.* — The meridian and prime vertical divide the horizon at four points, equally distant, and therefore separated by arcs of 90°. These points are called the CARDINAL POINTS. Those formed by the intersection of the meridian with the horizon are called the NORTH and SOUTH points, that which is nearest to the visible pole in the northern hemisphere being the north. Those formed by the intersection of the prime vertical with the horizon are called the EAST and WEST, that to the right of an observer looking towards the north being the east.

The cardinal points correspond with those marked on the card of a mariner's compass, allowance being made for the variation of the needle.

2344. *The azimuth.*—The direction of an object, whether terrestrial or celestial, in reference to the cardinal points, or to the plane of the meridian, is called its AZIMUTH. Thus it is said to have so many degrees of azimuth east or west, according as the vertical circle, whose plane passes through it, forms that angle east or west of the plane of the meridian.

2345. *Zenith distance and altitude.*—It is always possible to conceive a vertical circle which shall pass through any proposed object on the heavens. The arc of such a circle between the zenith and the object is called its ZENITH DISTANCE.

The remainder of the quadrant of the vertical between the object and the horizon is called its ALTITUDE.

It is evident, therefore, that the altitude of the zenith is 90°, and the zenith distance of every point on the horizon is also 90°.

The arc of the meridian between the zenith and the pole is the zenith distance of the pole, and the arc of the meridian between the pole and the horizon is the altitude of the pole.

2346. *Celestial equator.*—If a plane be imagined to pass through the place of the observer at right angles to the axis of the sphere, and to be continued to the heavens, it will meet the surface of the celestial vault in a circle which shall be 90° from the pole, and which will divide the sphere into two hemispheres, at the vertex of one of which is the visible or north pole, and at the vertex of the other the invisible or south pole.

This circle is called the CELESTIAL EQUATOR.

The several fixed points and circles described above will be more clearly conceived by the aid of the diagram, *fig.* 695.,

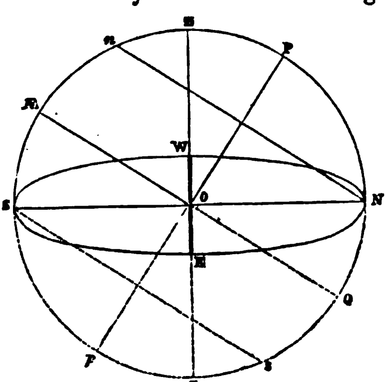


Fig. 695.

where O is the place of the observer, Z the zenith, P the pole, SZPN the visible, and spzN the invisible half of the meridian; SENW is the horizon seen by projection as an oval, being, however, really a circle; N and S are the north and south, and E and W the east and west cardinal points. The points of the several circles which are below the horizon are distinguished by dotted lines. The celestial equator is represented at ÆQ, and the prime vertical at ZWEz, both being looked at edgewise.

A plane Nn, drawn through the north cardinal point, cuts off a portion of the sphere, having the visible pole N at its centre, all of which is above the horizon; and a corresponding plane, Ss, through the south cardinal point, cuts off a part, leaving the invisible pole at its centre, all of which is below the horizon.

2347. *Apparent motion of the celestial sphere.*—Now, if the entire sphere be imagined to revolve on the line POp through the poles as a fixed axis, making one complete revolution, and in such a direction that it will pass over an observer at O, looking towards N from his right to his left, carrying with it all the objects on the firmament, without disturbing their relative position and arrangement, we shall form an exact notion of the apparent motion of the heavens. All objects rise upon the eastern half, SEN, of the horizon, and set upon the western half, SWN. The objects which are nearer to the visible pole P than the circle nN never set; and those which are nearer to the invisible pole P than the circle Ss never rise. Those which are between the equator ÆQ and the circle nN are longer above the horizon than below it; and those which are between the equator ÆQ and the circle Ss are longer below the equator than above it. Objects, in fine, which are upon the equator are equal times below and above the horizon.

When an object rises, it gradually increases its altitude until it reaches the meridian. It then begins to descend, and continues to descend until it sets.

CHAP. IV.

DIURNAL ROTATION OF THE EARTH.

2348. *Apparent diurnal rotation of the heavens—its possible causes.*—The apparent diurnal rotation of the celestial sphere being such as has been explained, it remains to determine what is the real motion which produces it. Now it is demonstrable that it may be caused indifferently, either by a real motion of the sphere round the observer corresponding in direction and velocity with the apparent motion, or by a real motion of the earth in the contrary direction, but with the same angular velocity upon that diameter of the globe which coincides with the direction of the axis of the celestial sphere, and that no other conceivable motion would produce that apparent rotation of the heavens which we witness. Between these two we are to decide which really exists.

2349. *Supposition of the real motion of the universe inadmissible.*—The fixity and absolute repose of the globe of the earth being assumed by the ancients as a physical maxim which did not even admit of being questioned, they perceived the inevitable character of the alternative which the apparent diurnal rotation of the heavens imposed upon them, and accordingly embraced the hypothesis, which now appears so monstrous, and which is implied in the term UNIVERSE*, which they have bequeathed to us.

It is true that, owing to the imperfect knowledge which prevailed as to the real magnitudes and distances of the bodies to which this common motion was so unhesitatingly ascribed, the improbability of the supposition would not have seemed so gross as it does to the more enlightened enquirers of our age. Nevertheless, in any view of it, and even with the most imperfect knowledge, the hypothesis which required the admission that

* UNUS, *one*, and VERSUM, *turning*, or rotation,—turning with one common motion of rotation.

the myriads of bodies which appear upon the firmament should have, besides the proper motions of several of them, such as the moon and planets, of which the ancients were not unaware, motions of revolution with velocities so prodigious and so marvellously related that all should, in the short interval of twenty-four hours, whirl round the axis of the earth with the unerring harmony and regularity necessary to explain the apparent diurnal rotation of the firmament, ought to have raised serious difficulties and doubts.

But with the knowledge which has been obtained by the labours of modern astronomers respecting the enormous magnitudes of the principal bodies of the physical universe, magnitudes compared with which that of the globe of the earth dwindles to a mere point, and their distances under the expression of which the very power of number itself almost fails, and recourse is had to colossal units in order to enable it to express even the smallest of them, the hypothesis of the immobility of the earth, and the diurnal rotation of the countless orbs of magnitudes so unconceivable filling the immensity of space once every twenty-four hours round this grain of matter composing our globe, becomes so preposterous that it is rejected, not as an improbability, but as an absurdity too gross to be even for a moment seriously entertained or discussed.

2350. *Simplicity and intrinsic probability of the rotation of the earth.*—But if any ground for hesitation in the rejection of this hypothesis existed, all doubt would be removed by the simplicity and intrinsic probability of the only other physical cause which can produce the phenomena. The rotation of the globe of the earth upon an axis passing through its poles, with an uniform motion from west to east once in twenty-four hours, is a supposition against which not a single reason can be adduced based on improbability. Such a motion explains perfectly the apparent diurnal rotation of the celestial sphere. Being uniform and free from irregularities, checks, or jolts, it would not be perceivable by any local derangement of bodies on the surface of the earth, all of which would partipate in it. Observers upon the surface of our globe would be no more conscious of it, than are the voyagers shut up in the cabin of a canal boat, or transported above the clouds in the car of a balloon.

2351. *Direct proofs of the earth's rotation.*—Irresistible, nevertheless, as this logical alternative is, the universality and

antiquity of the belief in the immobility of the earth, and the vast physical importance of the principle in question, have prompted enquirers to search for direct proofs of the actual motion of the earth upon its axis. Two phenomena have accordingly been produced as immediate and conclusive proof of this motion.

2352. *Proof by the descent of a body from a great height.*—It has been already (184) shown that a body descending from a great height does not fall in the true vertical line, which it would if the earth were at rest, but eastward of it, which it must, if the earth have a motion of rotation from west to east.

2353. *M. Leon Foucault's mode of demonstration.*—An ingenious expedient, by which the diurnal rotation of the earth is rendered visible, has been conceived and reduced to experiment by M. Leon Foucault. This contrivance is based upon the principle, that the direction of the plane of vibration of a pendulum is not affected by any motion of translation which may be given to its point of suspension. Thus, if a pendulum suspended in a room and put into vibration in a plane parallel to one of the walls be carried round a circular table, the plane of its vibration will continually be parallel to the same wall, and will therefore vary constantly in the angle it forms with the radius of the table which is directed to it.

Now, if a pendulum, suspended any where so near the pole of the earth that the circle round the pole may be considered a plane, be put in vibration in a plane passing through the pole, this plane, continuing parallel to its original direction as it is carried round the pole by the earth's rotation, will make a varying angle with the line drawn to the pole from the position it occupies. After being carried through a quarter of a revolution it will make an angle of 90° with the line to the pole, and so on. In fine, the direction of the pole will appear to be carried round the plane of vibration of the pendulum.

The same effects will be produced at greater distances from the pole, but the rate of variation of the angle under the plane of vibration and the plane of the meridian will be different, owing to the effects of the curvature of the meridian.

This phenomenon, therefore, being a direct effect of the rotation of the earth, supplies a proof of the existence of that motion, attainable without reference to objects beyond the limits of the globe.

2354. *Analogy supplies evidence of the earth's rotation.*—The obvious analogy of the planets to the earth, which will appear more fully hereafter, would supply strong evidence in favour of the earth's rotation, even if positive demonstration were wanting. All the planets are globes like the earth, receiving light and heat from the same luminary, and, like the earth, revolving round it. Now all the planets which we have been enabled to observe have motions of rotation on axes, in times not very different from that of the earth.

2355. *Figure of the earth supplies another proof.*—Besides these, it will be shown hereafter that another proof of the rotation of the earth is supplied by a peculiar departure from the strictly globular form.

2356. *How this rotation of the earth explains the diurnal phenomena.*—We are then to conclude that the earth, being a globe, has a motion of uniform rotation round a certain diameter. The universe around it is relatively stationary, and the bodies which compose it being at distances which mere vision cannot appreciate, appear as if they were situate on the surface of a vast celestial sphere in the centre of which the earth revolves. This rotation of the earth gives to the sphere the appearance of revolving in the contrary direction, as the progressive motion of a boat on a river gives to the banks an appearance of retrogressive motion; and since the apparent motion of the heavens is from east to west, the real rotation of the earth which produces that appearance must be from west to east.

How this motion of rotation explains the phenomena of the rising and setting of celestial objects is easily understood. An observer placed at any point upon the surface of the earth is carried round the axis in a circle in twenty-four hours, so that every side of the celestial sphere is in succession exposed to his view. As he is carried upon the side opposite to that in which the sun is placed, he sees the starry heavens visible in the absence of the splendour of that luminary. As he is turned gradually towards the side where the sun is placed, its light begins to appear in the firmament, the dawn of morning is manifested, and the globe continuing to turn, he is brought into view of the luminary itself, and all the phenomena of dawn, morning, and sunrise are exhibited. While he is directed towards the side of the firmament in which the sun is placed, the other bodies of inferior lustre are lost in the splendour of that luminary, and

all the phenomena of day are exhibited. When by the continued rotation of the globe the observer begins to be turned away from the direction of the sun, that luminary declines, and at length disappears, producing all the phenomena of evening and sunset.

Such, in general, are the effects which would attend the motion of a spectator placed upon the earth's surface, and carried round with it by its motion of rotation. He is the spectator of a gorgeous diorama exhibited on a vast scale, the earth which forms his station being the revolving stage by which he is carried round, so as to view in succession the spectacle which surrounds him.

These appearances vary with the position assumed by the observer on this revolving stage, or, in other words, upon his situation on the earth, as will presently appear.

2357. *The earth's axis.* — That diameter upon which it is necessary to suppose the earth to revolve in order to explain the phenomena is that which passes through the terrestrial poles.

2358. *The terrestial equator, poles, and meridians.* — If the globe of the earth be imagined to be cut by a plane passing through its centre at right angles to its axis, such a plane will meet the surface in a circle, which will divide it into two hemispheres, at the summits of which the poles are situate. This circle is called the TERRESTRIAL EQUATOR.

That hemisphere which includes the continent of Europe is called the NORTHERN HEMISPHERE, and the pole which it includes is called the NORTHERN TERRESTRIAL POLE; the other hemisphere being the SOUTHERN HEMISPHERE, and including the SOUTHERN TERRESTRIAL POLE.

If the surface of the earth be imagined to be intersected by planes passing through its axis, they will meet the surface in circles which, passing through the poles, will be at right angles to the equator. These circles are called TERRESTRIAL MERIDIANS, and will be seen delineated on any ordinary terrestrial globe.

2359. *Latitude and longitude.* — The positions of places upon the surface of the earth are expressed and indicated by stating their distance north or south of the equator, measured upon a meridian passing through them, and by the distance of such meridian east or west of some fixed meridian arbitrarily selected,

such as the meridian passing through the observatory at Greenwich. The former distance, expressed in degrees, minutes, and seconds, is called the LATITUDE, and the latter, similarly expressed, the LONGITUDE of the place.

2360. *Fixed meridians — those of Greenwich and Paris.* — As no natural phenomenon is found by which a fixed meridian from which longitude is measured can be determined, astronomers and geographers have not agreed in the arbitrary selection of one. The meridians of the Greenwich and Paris observatories have been taken, the former by English and the latter by French authorities, as the starting-point. To reduce the longitudes expressed by either to the other, it is only necessary to add or subtract the angle under the meridians of the two observatories, which has been ascertained to be 2° 20′ 22″, the meridian of Paris being east of that of Greenwich.

2361. *How the diurnal phenomena vary with the latitude.* — Let SÆNQ, *fig.* 696., represent the earth suspended in space, surrounded at an immeasurable distance by the stellar universe. The magnitude of the earth being absolutely insignificant compared with the distances of the stars, the aspect of these will be the same whether they are viewed from any point on its surface, or from its centre. The observer may therefore, whatever be his position on the earth, be considered as looking from the centre of the celestial sphere.

Let us suppose, in the first place, the observer to be at O, a point on its surface between the equator Æ and the north pole N, the latitude of which will therefore be OÆ, and will be measured by the angle OCÆ. If a line be imagined to be drawn from the centre C through the place O of the observer, and continued upwards to the firmament, it will arrive at the point *z*, which is the zenith of the observer. If the terrestrial axis SN be imagined to be continued to the firmament, it will arrive at the north celestial pole *n* and the south celestial pole *s*. If the plane of the terrestrial equator ÆQ be supposed to be continued to the heavens, it will intersect the surface of the celestial sphere at the celestial equator *æ q*.

The observer placed at O will see the entire hemisphere *h z h′* of which his zenith *z* is the summit; and the other hemisphere *h s h′* will be invisible to him, being in fact concealed from his view by the earth on which he stands.

It is evident that the arc of the heavens *z n* between his

zenith and the north celestial pole consists of the same number of degrees as the arc ON of the terrestrial meridian between his

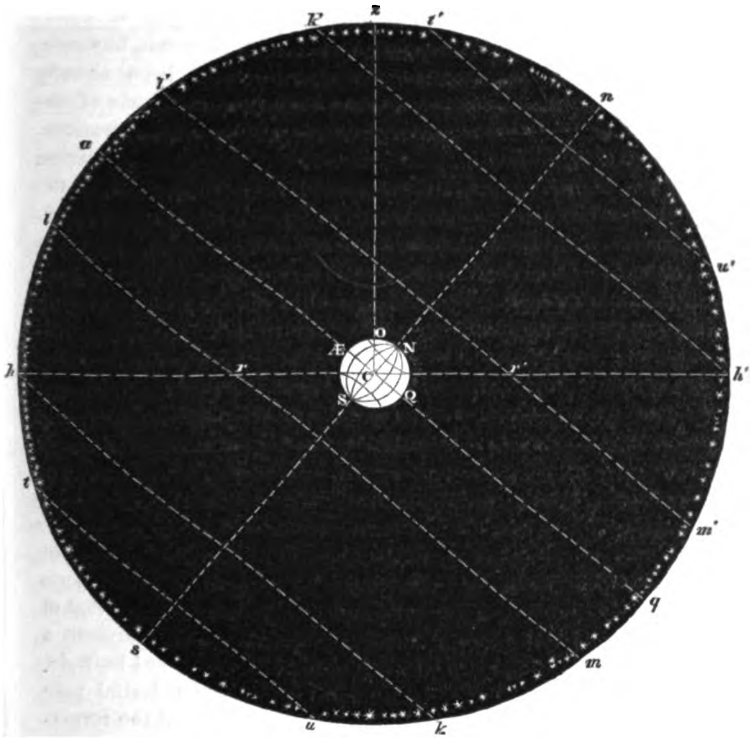


Fig. 696.

place of observation O and the north terrestrial pole *n*. The zenith distance therefore of the visible pole at any place is always equal to the actual distance expressed in degrees of that place from the terrestrial pole, and as this distance is the COMPLEMENT * OF THE LATITUDE, it follows that the zenith distance of the visible pole is the complement of the latitude, and that the altitude of the visible pole is equal to the latitude of the place.

* The complement of an angle or arc is that number of degrees by which it differs from 90°. Thus 30° is the complement of 60°.

2362. *Method of finding the latitude of the place.*—The latitude of the place of observation may therefore be always determined if the altitude of the celestial pole can be observed. If there were any star situate precisely at the pole, it would therefore be sufficient to observe its altitude. There is, however, no star exactly at the pole, although, as has been already observed, the POLE STAR is very near it. The altitude of the pole is found, therefore, not by one, but by two observations. The pole star, or any other star situate near the pole, is carried round it in a circle by the apparent diurnal motion of the sphere, and it necessarily crosses the meridian twice in each revolution, once *above*, and once *below* the pole. Its altitude in the latter position is the *least*, and in the former the *greatest* it ever has; and the pole itself is just midway between these two extreme positions of this circumpolar star. To find the actual altitude of the pole, it is only necessary therefore to take the *mean*, that is, *half the sum* of these two extreme altitudes. By making the same observations with several circumpolar stars, and taking a mean of the whole, still greater accuracy may be attained.

2363. *Position of celestial equator and poles varies with the latitude.*—Since the altitude of the celestial pole is everywhere equal to the latitude of the place, and since the position of the celestial equator and its parallels in which all celestial objects appear to be moved by the diurnal rotation, varies with that of the pole, it is evident that the celestial sphere must present a different appearance to the observer at every different latitude. In proceeding towards the terrestrial pole, the celestial pole will gradually approach the zenith, until we arrive at the terrestrial pole, when it will actually coincide with that point; and in proceeding towards the terrestrial equator the celestial pole will gradually descend towards the horizon, and on arriving at the Line it will be actually on the horizon.

2364. *Parallel sphere seen at the poles.*—At the poles, therefore, the celestial pole being in the zenith, the celestial equator will coincide with the horizon, and by the diurnal motion all objects will move in circles parallel to the horizon. Every object will therefore preserve during twenty-four hours the same altitude and the same zenith distance. No object will either rise or set, at least so far as the diurnal motion is concerned.

This aspect of the firmament is called a PARALLEL SPHERE, the motion being parallel to the horizon.

2365. *Right sphere seen at the equator.* — At the terrestrial equator, the poles being upon the horizon, the axis of the celestial sphere will coincide with a line drawn upon the plane of the horizon connecting the north and south points. The celestial equator and its parallels will be at right angles to the plane of the horizon; and since the plane of the horizon passes through the centre of all the parallels, it will divide them all into equal semicircles.

It follows, therefore, that all objects on the heavens will be equal times above and below the horizon, and that they will rise and set in planes perpendicular to the horizon.

This aspect of the firmament is called a RIGHT SPHERE, the diurnal motion being at right angles to the horizon.

2366. *Oblique sphere seen at intermediate latitudes.* — At latitudes between the equator and pole, the celestial pole holds a place between the horizon and the zenith determined by the latitude. The celestial equator *æq*, *fig.* 696., and its parallels, are inclined to the plane of the horizon at angles equal to the distance of the pole from the zenith, and therefore equal to the complement of the latitude. The centres of all parallels to the celestial equator *æq* which are between it and the visible pole are above the plane of the horizon, between C and N, and the centres of all parallels at the other side of the equator below it. The parallels, such as *l′m′* and *lm*, will therefore be all divided unequally by the plane of the horizon, the visible part *l′r′* being greater than the invisible part *m′r′* for the former, and the invisible part *mr* greater than the visible part *lr* for the latter.

It follows, therefore, that all objects between the celestial equator *æq* and the visible pole N will be longer above than below the horizon, and all objects on the other side of the equator will be longer below the horizon than above it.

A parallel *h′k′* to the celestial equator, whose distance from the visible pole is equal to the latitude, will be entirely above the horizon, just touching it at the point under the visible pole; and a corresponding parallel *hk*, at an equal distance from the invisible pole, will be entirely below the horizon, just touching it at the point above the invisible pole.

All parallels nearer to the visible pole than *h′k′* will be en-

tirely above the horizon, and all parallels nearer to the invisible pole than *h k* will be entirely below it.

Hence it is that, in European latitudes, stars within a certain limited distance of the north or visible celestial pole never set, and stars at a corresponding distance from the south or invisible celestial pole never rise.

The observer can only see these by going to places of observation having lower latitudes.

This aspect of the firmament is called an OBLIQUE SPHERE, the diurnal motion being oblique to the horizon.

2367. *Objects in celestial equator equal times above and below horizon.* — Whether the sphere be right or oblique, the centre of the celestial equator being on the plane of the horizon, one half of that circle will be below, and the other above the horizon. Every object upon it will therefore be equal times above and below the horizon, rising and setting exactly at the east and west points.

In the parallel sphere, the celestial equator coinciding with the horizon, an object upon it will be carried round the horizon by the diurnal rotation, without either rising or setting.*

2368. *Method of determining the longitude of places.* — This perfect uniformity of the earth's rotation, inferred from the observed uniformity of the apparent rotation of the firmament, is the basis of all methods of determining the longitude. The longitude of a place will be determined if the angle under the meridian of the place, and that of any other place whose longitude is known, can be found. But since, by the uniform rotation of the globe, the meridians of all places upon it are brought in regular succession under every part of the firmament, the moments at which the two meridians pass under the same star, or, what is the same, the moments at which the same star is seen to pass over the two meridians, being observed, the interval will bear the same ratio to the entire time of the earth's rotation as the difference of the longitudes of the two places bears to 360°.

* The teacher will find it advantageous to exercise the student in the subject of the preeeding paragraphs, aided by an armillary sphere, or, if that be not accessible, by a celestial globe, which will serve nearly as well. Many questions will suggest themselves, arising out of and deducible from what has been explained above, with respect to the various altitudes of the ph ere in different latitudes.

To make this more clear, let us take the case of two places P and P′, *fig.* 697., upon the equator. If C be the centre of the earth, the angle PCP′ will be the difference between the longitudes. Now, let the time be observed at each place at which any particular star S is seen upon the meridian. If the motion of the earth be in the direction of the arrow, the meridian of P will come to the star before the meridian of P′. This necessarily supposes P to be east of P′, since the earth revolves from west to east. Let the true interval of time between the passage of S over the two meridians be t, let T be the time of one complete revolution of the globe on its axis, and let L be the difference of the longitudes, or the angle PCP′; we shall then have

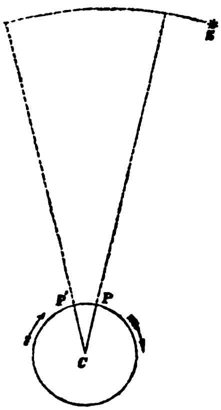


Fig. 697.

$$t : \mathrm{T} :: \mathrm{L} : 360°,$$

$$\mathrm{L} = \frac{t}{\mathrm{T}} \times 360°.$$

But in the practical solution of this problem a difficulty is presented which has conferred historical celebrity upon the question, and caused it to be referred to as the type of all difficult enquiries. It is supposed, in what has just been explained, that means are provided at the two places P and P′ by which the absolute moments of the transit of the star over the respective meridians may be ascertained, so as to give the exact interval between them. If these moments be observed by any form of chronometer, it would then be necessary that the two chronometers should be in exact accordance, or, what is the same, that their exact difference may be known. If a chronometer, set correctly by another which is stationary at one place P, be transported to the other place P′, this object will be attained, subject, however, to the error which may be incidental to the rate of the chronometer thus transported. If the distance between the places be not considerable, the chronometers may thus be brought into very exact accordance; but when the distance is great, and that a long interval must elapse during the trans-

port of the chronometer, this expedient is subject to errors too considerable to be tolerated in the solution of a problem of such capital importance.

It will be apparent that the real object to be attained is, to find some phenomenon sufficiently instantaneous in its manifestation to mark, with all the necessary precision, a certain moment of time. Such a phenomenon would be, for example, the sudden extinction of a conspicuous light seen at once at both places. The moment of such a phenomenon being observed by means of two chronometers at the places, the difference of the times indicated by them would be known, and they would then serve for the determination of the difference of the longitudes by the method explained above. Several phenomena, both terrestrial and celestial, have accordingly been used for this purpose. Among the former may be mentioned the sudden extinction of the oxyhydrogen or electric light, the explosion of a rocket, &c.; among the latter, the extinction of a star by the disk of the moon passing over it, and the eclipse of the satellites of certain planets, phenomena which will be more fully noticed hereafter.

2369. *Lunar method of finding the longitude.* — The change of position of the moon with relation to the sun and stars being very rapid, affords another phenomenon which has been found of great utility in the determination of the longitude, especially for the purposes of mariners. Tables are calculated in which the moon's apparent distances from the sun, and many of the most conspicuous fixed stars, are given for short intervals of time, and the exact times at Greenwich when the moon has these distances are given. If then the mariner, observing with proper instruments the position of the moon with relation to these objects, compares his observed distances with the tables which are supplied to him in the Nautical Almanack, he will find the time at Greenwich corresponding to the moment of his observation; and being always, by the ordinary methods, able to determine by observation the local time at the place of his observation, the difference gives him the time required for a star to pass from the meridian of Greenwich to the meridian of the place of his observation, or *vice versâ;* and this time gives the longitude, as already explained.

This last is known as the LUNAR METHOD OF DETERMINING THE LONGITUDE.

In practice, many details are necessary, and various calculations must be made, which cannot be explained here.

2370. *Method by electric telegraph.* — When two places are connected by a line of electric telegraph, their difference of longitude can be easily and exactly determined, inasmuch as instantaneous signals can be transmitted, by which the local clocks can be compared and regulated, and, if it be so desired, kept in exact accordance.

2371. *Parallels of latitude.*— A series of points on the earth which are at equal distances from the equator, or which have the same latitude, form a circle parallel to the equator, called a PARALLEL OF LATITUDE.

Thus all places which have the same latitude are on the same parallel.

All places which are on the same meridian have the same longitude.

CHAP. V.

SPHEROIDAL FORM, MASS, AND DENSITY OF THE EARTH.

2372. *Progress of physical investigation approximative.*— It is the condition of man, and probably of all other finite intelligences, to arrive at the possession of knowledge by the slow and laborious process of a sort of system of trial and error. The first conclusions to which, in physical enquiries, observation conducts us, are never better than very rough approximations to the truth. These being submitted to subsequent comparison with the originals, undergo a first series of corrections, the more prominent and conspicuous departures from conformity being removed. A second approximation, but still only an approximation, is thus obtained; and another and still more severe comparison with the phenomena under investigation is made, and another order of corrections is effected, and a closer approximation obtained. Nor does this progressive approach to perfect exactitude appear to have any limit. The best results of our intellectual labours are still only close resemblances to truth, the absolute perfection of which is probably reserved for a higher intellectual state.

The labours of the physical enquirer resemble those of the sculptor, whose first efforts produce from the block of marble a rude and uncouth resemblance of the human form, which only approaches the grace and beauty of nature by comparing it incessantly and indefatigably with the original; detaching from it first the grosser and rougher protuberances, and subsequently reducing its parts by the nicer and more delicate touches of the chisel to near conformity with the model.

It would however be a great mistake to depreciate on this account the results of our first efforts in the acquisition of a knowledge of the laws of nature. If the first conclusions at which we arrive are erroneous, they are not therefore the less necessary to the ultimate attainment of more exact knowledge. They prove, on the contrary, not only to be powerful agents in the discovery of those corrections to which they are themselves to be submitted, but to be quite indispensable to our progress in the work of investigation and discovery.

These observations will be illustrated by the process of instruction and discovery in every department of physical science, but in none so frequently and so forcibly as in that which now occupies us.

2373. *Figure of the earth an example of this.* — The first conclusions at which we have arrived respecting the form of the earth is that it is a globe; and with respect to its motion is, that it is in uniform rotation round one of its diameters, making one complete revolution in twenty-four hours sidereal time, or $23^{h.}\ 56^{m.}\ 4{\cdot}09^{s.}$ common or civil time.

2374. *Globular figure incompatible with rotation.* — The first question then which presents itself is, whether this form and rotation are compatible? It is not difficult to show, by the most simple principles of physics, that they are not; that with such a form such a rotation could not be maintained, and that with such a rotation such a form could not permanently continue. And if this can be certainly established, it will be necessary to retrace our steps, to submit our former conclusions to more rigorous comparison with the objects and phenomena from which they were derived, and ascertain which of them is inexact, and what is the modification and correction to which it must be submitted in order to be brought into harmony with the other.

2375. *Rotation cannot be modified — supposed form may.* —

The conclusion that the earth revolves on its axis with a motion corresponding to the apparent rotation of the firmament, is one which admits of no modification, and must from its nature be either absolutely admitted or absolutely rejected. The globular form imputed to the earth, however, has been inferred for observations of a general nature, unattended by any conditions of exact measurement, and which would be equally compatible with innumerable forms, departing to a very considerable and measurable extent from that of an exact geometrical sphere or globe.

2376. *How rotation would affect the superficial gravity on a globe.*—Let N Q S, *fig.* 698., represent a section of a globe supposed to have a motion of rotation round the diameter N S as an axis. Every point on its surface, such as P or P′, will revolve in a circle, the centre of which *o* or *o′* will be upon the axis, and the radius *o* P or *o′* P″ will gradually decrease in approaching the poles N and S, where no motion takes place, and will gradually increase in approaching the equator Q O Q, where the circle of rotation will be the equator itself.

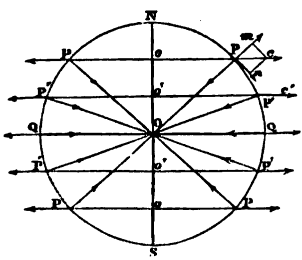


Fig. 698.

A body placed at any part of the surface, such as P, being thus carried round in a circle, will be affected by a centrifugal force, the intensity of which will be expressed by (314)

$$C = 1\cdot226 \times R \times N^2 \times W,$$

where $R = P\,o$, the radius of the circle, N the fraction of a revolution made in one second, and W the weight of the body, and the direction of which is P *c*.

This centrifugal force being expressed by P *c* is equivalent (170) to two forces expressed in intensity and direction by P *m* and P *n*. The component P *m* is directly opposed to the weight W of the body, which acts in the line P O directed to the centre, and has the effect of diminishing it. The component P *n* being directed towards the equator Q, has a tendency to cause the body to move towards the equator; and the body, if free, would necessarily so move.

Now it will be evident, by the mere inspection of the diagram,

that the nearer the point P is to the equator Q, the more directly will the centrifugal force P *c* be opposed to the weight, and consequently the greater will be that component of it, P *m*, which will have the effect of diminishing the weight.

But this diminution of the weight is further augmented by the increase of the actual intensity of the centrifugal force itself in approaching the equator. By the above formula, it appears that the intensity of the centrifugal force must increase in proportion as the radius R or P *o* increases. Now it is apparent that P *o* increases gradually in going from P to Q, since P′ *o*′ is greater, and Q O greater still than P *o*; and that, on the other hand, it decreases in going from P to N or S, where it becomes nothing.

Thus the effect of the centrifugal force in diminishing weight being nothing at the pole N or S, gradually increases in approaching the equator; *first*, because its absolute intensity gradually increases; and *secondly*, because it is more and more directly opposed to gravity until we arrive at the equator itself, where its intensity is greatest, and where it is directly opposed to gravity.

The effects, therefore, produced by the rotation of a globe, such as the earth has been assumed to be, are—1°. The decrease of the weights of bodies upon its surface, in going from the pole to the equator; and 2°. A tendency of all such bodies as are free to move from higher latitudes in either hemisphere towards the equator.

2377. *Amount of the diminution of weight produced at the equator by centrifugal force.* — This quantity may be easily computed by means of the formula

$$C = 1{\cdot}226 \times R \times N^2 \times W.$$

Taking the radius of the equator in round numbers (which are sufficient for this purpose) at 4000 miles, and reducing it to feet, and reducing the time of rotation $23^{h\cdot}\ 56^{m\cdot}\ 4{\cdot}09^{s\cdot}$ to seconds, we shall have

$$R = 21{,}120{,}000, \qquad N = \frac{1}{86{,}206}:$$

substituting these numbers we have

$$C = 1{,}226 \times 21{,}120{,}000 \times \frac{1}{(86{,}216)^2} \times W;$$

and executing the arithmetical operations here indicated, we find

$$C = \frac{1}{287} \times W.$$

The centrifugal force would therefore be the 287th part of the weight, and as it is directly opposed to gravity, the weight would sustain this entire loss.

2378. *Loss of weight at other latitudes.* — The centrifugal force at any latitude P would be less than at Q in the ratio of O Q to o P. But the part of this P m which is directly opposed to the weight is less than the whole P c, in the ratio of P c to P m, or, what is the same, of P O or O Q to P o. If then C′ express the whole centrifugal force at P, and C″ that part of it which is directly opposed to gravity, we shall have

$$C' = \frac{1}{287} \times \frac{Po}{OQ}, \qquad C'' = C' \times \frac{Po}{OQ} = \frac{1}{287} \times \left(\frac{Po}{OQ}\right)^2.$$

The number which expresses $\frac{Po}{OQ}$ is that which is called in Trigonometry the cosine of the arc P Q, that is, the cosine of the latitude. Therefore we have

$$C'' = \frac{1}{287} \times \cos.^2 \text{lat.}$$

The loss of weight, therefore, which would be sustained by reason of the centrifugal force at any proposed latitude, would be a fraction of the whole weight found by dividing the square of the cosine of latitude by 287.

2379. *Effect of centrifugal force on the geographical condition of the surface of the globe.* — In what precedes, we have only considered the effect of that one, P m, of the two components of the centrifugal force which is opposed to the weight. It remains to examine the effect of the other, P n, which is directed towards the equator.

If the surface of the globe were composed altogether of solid matter, of such coherence as to resist separation by the agency of this force, no other effect would take place except a *tendency* towards the equator, which would be neutralized by cohesion. But if the surface or any parts of it were fluid, whether liquid or gaseous, such parts, in virtue of their mobility, would yield to the impulse of the element P n of the centrifugal force, and

would flow towards the equator. The waters of the surface would thus flow from the higher latitudes in either hemisphere, and accumulating round the equator, the surface of the globe would be resolved into two great polar continents, separated by a vast equatorial ocean.

2380. *Such effects not existing, the earth cannot be an exact globe.*—But such is not the actual geographical condition of the surface of the globe. On the contrary, although about two-thirds of it are covered with water, no tendency of that fluid to accumulate more about the equator than elsewhere is manifested. Land and water, if not indifferently distributed over the surface, are certainly not apportioned so as to indicate any tendency such as that above described. If, therefore, the rotation of the earth be admitted, it follows that its figure must be such as to counteract the tendency of fluid matter to flow towards any one part of the surface rather than any other. In short, its figure must be such that gravity itself shall counteract that element P *n* of the centrifugal force which tends to move a body from the higher latitudes of either hemisphere towards the equator.

2381. *The figure must therefore be some sort of oblate spheroid.*—Now this condition would be fulfilled, if the earth, instead of being an exact sphere, were an oblate spheroid, having a certain definite ellipticity,—that is, a figure which would be produced by an ellipse revolving round its shorter axis. Such a figure would resemble an orange or a turnip. It would be more convex at the equator than at the poles. A globe composed of elastic materials would be reduced to such a figure by pressing its poles together, so as to flatten more or less the surface of these points, and produce a protuberance around the equator. The meridians of such a globe would be ellipses, having its axis as their lesser axis, and the diameters of the equator as their greater axes.

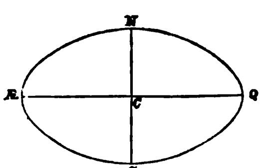


Fig. 699.

The form of the meridian would be such as is represented in *fig.* 699., N S being the axis of rotation, and Æ Q the equatorial diameter.

2382. *Its ellipticity must depend on gravity and centrifugal force.*—The protuberance around the equator may be more or less, according to the ellipticity of the spheroid; but since the distribution of land and water is indifferent on the surface,

having no prevalence about the equator rather than about the poles, or *vice versâ*, it is evident that the degree of protuberance must be that which counteracts, and no more than counteracts, the tendency of the fluids, in virtue of the centrifugal force, to flow towards the equator. This protuberance may be considered as equivalent in its effects to an acclivity of regulated inclination, rising from each pole towards the equator. To arrive at the equator the fluid must ascend this acclivity, to which ascent gravity opposes itself, with a force depending on its steepness, which increases with the magnitude of the protuberance, or, what is the same, with the ellipticity of the spheroid. If the ellipticity be less than is necessary to counteract the effect of the centrifugal force, the fluid will still flow to the equator, and the earth would consist, as before, of a great equatorial ocean separating two vast polar continents. If the ellipticity were greater than is necessary to counteract the effect of the centrifugal force, then gravity would prevail over the centrifugal force, and the waters would flow down the acclivities of the excessive protuberance towards the poles, and the earth would consist of a vast equatorial continent separating two polar oceans.

Since the geographical condition of the surface of the earth is not consistent with either of these consequences, it is evident that its figure must be an oblate spheroid, having an ellipticity exactly corresponding to the variation of gravity upon its surface, due to the combined effect of the attraction exerted by its constituent parts upon bodies placed on its surface, and the centrifugal force arising from its diurnal rotation.

It remains, therefore, to determine what this particular degree of ellipticity is, or, what is the same, to determine by what fraction of its whole length the equatorial diameter Æ Q exceeds the polar axis N S.

2383. *Ellipticity may be calculated and measured, and the results compared.*—The degree of ellipticity of the terrestrial spheroid may be found by theory, or ascertained by observation and measurement, or by both these methods, in which case the accordance or discrepancy of the results will either prove the validity of the reasoning on which the theoretical calculation is founded, or indicate the conditions or data in such reasoning which must be modified.

Both these methods have accordingly been adopted, and their results are found to be in complete harmony.

2384. *Ellipticity calculated.*—The several quantities which are involved in this problem are:—

1. The time of rotation = R.
2. The fraction of its whole length by which the equatorial exceeds the polar diameter = *e*.
3. The fraction of its whole weight by which the weight of a body at the pole exceeds the weight of the same body at the equator = *w*.
4. The mean density of the earth.
5. The law according to which the density of the strata varies in proceeding from the surface to the centre.

All these quantities have such a mutual dependance, that when some of them are given or known, the others may be found.

It whatever way the solution of the problem may be approached, it is evident that the form of the spheroid must be the same as it would be if the entire mass of the earth were fluid. If this were not so, the parts actually fluid would not be found, as they are always, in local equilibrium. The state of relative density of the strata proceeding from the surface to the centre is, however, not so evident. Newton investigated the question by ascertaining the form which the earth would assume if it consisted of fluid matter of uniform density from the surface to the centre; and the result of his analysis was that, in that case, assuming the time of rotation to be what it is, the equatorial diameter must exceed the polar by the 230th part of its whole length, and gravity at the pole must exceed gravity at the equator by the same fraction of its entire force.

As physical science progressed, and mathematical analysis was brought to a greater state of perfection, the same problem was investigated by Clairault and several other mathematicians, under more rigorous conditions. The uniform density of the constituents of the earth—a highly improbable supposition—was put aside, and it was assumed that the successive strata from the centre to the surface increased in density according to some undetermined conditions. It was assumed that the mutual attraction of all the constituent parts upon any one part, and the effect of the centrifugal force arising from the rotation, are in equilibrium; so that every particle composing the spheroid,

from its centre to its surface, is in repose, and would remain so were it free to move.

By a complicated and very abstruse, but perfectly clear and certain mathematical analysis, it has been proved that the quantities above mentioned have the following relation. Let r express a certain number, the amount of which will vary with R. We shall then have

$$e + w = r.$$

Now it has been shown that when $R = 23^{h.}\ 56^{m.}\ 4{\cdot}09^{s.}$, the number r will be $\frac{1}{115}$, so that in effect

$$e + w = \frac{1}{115}.$$

This result was shown to be true, whatever may be the law according to which the density of the strata varies.

It further results from these theoretical researches that the mean density of the entire terrestrial spheroid is about twice the mean density of its superficial crust.

It follows from this that the density of its central parts must greatly exceed twice the density of its crust.

It remains, therefore, to see how far these results of theory are in accordance with those of actual observation and measurement.

2385. *Ellipticity of terrestrial spheroid by observation and measurement.*—If a terrestrial meridian were an exact circle, as it would necessarily be if the earth were an exact globe, every part of it would have the same curvature. But if it were an ellipse, of which the polar diameter is the lesser axis, it would have a varying curvature, the convexity being greatest at the equator, and least at the poles. If, then, it can be ascertained by observation, that the curvature of a meridian is not uniform, but that on the contrary it increases in going towards the Line, and diminishes in going towards the poles, we shall obtain a proof that its form is that of an oblate spheroid.

To comprehend the method of ascertaining this, it must be considered that the curvature of circles diminishes as their diameters are augmented. It is evident that a circle of one foot in diameter has a less degree of curvature, and is less convex than a circle one inch in diameter. But an arc of a circle of a given angular magnitude, such for example as 1°,

has a length proportional to the diameter. Thus, an arc of 1° of a circle a foot in diameter, is twelve times the length of an arc of 1° of a circle an inch in diameter. The curvature, therefore, increases as the length of an arc of 1° diminishes.

If, therefore, a degree of the meridian be observed, and measured, by the process already explained (2317), at different latitudes, and it is found that its length is not uniformly the same as it would be if the meridian were a circle, but that it is less in approaching the equator, and greater in approaching the pole, it will follow that the convexity or curvature increases towards the equator, and diminishes towards the poles; and that consequently the meridian has the form, not of a circle, but of an ellipse, the lesser axis of which is the polar diameter.

Such observations have accordingly been made, and the lengths of a degree in various latitudes, from the Line to 66° N. and to 35° S., have been measured, and found to vary from 363,000 feet on the Line to 367,000 feet at lat. 66°.

From a comparison of such measurements, it has been ascertained that the equatorial diameter of the spheroid exceeds the polar by $\frac{1}{300}$th of its length. Thus (2384)

$$e = \frac{1}{300}.$$

2386. *Variation of gravity by observation.*—The manner in which the variation of the intensity of superficial gravity at different latitudes is ascertained by means of the pendulum, has been already explained (552). From a comparison of these observations it has been inferred that the effective weight of a body at the pole exceeds its weight at the equator by about the $\frac{1}{187}$th* part of the whole weight.

2387. *Accordance of these results with theory.*—By comparing these results with those obtained by Newton, on the supposition of the uniform density of the earth, a discrepancy will be found sufficient to prove the falsehood of that supposition. The value of e found by Newton is $\frac{1}{230}$, its actual value being $\frac{1}{300}$, and that of w $\frac{1}{230}$, its actual value being $\frac{1}{187}$.

On the other hand, the accordance of these results of observation and measurement with the more rigorous conclusions of later researches is complete and striking.

* Different values are assigned to this—Sir John Herschel prefers $\frac{1}{194}$, the Astronomer Royal $\frac{1}{180}$. We have taken a mean between these estimates.

If in the relation between e and w, explained in (2384),

$$e+w=\frac{1}{115},$$

we substitute for w the value $\frac{1}{187}$, obtained by observation, we find

$$e=\frac{1}{115}-\frac{1}{187}=\frac{1}{300},$$

which is the value of w obtained by computation founded on measurement.

2388. *Diminution of weight due to ellipticity.*—It has been already shown (2377) that the loss of weight at the equator due to the centrifugal force is the 287th of the entire weight. From what has been stated (2386), it appears that the actual loss of weight at the equator is greater than this, being the 187th part of the entire weight. The difference of these is

$$\frac{1}{187}-\frac{1}{287}=\frac{1}{537}.$$

It appears, therefore, that while the 287th part of the weight is balanced by the centrifugal force, the actual attraction exerted by the earth upon a body at the equator is less than at the pole by the 537th* part of the whole weight. This difference is due to the elliptical form of the meridian, by which the distance of the body from the centre of the earth is augmented.

2389. *Actual linear dimensions of the terrestrial spheroid.*—It is not enough to know the proportions of the earth. It is required to determine the actual dimensions of the spheroid. The following are the lengths of the polar and equatorial diameters, according to the computations of the most eminent and recent authorities:—

	Bessel.	Airy.
	Miles.	Miles.
Polar diameter	7899·114	7899·170
Equatorial diameter	7925·604	7925·648
Absolute difference	26·471	26·478
Excess of the equatorial expressed in a fraction of its entire length	$\frac{1}{299 \cdot 407}$	$\frac{1}{299 \cdot 330}$

The close coincidence of these results supplies a striking

* According to Herschel, the 590th part.

example of the precision to which such calculations have been brought.

The departure of the terrestrial spheroid from the form of an exact globe is so inconsiderable that, if an exact model of it turned in ivory were placed before us, we could not, either by sight or touch, distinguish it from a perfect billiard ball. A figure of a meridian accurately drawn on paper could only be distinguished from a circle by the most precise measurement. If the major axis of such an ellipse were equal in length to the page now under the eye of the reader, the lesser axis would fall short of the same length less than the fortieth of an inch.

2390. *Dimensions of the spheroidal equatorial excess.* — If a sphere N q S q be imagined to be inscribed within the terrestrial spheroid having the polar axis N S, *fig.* 700., for its diameter, a spheroidal shell will be included between its surface and that of the spheroid composed of the protuberant matter, having a thickness Q q of 26 miles at the equator, and becoming gradually thinner in proceeding to the poles, where its thickness vanishes. This shell, which constitutes the equatorial excess of the spheroid, and which, as will hereafter appear, has a density not more than half the mean density of the earth, the bulk of which, moreover, would be imperceptible upon a mere inspection of the spheroid, is nevertheless attended with most important effects, and by its gravitation is the origin of most striking phenomena not only in relation to the moon, but also to the far more distant mass of the sun.

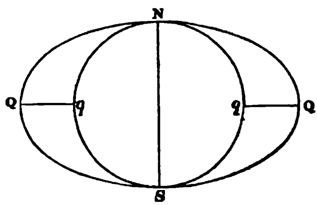


Fig. 700.

2391. *Density and mass of the earth by observation.* — The magnitude of the earth being known with great precision, the determination of its mass and that of its mean density become one and the same problem, since the comparison of its mass with its magnitude will give its mean density, and the comparison of its mean density with its magnitude will give its mass.

The methods of ascertaining the mass or actual quantity of matter contained in the earth are all based upon a comparison of the gravitating force or attraction which the earth exerts

upon an object with the attraction which some other body, whose mass is exactly known, exerts on the same object. It is assumed, as a postulate or axiom in physics, that two masses of matter which at equal distances exert equal attractions on the same body, must be equal. But as it is not always possible to bring the attracting and attracted bodies to equal distances, their attractions at unequal distances may be observed, and the attractions which they would exert at equal distances may be thence inferred by the general law of gravitation, by which the attraction exerted by the same body increases as the square of the distance from it is diminished.

Thus, if E be the mass of the earth, A the attraction it exerts at the distance D from its centre of gravity, and A′ the attraction it exerts at any other distance D′, we have —

$$\mathrm{A} : \mathrm{A}' :: \mathrm{D}'^2 : \mathrm{D}^2;$$

and therefore

$$\mathrm{A}' = \mathrm{A} \times \frac{\mathrm{D}^2}{\mathrm{D}'^2}.$$

If a be the attraction which any mass m of known quantity exerts at the distance D′ upon the same body upon which the earth exerts the attraction A′, we shall have —

$$\mathrm{E} : m :: \mathrm{A}' : a;$$

and therefore

$$\mathrm{E} = m \times \frac{\mathrm{A}'}{a} = m \times \frac{\mathrm{A}}{a} \times \frac{\mathrm{D}^2}{\mathrm{D}'^2}.$$

If, therefore, the mass m, the ratio of the attractions A and a, and the ratio of the distances D and D′, be respectively known, the mass E of the earth can be computed.

2392. *Dr. Maskelyne's solution by the attraction of Schehallien.* — This celebrated problem consisted in determining the ratio of the mean density of a mountain called Schehallien, in Perthshire, to that of the earth, by ascertaining the amount of the deviation of a plumb-line from the direction of the true vertical produced by the local attraction of the mountain.

To render this method practicable, it is necessary that the mountain selected be a solitary one, standing on an extensive plain, since otherwise the deviation of the plumb-line would be affected by neighbouring eminences to an extent which it might not be possible to estimate with the necessary precision. No

eminence sufficiently considerable exists near enough to Schehallien to produce such disturbance.

The mountain ranging east and west, two stations were selected on its northern and southern acclivities, so as to be in the same meridian, or very nearly so. A plumb-line, attached to an instrument called a zenith sector, adapted to measure with extreme accuracy small zenith distances, was brought to each of these stations, and the distance of the same star, seen upon the meridian from the directions of the plumb-line, were observed at both places.

The difference between those distances gave the angle under the two directions of the plumb-line. This will be more clearly understood by reference to *fig.* 701., where P and

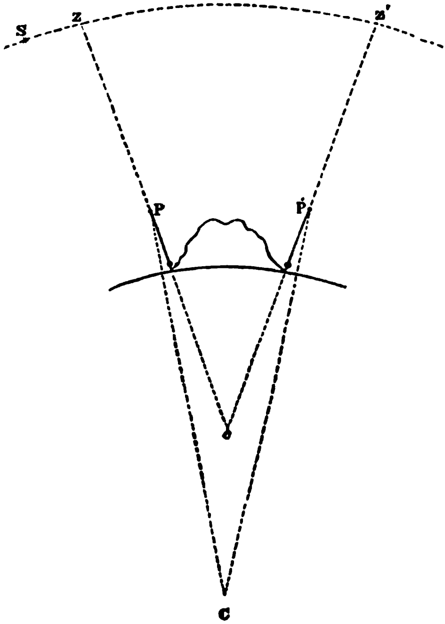


Fig. 701.

P′ represent the points of suspension of the two plumb-lines. If the mountain were removed, they would hang in the directions PC and P′C of the earth's centre, and their directions would be inclined at the angle PCP′. But the attraction exerted

by the interjacent mass produces on each side a slight deflection towards the mountain, so that the two directions of the plumb-line, instead of converging to the centre of the earth C, converge to a point *c* nearer to the surface, and form with each other an angle P*c*P′ greater than PCP′ by the sum of the two deflections CP*c* and CP′*c*.

Now by means of the zenith sector the distances SZ and SZ′ of the points Z and Z′ from any star such as S, can be observed with a precision so extreme as not to be subject to a greater error than a small fraction of a second. The difference of these distances will be —

$$SZ' - SZ = ZZ',$$

the apparent distance between the two points Z and Z′ on the heavens to which the plumb-line points at the two stations. This distance expressed in seconds gives the magnitude of the angle P*c*P′ formed by the directions of the plumb-line at the two stations, which is the sum of the deflection produced by the local attraction of the mountain.

If the mountain were not present, the angle PCP′ could be ascertained by the zenith sector; but as the indications of that instrument have reference to the direction of the plumb-line, it is rendered inapplicable in consequence of the disturbing effect of the mountain.

To determine the magnitude of the angle PCP′, therefore, the direct distance between the stations P and P′ is ascertained by making a survey of the mountain which, as will presently appear, is also necessary, in order to determine its exact volume. For every hundred feet in the distance between P and P′ there will be 1″ in the angle PCP′ (2319). Finding, therefore, the direct distance between P and P′ in feet, and dividing it by 100, we shall have the angle PCP′ in seconds.

In the case of the experiment of Dr. Maskelyne, which was made in 1774, the angle PCP′ was found to be 41″, and the angle P*c*P′ 53″. The sum of the two deflections was therefore 12″.

The survey of the mountain supplied the data necessary to determine its actual volume in cubic miles, or fraction of a cubic mile. An elaborate examination of its stratification, by means of sections, borings, and the other usual methods, supplied the data necessary to determine the weights of its com-

ponent parts, and thence the weight of its entire volume; and the comparison of this weight with its volume gave its mean density.

If the mean density of the earth were equal to that of the mountain, the entire weight of the earth would be greater than that of the mountain, in exactly the same proportion as the entire volume of the earth exceeds that of the mountain; and these volumes being known, the weight E of the earth on that supposition was computed, and by the formula given in (2391), or others based upon the same principles, the ratio $\frac{A}{a}$ of the attraction of the earth to that of the mountain was computed, and thence the deflections which the mountain would produce was found, which instead of 12″ was about 24″. It followed, therefore, that the density of the earth must be double, or, more exactly, eighteen-tenths of that of the mountain, in order to reduce the deflections to half their computed amount.

The mean density of the mountain having been ascertained to be about 2½ times that of water, it followed, therefore, that the mean density of the earth is about five times that of water.

2393. *Cavendish's solution.* — At a later period Cavendish made the experiment which bears his name, in which the attraction exerted by the earth upon a body on its surface was compared with the attraction exerted by a large metallic ball on the same body; and this experiment was repeated still more recently by Dr. Reich, and by the late Mr. Francis Baily, as the active member of a committee of the Royal Astronomical Society of London. All these several experimenters proceeded by methods which differed only in some of their practical details, and in the conditions and precautions adopted to obtain more accurate results.

In the apparatus used by Mr. Baily, the latest of them, the attracting bodies with which the globe of the earth was compared were two balls of lead, each a foot in diameter. The bodies upon which their attraction was manifested were small balls, about two inches in diameter. The former were supported on the ends of an oblong horizontal stage, capable of being turned round a vertical axis supporting the stage at a point midway between them. Let *fig.* 702. represent a plan of the apparatus. The large metallic balls B and B′ are supported upon a rectangular stage represented by the dotted lines, and

so mounted as to be capable of being turned round its centre C in its own plane. Two small balls *a*, *a′*, about two inches in

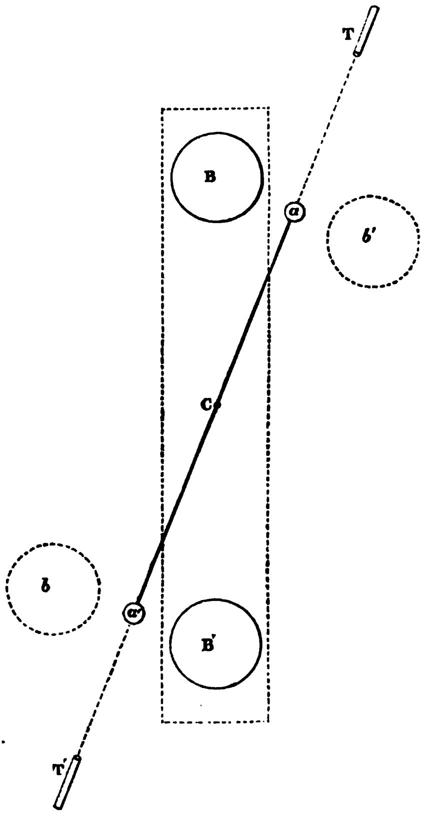


Fig. 702.

diameter, are attached to the ends of a rod, so that the distance between their centres shall be nearly equal to BB′. This rod is supported at C by two fine wires at a very small distance asunder, so that the balls will be in repose when the rod *aa′* is directed in the plane of the wires, and can only be turned from that plane by the action of a small and definite force, the intensity of which can always be ascertained by the angle of deflection of the rod *aa′*. The exact direction of the rod *aa′* is

observed, without approaching the apparatus, by means of two small telescopes T and T′, and the extent of its departure from its position of equilibrium may be measured with great precision by micrometers.

In the performance of the experiment a multitude of precautions were taken to remove or obviate various causes of disturbance, such as currents of air, which might arise from unequal changes of temperature which need not be described here.

The large balls being first placed at a distance from the small ones, the direction of the rod in its position of equilibrium was observed with the telescopes T T′. The stage supporting the large balls was then turned until they were brought near the small ones, as represented at B B′. It was then observed that the small balls were attracted by the large ones, and the amount of the deflection of the rod *a a′* was observed.

The frame supporting the large balls was then turned until B was brought to *b*, and B′ to *b′*, so as to attract the small balls on the other side, and the deflection of *a a′* was again observed. In each case the amount of the deflection being exactly ascertained, the intensity of the deflecting force, and its ratio to the weight of the balls, became known.

The properties of the pendulum supplied a very simple and exact means of comparing the attraction of the balls B and B′ with the attraction of the earth. The balls *a a′* were made to vibrate through a small arc on each side of the position which the attraction gave them, and the rate of their vibration was observed and compared with the rate of vibration of a common pendulum. The relative intensity of the two attractions was computed from a comparision of these rates by the principles established in (542). The precision of which this process of observation is susceptible may be inferred from the fact that the whole attraction of the balls B B′ upon *a a′* did not amount to the 20-millionth part of the weight of the balls *a a′*, and that the possible error of the result did not exceed 2 per cent. of its whole amount.

The attraction which the balls B B′ would exert on *a a′*, on the supposition that the mean density of the earth is equal to that of the metallic balls B B′, was then computed on the principles explained in (2381), and found to be less than the actual

attraction observed, and it was inferred that the density of the earth was less than that of the balls B B′ in the same ratio.

In fine, it resulted that the mean density of the earth is 5·67 times the density of water.

The accordance of this result with those of the Schehallien experiment, and the calculations upon the figure of the terrestrial spheroid, supply a striking proof of the truth of the theory of gravitation on which all these three independent investigations are based, and of the validity of the reasoning upon which they have been conducted.

2394. *Volume and weight of the earth.*—Having ascertained the linear dimensions and the mean density of the earth, it is a question of mere arithmetical labour to compute its volume and its weight. Taking the dimensions of the globe as already stated, its volume contains

259,800 millions of cubic miles.
382,425,60,000 billions of cubic feet.

The average weight of each cubic foot of the earth being 5·67 times the weight of a cubic foot of water, is 354·375 lbs., or 0·1587 of a ton. It follows, therefore, that the total weight of the earth is

6,069,094,272 billions of tons.

CHAP. VI.

THE OBSERVATORY.

2395. *Knowledge of the instruments of observation necessary.*—Having explained the dimensions, rotation, weight, and density of the earth, and described generally the aspect of the firmament and fixed lines and points upon it by which the relative position and motions of celestial objects are defined, it will be necessary, before proceeding to a further exposition of the astronomical phenomena, to explain the principal instruments with which an observatory is furnished, and to show the manner in which they are applied, so as to obtain those accurate data which supply the basis of those calculations from which has resulted our knowledge of the great laws of the universe. We shall therefore here explain the form and use of such of

the instruments of an observatory as are indispensably necessary for the observations by which such data are supplied.

2396. *The astronomical clock.*—Since the immediate objects of all astronomical observation are motions and magnitudes, and since motions are measured by the comparison of space and time, one of the most important instruments of observation is the time-piece or chronometer, which is constructed in various forms, according to the circumstances under which it is used and the degree of accuracy necessary to be obtained. In a stationary observatory, a pendulum clock is the form adopted.

The rate of the astronomical clock is so regulated that, if any of the stars be observed which are upon the celestial meridian at the moment at which the hands point to $0^{h.}\ 0^{m.}\ 0^{s.}$, they will again point to $0^{h.}\ 0^{m.}\ 0^{s.}$ when the same stars are next seen on the meridian. The interval, which is called a sidereal day, is divided into twenty-four equal parts, called SIDEREAL HOURS. The hour-hand moves over one principal division of its dial in this interval. In like manner the MINUTE and SECOND-HANDS move on divided circles, each moving over the successive divisions in the intervals of a minute and a second respectively.

The pendulum is the original and only real measure of time in this instrument. The hands, the dials on which they play, and the mechanism which regulates and proportions their movements, are only expedients for registering the number of vibrations which the pendulum has made in the interval which elapses between any two phenomena. Apart from this convenience a mere pendulum unconnected with wheel work or any other mechanism, the vibrations of which would be counted and recorded by an observer stationed near it, would equally serve as a measure of time.

And this, in fact, is the method actually used in all exact astronomical observations. The *eye* of the observer is occupied in watching the progress of the object moving over the wires (2302) in the field of view of the telescope. His *ear* is occupied in noting, and his mind in counting the successive beats of the pendulum, which in all astronomical clocks is so constructed as to produce a sufficiently loud and distinct sound, marking the close of each successive second. The practised observer is enabled with considerable precision in this way to subdivide a second, and determine the moment of the occurrence of a phenomenon within a small fraction of that interval. A star, for

example, is seen to the left of the wire $m\,m'$ at s, *fig.* 703., at one beat of the pendulum, and to the right of it at s' with the next. The observer estimates with great precision the proportion in which the wire divides the distance between the points s and s', and can therefore determine the fraction of a second after being at s, at which it was upon the wire $m\,m'$.

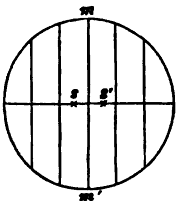


Fig. 703.

Although the art of constructing chronometers has attained a surprising degree of perfection, it is not perfect, and the RATE of even the best of such instruments is not absolutely uniform. It is therefore necessary from time to time to check the indications of the clock by observing its rate. If the clock were absolutely perfect, the pendulum would perform exactly $24 \times 60 \times 60 = 86{,}400$ vibrations in the interval between two successive returns of the same star to the meridian. Now a good astronomical clock will never make so many as 86,401 nor so few as 86,399 vibrations in the interval. In the one case its rate would be too fast, and in the other too slow by 1 in 86400. Even with such an erroneous rate the error thrown upon an observation of one hour would not exceed the 24th part of a second. If, however, the rate be observed, even this error may be allowed for, and no other will remain save the remote possibility of a change of rate since the rate was last ascertained.

2397. *The transit instrument.* — All the most important astronomical observations are made at the moment when the objects observed are upon the celestial meridian, and in a very extensive class of such observations the sole purpose of the observer is to determine with precision the time when the object is brought to the meridian by the apparent diurnal motion of the firmament.

This phenomenon of passing the meridian is called the TRANSIT; and an instrument mounted in such a manner as to enable an observer, supplied with a clock, to ascertain the exact time of the TRANSIT is called a TRANSIT INSTRUMENT.

Such an instrument consists of a telescope so mounted that the line of collimation will be successively directed to every point of the celestial meridian when the telescope is moved upon its axis through 180°.

This is accomplished by attaching the telescope to an axis at

right angles to its line of collimation, and placing the extremities of such axis on two horizontal supports, which are exactly at the same level, and in a line directed east and west. The line of collimation when horizontal will therefore be directed north and south; and if the telescope be turned on its axis through 180°, its line of collimation will move in the plane of the meridian, and will be successively directed to all points on the celestial meridian from the north to the pole, thence to the zenith, and thence to the south.

The instrument thus mounted is represented in *fig.* 704.

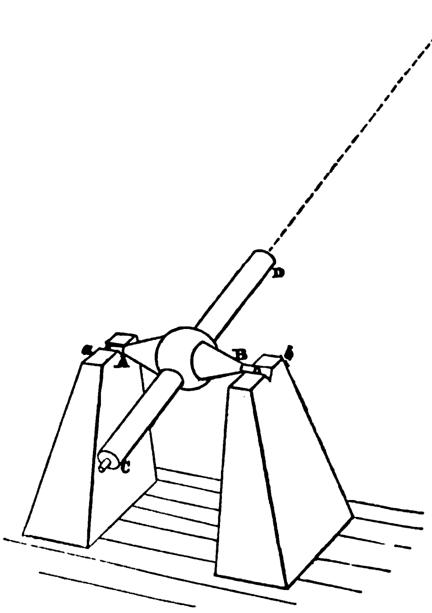


Fig. 704.

Two stone piers are erected on a solid foundation standing east and west. In the top of each of them is inserted a metallic support in the form of a Y to receive the cylindrical extremities of the transverse arms A, B of the instrument. The tube of the telescope CD consists of two equal parts inserted in a central globe, forming part of the transversal axis AB. Thus mounted, the telescope can be made to revolve like a wheel upon the axis AB, and while it thus revolves its line of collimation would be

directed successively to all the points of a vertical circle, the plane of which is at right angles to the axis A B. If the axis be exactly directed east and west, this vertical must be the meridian.

2398. *Its adjustments.* — This, however, supposes three conditions to be fulfilled with absolute precision :

1°. The axis A B must be level.

2°. The line of collimation must be perpendicular to it.

3°. It must be directed due east and west.

In the original construction and mounting of the instrument these three conditions are kept in view, and are nearly, but cannot be exactly, fulfilled in the first instance. In all astronomical instruments the conditions which they are required to fulfil are only approximated to in the making and mounting; but a class of expedients called ADJUSTMENTS are in all cases provided, by which each of the requisite conditions, only *nearly* attained at first, are fulfilled with infinitely greater precision.

In all such adjustments two provisions are necessary : *first*, a method of detecting and measuring the deviation from the exact fulfilment of the requisite condition ; and *secondly*, an expedient by which such deviation can be corrected.

2399. *To make the axis level.* — If the axis A B be not truly level, its deviation from this direction may be ascertained by suspending upon it a SPIRIT LEVEL.

This consists of a glass tube nearly filled with alcohol or ether, liquids selected for the purpose, in consequence of the absence of all viscidity, their perfect mobility, and because they are not liable to congelation. The tube A B, *fig.* 705., is formed

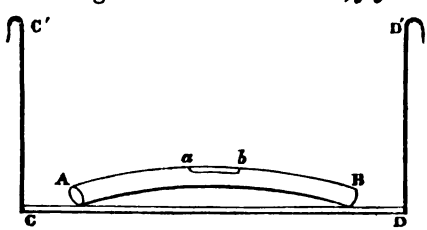


Fig. 705.

slightly convex, and when it is placed horizontally with its convexity upwards, the *bubble a b* produced by its deficient fulness will take the highest position, and therefore rest at the centre of its length. Marks are engraved on or attached to the

tube at *a* and *b* indicating the centre of its length. The tube is attached to a straight bar C D, or so mounted as to be capable of being suspended from two points C′D′, and is so adjusted that when the lower surface of the bar C D, or the line joining the two points of suspension C′D′, is exactly level, the bubble will rest exactly in the centre of the tube between the marks *a* and *b*.

To ascertain whether a surface, or the line joining two proposed points, be level, the instrument is applied upon the one, or suspended from the other. If the bubble rest between the marks *a* and *b*, they are level; if not, that direction towards which it deviates is the more elevated, and it must be lowered, or the other raised. The operation must be repeated until the bubble is found to rest between the central marks *a* and *b*, whichever way the level be placed.

A level is provided for the transit instrument with two loops of suspension corresponding with the cylindrical extremities of the axis A B, *fig.* 704., so that its points of suspension may rest on these cylinders. If it be found that, when the level is properly suspended thus upon the axis, the bubble rests nearer to one extremity than the other, it will be necessary to raise that end from which it is more remote, or to lower that to which it is nearer.

To accomplish this, one of the supports in which the extremity A of the axis rests is constructed so as to be moved through a small space vertically by a finely constructed screw. This support is therefore raised or lowered by such means, until the bubble of the level rests between the central marks *a* and *b*, whichever way the level be suspended.

2400. *To make the line of collimation perpendicular to the axis.* — It must be remembered, that the line of collimation is a line drawn from the centre of the object-glass to the intersection of the middle wires in the field of view of the telescope. The centre of the object-glass is fixed relatively to the telescope, but the wires are so mounted that the position of their intersection can be moved through a certain small space by means of a micrometer screw. One end of the line of collimation, therefore, being moveable, while the other is fixed, its direction may be changed at pleasure within limits determined by the construction of the eye-glass and its micrometer.

To ascertain whether the line of collimation is or is not at right angles to the line joining the points of support A and B,

fig. 704., let any distant point be observed upon which the intersection of the wires falls. Let the instrument be then reversed upon its supports, the end of the axis which rested on *a* being transferred to *b*, and that which rested on *b* to *a*, and let the same object be observed. If it still coincide with the intersection of the wires, the line of collimation is in the proper direction; but if not, its distance from the intersection of the wires will be twice the deviation of the line of collimation from the perpendicular, and the wires must be moved by the adjusting screw, until their intersection is moved towards the object through half of its apparent distance from it.

To render this more clear, let AB, *fig.* 706., represent the direction of the axis, CD that of a line exactly at right angles to it, or the direction which is to be given to the line of collimation, and let CD′ represent the erroneous direction which that line actually has. Let S be a distant object to which it is observed to be directed, this object being seen upon the intersection of the wires. If the instrument be reversed, the line CD′ will have the direction CD″, deviating as much from CD to the right as it before deviated to the left. The object S will now be seen at a distance to the left of the intersection of the wires which measures the angle D′CD″, which is twice the angle DCD′, or the deviation of the line of collimation from the perpendicular DC.

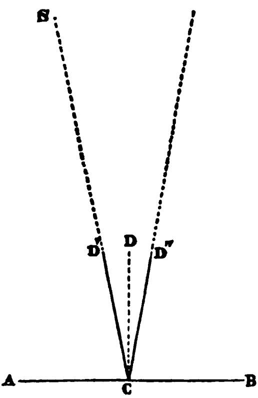


Fig. 706.

2401. *To render the direction of the supports due east and west.* — This is in some cases accomplished by a MERIDIAN MARK, which is a distinct object, such as a white vertical line painted on a black ground, erected at a sufficient distance from the instrument in the exact meridian of the observatory. If, on directing the telescope to it, it is seen on the one side or the other of the middle wire (which ought to coincide with the meridian), the direction of the axis AB, *fig.* 704., will deviate to the same extent from the true east and west, since it has been already, by the previous adjustments, rendered perpendicular to

the line of collimation. The entire instrument must therefore be shifted round, until the meridian mark coincides with the middle wire. This is accomplished by a provision made in the support on which the extremity of the axis B, *fig.* 704., rests, by which it has a certain play in the horizontal direction urged by a fine screw. In this way the axis A B is brought into the true direction east and west, and therefore the line of collimation into the true meridian.

It will be observed that, in explaining the second adjustment, it has been assumed that the deviations are not so great as to throw the object S out of the field of view after the instrument is reversed. This condition in practice is always fulfilled, the extent of deviation left to be corrected by the adjustments being always very small.

2402. *Micrometer wires — method of observing transit.* — In the focus of the eye-piece of the transit instrument, the system of micrometer wires (2302), already mentioned, is placed. This consists commonly of 5 or 7 equidistant wires, placed vertically at equal distances, and intersected at their middle points by a horizontal wire, as represented in *fig.* 703. When the instrument has been adjusted, the middle wire $m m'$ will be in the plane of the meridian, and when an object is seen upon it, such object will be on the celestial meridian, and the wire itself may be regarded as a small arc of the meridian rendered visible.

The fixed stars, as will be explained more fully hereafter, appear in the telescope, no matter how high its magnifying power be, as mere lucid points, having no sensible magnitude. By the diurnal motion of the firmament, the star passes successively over all the wires, a short interval being interposed between its passages. The observer, just before the star approaching the meridian enters the field of view, notes and writes down the *hours* and *minutes* indicated by the clock, and he proceeds to count the *seconds* by his ear. He observes, in the manner already explained, to a fraction of a second, the instant at which the star crosses each of the wires; and taking a mean of all these times, he obtains, with a great degree of precision, the instant at which the star passed the middle wire, which is the time of the transit.

By this expedient the result has the advantage of as many independent observations as there are parallel wires. The

errors of observation being distributed, are proportionally diminished.

When the sun, moon, or a planet, or, in general, any object which has a sensible disk, is observed, the time of the transit is the instant at which the centre of the disk is upon the middle wire. This is obtained by observing the instants which the western and eastern edges of the disk touch each of the wires. The middle of these intervals are the moments at which the centre of the disk is upon the wires respectively. Taking a mean of the contact of the western edges, the contact of the western edge with the middle wire will be obtained; and, in like manner, a mean of the contacts of the eastern edge will give the contact of that edge with the middle wire, and a mean of these two will give the moment of the transit of the centre of the disk, or a mean of all the contacts of both edges will give the same result.

By day the wires are visible, as fine black lines intersecting and spacing out the field of view. At night they are rendered visible by a lamp, by which the field of view is faintly illuminated.

2403. *Apparent motion of objects in field of view.* — Since the telescope reverses the objects observed, the motion in the field will appear to be from west to east, while that of the firmament is from east to west. An object will therefore enter the field of view on the west side, and, having crossed it, will leave it on the east side.

Since the sphere revolves at the rate of 15° per hour, 15′ per minute, or 15″ per second of time, an object will be seen to pass across the field of view with a motion absolutely uniform, the space passed over between two successive beats of the pendulum being invariably 15″.

Thus, if the moon or sun be in or near the equator, the disk will be observed to pass across the field with a visible motion, the interval between the moments of contact of the western and eastern edges with the middle wire being $2^{m \cdot}\ 8^{s \cdot}$, when the apparent diameter is 32′. Thus, the disk appears to move over a space equal to half its own diameter in $1^{m \cdot}\ 4^{s \cdot}$.

2404. *Circles of declination, or hour circles.* — Circles of the celestial sphere which pass through the poles are at right angles to the celestial equator, and are on the heavens exactly what meridians are upon the terrestrial globe. They divide

the celestial equator into arcs which measure the angles which such circles form with each other. Thus, two such circles which are at right angles include an arc of 90° of the celestial equator, and two which form with each other an angle of 1° include between them an arc of 1° of the celestial equator. These CIRCLES OF DECLINATION, or HOUR CIRCLES as they are called, are carried round by the diurnal motion of the heavens, and are brought in succession to coincide with the celestial meridian, the intervals between the moments of their coincidence with the meridian being always proportional to the angle they form with each other, or, what is the same, to the arc of the celestial equator included between them. Thus, if two circles of declination form with each other an angle of 30°, the interval between the moments of their coincidence with the meridian will be two sidereal hours.

The relative position of the circles of declination with respect to each other and to the meridian, and the successive positions assumed by any one such circle during a complete revolution of the sphere, will be perceived and understood without difficulty by the aid of a celestial globe, without which it is scarcely possible to obtain any clear or definite notion of the apparent motions of celestial objects.

2405. *Right ascension.* — The arc of the celestial equator between any circle of declination and a certain point on the equator called the FIRST POINT OF ARIES (which will be defined hereafter), is called the RIGHT ASCENSION of all objects through which the circle of declination passes. This arc is always understood to be measured from the point where the circle of declination meets the celestial equator *westward*, that is, in the direction of the apparent diurnal motion of the heavens, and it may extend, therefore, over any part whatever of the equator from 0° to 360°.

Right ascension is expressed sometimes according to angular magnitude, in degrees, minutes, and seconds; but since, according to what has been explained, these magnitudes are proportional to the time they take to pass over the meridian, right ascension is also often expressed immediately by this time. Thus, if the right ascension of an object is 15° 15′ 15″, it will be expressed also by $1^{h.}\ 1^{m.}\ 1^{s.}$.

In general, right ascension expressed in degrees, minutes, and seconds may be reduced to time by dividing it by 15; and if

it be expressed in time, it may be reduced to angular language by multiplying it by 15.

The difference of right ascensions of any two objects may be ascertained by the transit instrument and clock, by observing the interval which elapses between their transits over the meridian. This interval, whether expressed in time or reduced to degrees, is their difference of right ascension.

Hence, if the right ascension of any one object be known, the right ascension of all others can be found.

2406. *Sidereal clock indicates right ascension.* — If the hands of the sidereal clock be set to $0^{h.}\, 0^{m.}\, 0^{s.}$ when the first point of Aries is on the meridian, they will at all times (supposing the rate of the clock to be correct) indicate the right ascension of such objects as are on the meridian. For the motion of the hands in that case corresponds exactly with the apparent motion of the meridian on the celestial equator produced by the diurnal motion of the heavens. While 15° of the equator pass the meridian the hands move through $1^{h.}$, and other motions are made in the same proportion.

2407. *The mural circle.* — The transit instrument and sidereal clock supply means of determining with extreme precision the instant at which an object passes the meridian; but the instrument is not provided with any accurate means of indicating the point at which the object is seen on the meridian. A circle is sometimes, it is true, attached to the transit by which the position of this point may be roughly observed; but to ascertain it with a precision proportionate to that with which the transit instrument determines the right ascensions, requires an instrument constructed and mounted for this express object in a manner, and under conditions, altogether different from those by which the transit instrument is regulated. The form of instrument adopted in the most efficiently furnished observatories for this purpose is the MURAL CIRCLE.

This instrument is a graduated circle, similar in form and principle to the instrument described in (2304). It is centred upon an axis established in the face of a stone pier or *wall* (hence the name), erected in the plane of the meridian. The axis, like that of a transit instrument, is truly horizontal, and directed due east and west. Being by the conditions on which it is first constructed and mounted, *very nearly* in this position, it is rendered *exactly* so by two adjustments, one of which

moves the axis vertically, and the other horizontally, by means of screws, through spaces which, though small, are still large enough to enable the observer to correct the slight errors of position incidental to the workmanship and mounting.

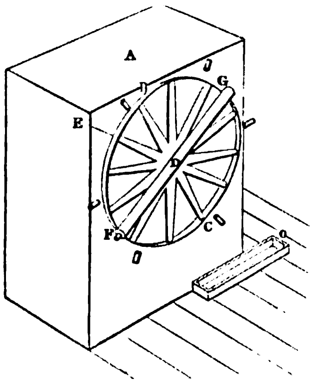


Fig. 707.

The instrument, as mounted and adjusted, is represented in perspective in *fig.* 707., where A is the stone wall to which the instrument is attached, D the central axis on which it turns; and F G the telescope, which does not move upon the circle, but is immoveably attached to it, so that the entire instrument, including the telescope, turns in the plane of the meridian upon the axis D.

A front view of the circle in the plane of the instrument is given in *fig.* 708.

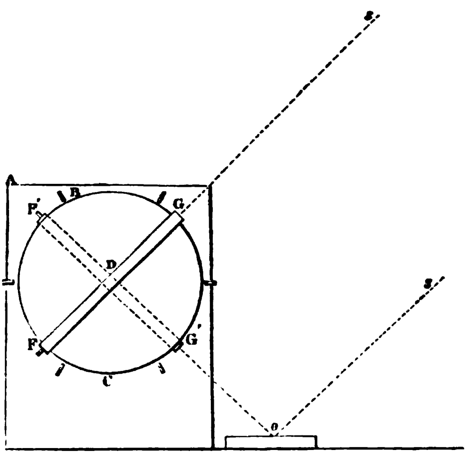


Fig. 708.

The graduation is usually made on the edge, and not on the face limb. The hoop of metal thus engraved forms, therefore, what may be called the tire of the wheel.

Troughs *o*, containing mercury, are placed on the floor in convenient positions in the plane of the instrument, in the surface of which are seen, by reflection, the objects as they pass over the meridian. The observer is thus enabled to ascertain the directions, as well of the images of the objects reflected in the mercury, as of the objects themselves, the advantage of which will presently appear.

Convenient ladders, chairs, and couches, capable of being adjusted by racks and other mechanical arrangements, at any desired inclinations, enable the observer, with the utmost ease and comfort, to apply his eye to the telescope, no matter what be its direction.

In the more important national observatories the mural circles are eight feet in diameter, and consequently 301·5 inches in circumference. Each degree upon the circumference measuring, therefore, above eight-tenths of an inch, admits of extremely minute subdivision.

The divisions on the graduated edge of the instrument are numbered as usual from 0° to 360° round the entire circle. The position which the direction of the line of collimation of the telescope has with relation to the 0° of the limb is indifferent. Nothing is necessary except that this line, in moving round the axis D of the instrument, shall remain constantly in the plane of the meridian. This condition being fulfilled, it is evident that, as the circle revolves, the line of collimation will be successively directed to every point of the meridian when presented upwards, and to every point of its reflected image in the mercury when presented downwards.

2408. *Method of observing with it.* — The position of the instrument when directed successively to two objects on the meridian, or to their images reflected in the mercury, being observed, the angular distance, or the arc of the meridian between them, will be found by ascertaining the arc of the graduated limb of the instrument, which passes before any fixed point or index, when the telescope is turned from the direction of the one object to the direction of the other.

2409. *Compound microscopes — their number and use.* — This arc is observed by a compound microscope (2307), attached to the wall or pier, and directed towards the graduated limb. The manner in which the fraction of a division of the limb is observed by this expedient has been already explained.

But to give greater precision to the observation, as well as to efface the errors which might arise, either from defective centreing, or from the small derangement of figure that might arise from the flexure produced by the weight of the instrument, several compound microscopes — generally six — are provided at nearly equal distances around the limb, so that the observer is enabled to note the position of six indices. The six arcs of the limb which pass under them being observed, are equivalent to six independent observations, the mean of which being taken, the errors incidental to them are reduced in proportion to their number.

2410. *Circle primarily a differential instrument.* — The observations, however, thus taken are, strictly speaking, only differential. The arc of the meridian between the two objects is determined, and this arc is the difference of their meridional distances from the zenith or from the horizon; but unless the positions which the six indexes have, when the line of collimation is directed to the zenith or horizon, be known, no positive result arises from the observations; nor can the absolute distance of any object, either from the horizon or the zenith, be ascertained.

2411. *Method of ascertaining the horizontal point.* — The "reading," as it is technically called, at each of the microscopes, in any proposed position of the instrument, is the distance of that microscope from the zero point of the limb. Now it is easy to show that half the sum of the two readings at any microscope, when the telescope is successively directed to an object and its image in the mercury, will be the reading at the same microscope when the line of collimation is horizontal.

Let a circle be imagined to be drawn upon the stone pier around the instrument, and let M, *fig.* 709., represent the position of any of the microscopes. Let CO be the position of the telescope when directed to the object, and let z be the position of the zero of the limb. Let CI be the position of the telescope when directed to the image of the same object in the mercury. If $z z'' = \text{OI}$, z'' will be then the place of the zero, because the zero will be

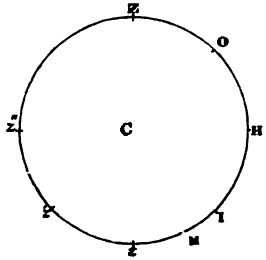


Fig. 709.

moved with the instrument through the same space as that through which the telescope is moved. Since the direction CI is as much below the horizon as CO is above it, the direction of the horizon must be that of the point H which bisects the arc OI. The telescope, when horizontal, will have therefore the direction CH, and when it has this position the zero will evidently be at z', the point which bisects the arc zz''.

The "readings" of the microscope M, when the telescope is directed to O and I, are Mz and Mz''. The "reading" of the same microscope when the telescope is horizontal would be Mz'. Now it is evident, from what has been stated above, that

$$\text{M}z' - \text{M}z = \text{M}z'' - \text{M}z';$$

and, therefore,

$$\text{M}z' = \tfrac{1}{2}(\text{M}z + \text{M}z'');$$

tha is, the reading for the horizontal direction of the telescope would be half the sum of the readings for an object and its image.

2412. *Method of observing altitudes and zenith distances.*— The readings of all the microscopes, when the telescope is directed to the horizon, being thus determined, are preserved as necessary data in all observations on the altitudes or zenith distances of objects. To determine the altitude of an object O, let the telescope be directed to it, so that it shall be seen at the intersection of the wires; and let the readings of the six microscopes be o_1, o_2, o_3, o_4, o_5, and o_6, and let their six horizontal readings be H_1, H_2, H_3, H_4, H_5, and H_6. We shall have six values for the altitudes:

$$\begin{aligned}
\text{A}_1 &= \text{H}_1 - o_1, \\
\text{A}_2 &= \text{H}_2 - o_2, \\
\text{A}_3 &= \text{H}_3 - o_3, \\
\text{A}_4 &= \text{H}_4 - o_4, \\
\text{A}_5 &= \text{H}_5 - o_5, \\
\text{A}_6 &= \text{H}_6 - o_6.
\end{aligned}$$

These will be nearly, but not precisely, equal, because they will differ by the small errors of observation, centreing, and form. A mean of the six being taken by adding them and dividing their sum by 6, these differences will be equalized, and the errors nearly effaced, so that we shall have the nearest approximation to the true altitude —

$$\text{A} = \tfrac{1}{6}\{\text{A}_1 + \text{A}_2 + \text{A}_3 + \text{A}_4 + \text{A}_5 + \text{A}_6\}.$$

The altitude of an object being known, its zenith distance may be found by subtracting the altitude from 90°: thus, if z express the zenith distance, we shall have

$$z = 90^\circ - \text{A}.$$

2413. *Method of determining the position of the pole and equator.* — The mural circle may be regarded as the celestial meridian reduced in scale, and brought immediately under the hands of the observer, so that all distances upon it may be submitted to exact examination and measurement. Besides the zenith and horizon, the positions of which, in relation to the microscopes, have just been ascertained, there are two other points of equal importance, the pole and the equator, which should also be established.

The stars which are so near the celestial pole that they never set, are carried by the diurnal motion of the heavens round the pole in small circles, crossing the visible meridian twice, once above and once below the pole. Of all these circumpolar stars, the most important and the most useful to the observer is the pole star, both because of its close proximity to the pole, from which its distance is only $1\frac{1}{2}^\circ$, and because its magnitude is sufficiently great to be visible with the telescope in the day. This star, then, crosses the meridian above the pole and below it, at intervals of twelve hours sidereal time, and the true position of the pole is exactly midway between the two points where the star thus crosses the meridian.

If, therefore, the readings of the six microscopes be taken when the pole star makes its transit above and below the pole, their readings for the pole itself will be half the sum of the former for each microscope.

The readings for the pole being determined, those which correspond to the point where the celestial equator crosses the meridian may be found by substracting the former from 90°.

When the positions of the microscopes in relation to the pole and equator are determined, the latitude of the observatory will be known, since it is equal to the altitude of the celestial pole (2362).

2414. *All circles of declination represented by the circle.* — Since the circles of declination, which are imagined to surround the heavens, are brought by the diurnal motion in succession to

coincide with the celestial meridian (2404), and since that meridian is itself represented by the mural circle, that circle may be considered as presenting successively a model of every circle of declination; and the position of any object upon the circle of declination is represented on the mural circle by the position of the telescope when directed to the point of the meridian at which the object crosses it.

If the object have a fixed position on the firmament, it is evident that it will always pass the meridian at the same point; and if the telescope be directed to that point and maintained there, the object will be seen at the intersection of the wires regularly after intervals of twenty-four hours sidereal time.

2415. *Declination and polar distance of an object.* — The distance of an object from the celestial equator, measured upon the circle of declination which passes through it, is called its DECLINATION, and is NORTH or SOUTH, according to the side of the equator at which the object is placed.

The declination of an object is ascertained with the mural circle in the same manner and by the same observation as that which gives its altitude. The readings of the microscopes for the object being compared with their readings for the pole (2413), give the polar distance of the object; and the difference between the polar distances and 90° gives the declination.

Thus the polar distance and declination of an object are to the equator exactly what its altitude and zenith distance are to the horizon. But since the equator maintains always the same position during the diurnal motion of the heavens, the declination and polar distance of an object are not affected by that motion, and remain the same, while the altitude and zenith distances are constantly changing.

2416. *Position of an object defined by its declination and right ascension.* — The position of an object on the firmament is determined by its declination and right ascension. Its declination expresses its distance north or south of the celestial equator, and its right ascension expresses the distance of the circle of declination upon which it is placed from a certain defined point upon the celestial equator.

It is evident, therefore, that declination and right ascension define the position of celestial objects in exactly the same manner

as latitude and longitude define the position of places on the earth. A place upon the globe may be regarded as being projected on the heavens into the point which forms its zenith; and hence it appears that the latitude of the place is identical with the declination of its zenith.

CHAP. VII.

ATMOSPHERIC REFRACTION.

2417. *Apparent position of celestial objects affected by refraction.* — It has been shown that the ocean of air which surrounds, rests upon, and extends to a certain limited height above the surface of the solid and liquid matter composing the globe, decreases gradually in density in rising from the surface (719); that when a ray of light passes from a rarer into a denser transparent medium, it is deflected towards the perpendicular to their common surface; and that the amount of such deflection increases with the difference of densities and the angle of incidence (978 *et seq.*). These properties, which air has in common with all transparent media, produce important effects on the apparent positions of celestial objects.

Let s*a*, *fig.* 710., be a ray of light coming from any distant object s, and falling on the surface of a series of layers of transparent matter, increasing in density downwards. The ray s*a*, passing into the first layer, will be deflected in the direction *aa'* towards the perpendicular; passing thence into the next, it will be again deflected in the direction *a' a''*, more towards the perpendicular; and, in fine, passing through the lowest layer, it will be still more deflected, and will enter the eye at *e*, in the direction *a'' e*: and since every object

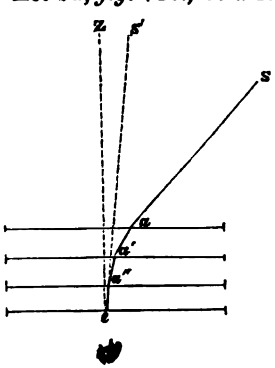


Fig. 710.

is seen in the direction from which the visual ray enters the eye, the object S will be seen in the direction *e* S′, instead of its true direction *a* S. The effect, therefore, is to make the object appear to be nearer to the zenithal direction than it really is.

And this is what actually occurs with respect to all celestial objects seen, as such objects always must be, through the atmosphere. The visual ray S D, *fig.* 711., passing through a

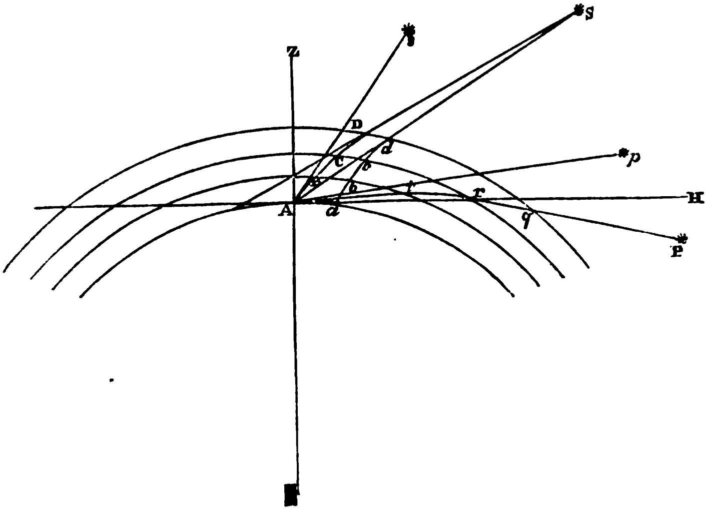


Fig. 711.

succession of strata of air, gradually and continually increasing in density, its path will be a curve bending from D towards A, and convex towards the zenithal line A Z. The direction in which the object will be seen, being that in which the visual ray enters the eye, will be the tangent A *s* to the curve at A. The object will therefore be seen in the direction A *s* instead of D S.

It has been shown that the deflection produced by refraction is increased with the increase of the angle of incidence. Now, in the present case, the angle of incidence is the angle under the true direction of the object and the zenithal line, or, what is the same, the zenith distance of the object. The extent, therefore, to which any celestial object is disturbed from its true place by the refraction of the atmosphere, increases with

its zenith distance. The refraction is, therefore, nothing in the zenith, and greatest in the horizon.

2418. *Law of atmospheric refraction.*—The extent to which a celestial object is displaced by refraction, therefore, depends upon and increases with its distance from the zenith; and it can be shown to be a consequence of the general principles of optics, that when other things are the same, the actual quantity of this displacement (except at very low altitudes) varies in the proportion of the tangent of the zenith distance.

Thus, if AZ, *fig.* 712., be the zenithal direction, and AO, AO′, AO″, &c., be the directions of celestial objects, their zenith distances being ZAO, ZAO′, ZAO″, &c., the quantities of refraction by which they will be severally affected, or, what is the same, the differences between their true and apparent directions, will be in the ratio of the tangents ZT, ZT′, ZT″, &c. of the zenith distances.*

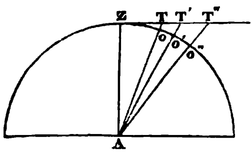


Fig. 712.

This law prevails with considerable exactitude, except at very low altitudes, where the refractions depart from it, and become uncertain.

2419. *Quantity of refraction.*—When the latitude of the observatory is known, the actual quantity of refraction at a given altitude may be ascertained by observing the altitudes of a circumpolar star, when it passes the meridian above and below the pole. The sum of these altitudes would be exactly

* This law may be demonstrated as follows:—The angle of incidence of the visual ray is equal to the zenith distance z of the object. If r express the refraction, the angle of refraction will be $z-r$. Let the index of refraction (980) be m. By the general law of refraction we have, therefore,

$$\sin. z = m \times \sin. (z-r) = m \times \sin. z \cos. r - m \times \cos. z \sin. r.$$

But since r is a very small angle, if it be expressed in seconds, we shall have

$$\cos. r = 1, \qquad \sin. r = \frac{r}{206265},$$

and, consequently,

$$\sin. z = m \times \sin. z - m \times \cos. z \times \frac{r}{206265},$$

and, therefore,

$$r = 206267'' \times \frac{m-1}{m} \times \frac{\sin. z}{\cos. z} = 206265'' \times \frac{m-1}{m} \times \tan. z.$$

equal to twice the latitude (2362) if the refraction did not exist, but since by its effects the star is seen at greater than its true altitudes, the sum of the altitudes will be greater than twice the latitude by the sum of the two refractions. This sum will therefore be known, and being divided between the two altitudes in the ratio of the tangents of the zenith distances, the quantity of refraction due to each altitude will be known.

The pole star answers best for this observation, especially in these and higher latitudes, where it passes the meridian within the limits of the more regular influence of refraction; and the difference of its altitudes being only 3°, no considerable error can arise in apportioning the total refraction between the two altitudes.

2420. *Tables of refraction.* — To determine with great exactitude the average quantity of refraction due to different altitudes, and the various physical conditions under which the actual refraction departs from such average, is an extremely difficult physical problem. These conditions are connected with phenomena subject to uncertain and imperfectly known laws. Thus, the quantity of refraction at a given altitude depends, not only on the density, but also on the temperature of the successive strata of air through which the visual ray has passed. Although, as a general fact, it is apparent that the temperature of the air falls as we rise in the atmosphere (2185), yet the exact law according to which it decreases is not fully ascertained. But even though it were, the refraction is also influenced by other agencies, among which the hygrometric condition of the air holds an important place.

From these causes, some uncertainty necessarily attends astronomical observations, and some embarrassment arises in cases where the quantities to be detected by the observations are extremely minute. Nevertheless, it must be remembered, that since the total amount of refraction is never considerable, and in most cases it is extremely minute, and since, small as it is, it can be very nearly estimated and allowed for, and in some cases wholly effaced, no serious obstacle is offered by it to the general progress of astronomy.

Tables of refraction have been constructed and calculated, partly from observation and partly from theory, by which the observer may at once obtain the average quantity of refraction

at each altitude; and rules are given by which this average refraction may be corrected according to the peculiar state of the barometer, thermometer, and other indicators of the physical state of the air.

2421. *Average quantity at mean altitudes.*—While the refraction is nothing in the zenith, and somewhat greater than the apparent diameter of the sun or moon in the horizon, it does not amount to so much as 1′, or the thirtieth part of this diameter, at the mean altitude of 45°.

2422. *Effect on rising and setting.*—Its mean quantity in the horizon is 33′, which being a little more than the mean apparent diameters of the sun and moon, it follows that these objects, at the moment of rising and setting, are visible above the horizon, the lower edge of their disks just touching it, when in reality they are below it, the upper edge of the disk just touching it.

The moments of rising of all objects are therefore accelerated, and those of setting retarded, by refraction. The sun and moon *appear* to rise *before* they have really risen, and to set *after* they have really set; and the same is true of all other objects.

2423. *General effect of the barometer on refraction.*—Since the barometer rises with the increased weight and density of the air, its rise is attended by an augmentation, and its fall by a decrease, of refraction. It may be assumed that the refraction at any proposed altitude is increased or diminished by 1·300th part of its mean quantity for every 10th of an inch by which the barometer exceeds or falls short of the height of 30 inches.

2424. *Effect of thermometer.*—As the increase of temperature causes a decrease of density, the effect of refraction is diminished by the elevation of the thermometer, the state of the barometer being the same. It may be assumed, that the refraction at any proposed altitude is diminished or increased by the 420th part of its mean amount for each degree by which Fahrenheit's thermometer exceeds or falls short of the mean temperature of 55°.

2425. *Twilight caused by the reflection of the atmosphere.*—The sun continues to illuminate the clouds and the superior strata of the air after it has set, in the same manner as it shines on the summits of lofty mountain peaks long after it

has descended from the view of the inhabitants of the adjacent plains. The air and clouds thus illuminated, reflect light to the surface below them; and thus, after sunset and before sunrise, produce that light, more or less feeble according to the depression of the sun, called TWILIGHT. Immediately after sunset the entire visible atmosphere, and all the clouds which float in it, are flooded with sunlight, and produce, by reflection, an illumination little less intense than before the sun had disappeared. According as the sun sinks lower and lower, less and less of the visible atmosphere receives his light, and less and less of it is transmitted by reflection to the surface, until at length, and by slow degrees, all reflection ceases, and night begins.

The same series of phenomena are developed in an opposite order before sunrise in the morning, commencing with the first feeble light of dawn, and ending with the full blaze of day when the disk of the sun becomes visible.

The general effect of the air, clouds, and vapours in diffusing light, and rendering more effectual the general illumination produced by the sun, has been already explained in (923, 924).

2426. *Oval form of disks of sun and moon explained.*—One of the most curious effects of atmospheric refraction is the oval form of the disks of the sun and moon, when near the horizon. This arises from the unequal refraction of the upper and lower limbs. The latter being nearer the horizon is more affected by refraction, and therefore raised in a greater degree than the upper limb, the effect of which is to bring the two limbs apparently closer together, by the difference between the two refractions. The form of the disk is therefore affected as if it were pressed between two forces, one acting above, and the other below, tending to compress its vertical diameter, and to give it the form of an ellipse, the lesser axis of which is vertical, and the greater horizontal.*

* For an explanation of the great apparent magnitude of the solar and lunar disks in rising and setting, see (1170).

CHAP. VIII.

ANNUAL MOTION OF THE EARTH.

2427. *Apparent motion of the sun in the heavens.*—Independently of the motion which the sun has in common with the entire firmament, and in virtue of which it rises, ascends to the meridian, and sets, it is observed to change its position from day to day with relation to the other celestial objects among which it is placed. In this respect, therefore, it differs essentially from the stars, which maintain their relative positions for months, years, and ages, unaltered.

If the exact position of the sun be observed from day to day and from month to month, through the year, with reference to the stars, it will be found that it has an apparent motion among them in a great circle of the celestial sphere, the plane of which forms an angle of 23° 28′ with the plane of the celestial equator.

2428. *Ascertained by the transit instrument and mural circle.*—This apparent motion of the sun was ascertained with considerable precision before the invention of the telescope and the subsequent and consequent improvement of the instruments of observation. It may, however, be made more clearly manifest by the transit instrument and mural circle.

If the transit of the sun be observed daily (2402), and its right ascension be ascertained (2405), it will be found that from day to day the right ascension continually increases, so that the circle of declination (2404) passing through the centre of the sun is carried with the sun round the heavens, making a complete revolution in a year, and moving constantly from west to east, or in a direction contrary to the apparent diurnal motion of the firmament.

If the point at which the sun's centre crosses the meridian daily be observed with the mural circle (2408), it will be found to change from day to day. Let its distance from the celestial equator, or its declination, be observed (2415) daily at noon. It will be found to be nothing on the 21st of March and 21st of September, on which days the polar distance of the sun's centre will be therefore 90°. The sun's centre is, then, on these days,

in the celestial equator. After the 21st March the sun's centre will be north of the equator, and its declination will continually increase, until it becomes 23° 28′ on the 21st June. It will then begin slowly to decrease, and will continue to decrease until 21st September, when the centre of the sun will again be in the equator. After that it will pass the meridian south of the equator, and will consequently have south declination. This will increase, until it becomes 23° 28′ on the 21st December; after which it will decrease until the centre of the sun returns to the equator on the 21st March.

By ascertaining the position of the centre of the sun's disk from day to day, by means of its right ascension and declination (2416), and tracing its course upon the surface of a celestial globe, its path is proved to be a great circle of the heavens, inclined to the equator at an angle of 23° 28′.

2429. *The ecliptic.*—This great circle in which the centre of the disk of the sun thus appears to move, completing its revolution in it in a year, is called the ECLIPTIC, because, for reasons which will be explained hereafter, solar and lunar eclipses can never take place except when the moon is in or very near it.

2430. *The equinoxial points.*—The ecliptic intersects the celestial equator at two points diametrically opposite to each other, dividing the equator, and being divided by it into equal parts. These are called the EQUINOXIAL POINTS, because, when the centre of the solar disk arrives at them, being then in the celestial equator, the sun will be equal times above and below the horizon (2367), and the days and nights will be equal.

2431. *The vernal and autumnal equinoxes.*—The equinoxial point at which the sun passes from the south to the north of the celestial equator is called the VERNAL, and that at which it passes from the north to the south is called the AUTUMNAL, equinoxial point. The TIMES at which the centre of the sun is found at these points are called, respectively, the VERNAL and AUTUMNAL EQUINOXES.

The vernal equinox, therefore, takes place on the 21st March, and the autumnal on the 21st September.

2432. *The seasons.*—That semicircle of the ecliptic through which the sun moves from the vernal to the autumnal equinox is north of the celestial equator; and during that interval the sun will therefore (2351) be longer above than below the hori-

zon, and will pass the meridian above the equator in places having north latitude. The days, therefore, during that half-year, will be longer than the nights.

That semicircle through which the centre of the sun moves from the autumnal to the vernal equinox being south of the celestial equator, the sun, for like reasons, will during that half-year be longer below than above the horizon, and the days will be shorter than the nights, the sun rising to a point of the meridian below the equator.

The three months which succeed the vernal equinox are called SPRING, and those which precede it WINTER; the three months which precede the autumnal equinox are called SUMMER, and those which succeed it WINTER.

2433. *The solstices.*—Those points of the ecliptic which are midway between the equinoxial points are the most distant from the celestial equator. The arcs of the ecliptic between these points and the equinoxial points are therefore 90°. These are called the SOLSTITIAL POINTS, and the times at which the centre of the solar disk passes through them are called the SOLSTICES.

The summer solstice, therefore, takes place on the 21st June, and the winter solstice on the 21st December.

This distance of the summer solstitial point north, and of the winter solstitial point south of the celestial equator is 23° 28′.

The more distant the centre of the sun is from the celestial equator, the more unequal will be the days and nights (2356), and consequently the longest day will be the day of the summer, and the shortest the day of the winter, solstice.

It will be evident that the seasons must be reversed in southern latitudes, since there the visible celestial pole will be the south pole. The summer solstice and the vernal equinox of the northern, are the winter solstice and autumnal equinox of the southern hemisphere. Nevertheless, as the most densely inhabited and civilized parts of the globe are in the northern hemisphere, the names in reference to the local phenomena are usually preserved.

2434. THE ZODIAC.—It will be shown hereafter that the apparent motions of the planets are included within a space of the celestial sphere extending a few degrees north and south of the ecliptic. The zone of the heavens included within these limits is called the ZODIAC.

2435. *The signs of the zodiac.*—The circle of the zodiac is divided into twelve equal parts, called SIGNS, each of which therefore measures 30°. They are named from principal constellations, or groups of stars, which are placed in or near them. Beginning from the vernal equinoxial point they are as follows:—

	Sign.		Sign.
1. Aries (the ram)	♈	7. Libra (the balance)	♎
2. Taurus (the bull)	♉	8. Scorpio (the scorpion)	♏
3. Gemini (the twins)	♊	9. Sagittarius (the archer)	♐
4. Cancer (the crab)	♋	10. Capricornus (the goat)	♑
5. Leo (the lion)	♄	11. Aquarius (the waterman)	♒
6. Virgo (the virgin)	♍	12. Pisces (the fishes)	♓

Thus, the position of the vernal equinoxial point is the FIRST POINT OF ARIES, and that of the autumnal the FIRST POINT OF LIBRA. The summer solstitial point is at the FIRST POINT OF CANCER, and the winter at the FIRST POINT OF CAPRICORN.

2436. *The tropics.*—The points of the ecliptic at which the centre of the sun is most distant from the celestial equator are also called the TROPICS,—the northern being the TROPIC OF CANCER, and the southern the TROPIC OF CAPRICORN.

This term TROPIC is also applied in geography to those parts of the earth whose distances from the terrestrial equator are equal to the greatest distance of the centre of the solar disk from the celestial equator. The NORTHERN TROPIC is, therefore, a parallel of latitude 23° 28′ north, and the SOUTHERN TROPIC a parallel of latitude 23° 28′ south of the terrestrial equator.

2437. *Celestial latitude and longitude.*—The terms latitude and longitude, as applied to objects on the heavens, have a signification different from that given to them when applied to places upon the earth. The latitude of an object on the heavens means its distance from the ecliptic, measured in a direction perpendicular to the ecliptic; and its longitude is the arc of the ecliptic, between the first point of Aries and the circle which measures its latitude, taken, like the right ascension, according to the order of the signs.

Thus since the centre of the sun is always on the ecliptic, its latitude is always 0°. At the vernal equinox its longitude is 0°, at the summer solstice it is 90°, at the autumnal equinox 180°, and at the winter solstice 270°.

2438. *Annual motion of the earth.*—The apparent annual motion of the sun, described above, is a phenomenon which can

only proceed from one or other of two causes. It may arise from a real annual revolution of the sun round the earth at rest, or from a real revolution of the earth round the sun at rest. Either of these causes would explain, in an equally satisfactory manner, all the circumstances attending the apparent annual motion of the sun around the firmament. There is nothing in the appearance of the sun itself which could give a greater probability to either of these hypotheses than to the other. If, therefore, we are to choose between them, we must seek the grounds of choice in some other circumstances.

It was not until the revival of letters that the annual motion of the earth was admitted. Its apparent stability and repose were until then universally maintained. An opinion so long and so deeply rooted must have had some natural and intelligible grounds. These grounds, undoubtedly, are to be found only in the general impression, that if the globe moved, and especially if its motion had so enormous a velocity as must be imputed to it, on the supposition that it moves annually round the sun, we must in some way or other be sensible of such movement.

All the reasons, however, why we are unconscious of the real rotation of the earth upon its axis (2350) are equally applicable to show why we must be unconscious of the progressive motion of the earth in its annual course round the sun. The motion of the globe through space being perfectly smooth and uniform, we can have no sensible means of knowing it, except those which we possess in the case of a boat moving smoothly along a river: that is, by looking abroad at some external objects which do not participate in the motion imputed to the earth. Now, when we do look abroad at such objects, we find that they appear to move exactly as stationary objects would appear to move, seen from a moveable station. It is plain, then, if it be true that the earth really has the annual motion round the sun which is contended for, that we cannot expect to be conscious of this motion from anything which can be observed on our own bodies or those which surround us on the surface of the earth: we must look for it elsewhere.

But it will be contended that the apparent motion of the sun, even upon the argument just stated, may equally be explained by the motion of the earth round the sun, or the motion of the sun round the earth; and that, therefore, this appearance can

still prove nothing positively on this question. We have, however, other proofs, of a very decisive character.

Newton showed that it was a general law of nature, and part, in fact, of the principle of gravitation, that any two globes placed at a distance from each other, if they are in the first instance quiescent and free, must move with an accelerated motion to their common centre of gravity, where they will meet and coalesce; but if they be projected in a direction not passing through this centre of gravity, they will both of them revolve in orbits around that point periodically.

Now it will appear hereafter that the common centre of gravity of the earth and sun, owing to the immense preponderance of the mass of the sun (309), is placed at a point very near the centre of the sun. Round that point, therefore, the earth must, according to this principle, revolve.

2439. *Motion of light proves the annual motion of the earth.* —Since the principle of gravitation itself might be considered as more or less hypothetical, it has been considered desirable to find other independent and more direct proofs of a phenomenon, so fundamentally important and so contrary to the first impressions of mankind, as the revolution of the earth and the quiescence of the sun. A remarkable evidence of this motion has been accordingly discovered in a vast body of apparently complicated phenomena which are the immediate effects of such a motion, which could not be explained if the earth were at rest and the sun in motion, and which, in fine, would be inexplicable on any other supposition save the revolution of the earth round the sun.

It has been ascertained, as has been already explained, that light is propagated through space with a certain great but definite velocity of about 192,000 miles per second. That light has this velocity is proved by the body of optical phenomena which cannot be explained without imputing to it such a motion, and which are perfectly explicable if such a motion be admitted. Independently of this, another demonstration that light moves with this velocity is supplied by an astronomical phenomenon which will be noticed in a subsequent part of this volume.

2440. *Aberration of light.*—Assuming, then, the velocity of light, and that the earth is in motion in an orbit round the sun with a velocity of about 19 miles per second, which must

be its speed if it move at all, as will hereafter appear, an effect would be produced upon the apparent places of all celestial objects by the combination of these two motions which we shall now explain.

It has been stated that the apparent direction of a visible object is the direction from which the visual ray enters the eye. Now this direction will depend on the actual direction of the ray if the eye which receives it be quiescent; but if the eye be in motion, the same effect is produced upon the organ of sense as if the ray, besides the motion which is proper to it, had another motion equal and contrary to that of the eye. Thus, if light moving from the north to the south with a velocity of 192,000 miles per second be struck by an eye moving from west to east with the same velocity, the effect produced by the light upon the organ will be the same as if the eye, being at rest, were struck by the light having a motion compounded of two equal motions, one from north to south, and the other from east to west. The direction of this compound effect would, by the principles of the composition of motion (176), be equivalent to a motion from the direction of the north-east. The object from which the light comes would, therefore, be apparently displaced, and would be seen at a point beyond that which it really occupies in the direction in which the eye of the observer is moved. This displacement is called accordingly the ABERRATION OF LIGHT.

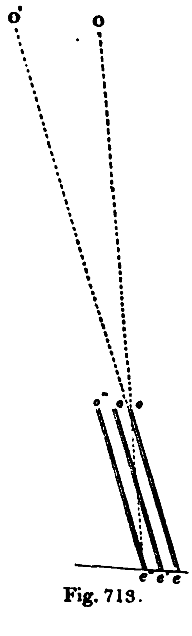


Fig. 713.

This may be made still more evident by the following mode of illustration. Let O, *fig.* 713., be the object from which light comes in the direction O *o* *e''*. Let *e* be the place of the eye of the observer when the light is at *o*, and let the eye be supposed to move from *e* to *e''* in the same time that the light moves from *o* to *e''*. Let a straight tube be imagined to be directed from the eye at *e* to the light at *o*, so that the light shall be in the centre of its opening, while the tube moves with the eye from O *e* to *o''* *e''*, maintaining constantly the same direction, and remaining parallel to itself: the light

in moving from o to e'', will pass along its axis, and will arrive at e'' when the eye arrives at that point. Now it is evident that in this case the direction in which the object would be visible, would be the direction of the axis of the tube, so that, instead of appearing in the direction o O, which is its true direction, it would appear in the direction o O′ advanced from O in the direction of the motion $e\,e''$ with which the observer is affected.

The motion of light being at the rate of 192,000 miles per second, and that of the earth (if it move at all) at the rate of 19 miles per second (both these velocities will be established hereafter), it follows, that the proportion of oe'' to ee'' must be 192,000 to 19, or 10,100 to 1.

The ANGLE OF ABERRATION OoO′ will vary with the obliquity of the direction ee'' of the observer's motion to that of the visual ray $o\,e''$. In all cases the ratio of oe'' to ee'' will be 10,100 to 1. If the direction of the earth's motion be at right angles to the direction oe'' of the object O, we shall have (2294) the aberration

$$a = \frac{206{,}265}{10{,}100} = 20''{\cdot}42.$$

If the angle $oe''e$ be oblique, it will be necessary to reduce ee'' to its component at right angles to oe'', which is done by multiplying it by the trigonometrical sine of the obliquity $oe''e$ of the direction of the object to that of the earth's motion. If this obliquity be expressed by O, we shall have for the aberrations in general

$$a = 20''{\cdot}42 \times \sin.\ \mathrm{O}.$$

According to this, the aberration would be greatest when the direction of the earth's motion is at right angles to that of the object, and would decrease as the angle O decreases, being nothing when the object is seen in the direction in which the earth is moving, or in exactly the contrary direction.

The phenomena may also be imagined by considering that the earth, in revolving round the sun, constantly changes the direction of its motion; that direction making a complete revolution with the earth, it follows that the effect produced upon the apparent place of a distant object would be the same as if that object really revolved once in a year round its true place in a circle whose plane would be parallel to that of the earth's orbit, and whose radius would subtend at the earth an angle of

20″·42, and the object would be always seen in such a circle 90° in advance of the earth's place in its orbit.

These circles would be reduced by projection to ellipses of infinitely various excentricities, according to the position of the object with relation to the plane of the earth's orbit. At a point perpendicularly above that plane, the object would appear to move annually in an exact circle. At points nearer to the ecliptic, its apparent path would be an ellipse, the excentricity of which would increase as the distance from the ecliptic would diminish, according to definite conditions.

Now, all these apparent motions are actually observed to affect all the bodies visible on the heavens, and to affect them in precisely the degree and direction which would be produced by the annual motion of the earth round the sun.

As the supposed motion of the earth round the sun completely and satisfactorily explains this complicated body of phenomena called aberration, while the motion of the sun round the earth would altogether fail to explain them, they afford another striking evidence of the annual motion of the earth.

2441. *Argument from analogy.* — In fine, another argument in favour of the earth's annual motion round the sun is taken from its analogy to the planets, to all of which, like the earth, the sun is a source of light and heat, and all of which revolve round the sun as a centre, having days, nights, and seasons in all respects similar to those which prevail upon the earth. It seems, therefore, contrary to all probability, that the earth alone, being one of the planets, and by no means the greatest in magnitude or physical importance, should be a centre round which not only the sun, but all the other planets, should revolve.

2442. *Annual parallax.* — If the earth be admitted to move annually round the sun, as a stationary centre in a circle whose diameter must have the vast magnitude of 200 millions of miles, all observers placed upon the earth, seeing distant objects from points of view so extremely distant one from the other as are opposite extremities of the same diameter of such a circle, must necessarily, as might be supposed, see these objects in very different directions.

To comprehend the effect which might be expected to be produced upon the apparent place of a distant object by such a motion, let E E′ E″ E‴, *fig.* 714., represent the earth's annual course round the sun as seen in perspective, and let O be any

distant object visible from the earth. The extremity E of the line EO, which is the visual direction of the object, being carried with the earth round the circle EE′E″E‴, will annually describe a cone of which the base is the path of the earth, and the vertex is the place of the object O. While the earth moves round the circle EE″, the line of visual direction would therefore have a corresponding motion, and the apparent place of the object would be successively changed with the change of direction of this line. If the object be imagined to be projected by the eye upon the firmament, it would trace upon it a path *oo′o″o‴*, which would be circular or elliptical, according to the direction of the object. When the earth is at E, the object would be seen at *o*; and when the earth is at E″, it would be seen at *o″*. The extent of this apparent displacement of the object would be measured by the angle EOE″, which the diameter EE″ of the earth's path or orbit would subtend at the object O.

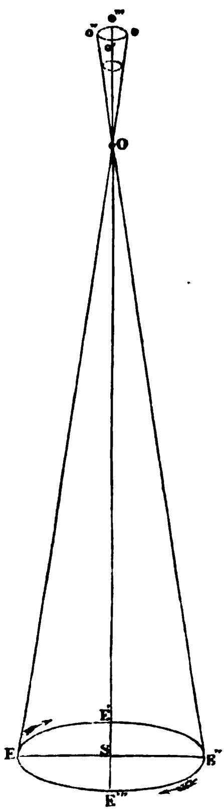


Fig. 714.

It has been stated that, in general, the apparent displacement of a distant visible object produced by any change in the station from which it is viewed is called PARALLAX. That which is produced by the change of position due to the diurnal motion of the earth being called DIURNAL PARALLAX, the corresponding displacement due to the annual motion of the earth is called the ANNUAL PARALLAX.

The greatest amount, therefore, of the annual parallax for any proposed object is the angle which the semidiameter of the earth's orbit subtends at such object, as the greatest amount of the diurnal parallax is the angle which the semidiameter of the earth itself subtends at the object.

Now, as the most satisfactory evidence of the annual motion of the earth would be the discovery of this displacement, and

successive changes of apparent position of all objects on the firmament consequent on such motion, the absence of any such phenomenon must be admitted to constitute, *primâ facie*, a formidable argument against the earth's motion.

2443. *Its effects upon the bodies of the solar system apparent.* — The effects of annual parallax are observable, and indeed are of considerable amount, in the case of all the bodies composing the solar system. The apparent annual motion of the sun is altogether due to parallax. The apparent motions of the planets and other bodies composing the solar system are the effects of parallax, combined with the real motions of these various bodies.

2444. *But erroneously explained by the ancients — Ptolemaic system.* — Until the annual motion of the earth was admitted, these effects of annual parallax on the apparent motions of the solar system were ascribed to a very complicated system of real motions of these bodies, of which the earth was assumed to be the stationary centre, the sun revolving round it, while at the same time the planets severally revolved round the sun as a moveable centre. This hypothesis, proposed originally by Apollonius of Perga, a Grecian astronomer, some centuries before the birth of Christ, received the name of the PTOLEMAIC SYSTEM, having been developed and explained by PTOLEMY, an Egyptian astronomer who flourished in the second century, and whose work, entitled "Syntax," obtained great celebrity, and for many centuries continued to be received as the standard of astronomical science.

Although Pythagoras had thrown out the idea that the annual motion of the sun was merely apparent, and that it arose from a real motion of the earth, the natural repugnancy of the human mind to admit a supposition so contrary to received notions prevented this happy anticipation of future and remote discovery from receiving the attention it merited; and Aristotle, less sagacious than Pythagoras, lent the great weight of his authority to the contrary hypothesis, which was accordingly adopted universally by the learned world, and continued to prevail, until it was overturned in the middle of the sixteenth century by the celebrated Copernicus, who revived the Pythagorean hypothesis of the stability of the sun and the motion of the earth.

2445. *Copernican system.* — The hypothesis proposed by him

in a work entitled "De Revolutionibus Orbium Cœlestium," published in 1543, at the moment of his death, is that since known as the Copernican System, and, being now established upon evidence sufficiently demonstrative to divest it of its hypothetical character, is admitted as the exposition of the actual movements by which that part of the universe called the solar system is affected.

2446. *Effects of annual parallax of the stars.*—The greatest difficulty against which the Copernican system has had to struggle, even among the most enlightened of its opponents, has been the absence of all apparent effects of parallax among the fixed stars, those objects which are scattered in such countless numbers over every part of the firmament. From what has been explained, it will be perceived that, supposing these bodies to be, as they evidently must be, placed at vast distances outside the limits of the solar system and in every imaginable direction around it, the effects of annual parallax would be to give to each of them an apparent annual motion in a circle or ellipse, according to their direction in relation to the position of the earth in its orbit, the ellipse varying in its eccentricity with this position, and the diameter of the circle or major axis of the ellipse being determined by the angle which the diameter E E″ (*fig.* 714.) of the earth's orbit subtends at the star, being less the greater the distance of the star, and *vice versâ.* The apparent position of the star in this circle or ellipse would be evidently always in the plane passing through the star and the line joining the sun and earth.

2447. *Close resemblance of these to aberration.*—Now, it will be apparent, that such phenomena bear a very close resemblance to those of aberration already described (2440.). In both the stars appear to move annually in small circles when situate 90° from the ecliptic; in both they appear to move in small ellipses between that position and the ecliptic; in both the eccentricities of the ellipses increase in approaching the ecliptic; and in both the ellipses flatten into their transverse axis when the object is actually in the ecliptic.

2448. *Yet aberration cannot arise from parallax.*—Notwithstanding this close correspondence, the phenomena of aberration are utterly incompatible with the effects of annual parallax. The apparent displacement produced by aberration is always in the direction of the earth's motion, that is to say,

in the direction of the tangent to the earth's orbit at the point where the earth happens to be placed. The apparent displacement due to parallax would, on the contrary, be in the direction of the line joining the earth and sun. The apparent axis of the ellipse or diameter of the circle of aberration is exactly the same, that is $20''\cdot42$, for all the stars; while the apparent axis of the ellipse or diameter of the circle due to annual parallax would be different for stars at different distances, and would vary, in fact, in the inverse ratio of the distance of the star, and could not therefore be the same for all stars whatever, except on the supposition that all stars are at the same distance from the solar system, a supposition that cannot be entertained.

2449. *General absence of parallax explained by great distance.*—Since, then, with two or three exceptions, which will be noticed hereafter, no traces of the effects of annual parallax have been discovered among the innumerable fixed stars by which the solar system is surrounded, and since, nevertheless, the annual motion of the earth in its orbit rests upon a body of evidence and is supported by arguments which must be regarded as conclusive, the absence of parallax can only be ascribed to the fact that the stars generally are placed at distances from the solar system compared with which the orbit of the earth shrinks into a point, and therefore that the motion of an observer round this orbit, vast as it may seem compared with all our familiar standards of magnitude, produces no more apparent displacement of a fixed star than the motion of an animalcule round a grain of mustard seed would produce upon the apparent direction of the moon or sun.

We shall return to the subject of the annual parallax of the stars in a subsequent chapter.

2450. *The diurnal and annual phenomena explained by the two motions of the earth.*—Considering, then, the annual revolution of the earth, as well as its diurnal rotation, established, it remains to show how these two motions will explain the various phenomena manifested in the succession of seasons.

While the earth revolves annually round the sun, it has a motion of rotation at the same time upon a certain diameter as an axis, which is inclined from the perpendicular to its orbit at an angle of 23° 28′. During the annual motion of the earth this diameter keeps continually parallel to the same direction, and the earth completes its revolution upon it in twenty-three

hours and fifty-six minutes. In consequence of the combination of this motion of rotation of the earth upon its axis with its annual motion round the sun, we are supplied with the alternations of day and night, and the succession of seasons.

When the globe of the earth is in such a position that its north pole leans toward the sun, the greater portion of its northern hemisphere is enlightened, and the greater portion of the southern hemisphere is dark. This position is represented in *fig.* 715., where N is the north pole, and S the south pole.

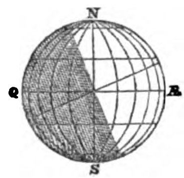


Fig. 715.

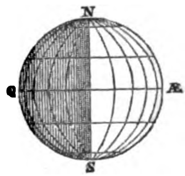


Fig. 716.

The days are therefore longer than the nights in the northern hemisphere. The reverse is the case with the southern hemisphere, for there the greater segments of the parallels are dark, and the lesser segments enlightened; the days are therefore shorter than the nights. Upon the equator, however, at Æ, the circle of the earth is equally divided, and the days and nights are equal. When the south pole leans toward the sun, which it does exactly at the opposite point of the earth's annual orbit, circumstances are reversed : then the days are longer than the nights in the southern hemisphere, and the nights are longer than the days in the northern hemisphere. At the intermediate points of the earth's annual path, when the axis assumes a position perpendicular to the direction of the sun, *fig.* 716., then the circle of light and darkness passes through the poles; all parallels in every part of the earth are equally divided, and there is consequently equal day and night all over the globe.

In the annexed perspective diagram, *fig.* 717., these four positions of the earth are exhibited in such a manner as to be clearly intelligible.

On the day of the 21st of June, the north pole is turned in the direction of the sun ; on the 21st of December, the south pole is turned in that direction. On the days of the equinoxes, the axis of the earth is at right angles to the direction of the sun, and it is equal day and night everywhere on the earth.

The annual variation of the position of the sun with reference to the equator, or the changes of its declination, are explained

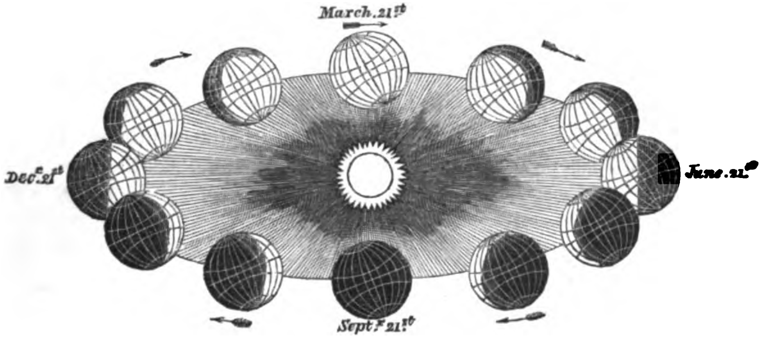


Fig. 717.

by these motions. The summer solstice—the time when the sun's distance from the equator is the greatest — takes place when the north pole leans toward the sun; and the winter solstice — or the time when the sun's distance south of the equator is greatest — takes place when the south pole leans toward the sun.

In virtue of these motions, it follows that the sun is twice a year vertical at all places between the tropics; and at the tropics themselves it is vertical once a year. In all higher latitudes the point at which the sun passes the meridian daily alternately approaches to and recedes from the zenith. From the 21st of December until the 21st of June, the point continually approaches the zenith. It comes nearest to the zenith on the 21st of June; and from that day until the 21st of December, it continually recedes from the zenith, and attains its lowest position on the latter day. The difference, therefore, between the meridional altitudes of the sun on the days of the summer and winter solstices at all places will be twice twenty-three degrees and twenty-eight minutes, or forty-six degrees and fifty-six minutes. In all places beyond the tropics in the northern hemisphere, therefore, the sun rises at noon on the 21st of June, forty-six degrees and fifty-six minutes higher than it rises on the 21st of December. These are the limits of meridional altitude which determine the influence of the sun in different places.

2451. *Mean solar or civil time.* — It has been explained that the rotation of the earth upon its axis is rigorously uniform, and is the only absolutely uniform motion among the many and complicated motions observable on the heavens. This quality would render it a highly convenient measure of time, and it is accordingly adopted for that purpose in all observatories. The hands of a sidereal clock move in perfect accordance with the apparent motion of the firmament.

But for civil purposes, uniformity of motion is not the only condition which must be fulfilled by a measure of time. It is equally indispensable that the intervals into which it divides duration should be marked by conspicuous and universally observable phenomena. Now it happens that the intervals into which the diurnal revolution of the heavens divides duration, are marked by phenomena which astronomers alone can witness and ascertain, but of which mankind in general are, and must remain, altogether unconscious.

2452. *Civil day — noon* and *midnight.* — For the purposes of common life, mankind by general consent has therefore adopted the interval between the successive returns of the centre of the sun's disk to the meridian, as the unit or standard measure of time. This interval, called a CIVIL DAY, is divided into 24 equal parts called HOURS, which are again subdivided into minutes and seconds as already explained in relation to sidereal time. The hours of the civil day, however, are not counted from 0 to 24 as in sidereal time, but are divided into two equal parts of 12 hours, one commencing when the centre of the sun is on the meridian, the moment of which is called NOON or MIDDAY, and the other 12 hours later when the centre of the sun must pass the meridian below the horizon, the moment of which is MIDNIGHT.

For civil purpose, this latter moment has been adopted as the commencement of one day, and the end of the other.

2453. *Difference between mean solar and sidereal time.* — A solar day is evidently longer than a sidereal day. If the sun did not change its position on the firmament, its centre would return to the meridian after the same interval that elapses between the successive transits of a fixed star. But since the sun, as has been explained, moves at the rate of about 1° per day from west to east, and since this motion takes place upon the ecliptic, which is inclined to the equator at an angle of 23° 28′, the centre

of the sun increases its right ascension from day to day, and this increase varies according to its position on the ecliptic. When the circle of declination on which the centre of the sun is placed at noon on one day returns to the meridian the next day, the centre of the sun will have left it, and will be found upon another circle of declination to the east of it; and it will not consequently come to the meridian until a few minutes later, when this other circle of declination, by the diurnal motion of the heavens, shall come to coincide with the meridian.

Hence the solar day is longer than the sidereal day.

2454. *Difference between apparent noon and mean noon.* — But since, from the cause just stated and another which will be presently explained, the daily increase of the sun's right ascension is variable, the difference between a sidereal day and the interval between the successive transits of the sun is likewise variable, and thus it would follow that the solar days would be more or less unequal in length.

2455. *Mean solar time — Equation of time.* — Hence has arisen an expedient adopted for civil purposes to efface this inequality. An imaginary sun is conceived to accompany the true sun, making the complete revolution of the heavens with a rigorously uniform increase of right ascension from hour to hour, while the increase of right ascension of the true sun thus varies. The time measured by the motion of this imaginary sun is called MEAN SOLAR TIME, and the time measured by the motion of the true sun is called APPARENT SOLAR TIME.

The difference between the apparent and mean solar time is called the "EQUATION OF TIME."

The variation of the increase of the sun's right ascension being confined within narrow limits, the true and imaginary suns can never be far asunder, and consequently the difference between mean and apparent time is never considerable.

The time indicated by a sun-dial is apparent time, that indicated by an exactly regulated clock or watch is mean time.

The correction to be applied to apparent time, to reduce it to mean time, is often engraved on sun-dials, where it is stated how much "the sun is too fast or too slow."

2456. *Distance of the sun.* — Although the problem to determine with the greatest practicable precision the distance of the sun from the earth is attended with great difficulties, many

phenomena of easy observation supply the means of ascertaining that this distance must bear a very great proportion to the earth's diameter, or must be such that, by comparison with it, a line 8000 miles in length is almost a point. If, for example, the apparent distance of the centre of the sun from any fixed star be observed simultaneously from two places upon the earth, no matter how far they are apart, no difference will be discovered between them, unless means of observation susceptible of extraordinary precision be resorted to. The expedients by which the apparent displacement of the sun's centre by a change of position of the observer from one extremity of a diameter of the earth to the other, or, what is the same, the apparent magnitude of the diameter of the earth as it would be seen from the sun, has been ascertained, will be explained hereafter. Meanwhile, however, it may be stated that this visual angle amounts to no more than $17''{\cdot}2$, or about the hundredth part of the apparent diameter of the sun as seen from the earth.

Supplied with this datum, and the actual magnitude of the diameter of the earth, we can calculate the distance of the sun by the rule explained in 2298. If r express the distance of the sun, and a the diameter of the earth, we shall have

$$r = \frac{2{,}062{,}650}{172} \times a = 11{,}992 \times a.$$

It appears, therefore, that the distance of the sun is equal to 11,992 diameters of the earth, and since the diameter of the earth measures about 7900 miles (2389), the distance of the sun must be

$$11{,}992 \times 7900 = 94{,}736{,}800 \text{ miles,}$$

or very nearly NINETY-FIVE MILLIONS OF MILES.

Since the mean distance of the earth from the sun has been adopted as the unit or standard, with reference to which astronomical distances generally are expressed, it is of the highest importance to ascertain its value with the greatest precision which our means of observation and measurement admit. By elaborate calculations, based upon the observations made, in 1769, on the transit of Venus, it has accordingly been shown by Professor Encké that when the earth is at its mean distance from the sun,

the semidiameter of the terrestrial equator subtends at the sun an angle of $8''\cdot5776$. This is therefore the mean equatorial horizontal parallax of the sun; and if r express the semidiameter of the equator, and D the mean distance of the earth from the sun, we shall therefore have

$$D = \frac{206265}{8\cdot5776} \times r = 24047 \times r,$$

and since the semidiameter of the equator measures 3962·8 miles (2389), it follows that

$$D = 95{,}293{,}452.$$

Since all the numerical results of observation and measurement are liable to some amount of error, it is important, when precision is required, to know the limit of this error, in order to appreciate the extent to which such results are to be relied upon. In all cases this is possible, a major and minor limit of the computed or observed quantity being assignable, which cannot be exceeded. In the present case the value of D cannot vary from the truth by more than its three-hundredth part; that is to say, the actual mean distance of the earth from the sun, or the semiaxis major of the orbit, cannot be greater than

$$95{,}293{,}452 + 117{,}645 = 95{,}411{,}097 \text{ miles},$$

or less than

$$95{,}293{,}452 - 117{,}645 = 95{,}175{,}807 \text{ miles}.$$

2457. *Linear value of* $1''$ *at the sun's distance.*—By what has been explained in 2298, it appears that the linear value of $1''$ at the sun's distance is

$$\frac{95{,}000{,}000}{206{,}265} = 466 \text{ miles}.$$

2458. *Daily and hourly apparent motion of the sun, and real motion of the earth.* — Since the sun moves over 360° of the heavens in 365¼ days, its daily apparent motion must be $59'\cdot14$, or $3548''$, which being about twice the sun's apparent diameter, it is easy to remember that the disk of the sun appears to move in the firmament daily over a space nearly equal to twice its own apparent diameter. Its hourly apparent motion is

$$\frac{3548''}{24} = 147''\cdot8.$$

Since 1″ at the sun's distance is equal to 466 miles, and since the real orbitual motion is equal to that which the sun would have if it moved round the earth in a year, it follows that the daily orbitual motion of the earth is

$$3548 \times 466 = 1{,}653{,}368 \text{ miles},$$

and its motions per hour, minute, and second, are

68,890 miles per hour,
1,148 miles per minute,
19·1 miles per second.

2459. *Orbit of the earth elliptical.* — In what precedes, we have considered the path of the earth around the sun, called by astronomers its ORBIT, to be a circle, in the centre of which the centre of the sun is placed. This is nearly true, but not exactly so, as will appear from the following observed phenomena.

Let a telescope supplied with the micrometric wires described in 2317, be directed to the sun, and the wires so adjusted that they shall exactly touch the upper and lower limbs, as in *fig.* 718. Let the observer then watch from day to day the appear-

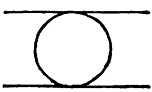


Fig. 718.

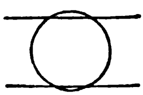


Fig. 719.

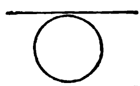


Fig. 720.

ance of the sun and the position of the wires; he will find that, after a certain time, the wires will no longer touch the sun, but will perhaps fall a little within it, as represented in *fig.* 719. And after a further lapse of time, he will find, on the other hand, that they fall a little without it, as in *fig.* 720.

Now, as the wires throughout such a series of observations are maintained always in the same position, it follows that the disk of the sun must appear smaller at one time, and larger at another—that, in fact, the apparent magnitude of the sun must be variable. It is true that this variation is confined within very small limits, but still it is distinctly perceptible. What, then, it may be asked, must be its cause? Is it possible to imagine that the sun *really undergoes a change in its size?* This idea would, under any circumstances, be absurd;

but when we have ascertained, as we may do, that the change of apparent magnitude of the sun is regular and periodical — that for one half of the year it continually diminishes until it attains a minimum, and then for the next half year it increases until it attains a maximum — such a supposition as that of a real periodical change in the globe of the sun becomes altogether incredible.

If then, an actual change in the magnitude of the sun be impossible, there is but one other conceivable cause for the change in its apparent magnitude — which is, a corresponding change in the earth's distance from it. If the earth at one time be more remote than at another, the sun will appear proportionally smaller. This is an easy and obvious explanation of the changes of appearance that are observed, and it has been demonstrated accordingly to be the true one.

On examining the change of the apparent diameter of the sun, it is found that it is least on the 1st of July, and greatest on the 31st of December; that from December to July, it regularly decreases; and from July to December, it regularly increases.

Since the distance of the earth from the sun must increase in the same ratio as the apparent diameter of the sun decreases, and *vice versâ* (1118), the variation of the distance of the earth from the sun in every position which it assumes in its orbit can be exactly ascertained. A plan of the form of the orbit may therefore be laid down, having the point occupied by the centre of the sun marked in it. Such a plan proves on geometric examination to be an ellipse, the place of the sun being one of the foci.

2460. *Method of describing an ellipse — its foci, axis, and eccentricity.* — If the ends of a thread be attached to two points less distant from each other than its entire length, and a pencil be looped in the thread, and moved round the points, so as to keep the thread tight, it will trace an ellipse, of which the two points are the FOCI.

The line drawn joining the foci, continued in both directions to the ellipse, is called its TRANSVERSE, OR MAJOR AXIS.

Another line, passing through the middle point of this at right angles to it, is called its MINOR AXIS.

The middle point of the major axis is called the CENTRE of the ellipse.

The fractional or decimal number which expresses the distance of the focus from the centre, the semiaxis major being taken as the unit, is called the eccentricity of the ellipse.

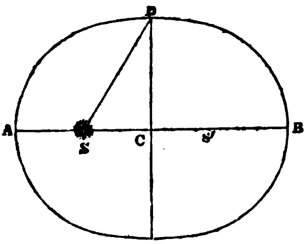


Fig. 721.

In *fig.* 721., C is the centre, S and S′ the foci, A B the transverse axis.

The less the ratio of S S′ to A B, or, what is the same, the less the eccentricity is, the more nearly the form of the ellipse approaches to that of a circle, and when the foci actually coalesce, the ellipse becomes an exact circle.

2461. *Eccentricity of the earth's orbit.*—The eccentricity of the elliptic orbit of the earth is so small, that if an ellipse, representing truly that orbit, were drawn upon paper, it would be distinguishable from a circle only by submitting it to exact measurement. The eccentricity of the orbit has been ascertained to be only 0·01679. The semiaxis major, or mean distance, being 1·0000, the greatest and least distances of the earth from the sun will be—

$$B\,S = 1{\cdot}0000 + 0{\cdot}01679 = 1{\cdot}01679$$
$$A\,S = 1{\cdot}0000 - 0{\cdot}01679 = 0{\cdot}98321.$$

The difference between these extreme distances is, therefore, only 0·03358. So that the difference between the greatest and least distances does not amount to so much as four hundredths of the mean distance.

2462. *Perihelion and aphelion of the earth.*—The positions A and B, where the earth is nearest to, and most distant from, the sun, are called PERIHELION and APHELION.

The positions of these points are ascertained by observing the places of the sun when its apparent diameter is greatest and least.

It is evident from what has been stated that the earth is in aphelion on 1st July, and in perihelion on 1st January.

Contrary to what might be expected, therefore, the earth is more distant from the sun in summer than in winter.

2463. *Variations of temperature through the year.*—The succession of spring, summer, autumn, and winter, and the

variations of temperature of the seasons — so far as these variations depend on the position of the sun — will now require to be explained.

The influence of the sun in heating a portion of the earth's surface, will depend partly on its altitude above the horizon. The greater that altitude is, the more perpendicularly the rays will fall, and the greater will be their calorific effect.

To explain this, let us suppose ABCD, *fig.* 722., to represent a beam of the solar light; let CD represent a portion of the earth's surface, upon which the beam would fall perpendicularly; and let CE represent that portion on which it would fall obliquely; the same number of rays will strike the surfaces CD and CE; but the surface CE being obviously greater than CD, the rays will necessarily fall more densely on the latter: and as the heating power must be in proportion to the density of the rays, it follows that CD will be heated more than CE in just the same proportion as CE is greater than CD. But if we would compare two surfaces on neither of which the sun's rays fall perpendicularly, let us take CE and CF. They fall on CE with more obliquity than on CF; but CE is evidently greater than CF, and therefore the rays, being diffused over a larger surface, are less dense, and therefore less effective in heating.

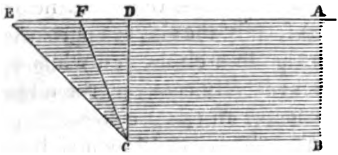


Fig. 722.

The calorific effect of the sun's rays on a surface more oblique to their direction than another will then be proportionably less.

If the sun be in the zenith, its rays will strike the surface perpendicularly, and the heating effect will therefore be greater than when the sun is in any other position.

The greater the altitude to which the sun rises, the less obliquely will be the direction in which its rays will strike the surface at noon, and the more effective will be their heating power. So far, then, as the heating power depends on the altitude of the sun, it will be increased with every increase of its meridian altitude.

Hence it is that the heat of summer increases as we approach the equator. The lower the latitude is, the greater will be the height to which the sun will rise. The meridian altitude of

the sun at the summer solstice being everywhere outside the tropics forty-six degrees and fifty-six minutes more than at the winter solstice, the heating effect will be proportionately greater.

But this is not the only cause which produces the greatly superior heat of summer as compared with winter, especially in the higher latitudes. The heating effect of the sun depends not alone on its altitude at midday; it also depends on the length of time which it is above the horizon and below it. While the sun is above the horizon, it is continually imparting heat to the air and to the surface of the earth; and while it is below the horizon, the heat is continually being dissipated. The longer, therefore,—other things being the same,—the sun is above the horizon, and the shorter time it is below it, the greater will be the amount of heat imparted to the earth every twenty-four hours. Let us suppose that between sunrise and sunset, the sun, by its calorific effect, imparts a certain amount of heat to the atmosphere and the surface of the earth, and that from sunset to sunrise a certain amount of this heat is lost: the result of the action of the sun will be found by deducting the latter from the former.

Thus, then, it appears that the influence of the sun upon the seasons depends as much upon the length of the days and nights as upon its altitude; but it so happens that one of these circumstances depends upon the other. The greater the sun's meridional altitude is, the longer will be the days, and the shorter the nights; and the less it is, the longer will be the nights, and the shorter the days. Thus both circumstances always conspire in producing the increased temperature of summer, and the diminished temperature of winter.

2464. *Why the longest day is not also the hottest.—The dog-days.*—A difficulty is sometimes felt when the operation of these causes is considered, in understanding how it happens that, notwithstanding what has been stated, the 21st of June—when the sun rises the highest, when the days are longest and the nights shortest—is not the hottest day, but that, on the contrary, the dog-days, as they are called, which comprise the hottest weather of the year, occur in August; and in the same manner, the 21st of December—when the height to which the sun rises is least, the days shortest, and the nights longest—is not usually the coldest day, but that, on the other hand, the most inclement weather occurs at a later period.

To explain this, so far as it depends on the position of the sun and the length of the days and nights, we are to consider the following circumstances:—

As midsummer approaches, the gradual increase of the temperature of the weather has been explained thus: The days being considerably longer than the nights, the quantity of heat imparted by the sun during the day is greater than the quantity lost during the night; and the entire result during the twenty-four hours gives an increase of heat. As this augmentation takes place after each successive day and night, the general temperature continues to increase. On the 21st of June, when the day is longest, and the night is shortest, and the sun rises highest, this augmentation reaches its maximum; but the temperature of the weather does not therefore cease to increase. After the 21st of June, there continues to be still a daily augmentation of heat, for the sun still continues to impart more heat during the day than is lost during the night. The temperature of the weather will therefore only cease to increase when, by the diminished length of the day, the increased length of the night, and the diminished meridional altitude of the sun, the heat imparted during the day is just balanced by the heat lost during the night. There will be, then, no further increase of temperature, and the heat of the weather will have attained its maximum.

But it might occur to a superficial observer, that this reasoning would lead to the conclusion that the weather would continue to increase in its temperature, until the length of the days would become equal to the length of the nights; and such would be the case, if the loss of heat per hour during the night were equal to that gain of heat per hour during the day. But such is not the case; the loss is more rapid than the gain, and the consequence is, that the hottest day usually comes within the month of July, but always long before the day of the autumnal equinox.

The same reasoning will explain why the coldest weather does not usually occur on the 21st of December, when the day is shortest and the night longest, and when the sun attains the lowest meridional altitude. The decrease of the temperature of the weather depends upon the loss of heat during the night being greater than the gain during the day; and until, by the

increased length of the day and the diminished length of the night, these effects are balanced, the coldest weather will not be attained.

These observations must be understood as applying only so far as the temperature of the weather is affected by the sun, and by the length of the days and nights. There are a variety of other local and geographical causes which interfere with these effects, and vary them at different times and places.

On referring to the annual motion of the earth round the sun, it appears that the position of the sun within the elliptic orbit of the earth is such that the earth is nearest to the sun about the 1st of January, and most distant from it about the 1st of July. As the calorific power of the sun's rays increases as the distance from the earth diminishes, in even a higher proportion than the change of distances, it might be expected that the effect of the sun in heating the earth on the 1st of January would be considerably greater than on the 1st of July. If this were admitted, it would follow that the annual motion of the earth in its elliptic orbit would have a tendency to diminish the cold of the winter in the northern hemisphere, and mitigate the heat of summer, so as to a certain extent to equalise the seasons; and, on the contrary, in the southern hemisphere, where the 1st of January is in the middle of summer and the 1st of July the middle of winter, its effects would be to aggravate the cold in winter and the heat in summer. The investigations, however, which have been made in the physics of heat, have shown that that principle is governed by laws which counteract such effects. Like the operation of all other physical agencies, the sun's calorific power requires a definite time to produce a given effect, and the heat received by the earth at any part of its orbit will depend conjointly on its distance from the sun and the length of time it takes to traverse that portion of its orbit. In fact, it has been ascertained that the heating power depends as much on the rate at which the sun changes its longitude as upon the earth's distance from it. Now it happens that, in consequence of the laws of the planetary motions, discovered by Kepler, and explained by Newton, when the earth is most remote from the sun, its velocity is least, and consequently the hourly changes of longitude of the sun will be proportionally less. Thus it appears that

what the heating power loses by augmented distance, it gains by diminished velocity; and again, when the earth is nearest to the sun, what it gains by diminished distance, it loses by increased speed. There is thus a complete compensation produced in the heating effect of the sun, by the diminished velocity of the earth which accompanies its increased distance.

This period of the year, during which the heat of the weather is usually most intense, was called the CANICULAR DAYS, or DOG DAYS. These days were generally reckoned as forty, commencing about the 3rd of July, and received their name from the fact, that in ancient times the bright star Sirius, in the constellation of Canis major, or the great dog, at that time rose a little before the sun, and it was to the sinister influence of this star that were ascribed the bad effects of the inclement heat, and especially the prevalence of madness among the canine race. Owing to a cause which will be explained hereafter (the precession of the equinoxes), this star no longer rises with the sun during the hot season.

CHAP. IX.

THE MOON.

2465. *The moon an object of popular interest.*—Although it be in mere magnitude, and physically considered, one of the most insignificant bodies of the solar system, yet for various reasons the MOON has always been regarded by mankind with feelings of profound interest, and has been invested by the popular mind with various influences, affecting not only the physical condition of the globe, but also the phenomena of the organised world. It has been as much an object of popular superstition as of scientific observation. These circumstances doubtless are in some degree owing to its striking appearance in the firmament, to the various changes of form to which it is subject, and above all to its proximity to the earth, and the close alliance existing between it and our planet.

2466. *Its distance.*—The distance of the moon is computed,

by the method explained in 2328, by first ascertaining its horizontal parallax.

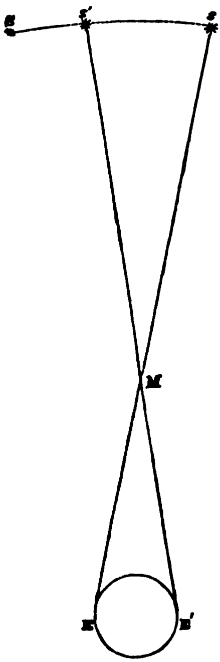


Fig. 723.

Let E and E′, *fig.* 723., be the opposite ends of a diameter of the earth, and let M be the place of the moon's centre. Let S be any conspicuous star seen near the moon in the heavens, in the plane of the points E, E′, and M. The apparent distance of this star from the moon's centre is S*s* to an observer at E, and it is S*s*′ to an observer at E′. The difference of these distances *s*′*s* is the arc of the heavens which measures the angle *s*M*s*′ or, what is the same, the angle EME′, under which the diameter EE′ of the earth would be seen from the moon.

Now the arcs S*s* and S*s*′ can be and have been measured, and their mean difference *ss*′ has been ascertained to be 114′ 12″ = 6852″, subject to a slight variation from a cause which will presently be explained.

It appears, from what has been explained in 2327, that half the angle EME′ is the moon's horizontal parallax, which is therefore 57′ 6″ = 3426″.

The moon's distance therefore, computed by the formula explained in 2328, is

$$\text{ME} = \frac{206265}{3425} \times r = 60{\cdot}2 \times r.$$

It follows, therefore, that the moon's distance is about thirty times the earth's diameter; and since the value of the latter is 7900 miles, the moon's distance is

$$7900 \times 30 = 237{,}000 \text{ miles},$$

or, as appears by more exact computation, 237,630 miles.

2467. *Linear value of* 1″ *on it.* — Having thus ascertained the moon's distance, we are enabled, by the method explained in 2319, to ascertain the actual length measured transversely to the line of vision on the moon which corresponds to the visual angle of 1″. This length is

$$\frac{237630}{206265} = 1{\cdot}15 \text{ mile.}$$

By this formula any space upon the moon, measured by its visual angle, can be reduced to its actual linear value, provided its direction be at right angles to the visual ray, which it will be if it be at the centre of the lunar disk. If it be between the centre and the edges it will be foreshortened by the obliquity of the moon's surface to the line of vision, and, consequently, the linear value thus computed will be the real linear value diminished by projection, which, however, can be easily allowed for so that the true linear value can be obtained for every part of the lunar disk.

2468. *Its apparent and real diameter.*—The apparent diameter of the moon is subject to a slight variation, owing to a corresponding variation due to the small ellipticity of its orbit. Its mean value is found to be $31' \, 7''$ or $1867''$.

By what has just been established (2392), therefore, its real diameter must be

$$1867 \times 1{\cdot}15 = 2147 \text{ miles.}$$

More exact methods give 2153 miles.

Since the superficial magnitude of spheres is as the squares, and their volume or solid bulk as the cubes, of their diameters, it follows that the superficial extent of the moon is about the fourteenth part of the surface, and its volume about the forty-ninth part of the bulk, of our globe.

2469. *Apparent and real motion.*—The moon, like the sun, appears to move upon the celestial sphere in a direction contrary to that of the diurnal motion. Its apparent path is a great circle of the sphere, inclined to the ecliptic at an angle of about $5° \, 8' \, 48''$. It completes its revolution of the heavens in $27^{d.}$ $7^{h.}$ $44^{m.}$

This apparent motion is explained by a real motion of the moon round the earth at the mean distance above mentioned, and in the time in which the apparent revolution is completed.

2470. *Hourly motion, apparent and real.*—Since the time taken by the moon to make a complete revolution, or 360° of the heavens, is $27^{d.}$ $7^{h.}$ $44^{m.}$ or $655^{h}{\cdot}73$, it follows, that her mean apparent motion per day is $13° \, 10' \, 35''$, and per hour is $32' \, 42''$, which is a little more than her mean apparent diameter. The rate of the moon's apparent motion on the firmament may

therefore be remembered by the fact, that she moves over the length of her own apparent diameter in an hour.

Since the linear value of $1''$ at the moon's distance is 1·15 mile, the linear value of $1'$ is 6·9 miles, and, consequently, the real motion of the moon per hour in her orbit, is

$$6{\cdot}9 \times 32{\cdot}9 = 227 \text{ miles.}$$

Her orbitual motion is therefore at the rate of 3·8 miles per minute.

2471. *Orbit elliptical.*— Although in its general form and character, the path of the moon round the earth is, like the orbits of the planets and satellites, circular, yet when submitted to accurate observation, we find that it is strictly an ellipse or oval, the centre of the earth occupying one of its *foci.* This fact can be ascertained by immediate observation upon the apparent magnitude of the moon. It will be easily comprehended that any change which the apparent magnitude, as seen from the earth, undergoes, must arise from corresponding changes in the moon's distance from us. Thus, if at one time the disk of the moon appears larger than at another time, as it cannot be supposed that the actual size of the moon itself could be changed, we can only ascribe the increase of the apparent magnitude to the diminution of its distance. Now we find by observation that such apparent changes are actually observed in its monthly course around the earth. The moon is subject to a small though perceptible variation of apparent size. We find that it diminishes until it reaches a minimum, and then gradually increases until it reaches a maximum.

When the apparent magnitude is least, it is at its greatest distance, and when greatest, at its least distance. The positions in which these distances lie are directly opposite. Between these two positions the apparent size of the moon undergoes a regular and gradual change, increasing continually from its minimum to its maximum, and consequently between these positions its distance must gradually diminish from its maximum to its minimum. If we lay down on a chart or plan a delineation of the course or path thus determined, we shall find that it will represent an oval, which differs however very little from a circle; the place of the earth being nearer to one end of the oval than the other.

2472. *Moon's apsides — apogee and perigee — progression of*

the apsides. — The point of the moon's path in the heavens at which its magnitude appears the greatest, and when, therefore, it is nearest the earth, is called its *perigee;* and the point where its apparent size is least, and where, therefore, its distance from the earth is greatest, is called its *apogee.* These two points are called the *moon's apsides.*

If the positions of these points in the heavens be observed accurately for a length of time, it will be found that they are subject to a regular change; that is to say, the place where the moon appears smallest will every month shift its position; and a corresponding change will take place in the point where it appears largest. The movement of these points in the heavens is found to be in the same direction as the general movement of the planets; that is, from west to east, or progressive. This phenomenon is called the PROGRESSION OF THE MOON'S APSIDES.

The rate of this progression of the moon's apsides is 40° 68′ in a tropical or common year, being equivalent to 6′ 41″ per day. They consequently make a complete revolution in 8·85 years.

2473. *Moon's nodes — ascending and descending node—their retrogression.* — If the position of the moon's centre in the heavens be observed from day to day, it will be found that its apparent path is a great circle, making an angle of about 5° with the ecliptic. This path consequently crosses the ecliptic at two points in opposite quarters of the heavens. These points are called the *moon's nodes.* Their positions are ascertained by observing from time to time the distance of the moon's centre from the ecliptic, which is the moon's latitude; by watching its gradual diminution, and finding the point at which it becomes nothing; the moon's centre is then in the ecliptic, and its position is the *node.* The node at which the moon passes from the south to the north of the ecliptic is called the *ascending node,* and that at which it passes from the north to the south is called the *descending note.*

These points, like the apsides, are subject to a small change of position, but in a retrogade direction. They make a complete revolution of the ecliptic in a direction contrary to the motion of the sun in 18·6 years, being at the rate of 3′ 10″·6 per day.

2474. *Rotation on its axis*—While the moon moves round the

earth thus in its monthly course, we find, by observations of its appearance, made even without the aid of telescopes, that the same hemisphere is always turned towards us. We recognise this fact by observing that the same marks are always seen in the same positions upon it. Now in order that a globe which revolves in a circle around a centre should turn continually the same hemisphere toward that centre, it is necessary that it should make one revolution upon its axis in the time it takes so to revolve. For let us suppose that the globe, in any one position, has the centre round which it revolves north of it, the hemisphere turned toward the centre is turned toward the north. After it makes a quarter of a revolution, the centre is to the east of it, and the hemisphere which was previously turned to the north must now be turned to the east. After it has made another quarter of a revolution the centre will be south of it, and it must be now turned to the south. In the same manner, after another quarter of a revolution, it must be turned to the west. As the same hemisphere is successively turned to all the points of the compass in one revolution, it is evident that the globe itself must make a single revolution on its axis in that time.

It appears, then, that the rotation of the moon upon its axis, being equal to that of its revolution in its orbit, is $27^{d.}\ 7^{h.}\ 44^{m.}$ or $655^{h.}\ 44^{m.}$ The intervals of light and darkness to the inhabitants of the moon, if there were any, would then be altogether different from those provided in the planets; there would be about $327^{h.}\ 52^{m.}$ of continued light alternately with $327^{h.}\ 52^{m.}$ of continued darkness; the analogy, then, which, as will hereafter appear, prevails among the planets with regard to days and nights, and which forms a main argument in favour of the conclusion that they are inhabited globes like the earth, does not hold good in the case of the moon.

2475. *Inclination of axis of rotation.*—Although as a general proposition it be true that the same hemisphere of the moon is always turned toward the earth, yet there are small variations at the edge called librations, which it is necessary to notice. The axis of the moon is not exactly perpendicular to its orbit, but is inclined at the small angle of $1° 30' 10''\cdot 8$. By reason of this inclination, the northern and southern poles of the moon lean alternately in a slight degree to and from the earth.

2476. *Libration in latitude.*—When the north pole leans towards the earth, we see a little more of that region, and a little

less when it leans the contrary way. This variation in the northern and southern regions of the moon visible to us, is called the LIBRATION IN LATITUDE.

2477. *Libration in longitude.*—In order that in a strict sense the same hemisphere should be continually turned toward the earth, the time of rotation upon its axis must not only be equal the time of rotation in its orbit, which in fact it is, but its angular velocity on its axis in every part of its course, must be exactly equal to its angular velocity in its orbit. Now it happens that while its angular velocity on its axis is rigorously uniform throughout the month, its angular velocity in its orbit is subject to a slight variation; the consequence of this is that a little more of its eastern or western edge is seen at one time than at another. This is called the LIBRATION IN LONGITUDE.

2478. *Diurnal libration.*—By the diurnal motion of the earth, we are carried with it round its axis; the stations from which we view the moon in the morning and evening, or rather when it rises and when it sets, are then different according to the latitude of the earth in which we are placed. By thus viewing it from different places, we see it under slightly different aspects. This is another cause of a variation, which we see in its eastern and western edges; this is called the DIURNAL LIBRATION.

2479. *Phases of the moon.*—While the moon revolves round the earth, its illuminated hemisphere is always presented to the sun; it therefore takes various positions in reference to the earth. In *fig.* 724. the effects of this are exhibited. Let E S

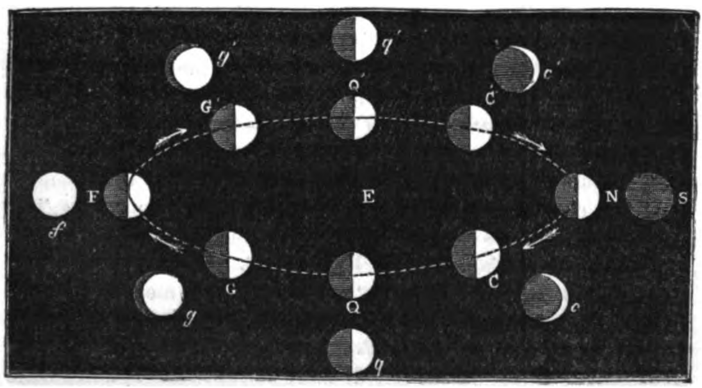


Fig. 724.

represent the direction of the sun, and E the earth; when the moon is at N, between the sun and the earth, its illuminated hemisphere being turned toward the sun, its dark hemisphere will be presented toward the earth; it will therefore be invisible. In this position the moon is said to be in CONJUNCTION.

When it moves to the position C, the enlightened hemisphere being still presented to the sun, a small portion of it only is turned to the earth, and it appears as a thin crescent, as represented at *c*.

When the moon takes the position of Q, at right angles to the sun, it is said to be in QUADRATURE; one half of the enlightened hemisphere only is then presented to the earth, and the moon appears halved, as represented at *q*.

When it arrives at the position G, the greater part of the enlightened portion is turned to the earth, and it is gibbous, appearing as represented at *g*.

When the moon comes in OPPOSITION to the sun, as seen at F, the enlightened hemisphere is turned full toward the earth, and the moon will appear full as at *f*, unless it be obscured by the earth's shadow, which rarely happens. In the same manner it is shown that at G′ it is again gibbous; at Q′ it is halved, and at C′ it is a crescent.

When the moon is full, being in opposition to the sun, it will necessarily be in the meridian at midnight, and will rise as the sun sets, and set as the sun rises; and thus, whenever the enlightened hemisphere is turned toward us, and when, therefore, it is the most capable of benefiting us, it is up in the firmament all night; whereas, when it is in conjunction, as at N, and the dark hemisphere is turned toward us, it would then be of no use to us, and is accordingly up during the day. The position at Q is called the "first quarter," and at Q′ the "last quarter." The position at C is called the first octant; G the second octant; G′ the third octant; and C′ the fourth octant. At the first and fourth octants it is a crescent, and at the second and third octants it is gibbous.

2480.—*Synodic period or common month.*—The apparent motion of the moon in the heavens is much more rapid than that of the sun; for while the sun makes a complete circuit of the ecliptic in 365·25 days, and therefore moves over it at about 61′ per day, the moon moves at the rate of 13° 10′ 35″ (2470) per day. As the sun and moon appear to move in the same direction in the

firmament, both proceeding from west to east, the moon will, after conjunction, depart from the sun toward the east at the rate of about 12° 9′ per day. If then, the moon be in conjunction with the sun on any given day, it will be 12° 9′ east of it at the same time on the following day; 24° 18′ east of it after two days, and so on. If, then, the sun set with the moon on any evening, it will, at the moment of sunset on the following evening, be 12° 9′ east of it, and at sunset will appear as a thin crescent, at a considerable altitude; on the succeeding day it will be 24° 18′ east of the sun, and will be at a still greater altitude at sunset, and will be a broader crescent. After seven days, the moon will be removed nearly 90° from the sun; it will be at or near the meridian at sunset. It will remain in the heavens for about six hours after sunset, and will be seen in the west as the half-moon. Each successive evening increasing its distance from the sun, and also increasing its breadth, it will be visible in the meridian at a later hour, and will consequently be longer apparent in the firmament during the night—it will then be gibbous. After about fifteen days, it will be 180° removed from the sun, and will be full, and consequently will rise when the sun sets, and set when the sun rises—being visible the entire night. After the lapse of about twenty-two days, the distance of the moon from the sun being about 270°, it will not reach the meridian until nearly the hour of sunrise; it will then be visible during the last six hours of the night only. The moon will then be waning, and toward the close of the month will only be seen in the morning before sunrise, and will appear as a crescent.

If the earth and sun were both stationary while the moon revolves round the former, the period of the phases would be the same as the period of the moon. But from what has been explained, it will be evident that while the moon makes its apparent revolution of the heavens in about 27·3 days, the sun advances through somewhat more than 27° of the heavens, *in the same direction*. Before the moon can reassume the same phase, it must have the same position relative to the sun, and must, therefore, overtake it. But since it moves at the rate of about 1° in two hours, it will take more than two days to move over 27°. Hence the *synodic period*, or lunar month, or the interval between two successive conjunctions, is about two days longer than the sidereal period of our satellite.

The exact length of the synodic period is $29^{d.}\ 12^{h.}\ 14^{m.}\ 2^{s}{\cdot}87$, or 29·53059 mean solar days.

2481. *Mass and density.* — The methods by which the mass or weight of the moon has been ascertained will be explained hereafter; meanwhile it may be stated here that the result of the most recent solutions of this problem, by various methods and on different data, proves that the mass or quantity of matter composing the globe of the moon, is a little more than the 90th part of the mass of the earth; or, more exactly, if the mass of the earth consist of a million of equal parts, the mass of the moon will be equal to 11,399 of these parts.

Since the volume or bulk of the moon is about the 50th part of that of the earth, while its mass or weight is little more than the 90th part of that of the earth, it follows that its mean density must be little more than half the density of the earth, and therefore (2393.) about 2·83 times that of water.

2482. *No air upon the moon.* — In order to determine whether or not the globe of the moon is surrounded with any gaseous envelope like the atmosphere of the earth, it is necessary first to consider what appearances such an appendage would present, seen at the moon's distance, and whether any such appearances are discoverable.

According to ordinary and popular notions, it is difficult to separate the idea of an atmosphere from the existence of clouds; yet to produce clouds something more is necessary than air. The presence of water is indispensable, and if it be assumed that no water exist, then certainly the absence of clouds is no proof of the absence of an atmosphere. Be this as it may, however, it is certain that there are no clouds upon the moon, for if there were, we should immediately discover them, by the variable lights and shadows they would produce. If there is, then, an atmosphere upon the moon, it is one entirely unaccompanied by clouds.

One of the effects produced by a distant view of an atmosphere surrounding a globe, one hemisphere of which is illuminated by the sun, is, that the boundary, or line of separation between the hemisphere enlightened by the sun and the dark hemisphere, is not sudden and sharply defined, but is gradual — the light fading away by slow degrees into the darkness.

It is to this effect upon the globe of the earth that twilight is owing, and as we shall see hereafter, such a gradual fading away of the sun's light is discoverable on some of the planets, upon which an atmosphere is observed.

Now, if such an effect of an atmosphere were produced upon the moon, it would be perceived by the naked eye, and still more distinctly with the telescope. When the moon appears as a crescent, its concave edge is the boundary which separates the enlightened from the dark hemisphere. When it is in the quarters, the diameter of the semicircle is also that boundary. In neither of these cases, however, do we ever discover the slightest indication of any such appearance as that which has just been described. There is no gradual fading away of the light into the darkness; on the contrary, the boundary, though serrated and irregular, is nevertheless perfectly well defined and sudden.

All these circumstances conspire to prove that there does not exist upon the moon an atmosphere capable of reflecting light in any sensible degree.

2483. *Absence of air indicated by absence of refraction.* — But it may be contended that an atmosphere may still exist, though too attenuated to produce a sensible twilight. Astronomers, however, have resorted to another test of a much more decisive and delicate kind, the nature of which will be understood by explaining a simple principle of optics.

Let *m m'*, *fig.* 725., represent the disk of the moon. Let *a a'*

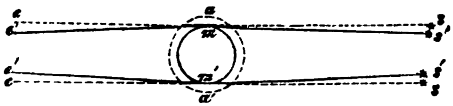


Fig. 725.

represent the atmosphere which surrounds it. Let *s m e* and *s m' e* represent two lines touching the moon at *m* and *m'*, and proceeding towards the earth. Let *s s* be two stars seen in the direction of these lines. If the moon had no atmosphere, these stars would appear to touch the edge of the moon at *m* and *m'*, because the rays of light from them would pass directly

towards the earth; but if the moon have an atmosphere, then that atmosphere will possess the property which is common to all transparent media of refracting light, and, in virtue of such property, stars in such positions as *s′ s′*, behind the edge of the moon, would be visible at the earth, for the ray *s′ m*, *s′ m′*, in passing through the atmosphere, would be bent at an angle in the direction *m e′*, and in like manner the ray *s′ m′* would be bent at the angle *m′ e′*— so that the stars *s′ s′* would be visible at *e′ e′*, notwithstanding the interposition of the edges of the moon.

This reasoning leads to the conclusion that as the moon moves over the face of the firmament, stars will be continually visible at its edge which are really behind it if it have an atmosphere, and the extent to which this effect will take place will be in proportion to the density of the atmosphere.

The magnitude and motion of the moon and the relative positions of the stars are so accurately known that nothing is more easy, certain, and precise, than the observations which may be made with the view of ascertaining whether any stars are ever seen which are sensibly behind the edge of the moon. Such observations have been made, and no such effect has ever been detected. This species of observation is susceptible of such extreme accuracy, that it is certain that if an atmosphere existed upon the moon a thousand times less dense than our own, its presence must be detected.

Bessel has calculated that if the difference between the apparent diameter of the moon, and the arc of the firmament moved over by the moon's centre during the occultation of a star, centrically occulted, were admitted to amount to so much as 2″, and allowing for the possible effect of mountains, by which the edge of the disk is serrated, taking these at the extreme height of 24,000 feet, the density of the lunar atmosphere, whose refraction would produce such an effect, would not exceed the 968th part of the density of the earth's atmosphere, supposing the two fluids to be similarly constituted. Nor would this conclusion be materially modified by any supposition of an atmosphere composed of gases different from the constituents of the earth's atmosphere.

The earth's atmosphere supports a column of 30 inches of mercury: an atmosphere 1000 times less dense would support

a column of three-tenths of an inch only. We may therefore consider it as an established fact, that no atmosphere exists on the moon having a density even as great as that which remains under the receiver of the most perfect air-pump, after that instrument has withdrawn from it the air to the utmost extent of its power.

If further proofs of the nonexistence of a lunar atmosphere were required, Sir J. Herschel indicates several which are found in the phenomena of eclipses. In a solar eclipse the existence of an atmosphere having any sensible refraction, would enable us to trace the limb of the moon beyond the cusps externally to the sun's disk, by a *narrow but brilliant* line of light extending to some distance along its edge. No such phenomenon has, however, been seen.

If there were any appreciable quantity of vapour suspended over the moon's surface, *very* faint stars ought to disappear behind it before the moment of their occultation by the interposition of the moon's edge. Such, however, is not the case. When occulted at the enlightened edge of the lunar disk, the light of the moon overpowers them and renders them invisible, and even at the dark edge the glare in the sky, caused by the proximity of the enlightened part of the disk, renders the occultation of extremely minute stars incapable of observation. But these obstacles are removed in the case of total solar eclipses, on which occasions stars, so faint as to be only seen by the aid of a telescope, come up close to the limb without any sensible diminution of their brightness, and undergo an extinction as instantaneous as the largest and brightest by the interposition of the moon's limb.

2484. *Moonlight not sensibly calorific.* — It has long been an object of inquiry whether the light of the moon has any heat, but the most delicate experiments and observations have failed to detect this property in it. The light of the moon was collected into the focus of a concave mirror of such magnitude as would have been sufficient, if exposed to the sun's light, to evaporate gold or platinum. The bulb of a differential thermometer, sensitive enough to show a change of temperature amounting to the 500th part of a degree, was placed in its focus so as to receive upon it the concentrated rays. Yet no sensible effect was produced. We must, therefore, conclude that the light of the moon does not possess the calorific property

in any sensible degree. But if the rays of the moon be not warm, the vulgar impression that they are cold is equally erroneous. We have seen that they produce no effect either way on the thermometer.

2485. *No liquids on the moon.* — The same physical tests which show the nonexistence of an atmosphere of air upon the moon are equally conclusive against an atmosphere of vapour. It might, therefore, be inferred that no liquids can exist on the moon's surface, since they would be subject to evaporation. Sir John Herschel, however, ingeniously suggests that the non-existence of vapour is not conclusive against evaporation. One hemisphere of the moon being exposed continuously for 328 hours to the glare of sunshine of an intensity greater than a tropical noon, because of the absence of an atmosphere and clouds to mitigate it, while the other is for an equal interval exposed to a cold far more rigorous than that which prevails on the summits of the loftiest mountains or in the polar region, the consequence would be the immediate evaporation of all liquids which might happen to exist on the one hemisphere, and the instantaneous condensation and congelation of the vapour on the other. The vapour would, in short, be no sooner formed on the enlightened hemisphere than it would rush to the vacuum over the dark hemisphere, where it would be instantly condensed and congealed, an effect which Herschel aptly illustrates by the familiar experiment of the CRYOPHOROUS. The consequence, as he observes, of this state of things would be absolute aridity below the vertical sun, constant accretion of hoar frost in the opposite region, and perhaps a narrow zone of running water at the borders of the enlightened hemisphere. He conjectures that this rapid alternation of evaporation and condensation may to some extent preserve an equilibrium of temperature, and mitigate the severity of both the diurnal and nocturnal conditions of the surface. He admits nevertheless that such a supposition could only be compatible with the tests of the absence of a transparent atmosphere even of vapour within extremely narrow limits; and it remains to be seen whether the general physical condition of the lunar surface as disclosed by the telescope be not more compatible with the supposition of the total absence of all liquid whatever.

It appears to have escaped the attention of those who assume

the possibility of the existence of water in the liquid state on the moon, that, in the absence of an atmosphere, the temperature must necessarily be, not only far below the point of congelation of water, but even that of most other known liquids. Even within the tropics, and under the line with a vertical sun, the height of the snow line does not exceed 16,000 feet (2187), and nevertheless at that elevation, and still higher, there prevails an atmosphere capable of supporting a considerable column of mercury. At somewhat greater elevations, but still in an atmosphere of very sensible density, mercury is congealed. Analogy, therefore, justifies the inference that the total, or nearly total, absence of air upon the moon is altogether incompatible with the existence of water, or probably any other body in the liquid state, and necessarily infers a temperature altogether incompatible with the existence of organised beings in any respect analogous to those which inhabit the earth.

But another conclusive evidence of the nonexistence of liquids on the moon is found in the form of its surface, which exhibits none of those well understood appearances which result from the long-continued action of water. The mountain formations with which the entire visible surface is covered are, as will presently appear, universally so abrupt, precipitous, and unchangeable, as to be utterly incompatible with the presence of liquids.

2486. *Absence of air deprives solar light and heat of their utility.* — The absence of air also prevents the diffusion of the solar light. It has been already shown (923) that the general diffusion of the sun's light upon the earth is mainly due to the reflection and refraction of the atmosphere, and to the light reflected by the clouds; and that without such means of diffusion the solar light would only illuminate those places into which its rays would directly penetrate. Every place not in full sunshine, or exposed to some illuminated surface, would be involved in the most pitchy darkness. The sky at noon-day would be intensely black, for the beautiful azure of our firmament in the day-time is due to the reflected colour of the air.

Thus it appears that the absence of air must deprive the sun's illuminating and heating agency of nearly all its utility. If no diffusion of light and no retention and accumulation of

heat, such as an atmosphere supplies, prevail, it is impossible to conceive the existence and maintenance of an organised world having any analogy to the earth.

2487. *As seen from the moon, appearance of the earth and the firmament.* — If the moon were inhabited, observers placed upon it would witness celestial phenomena of a singular description, differing in many respects from those presented to the inhabitants of our globe. The heavens would be perpetually serene and cloudless. The stars and planets would shine with extraordinary splendour during the long night of 328 hours. The inclination of her axis being only 5°, there would be no sensible changes of season. The year would consist of one unbroken monotony of equinox. The inhabitants of one hemisphere would never see the earth; while the inhabitants of the other would have it constantly in their firmament by day and by night, and always in the same position. To those who inhabit the central part of the hemisphere presented to us, the earth would appear stationary in the zenith, and would never leave it, never rising nor setting, nor in any degree changing its position in relation to the zenith or horizon. To those who inhabit places intermediate between the central part of that hemisphere and those places which are at the edge of the moon's disk, the earth would appear at a fixed and invariable distance from the zenith, and also at a fixed and invariable azimuth, the distance from the zenith being everywhere equal to the distance of the observer from the middle point of the hemisphere presented to the earth. To an observer at any of the places which are at the edge of the lunar disk, the earth would appear perpetually in a fixed direction on the horizon.

The earth shone upon by the sun would appear as the moon does to us; but with a disk having an apparent diameter greater than that of the moon in the ratio of 79 to 21, and an apparent superficial magnitude about fourteen times greater, and it would consequently have a proportionately illuminating power.

Earth light at the moon would, in fine, be about fourteen times more intense than *moonlight* at the earth. The earth would go through the same phases and complete the series of them in the same period as that which regulates the succession of the lunar phases, but the corresponding phases would be separated

by the interval of half a month. When the moon is *full* to the earth, the earth is *new* to the moon, and *vice versâ:* when the moon is a crescent, the earth is gibbous, and *vice versâ.*

The features of light and shade would not, as on the moon, be all permanent and invariable. So far as they would arise from the clouds floating in the terrestrial atmosphere, they would be variable. Nevertheless, their arrangement would have a certain relation to the equator, owing to the effect of the prevailing atmospheric currents parallel to the line.* This cause would produce streaks of light and shade, the general direction of which would be at right angles to the earth's axis, and the appearance of which would be in all respects similar to the BELTS which, as will appear hereafter, are observed upon some of the planets, and which are ascribed to a like physical cause.

Through the openings of the clouds the permanent geographical features of the surface of the earth would be apparent, and would probably exhibit a variety of tints according to the prevailing characters of the soil, as is observed to be the case with the planet Mars even at an immensely greater distance. The rotation of the earth upon its axis would be distinctly observed and its time ascertained. The continents and seas would be seen to disappear in succession at one side and to reappear at the other, and to pass across the disk of the earth as carried round by the diurnal rotation.

2488. *Why the full disk of the moon is faintly visible at new moon.* — Soon after conjunction, when the moon appears as a thin crescent, but is so removed from the sun as to be seen at a sufficient altitude after sunset, the entire lunar disk appears faintly illuminated within the horns of the crescent. This phenomenon is explained by the effect of the earth shining upon the moon and illuminating it by reflected light as the moon illuminates the earth, but with a degree of intensity greater in the ratio of about 14 to 1. According to what has just been explained, the earth appears to the moon nearly full at the time when the moon appears to the earth as a thin crescent, and it therefore receives then the strongest possible illumination. As the lunar crescent increases in breadth, the phase of the earth as seen from the moon becomes less and less full, and the intensity of the illu-

* See Chapter on the tides and trade winds.

mination is proportionately diminished. Hence we find, that as the lunar crescent passes gradually to the quarter, the complement of the lunar disk becomes gradually more faintly visible, and soon disappears altogether.

2489. *Physical condition of the moon's surface.*—If we examine the moon carefully, even without the aid of a telescope, we shall discover upon it distinct and definite lineaments of light and shadow. These features never change; there they remain, always in the same position upon the visible orb of the moon. Thus the features that occupy its centre now have occupied the same position throughout all human record. We have already stated that the first and most obvious inference which this fact suggests, is that the same hemisphere of the moon is always presented toward the earth, and consequently, the other hemisphere is never seen. This singular characteristic which attaches to the motion of the moon round the earth, seems to be a general characteristic of all other moons in the system. Sir William Herschel, by the aid of his powerful telescopes, observed indications which render it probable that the moons of Jupiter revolve in the same manner, each presenting continually the same hemisphere to the planet. The cause of this peculiar motion has been attempted to be explained by the hypothesis that the hemisphere of the satellite which is turned toward the planet, is very elongated and protuberant, and it is the excess of its weight which makes it tend to direct itself always toward the primary, in obedience to the universal principle of attraction. Be this as it may, the effect is, that our selenographical knowledge is necessarily limited to that hemisphere which is turned toward us.

But what is the condition and character of the surface of the moon? What are the lineaments of light and shade which we see upon it? There is no object outside the earth with which the telescope has afforded us such minute and satisfactory information.

If, when the moon is a crescent, we examine with a telescope, even of moderate power, the concave boundary, which is that part of the surface where the enlightened hemisphere ends and the dark hemisphere begins, we shall find that this boundary is not an even and regular curve, which it undoubtedly would be if the surface were smooth and regular, or nearly so. If, for

example, the lunar surface resembled in its general characteristics that of our globe, supposing that the entire surface is land, having the general characteristics of the continents of the earth, the inner boundary of the lunar crescent would still be a regular curve broken or interrupted only at particular points. Where great mountain ranges, like those of the Alps, the Andes, or the Himalaya, might chance to cross it, these lofty peaks would project vastly elongated shadows along the adjacent plain; for it will be remembered that, being situated, at the moment in question, at the boundary of the enlightened and darkened hemispheres, the shadows would be those of evening or morning; which are prodigiously longer than the objects themselves. The effect of these would be to cause gaps or irregularities in the general outline of the inner boundary of the crescent. With these rare exceptions, the inner boundary of the crescent produced by a globe like the earth would be an even and regular curve.

Such, however, is not the case with the inner boundary of the lunar crescent, even when viewed by the naked eye, and still less so when magnified by a telescope.

It is found, on the contrary, rugged and serrated, and brilliantly illuminated points are seen in the dark parts at some distance from it, while dark shadows of considerable length appear to break into the illuminated surface. The inequalities thus apparent indicate singular characteristics of the surface. The bright points seen within the dark hemisphere are the peaks of lofty mountains tinged with the sun's light. They are in the condition with which all travellers in Alpine countries are familiar; after the sun has set, and darkness has set in over the valleys at the foot of the chain, the sun still continues to illuminate the peaks above.

The sketch of the lunar crescent given in *fig.* 726. will illustrate these observations.

The visible hemisphere of our satellite has, within the last quarter of a century, been subjected to the most rigorous examination which unwearied industry, aided by the vast improvement which has been effected in the instruments of telescopic observation, rendered possible; and it is no exaggeration now to state that we possess a chart of that hemisphere which in accuracy of detail far exceeds any similar representation of the earth's surface.

Among the selenographical observers the Prussian astronomers, MM. Beer and Mädler, stand pre-eminent. Their descriptive work entitled Der Monde contains the most complete collection of observations on the physical condition of our satellite, and the chart, measuring 37 inches in diameter, exhibits the most complete representation of the lunar surface extant. Besides this great work, a selenographic chart was produced by Mr. Russell, from observations made with a seven-foot reflector, a similar delineation by Lohrmann, and, in fine, a very complete model in relief of the visible hemisphere by Madame Witte, an Hanoverian Lady.

Fig. 726.

To convey to the student any precise or complete idea of the mass of information collected by the researches and labours of these eminent observers, would be altogether incompatible with the necessary limits of a work like that which we have undertaken. We shall therefore confine ourselves to a selection from some of the most remarkable results of those works, aided by the telescopic chart of the south-eastern quadrant of the moon's disk, given in Plate I., which has been reduced from the great chart of Beer and Mädler, the scale being exactly one half of that of the original.

2490. General Description of the Moon's Surface.

(*a*) *Description of the chart, Plate I.* — The entire surface of the visible hemisphere of the moon is thickly covered with mountainous masses and ranges of various forms, magnitudes, and heights, in which, however, the prevalence of a circular or crater-like form is conspicuous. The mere inspection of the chart of the S. E. quadrant, Plate I., will render this evident; and the other three quadrants of the disk do not differ from this in their general character.*

* It must be observed that the chart represents the moon's disk as it is seen on the south meridian in an astronomical telescope. As that instrument produces an inverted image, the south pole appears at the highest and

(*b*) *Causes of the tints of white and gray on the moon's disk.*—The various tints of white and gray which mark the lineaments observed upon the disk of the full moon arise partly from the different reflecting powers of the matter composing different parts of the lunar surface, and partly from the different angles at which the rays of the solar light are incident upon them. If the surface of the lunar hemisphere were uniformly level, or nearly so, these angles of incidence would be determined by the position of each point with relation to the centre of the illuminated hemisphere; and, in that case, the tints would be more regular and would vary in relation principally to the centre of the disk; but, owing to the great inequalities of level, and the vast and complicated mountainous masses which project from every part of the surface, and the great depths of the cavities and plains which are surrounded by the circular mountain ranges, the angles of incidence of the solar rays are subject to extreme and irregular variation, which produce those lineaments and forms tinted with various shades of gray and white with which every eye is familiar.

(*c*) *Shadows visible only in the phases—they supply measures of heights and depths.*—When the moon is full no shadows upon it can be seen, because, in that position, the visual ray coinciding with the luminous ray, each object is directly interposed between the observer and its shadow. As the phases progress, however, the shadows gradually come into view, because the visual ray is inclined at a gradually increasing angle to the solar ray, and, in the quarters, this angle having increased to 90°, and the boundary of the enlightened hemisphere being then in the centre of the hemisphere presented to the observer, the position is most favourable for the observation of the shadows by which chiefly, not only the forms and dispositions of the mountainous masses and the intervening and enclosed valleys and ravines are ascertained, but their heights and depths are measured. This latter problem is solved by the well-understood principles of geometrical projection when the directions of the visual and solar rays, the position of the object, and of the surface on which the shadows are projected, are severally given.

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the north pole at the lowest point of the disk, and the eastern limit is on the right and the western on the left of the observer, all of which positions are the reverse of those which the same points have when viewed without a telescope, or with one which does not invert. The longitudes are measured east and west of the meridian which bisects the visible disk. The original chart is engraved in four separate sheets, each representing a quadrant of the visible hemisphere. The names of the various selenographical regions and more prominent mountains are indicated on the chart, and have been taken generally from those of eminent scientific men. The meridians drawn on the chart divide the surface into zones, each of which measures five degrees of longitude, and the parallels to the equator divide it into zones, having each the width of five degrees of latitude. The moon's diameter being less than that of the earth in the ratio of 100 to 182, a degree of lunar latitude is less than 60 geographical miles in the same proportion, and is, therefore, equal to 33 geographical miles. This supplies a scale by which the magnitudes on the chart, Plate I., may be approximately estimated.

(*d*) *Uniform patches, called oceans, seas, &c., proved to be irregular land surface.* — Uniform patches of greater or less extent, each having an uniform gray tint more or less dark, having been supposed, by early observers, to be large collections of water, were designated by the names, OCEANUS, MARE, PALUS, LACUS, SINUS, &c. These names are still retained, but the increased power of the telescope has proved that such regions are diversified, like the rest of the lunar surface, by inequalities and undulations of permanent forms, and are therefore not, as was imagined, water or other liquid. They differ from other regions only in the magnitude of the mountain masses which prevail upon them. About two-thirds of the visible hemisphere of the moon consists of this character of surface. Examples of these are presented by the Mare Nubium, Oceanus Procellarum, Mare Humorum, &c., on the chart.

(*e*) *Whiter spots, mountains.* — The more intensely white parts are mountains of various magnitude and form, whose height, relatively to the moon's magnitude, greatly exceeds that of the most stupendous terrestrial eminences; and there are many, characterised by an abruptness and steepness which sometimes assume the position of a vast vertical wall, altogether without example upon the earth. These are generally disposed in broad masses, lying in close contiguity, and intersected with vast and deep valleys, gullies, and abysses, none of which, however, have any of the characters which betray the agency of water.

(*f*) *Classes of circular mountain ranges.*—Circular ranges of mountains which, were it not for their vast magnitude, might be inferred from their form to have been volcanic craters, are by far the most prevalent arrangement. These have been denominated, according to their magnitudes, BULWARK PLAINS, RING MOUNTAINS, CRATERS, and HOLES.

(*g*) *Bulwark plains.*—These are circular areas, varying from 40 to 120 miles in diameter, enclosed by a ring of mountain ridges, mostly continuous, but in some cases intersected at one or more points by vast ravines. The enclosed area is generally a plain on which mountains of less height are often scattered. The surrounding circular ridge also throws out spurs, both externally and internally, but the latter are generally shorter than the former. In some cases, however, internal spurs, which are diametrically opposed, unite in the middle so as to cut in two the enclosed plain. In some rare cases the enclosed plain is uninterrupted by mountains, and it is almost invariably depressed below the general level of the surrounding land. A few instances are presented of the enclosed plain being convex.

The mountainous circle enclosing these vast areas is seldom a single ridge. It consists more generally of several concentric ridges, one of which, however, always dominates over the rest and exhibits an unequal summit, broken by stupendous peaks, which here and there shoot up from it to vast heights. Occasionally it is also interrupted by smaller mountains of the circular form.

Examples of bulwark plains are presented in the cases of Clavius, Walther, Regiomontanus, Purbach, Alphonse, and Ptolemæus.

The diameter of Clavius is 124 miles*, and the enclosed area is 12,000

* The geographical mile, or the sixtieth part of a degree of the earth's meridian.

square miles. One of the peaks of the surrounding ridge shoots up to the height of 16,000 feet.

The diameter of Ptolemæus is 100 miles, and it encloses an area of 6,400 square miles. This area is intersected by numerous small ridges, not above a mile in breadth and 100 feet in height. Ptolemæus is surrounded by very high mountains, and is remarkable for the precipitous character of its inner sides.

The other bulwark plains above named have nearly the same character, but less dimensions.

(*h*) *Ring mountains.*—These circular formations are on a smaller scale than the bulwark plains, varying from 10 to 50 miles in diameter, and they are generally more regular and more exactly circular in their form. They are sometimes found upon the ridge which encloses a bulwark plain, thus interrupting the continuity of its boundary, and sometimes they are seen within the enclosed area. Sometimes they stand in the midst of the *maria*. Their inner declivity is always steep, and the enclosed area, which is always concave, often includes a central mountain, presenting thus the general character of a volcanic crater, but on a scale of magnitude without example in terrestrial volcanoes. The surface enclosed is always lower than the region surrounding the enclosing ridge, and the central mountain often rises to such a height that, if it were levelled, it would fill the depression.

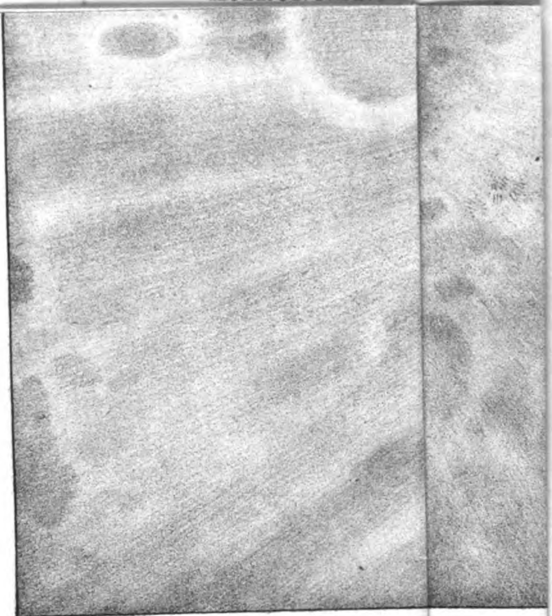
(*i*) *Tycho, a ring mountain.*—The most remarkable example of this class is Tycho (see chart, lat. 42°, long. 12°). This object is distinguishable without a telescope on the lunar disk when full; but, owing to the multitude of other features which become apparent around it in the phases, it can then be only distinguished by a perfect knowledge of its position, and with a good telescope. The enclosed area, which is very nearly circular, is 47 miles in diameter, and the inside of the enclosing ridge has the steepness of a wall. Its height above the level of the enclosed plain is 16,000 feet, and above that of the external regions 12,000 feet. There is a central mount, height 4,700 feet, besides a few lesser hills within the enclosure.

(*k*) *Craters and holes.*—These are the smallest formations of the circular class. Craters enclose a visible area, containing generally a central mound or peak, exhibiting in a striking manner the volcanic character. Holes include no visible area, but may possibly be craters on a scale too small to be distinguished by the telescope.

Formations of this class are innumerable on every part of the visible surface of the moon, but are no where more prevalent than in the region around Tycho, which may be seen on a very enlarged scale in Plate II., which represents that ring mountain, and the adjacent region extending over sixteen degrees of latitude, and from sixteen to twenty degrees of longitude.

(*l*) *Other mountain formations.*—Besides the preceding, which are the most remarkable, the most characteristic, and the most prevalent, there are various other forms of mountain, classified by Beer and Mädler, but which our limits compel us to omit.

(*m*) *Singular and unexplained optical phenomenon of radiating streaks.*—Among the most remarkable phenomena presented to lunar observers, is the systems of streaks of light and shade, which radiate from the borders of some



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of the largest of the ring mountains, spreading to distances of several hundred miles around them. Seven of the mountains of this class, viz., Tycho, Copernicus, Kepler, Pyrgius, Anaxagoras, Aristarchus, and Olbers are severally the centres round which this extraordinary radiation is manifested. Similar phenomena, less conspicuously developed, however, are visible around Meyer, Euler, Proclus, Aristillus, Timocharis, and some others.

These phenomena, as displayed when the moon is full around Tycho, are represented in Plate III. on the same scale as Plate II.

These radiating streaks commence at a distance of about 20 miles outside the circular ridge of Tycho. From that limit they diverge and overspread fully a fourth part of the visible hemisphere. On the S. they extend to the edge of the disk; on the E. to Hainzel and Capuanus; on the S.E. to the Mare Nubium; on the N. to Alphonse; on the N.W. to the Mare Nectaris, and to the W., so as to cover nearly the entire south-western quadrant.

They are only visible when the sun's rays fall upon the region of Tycho at an incidence greater than 25°, and the more perpendicularly the rays fall upon it, the more fully developed the phenomena will be. They are, therefore, only seen in their splendour, as represented in Plate III., when the moon is full. As the moon moves from opposition to the last quarter, the streaks therefore gradually disappear, and the shadows of the mountain formations are at the same time gradually brought into view, so that the aspect of the moon undergoes a complete transformation. This change may be very well exhibited by holding the Plate III. before a window to which the back of the observer is turned. He will then see the phenomena as they are presented on the full moon. Let him then turn slowly upon his heel until his face is presented to the window, holding the paper between his eyes and the light. The Plate II. will then be seen by means of the transparency of the paper, and it will gradually become more and more distinctly apparent as he turns more directly towards the light.*

Although the mountain formations generally disappear under the splendour of these radiating streaks, some few, as will be perceived on Plate III., continue to be visible through them.

None of the numerous selenographic observers have proposed any satisfactory explanation of these phenomena, which are exhibited nearly in the same manner around the other ring mountains above named. Schröter supposed them to be mountains, an hypothesis overturned by the observations since made with more powerful instruments. Herschel, the elder, suggested the idea of streams of lava; Cassini imagined they might be clouds; and others even suggested the possibility of their being roads! Mädler imagines that these ring mountains may have been among the first selenological formations; and, consequently, the points to which all the

* This ingenious expedient is suggested by Mädler. It must be remembered, however, that, while Plate II. represents the region as it appears in a telescope which inverts, Plate III. represents it as if it were reflected in a mirror, or as it would be seen with a telescope having a prismatic eyepiece.

gases evolved in the formation of our satellite would have been attracted. These emanations produced effects, such as vitrification or oxydation, which modified the reflective powers of the surface. We must, however, dismiss these conjectures, however ingenious and attractive, referring those who desire to pursue the subject to the original work.

(*n*) *Environs of Tycho.*—This region is crowded with hundreds of peaks, crests, and craters (see Plate II.); not the least vestige of a plain can anywhere be discovered. Towards the E. and S.E. craters predominate, while to the W. chains parallel to the ring are more numerous. On the S. the mountains are thickly scattered in confused masses. At a distance of 15 to 25 miles craters and small ring mountains are seen, few being circular, but all approaching to that form. All are surrounded by steep ramparts.

(*o*) *Wilhelm I.*—This is a considerable ring mountain S.E. of Tycho. The altitude of its eastern parapet is 10,000 feet, that of its western being only 6,000. Its crest is studded with peaks; and craters of various magnitudes, heights, and depths, surrounding it in great numbers, and giving a varied appearance to the adjacent region.

(*p*) *Longomontanus.*—A large circular range, having a diameter of 80 miles, enclosing a plain of great depth. The eastern and western ridges rise to the height of 12,000 to 13,000 feet above the level of the enclosed plain. Its shadow sometimes falls upon and conceals the numerous craters and promontories which lie near it. The whole surrounding region is savage and rugged in the highest degree, and must, according to Mädler, have resulted from a long succession of convulsions. The principal, and apparently original, crater has given way in course of time to a series of new and less violent eruptions. All these smaller formations are visible on the full moon, but not the principal range, which then disappears, though its place may still be ascertained by its known position in relation to Tycho.

(*q*) *Maginus.*—This range N.W. of Tycho (see Plate I.) has the appearance of a vast and wild ruin. The wide plain enclosed by it lies in deep shade even when the sun has risen to the meridian. Its general height is 13,000 feet. A broad elevated base connects the numberless peaks, terraces, and groups of hills constituting this range, and small craters are numerous among these wild and confused masses. The central peak A is a low but well-defined hill, close to which is a crater-like depression, and other less considerable hills.

(*r*) *Analogy to terrestrial volcanoes more apparent than real—enlarged view of Gassendi.*—The volcanic character observed in the selenographic formations loses much of its analogy to like formations on the earth's surface when higher magnifying powers enable us to examine the details of what appear to be craters, and to compare their dimensions with even the most extensive terrestrial craters. Numerous examples may be produced to illustrate this. We have seen that Tycho, which, viewed under a moderate magnifying power, appears to possess in so eminent a degree the volcanic character, is, in fact, a circular chain enclosing an area upwards of fifty miles in diameter. Gassendi, another system of like form, and of still more stupendous dimensions, is delineated in *fig.* 1. Plate IV., as seen with high magnifying powers. This remarkable object consists of two enormous cir-

Plan of the Lunar Mountain Gassendi by Mädler.

From observations with the Dorpat Telescope.

cular chains of mountains, the lesser, which lies to the north, measuring 16½ miles in diameter, and the greater, lying to the south, enclosing an area 60 miles in diameter. The area enclosed by the former is therefore 214, and by the latter 2,827 square miles. The height of the lesser chain is about 10,000 feet, while that of the greater varies from 3,500 to 5,000 feet. The vast area thus enclosed by the greater chain includes, at or near its centre, a principal central mountain, having eight peaks and an height of 2,000 feet, while scattered over the surrounding enclosure upwards of a hundred mountains of less considerable elevation have been counted.

It is easy to see how little analogy to a terrestrial volcanic crater is presented by these characters.

The preceding selections, combined with the charts, Plates I., II., III., and IV., will serve to show the general physical character of the lunar surface, and the elaborate accuracy with which it has been submitted to telescopic examination. In the work of Beer and Mädler a table of the heights of above 1000 mountains is given, several of which attain to an elevation of 23,000 feet, equal to that of the highest summits of terrestrial mountains, while the diameter of the moon is little more than half that of the earth.

2491. *Observations of Herschel.*— Sir John Herschel says, that among the lunar mountains may be observed in its highest perfection the true volcanic character, as seen in the crater of Vesuvius and elsewhere ; but with the remarkable peculiarity that the bottoms of many of the craters are very deeply depressed below the general surface of the moon, the internal depth being in many cases two or three times the external height. In some cases, he thinks, decisive marks of volcanic stratification, arising from a succession of deposits of ejected matter, and evident indications of currents of lava streaming outwards in all directions, may be clearly traced with powerful telescopes.

2492. *Observations of the Earl of Rosse.*— By means of the great reflecting telescope of Lord Rosse, the flat bottom of the crater called Albategnius is distinctly seen to be strewed with blocks, not visible with less powerful instruments; while the exterior of another (Aristillus) is intersected with deep gullies radiating from its centre.

2493. *Supposed influence of the moon on the weather.*— Among the many influences which the moon is supposed, by the world in general, to exercise upon our globe, one of those, which has been most universally believed, in all ages and in all

countries, is that which it is presumed to exert upon the changes of the weather. Although the particular details of this influence are sometimes pretended to be described, the only general principle, or rule, which prevails with the world in general is, that a change of weather may be looked for at the epochs of new and full moon: that is to say, if the weather be previously fair it will become foul, and if foul will become fair. Similar changes are also, sometimes, though not so confidently, looked for at the epochs of the quarters.

A question of this kind may be regarded either as a question of science, or a question of fact.

If it be regarded as a question of science, we are called upon to explain how and by what property of matter, or what law of nature or attraction, the moon, at a distance of a quarter of a million of miles, combining its effects with the sun, at four hundred times that distance, can produce those alleged changes. To this it may be readily answered that no known law or principle has hitherto explained any such phenomena. The moon and sun must, doubtless, affect the ocean of air which surrounds the globe, as they affect the ocean of water — producing effects analogous to tides; but when the quantity of such an effect is estimated, it is proved to be such as could by no means account for the meteorological changes here adverted to.

But in conducting investigations of this kind we proceed altogether in the wrong direction, and begin at the wrong end, when we commence with the investigation of the physical cause of the supposed phenomena. Our first business is carefully and accurately to observe the phenomena of the changes of the weather, and then to put them in juxtaposition with the contemporaneous changes of the lunar phases. If there be any discoverable correspondence, it then becomes a question of physics to assign its cause.

Such a course of observation has been made in various observatories with all the rigour and exactitude necessary in such an inquiry, and has been continued over periods of time so extended, as to efface all conceivable effects of accidental irregularities.

We can imagine, placed in two parallel columns, in juxtaposition, the series of epochs of the new and full moons, and the quarters, and the corresponding conditions of the weather

at these times, for fifty or one hundred years back, so that we may be enabled to examine, as a mere matter of fact, the conditions of the weather for one thousand or twelve hundred full and new moons and quarters.

From such a mode of observation and inquiry, it has resulted conclusively that the popular notions concerning the influence of the lunar phases on the weather have no foundation in theory, and no correspondence with observed facts. That the moon, by her gravitation, exerts an attraction on our atmosphere cannot be doubted; but the effects which that attraction would produce upon the weather are not in accordance with observed phenomena; and, therefore, these effects are either too small in amount to be appreciable in the actual state of meteorological instruments, or they are obliterated by other more powerful causes, from which hitherto they have not been eliminated. It appears, however, by some series of observations, not yet confirmed or continued through a sufficient period of time, that a slight correspondence may be discovered between the periods of rain and the phases of the moon, indicating a very feeble influence, depending on the relative position of that luminary to the sun, but having no discoverable relation to the lunar attraction. This is not without interest as a subject of scientific inquiry, and is entitled to the attention of meteorologists; but its influence is so feeble that it is altogether destitute of popular interest as a weather prognostic. It may, therefore, be stated that, as far as observation combined with theory has afforded any means of knowledge, there are no grounds for the prognostications of weather erroneously supposed to be derived from the influence of the sun and moon.

Those who are impressed with the feeling that an opinion so universally entertained even in countries remote from each other, as that which presumes an influence of the moon over the changes of the weather, will do well to remember that against that opinion we have not here opposed mere theory. Nay, we have abandoned for the occasion the support that science might afford, and the light it might shed on the negative of this question, and have dealt with it as a mere question of fact. It matters little, so far as this question is concerned, in what manner the moon and sun may produce an effect on the weather, nor even whether they be active causes in producing such effect at all.

The point, and the only point of importance, is, whether, regarded as a mere *matter of fact*, any correspondence between the changes of the moon and those of the weather exists? And a short examination of the recorded facts proves that IT DOES NOT.

2494. *Other supposed lunar influences.*—But meteorological phenomena are not the only effects imputed to our satellite; that body, like comets, is made responsible for a vast variety of interferences with organised nature. The circulation of the juices of vegetables, the qualities of grain, the fate of the vintage, are all laid to its account; and timber must be felled, the harvest cut down and gathered in, and the juice of the grape expressed, at times and under circumstances regulated by the aspects of the moon, if excellence be hoped for in these products of the soil.

According to popular belief, our satellite also presides over human maladies; and the phenomena of the sick chamber are governed by the lunar phases; nay, the very marrow of our bones, and the weight of our bodies, suffer increase or diminution by its influence. Nor is its imputed power confined to physical or organic effects; it notoriously governs mental derangement.

If these opinions respecting lunar influences were limited to particular countries, they would be less entitled to serious consideration; but it is a curious fact that many of them prevail and have prevailed in quarters of the earth so distant and unconnected, that it is difficult to imagine the same error to have proceeded from the same source.

Our limits, and the objects to which this volume is directed, render it impossible here to notice more than a few of the principal physical and physiological influences imputed to the moon; nor even with respect to these can we do more than indicate the kind of examination to which they have been submitted, and the conclusions which have been deduced from it.

2495. *The red moon.*—Gardeners give the name of *Red Moon* to that moon which is full between the middle of April and the close of May. According to them the light of the moon at that season exercises an injurious influence upon the young shoots of plants. They say that when the sky is clear the leaves and buds exposed to the lunar light redden and are killed as if by frost, at a time when the thermometer exposed to the atmosphere stands at many degrees above the freezing point. They

say also that if a clouded sky intercepts the moon's light it prevents these injurious consequences to the plants, although the circumstances of temperature are the same in both cases. Nothing is more easy than the explanation of these effects. The leaves and flowers of plants are strong and powerful radiators of heat; when the sky is clear they therefore lose temperature and may be frozen; if, on the other hand, the sky be clouded, their temperature is maintained for the reasons above stated.

The moon, therefore, has no connection whatever with this effect, and it is certain that plants would suffer under the same circumstances whether the moon is above or below the horizon. It equally is quite true that if the moon be above the horizon, the plants cannot suffer unless it be visible; because a *clear sky* is indispensable as much to the production of the injury to the plants as to the visibility of the moon; and, on the other hand, the same clouds which veil the moon and intercept her light give back to the plants that warmth which prevents the injury here adverted to. The popular opinion is therefore right as to the *effect*, but wrong as to the *cause;* and its error will be at once discovered by showing that on a clear night, when the moon is new, and, therefore, not visible, the plants may nevertheless suffer.

2496. *Supposed influence on timber.*—An opinion is generally entertained that timber should be felled only during the decline of the moon; for if it be cut down during its increase, it will not be of a good and durable quality. This impression prevails in various countries. Is is acted upon in England, and is made the ground of legislation in France. The forest laws of the latter country interdict the cutting of timber during the increase of the moon. The same opinion prevails in Brazil. Signor Francisco Pinto, an eminent agriculturist in the province of Espirito Santo, affirmed, as the result of his experience, that the wood which was not felled at the full of the moon was immediately attacked by worms and very soon rotted. In the extensive forests of Germany, the same opinion is entertained and acted upon. M. Duhamel Monceau, a celebrated French agriculturist, has made direct and positive experiments for the purpose of testing this question; and has clearly and conclusively shown that the qualities of timber felled in different parts of the lunar month are the same.

2497. *Supposed lunar influence on vegetables.*—It is an

aphorism received by all gardeners and agriculturists in Europe, that vegetables, plants, and trees, which are expected to flourish and grow with vigour, should be planted, grafted, and pruned, during the increase of the moon. This opinion is altogether erroneous. The increase or decrease of the moon has no appreciable influence on the phenomena of vegetation; and the experiments and observations of several French agriculturists, and especially of M. Duhamel du Monceau (already alluded to) have clearly established this.

2498. *Supposed lunar influence on wine-making.* — It is a maxim of wine-growers, that wine which has been made in two moons is never of a good quality, and cannot be clear.

To this we need only answer, that the moon's rays do not affect the temperature of the air to the extent of one thousandth part of a degree of the thermometer, and that the difference of temperatures of any two neighbouring places in which the process of making the wine of the same soil and vintage might be conducted, may be a thousand times greater at any given moment of time, and yet no one ever imagines that such a circumstance can affect the quality of the wine.

2499. *Supposed lunar influence on the complexion.* — It is a prevalent popular notion in some parts of Europe, that the moon's light is attended with the effect of darkening the complexion.

That light has an effect upon the colour of material substances is a fact well known in physics and in the arts. The process of bleaching by exposure to the sun is an obvious example of this class of facts. Vegetables and flowers which grow in a situation excluded from the light of the sun are different in colour from those which have been exposed to its influence. The most striking instance, however, of the effect of certain rays of solar light in blackening a light coloured substance, is afforded by chloride of silver, which is a white substance, but which immediately becomes black when acted upon by the rays near the red extremity of the spectrum. This substance, however, highly susceptible as it is of having its colour affected by light, is, nevertheless, found not to be changed in any sensible degree when exposed to the light of the moon, even when that light is condensed by the most powerful burning lenses. It would seem, therefore, that as far as any analogy can be derived from the qualities of this substance, the popular impression of

the influence of the moon's rays in blackening the skin receives no support.

2500. *Supposed lunar influence on putrefaction.* — Pliny and Plutarch have transmitted it as a maxim, that the light of the moon facilitates the putrefaction of animal substances, and covers them with moisture. The same opinion prevails in the West Indies, and in South America. An impression is prevalent, also, that certain kinds of fruit exposed to moonlight lose their flavour and become soft and flabby; and that if a wounded mule be exposed to the light of the moon during the night, the wound will be irritated, and frequently become incurable.

Such effects, if real, may be explained upon the same principles as those by which we have already explained the effects imputed to the red moon. Animal substances exposed to a clear sky at night, are liable to receive a deposition of dew, which humidity has a tendency to accelerate putrefaction. But this effect will be produced if the sky be clear, whether the moon be above the horizon or not. The moon, therefore, in this case, is a witness and not an agent; and we must acquit her of the misdeeds imputed to her.

Supposed lunar influence on shell-fish. — It is a very ancient remark, that oysters and other shell-fish become larger during the increase than during the decline of the moon. This maxim is mentioned by the poet Lucilius, by Aulus Gellius, and others; and the members of the academy *del Cimento* appear to have tacitly admitted it, since they endeavour to give an explanation of it. The fact, however, has been carefully examined by Rohault, who has compared shell-fish taken at all periods of the lunar month, and found that they exhibit no difference of quality.

2501. *Supposed lunar influence on the marrow of animals.*— An opinion is prevalent among butchers that the marrow found in the bones of animals varies in quantity according to the phase of the moon in which they are slaughtered. This question has also been examined by Rohault, who made a series of observations which were continued for twenty years with a view to test it; and the result was that it was proved completely destitute of foundation.

2502. *Supposed lunar influence on the weight of the human body.* — Sanctorius, whose name is celebrated in physics for the invention of the thermometer, held it as a principle that a

healthy man gained two pounds weight at the beginning of every lunar month, which he lost toward its completion. This opinion appears to be founded on experiments made upon himself; and affords another instance of a fortuitous coincidence hastily generalised. The error would have been corrected if he had continued his observations a sufficient length of time.

2503. *Supposed lunar influence on births.* — It is a prevalent opinion that births occur more frequently in the decline of the moon than in her increase. This opinion has been tested by comparing the number of births with the periods of the lunar phases; but the attention directed to statistics, as well in this country as abroad, will soon lead to the decision of this question.*

2504. *Supposed lunar influence on incubation.* — It is a maxim handed down by Pliny, that eggs should be put to cover when the moon is new. In France it is a maxim generally adopted, that the fowls are better and more successfully reared when they break the shell at the full of the moon. The experiments and observations of M. Girou de Buzareingues have given countenance to this opinion. But such observations require to be multiplied before the maxim can be considered as established. M. Girou inclines to the opinion that during the dark nights about new moon the hens sit so undisturbed that they either kill their young or check their development by too much heat; while in moonlight nights, being more restless, this effect is not produced.

2505. *Supposed lunar influence on mental derangement and other human maladies.* — The influence on the phenomena of human maladies imputed to the moon is very ancient. Hippocrates had so strong a faith in the influence of celestial objects upon animated beings, that he expressly recommends no physician to be trusted who is ignorant of astronomy. Galen, following Hippocrates, maintained the same opinion, especially of the influence of the moon. Hence in diseases the lunar periods were said to correspond with the succession of the sufferings of the patients. The critical days or *crises* (as they were afterward called), were the seventh, fourteenth, and twenty-first of the disease, corresponding to the intervals between the moon's prin-

* Other sexual phenomena, such as the period of gestation, vulgarly supposed to have some relation to the lunar month, have no relation whatever to that period.

cipal phases. While the doctrine of alchymists prevailed, the human body was considered as a microcosm; the heart representing the sun, the brain the moon. The planets had each its proper influence: Jupiter presided over the lungs, Mars over the liver, Saturn over the spleen, Venus over the kidneys, and Mercury over the organs of generation. Of these grotesque notions there is now no relic, except the term *lunacy*, which still designates unsoundness of mind. But even this term may in some degree be said to be banished from the terminology of medicine, and it has taken refuge in that receptacle of all antiquated absurdities of phraseology — the law. Lunatic, we believe, is still the term for the subject who is incapable of managing his own affairs.

Although the ancient faith in the connection between the phases of the moon and the phenomena of insanity appears in a great degree to be abandoned, yet it is not altogether without its votaries; nor have we been able to ascertain that any series of observations conducted on scientific principles, has ever been made on the phenomena of insanity, with a view to disprove this connection. We have even met with intelligent and well-educated physicians who still maintain that the paroxysms of insane patients are more violent when the moon is full than at other times.

2506. *Examples produced by Faber and Ramazzini.* — Mathiolus Faber gives an instance of a maniac who, at the very moment of an eclipse of the moon, became furious, seized upon a sword, and fell upon every one around him. Ramazzini relates that, in the epidemic fever which spread over Italy in the year 1693, patients died in an unusual number on the 21st of January, at the moment of a lunar eclipse.

Without disputing this fact (to ascertain which, however, it would be necessary to have statistical returns of the daily deaths), it may be objected that the patients who thus died in such numbers at the moment of the eclipse, might have had their imaginations highly excited, and their fears wrought upon by the approach of that event, if popular opinion invested it with danger. That such an impression was not unlikely to prevail is evident from the facts which have been recorded.

At no very distant period from that time, in August, 1654, it is related that patients in considerable numbers were, by order of the physicians, shut up in chambers well closed,

warmed, and perfumed, with a view to escape the injurious influence of the solar eclipse, which happened at that time; and such was the consternation of persons of all classes, that the numbers who flocked to confession were so great that the ecclesiastics found it impossible to administer that rite. An amusing anecdote is related of a village curate near Paris, who, with a view to ease the minds of his flock, and to gain the necessary time to get through his business, seriously assured them that the eclipse was postponed for a fortnight.

2507. *Examples of Vallisnieri and Bacon.*—Two of the most remarkable examples recorded, of the supposed influence of the moon on the human body, are those of Vallisnieri and Bacon. Vallisnieri declares that, being at Padua, recovering from a tedious illness, he suffered, on the 12th of May, 1706, during the eclipse of the sun, unusual weakness and shivering. Lunar eclipses never happened without making Bacon faint; and he did not recover his senses till the moon recovered her light.

That these two striking examples should be admitted in proof of the existence of lunar influence, it would be necessary, says M. Arago, to establish the fact, that feebleness and pusillanimity of character are never connected with high qualities of mind.

2508. *Supposed influence on cutaneous affections.*—Menuret considered that cutaneous maladies had a manifest connection with the lunar phases. He says that he himself observed, in the year 1760, a patient afflicted with a scald head (*teigne*), who, during the decline of the moon, suffered from a gradual increase of the malady, which continued until the epoch of the new moon, when it had covered the face and breast, and produced insufferable itching. As the moon increased, these symptoms disappeared by degrees; the face became free from the eruption; but the same effects were reproduced after the full of the moon. These periods of the disease continued for three months.

Menuret also stated that he witnessed a similar correspondence between the lunar phases and the distemper of the itch; but the circumstances were the reverse of those in the former case; the malady attaining its maximum at the full of the moon, and its minimum at the new moon.

Without disputing the accuracy of these statements, or throwing any suspicion on the good faith of the physician who

has made them, we may observe that such facts prove nothing except the fortuitous coincidence. If the relation of cause and effect had existed between the lunar phases and the phenomena of these distempers the same cause would have continued to produce the same effect in like circumstances; and we should not be left to depend for the proof of lunar influence on the statements of isolated cases, occurring under the observation of a physician who was himself a believer.

2509. *Remarkable case adduced by Hoffman.* — Maurice Hoffman relates a case which came under his own practice, of a young woman, the daughter of an epileptic patient. The abdomen of this girl became inflated every month as the moon increased, and regularly resumed its natural form with the decline of the moon.

Now if this statement of Hoffman were accompanied by all the necessary details, and if, also, we were assured that this strange effect continued to be produced for any considerable length of time, the relation of cause and effect between the phases of the moon and the malady of the girl could not legitimately be denied; but receiving the statement in so vague a form, and not being assured that the effect continued to be produced beyond a few months, the legitimate conclusion at which we must arrive is, that this is another example of fortuitous coincidence, and may be classed with the fulfilment of dreams, prodigies, &c., &c.

2510. *Cases of nervous diseases.* — As may naturally be expected, nervous diseases are those which have presented the most frequent indications of a relation with the lunar phases. The celebrated Mead was a strong believer, not only in the lunar influence, but in the influence of all the heavenly bodies on all the human. He cites the case of a child who always went into convulsions at the moment of full moon. Pyson, another believer, cites another case of a paralytic patient whose disease was brought on by the new moon. Menuret records the case of an epileptic patient whose fits returned with the full moon. The transactions of learned societies abound with examples of giddiness, malignant fever, somnambulism, &c., having in their paroxysms more or less corresponded with the lunar phases. Gall states, as a matter having fallen under his own observation, that patients suffering under weakness of intellect had two periods in the month of peculiar excitement; and, in a work

published in London so recently as 1829, we are assured that these epochs are between the new and full moon.

2511. *Observations of Dr. Olbers on insane patients.*—Against all these instances of the supposed effect of lunar influence, we have little direct proof to offer. To establish a negative is not easy. Yet it were to be wished that in some of our great asylums for insane patients, a register should be preserved of the exact times of the access of all the remarkable paroxysms; a subsequent comparison of this with the age of the moon at the time of their occurrence would furnish the ground for legitimate and safe conclusions. We are not aware of any scientific physician who has expressly directed his attention to this question, except Dr. Olbers of Bremen, celebrated for his discovery of the planets Pallas and Vesta. He states that, in the course of a long medical practice, he was never able to discover the slightest trace of any connection between the phenomena of disease and the phases of the moon.

2512. *Influence not to be hastily rejected.* — In the spirit of true philosophy, M. Arago, nevertheless, recommends caution in deciding against this influence. The nervous system, says he, is in many instances an instrument infinitely more delicate than the most subtle apparatus of modern physics. Who does not know that the olfactory nerves inform us of the presence of odoriferous matter in air, the traces of which the most refined physical analysis would fail to detect? The mechanism of the eye is highly affected by that lunar light which, even condensed with all the power of the largest burning lenses, fails to affect by its heat the most susceptible thermometers, or by its chemical influence, the chloride of silver; yet a small portion of this light introduced through a pin-hole will be sufficient to produce an instantaneous contraction of the pupil; nevertheless the integuments of this membrane, so sensible to light, appear to be completely inert when otherwise affected. The pupil remains unmoved, whether we scrape it with the point of a needle, moisten it with liquid acids, or impart to its surface electric sparks. The retina itself, which sympathises with the pupil, is insensible to the influence of the most active mechanical agents. Phenomena so mysterious should teach us with what reserve we should reason on analogies drawn from experiments made upon inanimate substances, to the far different and more difficult case of organised matter endowed with life.

CHAP. X.

THE TIDES AND TRADE WINDS.

2513. *Correspondence between the recurrence of the tides and the diurnal appearance of the moon.*—The phenomena of the tides of the ocean are too remarkable not to have attracted notice at an early period in the progress of knowledge. The intervals between the epochs of high and low water everywhere corresponding with the intervals between the passage of the moon over the meridian above and below the horizon, suggested naturally the physical connection between these two effects, and indicated the probability of the cause of the tides being found in the motion of the moon.

2514. *Erroneous notions of the lunar influence.*—There are few subjects in physical science about which more erroneous notions prevail among those who are but a little informed. A common idea is, that the attraction of the moon draws the waters of the earth toward that side of the globe on which it happens to be placed, and that consequently they are heaped up on that side, so that the oceans and seas acquire there a greater depth than elsewhere; and that high water will thus take place under, or nearly under, the moon. But this does not correspond with the fact. High water is not produced merely under the moon, but is equally produced upon those parts most removed from the moon. Suppose a meridian of the earth so selected, that if it were continued beyond the earth, its plane would pass through the moon; we find that, subject to certain modifications, a great tidal wave, or what is called *high water*, will be formed on both sides of this meridian; that is to say, on the side next the moon, and on the side remote from the moon. As the moon moves, these two great tidal waves follow her. They are of course separated from each other by half the circumference of the globe. As the globe revolves with its diurnal motion upon its axis, every part of its surface passes successively under these tidal waves; and at all such parts, as they pass under them, there is the phenomenon of high water.

Hence it is that in all places there are two tides daily, having an interval of about twelve hours between them. Now, if the common notion of the cause of the tides were well founded, there would be only one tide daily—viz., that which would take place when the moon is at or near the meridian.

2515. *The moon's attraction alone will not explain the tides.* —That the moon's attraction upon the earth simply considered would not explain the tides is easily shown. Let us suppose that the whole mass of matter on the earth, including the waters which partially cover it, were attracted equally by the moon; they would then be equally drawn toward that body, and no reason would exist why they should be heaped up under the moon; for if they were drawn with the same force as that with which the solid globe of the earth under them is drawn, there would be no reason for supposing that the waters would have a greater tendency to collect toward the moon than the solid bottom of the ocean on which they rest. In short, the whole mass of the earth, solid and fluid, being drawn with the same force, would equally tend toward the moon; and its parts, whether solid or fluid, would preserve among themselves the same relative position as if they were not attracted at all.

2516. *Tides caused by the difference of the attractions on different parts of the earth.*—When we observe, however, in a mass composed of various particles of matter, that the relative arrangement of these particles is disturbed, some being driven in certain directions more than others, the inference is, that the component parts of such a mass must be placed under the operation of different forces; those which tend more than others in a certain direction being driven with a proportionally greater force. Such is the case with the earth, placed under the attraction of the moon. And this is, in fact, what must happen under the operation of an attractive force like that of gravitation, which diminishes in its intensity as the square of the distance increases.

Let A, B, C, D, E, F, G, H, *fig.* 727., represent the globe of the earth, and, to simplify the explanation, let us first suppose the entire surface of the globe to be covered with water. Let M, the moon, be placed at the distance MH from the nearest point of the surface of the earth. Now it will be apparent that the various points of the earth's surface are at different distances from the moon M. A and G are more remote than H;

BF still more remote; C and E more distant again, and D more remote than all. The attraction which the moon exercises at

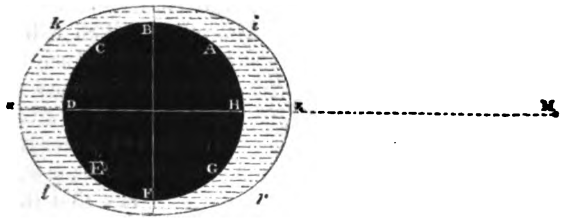


Fig. 727.

H is, therefore, greater than that which it exercises at A and G, and still greater than that which it produces at B and F; and the attraction which it exercises at D is least of all. Now this attraction equally affects matter in every state and condition. It affects the particles of fluid as well as solid matter; but there is this difference, that where it acts upon solid matter, the component parts of which are at different distances from it, and therefore subject to different attractions, it will not disturb the relative arrangement, since such disturbances or disarrangements are prevented by the cohesion which characterises a solid body; but this is not the case with fluids, the particles of which are mobile.

The attraction which the moon exercises upon the shell of water, which is collected immediately under it near the point Z, is greater than that which it exercises upon the solid mass of the globe; consequently there will be a greater tendency of this attraction to draw the fluid which rests upon the surface at H toward the moon, than to draw the solid mass of the earth which is more distant.

As the fluid, by its nature, is free to obey this excess of attraction, it will necessarily heap itself up in a pile or wave over H, forming a convex protuberance, as represented between *r* and *i*. Thus high water will take place at H, immediately under the moon. The water which thus collects at H will necessarily flow from the regions B and F, where therefore there will be a diminished quantity in the same proportion.

But let us now consider what happens to that part of the earth D. Here the waters, being more remote from the moon than the solid mass of the earth under them, will be less at-

tracted, and consequently will have a less tendency to gravitate toward the moon. The solid mass of the earth, DH, will, as it were, recede from the waters at *n*, in virtue of the excess of attraction, leaving these waters behind it, which will thus be heaped up at *n*, so as to form a convex protuberance between *l* and *k*, similar exactly to that which we have already described between *r* and *i*. As the difference between the attraction of the moon on the waters at *z* and the solid earth under the waters is nearly the same as the difference between its attraction on the latter and upon the waters at *n*, it follows that the height of the fluid protuberances at *z* and *n* are equal. In other words, the height of the tides on opposite sides of the earth, the one being under the moon and the other most remote from it, are equal.

It appears, therefore, that the cause of the tides, so far as the action of the moon is concerned, is not, as is vulgarly supposed, the mere attraction of the moon; since, if that attraction were equal on all the component parts of the earth, there would assuredly be no tides. We are to look for the cause, not in the attraction of the moon, but in the *inequality* of its attraction on different parts of the earth. The greater this inequality is, the greater will be the tides. Hence, as the moon is subject to a slight variation of distance from the earth, it will follow, that when it is at its least distance, or at the point called *perigee*, the tides will be greatest; and when it is at the greatest distance, or at the point called *apogee*, the tides will be least; not because the entire attraction of the moon in the former case is greater than in the latter, but because the diameter of the globe bearing a greater proportion to the lesser distance than the greater, there will be a greater *inequality* of attraction.

2517. *Effects of sun's attraction.* — It will occur to those who bestow on these observations a little reflection, that all which we have stated in reference to the effects produced by the attraction of the moon upon the earth, will also be applicable to the attraction of the sun. This is undoubtedly true; but in the case of the sun the effects are modified in some very important respects. The sun is at 400 times a greater distance than the moon, and the actual amount of its attraction on the earth would, on that account, be 160,000 times less than that of the moon; but the mass of the sun exceeds that of the moon in a much greater ratio than that of 160,000 to 1. It therefore possesses a much greater attracting power in virtue of its

mass, compared with the moon, than it loses by its greater distance. It exercises, therefore, upon the earth an attraction enormously greater than the moon exercises. Now, if the simple amount of its attraction were, as is commonly supposed, the cause of the tides, the sun ought to produce a vastly greater tide than the moon. The reverse is, however, the case, and the cause is easily explained. Let it be remembered that the tides are due solely to the inequality of the attraction on different sides of the earth, and the greater that inequality is, the greater will be the tides, and the less that inequality is, the less will be the tides.

In the case of the sun, the total distance is 12,000 diameters of the earth, and consequently the difference between its distances from the one side and the other of the earth will be only the 12,000th part of the whole distance, while in the case of the moon, the total distance being only 30 diameters of the earth, the difference of the distances from one side and the other is the 30th part of the whole distance. The inequality of the attraction, upon which alone, and not on its whole amount, the production of the tidal wave depends, is therefore much greater in the case of the moon. According to Newton's calculation, the tidal wave due to the moon is greater in height than that due to the sun in the ratio of 58 to 23, or $2\frac{1}{2}$ to 1 very nearly.

2518. *Cause of spring and neap tides.* — There is, therefore, a solar as well as a lunar tide wave, the former being much less elevated than the latter, and each following the luminary from which it takes its name. When the sun and moon, therefore, are either on the same side of the earth, or on the opposite sides of the earth — in other words, when it is new or full moon — their effects in producing tides are combined, and the spring tide is produced, the height of which is equal to the solar and lunar tides taken together.

On the other hand, when the sun and moon are separated from each other by a distance of one fourth of the heavens, that is, when the moon is in the quarters, the effect of the solar tide has a tendency to diminish that of the lunar tide.

The tides produced by the combination of the lunar and solar tide waves at the time of new and full moon are called SPRING TIDES; and those produced by the lunar wave diminished by the effect of the solar wave at the quarters are called NEAP TIDES.

2519. *Why the tides are not produced directly under the*

moon.—If physical effects followed immediately, without any appreciable interval of time, the operation of their causes, then the tidal wave produced by the moon would be on the meridian of the earth directly under and opposite to that luminary; and the same would be true of the solar tides. But the waters of the globe have, in common with all other matter, the property of inertia, and it takes a certain interval of time to impress upon them a certain change of position. Hence it follows that the tidal wave produced by the moon is not formed immediately under that body, but follows it at a certain distance. In consequence of this, the tide raised by the moon does not take place for two or three hours after the moon passes the meridian; and as the action of the sun is still more feeble, there is a still greater interval between the transit of the sun and occurrence of the solar tide.

2520. *Priming and lagging of the tides.*—But besides these circumstances, the tide is affected by other causes. It is not to the separate effect of either of these bodies, but to the combined effect of both, that the effects are due; and at every period of the month, the time of actual high water is either accelerated or retarded by the sun. In the first and third quarters of the moon, the solar tide is westward of the lunar one; and, consequently, the actual high water, which is the result of the combination of the two waves, will be to the westward of the place it would have if the moon acted alone, and the time of high water will therefore be accelerated. In the second and fourth quarters the general effect of the sun is, for a similar reason, to produce a retardation in the time of high water. This effect, produced by the sun and moon combined, is what is commonly called the *priming* and *lagging* of the tides. The highest spring tides occur when the moon passes the meridian about an hour after the sun; for then the maximum effect of the two bodies coincides.

2521. *Researches of Whewell and Lubbock.*—The subject of the tides has of late years received much attention from several scientific investigators in Europe. The discussions held at the annual meetings of the British Association for the Advancement of Science, on this subject, have led to the development of much useful information. The labours of Professor Whewell have been especially valuable on these questions. Sir John Lubbock has also published a valuable treatise upon it. To trace the results of these investigations in all the details which would

render them clear and intelligible, would greatly transcend the necessary limits of this volume. We shall, however, briefly advert to a few of the most remarkable points connected with these questions.

2522. *Vulgar and corrected establishment.* — The apparent time of high water at any port in the afternoon of the day of new or full moon, is what is usually called the *establishment of the port.* Professor Whewell calls this the vulgar establishment, and he calls the *corrected establishment* the mean of all the intervals of the tides and transit of half a month. This corrected establishment is consequently the luni-tidal interval corresponding to the day on which the moon passes the meridian at noon or midnight.

2523. *Diurnal inequality.* — The two tides immediately following one another, or the tides of the day and night, vary, both in height and time of high water at any particular place, with the distance of the sun and moon from the equator. As the vertex of the tide wave always tends to place itself vertically under the luminary which produces it, it is evident that of two consecutive tides that which happens when the moon is nearest the zenith or nadir will be greater than the other; and, consequently, when the moon's declination is of the same denomination as the latitude of the place, the tide which corresponds to the upper transit will be greater than the opposite one, and *vice versâ*, the difference being greatest when the sun and moon are in opposition, and in opposite tropics. This is called the DIURNAL INEQUALITY, because its cycle is one day; but it varies greatly at different places, and its laws, which appear to be governed by local circumstances, are very imperfectly known.

2524. *Local effects of the land upon the tides.* — We have now described the principal phenomena that would take place were the earth a sphere, and covered entirely with a fluid of uniform depth. But the actual phenomena of the tides are infinitely more complicated. From the interruption of the land, and the irregular form and depth of the ocean, combined with many other disturbing circumstances, among which are the inertia of the waters, the friction on the bottom and sides, the narrowness and length of the channels, the action of the wind, currents, difference of atmospheric pressure, &c., &c., great variation takes place in the mean times and heights of high water at places differently situated.

2525. *Velocity of tidal wave.* — In the open ocean the crest of tide travels with enormous velocity. If the whole surface were uniformly covered with water, the summit of the tide wave, being mainly governed by the moon, would everywhere follow the moon's transit at the same interval of time, and consequently travel round the earth in a little more than twenty-four hours. But the circumference of the earth at the equator being about 25,000 miles, the velocity of propagation would therefore be about 1,000 miles per hour. The actual velocity is, perhaps, nowhere equal to this, and is very different at different places. In latitude 60° south, where there is no interruption from land (excepting the narrow promontory of Patagonia), the tide wave will complete a revolution in a lunar day, and consequently travel at the rate of 670 miles an hour. On examining Mr. Whewell's map of cotidal lines, it will be seen that the great tide wave from the Southern Ocean travels from the Cape of Good Hope to the Azores in about twelve hours, and from the Azores to the southernmost part of Ireland in about three hours more. In the Atlantic, the hourly velocity in some cases appears to be 10° latitude, or near 700 miles, which is almost equal to the velocity of sound through the air. From the south point of Ireland to the north point of Scotland, the time is eight hours, and the velocity about 160 miles an hour along the shore. On the eastern coast of Britain, and in shallower water, the velocity is less. From Buchanness to Sunderland it is about 60 miles an hour; from Scarborough to Cromer, 35 miles; from the North Foreland to London, 30 miles; from London to Richmond, 18 miles an hour in that part of the river. (Whewell, *Phil. Trans.* 1833 and 1836.) It is scarcely necessary to remind the reader that the above velocities refer to the transmission of the undulation, and are entirely different from the velocity of the current to which the tide gives rise in shallow water.

2526. *Range of the tides.* — The difference of level between high and low water is affected by various causes, but chiefly by the configuration of the land, and is very different at different places. In deep inbends of the shore, open in the direction of the tide wave and gradually contracting like a funnel, the convergence of water causes a very great increase of the range. Hence the very high tides in the Bristol Channel, the Bay of St. Malo, and the Bay of Fundy, where the tide is said to rise sometimes to the height of one hundred feet. Promontories, under

certain circumstances, exert an opposite influence, and diminish the magnitude of the tide. The observed ranges are also very anomalous. At certain places on the south-east coast of Ireland, the range is not more than three feet, while at a little distance on each side it becomes twelve or thirteen feet; and it is remarkable that these low tides occur directly opposite the Bristol Channel, where (at Chepstow) the difference between high and low water amounts to sixty feet. In the middle of the Pacific it amounts to only two or three feet. At the London Docks, the average range is about 22 feet; at Liverpool, 15·5 feet; at Portsmouth, 12·5 feet; at Plymouth, also 12·5 feet; at Bristol, 33 feet.

2527. *Tides affected by the atmosphere.* — Besides the numerous causes of irregularity depending on the local circumstances, the tides are also affected by the state of the atmosphere. At Brest, the height of high water varies inversely as the height of the barometer, and rises more than eight inches for a fall of about half an inch of the barometer. At Liverpool, a fall of one-tenth of an inch in the barometer corresponds to a rise in the river Mersey of about an inch; and at the London Docks, a fall of one-tenth of an inch corresponds to a rise in the Thames of about seven-tenths of an inch. With a low barometer, therefore, the tide may be expected to be high, and *vice versâ*. The tide is also liable to be disturbed by winds. Sir John Lubbock states that, in the violent hurricane of January 8. 1839, there was no tide at Gainsborough, which is twenty-five miles up the Trent — a circumstance unknown before. At Saltmarsh, only five miles up the Ouse from the Humber, the tide went on ebbing, and never flowed until the river was dry in some places; while at Ostend, toward which the wind was blowing, contrary effects were observed. During strong north-westerly gales the tide marks high water earlier in the Thames than otherwise, and does not give so much water, while the ebb tide runs out late, and marks lower; but upon the gales abating and weather moderating, the tides put in and rise much higher, while they also run longer before high water is marked, and with more velocity of current: nor do they run out so long or so low.

2528. *The trade winds.*—The great atmospheric currents thus denominated, from the advantages which navigation has derived from them, as well as other currents arising from the same causes, are produced by the unequal exposure of the atmospheric ocean, which coats the terrestial globe, to the action of

solar heat; the expansion and contraction that air, in common with all gaseous bodies, suffers from increase and diminution of temperature; the tendency which lighter fluids have to rise through heavier; and, in fine, the rotation of the earth upon its axis.

The regions in which the TRADES prevail are two great tropical belts extending through a certain limited number of degrees north and south of the line, but not prevailing on the line itself, the atmospherical character of which is an almost constant calm. The permanent currents blow in the northern tropical belt from the north-east, and in the southern from the south-east.

On the other hand, in the higher latitudes of both hemispheres the prevalent atmospheric currents are directed from west to east, redressing, as it were, the disturbance produced by the trades.

To understand the cause of these phenomena, it is necessary to remember that the sun, never departing more than 23½° from the celestial equator, is vertical daily to different points around the tropical regions, the rotation of the earth bringing these points successively under his disk. The sun, at noon, for places within the tropics, is never so much as 23½° from the zenith. The intertropical zone from these causes becomes much more intensely heated upon its surface than the parts of either hemisphere at higher latitudes. This heat, reflected and radiated upon the incumbent atmosphere, causes it to expand and become specifically lighter, and it ascends as smoke and heated air do in a chimney. The space it deserts is filled by colder and therefore heavier air, which rushes in from the higher parts of either hemisphere; while the air thus displaced, raised by its buoyancy above its due level, and unsustained by any lateral pressure, flows down towards either pole, and, being cooled in its course and rendered heavier, it descends to the surface of the globe at those upper latitudes from which the air had been sucked in towards the line by its previous ascent.

A constant circulation and an interchange of atmosphere between the intertropical and extratropical regions of the earth would thus take place, the air ascending from the intertropical surface and then flowing towards the extratropical regions, where it descends to the surface to be again sucked towards the line.

But in this view of the effects, the rotation of the earth on its

axis is not considered. In that rotation the atmosphere participates. The air which rises from the intertropical surfaces carries with it the velocity of that surface, which is at the rate of about 1,000 miles an hour from west to east. This velocity it retains to a considerable extent after it has passed to the higher latitudes and descended to the surface, which moving with much less velocity from west to east, there is an effective current produced in that direction equivalent to the excess of the eastward motion of the air over the eastward motion of the surface of the earth. Hence arises the prevalent westward winds, especially at sea, where causes of local disturbance are not frequent, which are so familiar, and one of the effects of which has been, that, while the average length of the trip of good sailing vessels from New York to Liverpool has been only twenty days, that of the trip from Liverpool to New York has been thirty-five days.

By the friction of the earth and other causes, the air, however, next the surface, at length acquires a common velocity with it, and when it is, as above described, sucked towards the line to fill the vacuum produced by the air drawn upwards by the solar heat, it carries with it the motion from west to east which it had, in common with the surface, at the higher latitudes. But the surface at the line has a much greater velocity than this from west to east. The surface, therefore, and all objects upon it, are carried against the air with the *relative velocity* of the surface and the air, that is to say, with the effect of the *difference* of their velocities. Since the surface, and the objects upon it, are carried eastward at a much less rate than the air which has just descended from the higher latitudes, they will strike against the air with a force proportional to the difference of their velocities, and this force will have a direction contrary to that of the motion of the surface, that is to say, from east to west.

But it must be considered that this eastward force, due to the motion of the earth's surface, is combined with the force with which the air moves from the extratropical regions towards the line. Thus, in the northern hemisphere, the force eastward is combined with the motion of the air from north to south, and the resultant of these forces is that north-east current which actually prevails; while, for like reasons, south of the line, the motion of the air from south to north, being combined with the force eastward, produces the south-eastern current which prevails south of the line.

Were any considerable mass of air, as Sir J. Herschel observes, to be *suddenly* transferred from beyond the tropics to the equator, the difference of the rotatory velocities proper to the two situations would be so great, as to produce, not merely a wind, but a tempest of the most destructive violence; and the same observation would be equally applicable to masses of air transported in the contrary direction. But this is not the case; the advance of the air is gradual, and all the while the earth is continually acting on the air, and by the friction of its surface accelerating or retarding its velocity. Supposing its progress to cease at any point, this cause would almost immediately communicate to it the deficient or deprive it of the excessive motion of rotation, after which it would revolve quietly with the earth and be at relative rest. We have only to call to mind the comparative *thinness* of the coating of air with which the globe is invested (2323) and its immense mass, exceeding, as it does, the weight of the atmosphere at least 100,000,000 times, to appreciate the absolute command of any extent of territory of the earth over the atmosphere immediately incumbent upon it.

It appears, therefore, that these currents, as they approach the equator on the one side and the other, must gradually lose their force; their exciting cause being the difference of the magnitude of the parallels of latitude; and this difference being evanescent near the line, and very inconsiderable within many degrees of it, the equalising force of the earth above described is allowed to take full effect: but, besides this, the currents directed from the two poles encounter each other at the line, and destroy each other's force. Hence arises the prevalence of those calms which characterise the line.

CHAP. XI.

THE SUN.

2529. *Apparent and real magnitude.*—Owing to the ellipticity of the earth's orbit, the distance of the sun is subject to a periodical variation, which causes, as has been already explained, a corresponding variation in its apparent magnitude. Its greatest apparent diameter, when in perihelion, is 32′ 35″·6, or

1955″·6, and its least apparent magnitude, when in aphelion, is 31′ 30″, or 1890″. Its mean apparent diameter is therefore 1923″.

It has been already (2457) shown that the linear value of 1″ at the sun's distance is 466 miles. It follows, therefore, that the actual length of the diameter of the globe of the sun is

$$1923 \times 466 = 896{,}118 \text{ miles.}$$

The real magnitude of the sun may also be easily inferred in round numbers from that of the moon. The apparent diameter of the moon being equal in round numbers to that of the sun, and the distance of the sun being 400 times greater than that of the moon, it follows that the real diameter of the sun must be 400 times greater than that of the moon. It must, therefore, be

$$2153 \times 400 = 861{,}200 \text{ miles.}$$

By methods of calculation susceptible of closer approximation than these, it has been found that the magnitude is 882,000 miles, or $111\frac{7}{10}$ times the diameter of the earth.

2530. *Magnitude of the sun illustrated.*—Magnitudes such as that of the sun so far transcend all standards with which the mind is familiar, that some stretch of imagination, and some effort of the understanding, are necessary to form a conception, however imperfect, of them. The expedient which best serves to obtain some adequate idea of them is, to compare them with some standard, stupendous by comparison with all ordinary magnitudes, yet minute when compared with them.

The earth itself is a globe 8000 miles in diameter. If the sun be represented by a globe nine feet four inches in diameter, the earth would be represented by a globe an inch in diameter. If the orbit of the moon, which measures 474,000 miles in diameter, were filled by a sun, such a sun might be placed within the actual sun, leaving between their surfaces a distance of 200,000 miles. Such a sun, seen from the earth, would have an apparent diameter little more than half the diameter of the actual sun.

2531. *Surface and volume.*—Since the surfaces of globes are as the squares, and their volumes as the cubes, of their diameters, it follows that the surface of the sun must be 12,500 times, and its volume 1,400,000 times, greater than those of the earth.

Thus, to form a globe like the sun it would be necessary to roll nearly fourteen hundred thousand globes like the earth into one.

It is found, by considering the bulks of the different planets, that if all the planets and satellites in the solar system were moulded into a single globe, that globe would still not exceed the five-hundredth part the globe of the sun: in other words, the bulk of the sun is five hundred times greater than the aggregate bulk of all the rest of the bodies of the system.

2532. *Its mass and density.*—By methods of calculation and observation, which will be explained hereafter, the ratio of the mass of matter composing the globe of the sun, to the mass of matter composing the earth, has been ascertained to be 354,936 to 1.

By comparing this proportion of the quantities of ponderable matter in the sun and earth with their relative volumes, it will be evident that the mean density of the matter composing the sun must be about four times less than the mean density of the matter composing the earth; for although the volume of the sun exceeds that of the earth in the ratio of 1,400,000 to 1, its weight or mass exceeds that of the earth in the lesser ratio of 355,000 to 1, the latter ratio being four times less than the former. Bulk for bulk, therefore, the sun is four times lighter than the earth.

Since the mean density of the earth is 5·67 times that of water (2393), it follows that the mean density of the sun is 1·42 times, or about one half, greater than that of water.

From the comparative lightness of the matter composing it, Herschel infers the probability that an intense heat prevails in its interior, by which its elasticity is reinforced, and rendered capable of resisting the almost inconceivable pressure due to its intrinsic gravitation, without collapsing into smaller dimensions.

2533. *Form and rotation — axis of rotation.*—Although to minds unaccustomed to the rigour of scientific research, it might appear sufficiently evident, without further demonstration, that the sun is globular in its form, yet the more exact methods pursued in the investigation of physics demand that we should find more conclusive proof of the sphericity of the solar orb than the mere fact that the disk of the sun is always circular. It is barely possible, however improbable, that a flat circular disk of

matter, the face of which should always be presented to the earth, might be the form of the sun; and indeed there are a great variety of other forms which, by a particular arrangement of their motions, might present to the eye a circular appearance as well as a globe or sphere. To prove, then, that a body is globular, something more is necessary than the mere fact that it always appears circular.

When a telescope is directed to the sun, we discover upon it certain marks or spots, of which we shall speak more fully presently. We observe that these marks, while they preserve the same relative position with respect to each other, move regularly from one side of the sun to the other. They disappear, and continue to be invisible for a certain time, come into view again on the other side, and so once more pass over the sun's disk. This is an effect which would evidently be produced by marks on the surface of a globe, the globe itself revolving on an axis, and carrying these marks upon it. That this is the case, is abundantly proved by the fact that the periods of rotation for all these marks are found to be exactly the same, viz., about twenty-five days and a quarter, or more exactly $25^{d.}\ 7^{h.}\ 48^{m.}$ Such is, then, the time of rotation of the sun upon its axis, and that it is a globe remains no longer doubtful, since a globe is the only body which, while it revolves with a motion of rotation, would always present the circular appearance to the eye. The axis on which the sun revolves is very nearly perpendicular to the plane of the earth's orbit, and the motion of rotation is in the same direction as the motion of the planets round the sun, that is to say, from west to east.

2534. *Spots.*—One of the earliest fruits of the invention of the telescope was the discovery of the spots upon the sun; and the examination of these has gradually led to some knowledge of the physical constitution of the centre of attraction and the common fountain of light and heat of our system.

When we submit a solar spot to telescopic examination, we discover its appearance to be that of an intensely black irregularly shaped patch, edged with a penumbral fringe. When watched for a considerable time, it is found to undergo a gradual change in its form and magnitude; at first increasing gradually in size, until it attain some definite limit of magnitude, when it ceases to increase, and soon begins, on the contrary, to diminish; and its diminution goes on gradually, until at length, the bright

sides closing in upon the dark patch, it dwindles first to a mere point, and finally disappears altogether. The period which elapses between the formation of the spot, its gradual enlargement, subsequent diminution, and final disappearance, is very various. Some spots appear and disappear very rapidly, while others have lasted for weeks and even for months.

The magnitude of the spots, and the velocities with which the matter composing their edges and fringes moves, as they increase and decrease, are on a scale proportionate to the dimensions of the orb of the sun itself. When it is considered that a space upon the sun's disk, the apparent breadth of which is only a minute, actually measures (2457)

$$466 \times 60 = 27{,}960 \text{ miles},$$

and that spots have been frequently observed, the apparent length and breadth of which have exceeded 2′, the stupendous magnitude of the regions they occupy may be easily conceived.

The velocity with which the luminous matter at the edges of the spots occasionally moves, during the gradual increase or diminution of the spot, has been in some cases found to be enormous. A spot, the apparent breadth of which was 90″, was observed by Mayer to close in about 40 days. Now, the actual linear dimensions of such a spot must have been

$$466 \times 90 = 41{,}940 \text{ miles},$$

and consequently, the average daily motion of the matter composing its edges must have been 1050 miles, a velocity equivalent to 44 miles an hour.

2535. *Cause of the spots—physical state of the sun's surface.*—Two, and only two, suppositions have been proposed to explain the spots. One supposes them to be scoriæ, or dark scales of incombustible matter, floating on the general surface of the sun. The other supposes them to be excavations in the luminous matter which coats the sun, the dark part of the spot being a part of the solid non-luminous nucleus of the sun. In this latter hypothesis it is assumed that the sun is a solid non-luminous globe, covered with a coating of a certain thickness of luminous matter.

That the spots are excavations, and not mere black patches on the surface, is proved by the following observations: If we select a spot which is at the centre of the sun's disk, having

some definite form, such as that of a circle, and watch its changes of appearance, when, by the rotation of the sun, it is carried toward the edge, we find, first, that the circle becomes an oval. This, however, is what would be expected, even if the spot were a circular patch, inasmuch as a circle seen obliquely is foreshortened into an oval. But we find that as the spot moves toward the side of the sun's limb, the black patch gradually disappears, the penumbral fringe on the inside of the spot becomes invisible, while the penumbral fringe on the outside of the spot increases in apparent breadth, so that when the spot approaches the edge of the sun, the only part that is visible is the external penumbral fringe. Now, this is exactly what would occur if the spot were an excavation. The penumbral fringe is produced by the shelving of the sides of the excavation, sloping down to its dark bottom. As the spot is carried toward the edge of the sun, the height of the inner side is interposed between the eye and the bottom of the excavation, so as to conceal the latter from view. The surface of the inner shelving side also taking the direction of the line of vision or very nearly, diminishes in apparent breadth, and ceases to be visible, while the surface of the shelving side next the edge of the sun becoming nearly perpendicular to the line of vision, appears of its full breadth.

In short, all the variations of appearance which the spots undergo, as they are carried round by the rotation of the sun, changing their distances and positions with regard to the sun's centre, are exactly such as would be produced by an excavation, and not at all such as a dark patch on the solar surface would undergo.

2536. *Sun invested by two atmospheres, one luminous and the other non-luminous.*—It may be considered then as proved, that the spots on the sun are excavations; and that the apparent blackness is produced by the fact that the part constituting the dark portion of the spot is either a surface totally destitute of light, or by comparison so much less luminous than the general surface of the sun as to appear black. This fact, combined with the appearance of the penumbral edges of the spots, has led to the supposition, advanced by Sir W. Herschel, which appears scarcely to admit of doubt, that the solid, opaque nucleus, or globe of the sun, is invested with at least two atmospheres, that which is next the sun being, like our own, non-luminous, and

and the superior one being that alone in which light and heat are evolved; at all events, whether these strata be in the gaseous state or not, the existence of two such, one placed above the other, the superior one being luminous, seems to be exempt from doubt.

2537. *Spots may not be black.*—We are not warranted in assuming that the black portion of the spots are surfaces really deprived of light, for the most intense artificial lights which can be produced, such, for example, as that of a piece of quick-lime exposed to the action of the compound blow-pipe, when seen projected on the sun's disk, appear as dark as the spots themselves; an effect which must be ascribed to the infinitely superior splendour of the sun's light. All that can be legitimately inferred respecting the spots, then, is, not that they are destitute of light, but that they are incomparably less brilliant than the general surface of the sun.

2538. *Spots variable.*—The prevalence of spots on the sun's disk is both variable and irregular. Sometimes the disk will be completely divested of them, and will continue so for weeks or months; sometimes they will be spread over certain parts of it in profusion. Sometimes the spots will be small, but numerous; sometimes individual spots will appear of vast extent; sometimes they will be manifested in groups, the penumbræ or fringes being in contact.

The duration of each spot is also subject to great and irregular variation. A spot has appeared and vanished in less than twenty-four hours, while some have maintained their appearance and position for nine or ten weeks, or during nearly three complete revolutions of the sun upon its axis.

A large spot has sometimes been observed suddenly to crumble into a great number of small ones.

2539. *Prevail generally in two parallel zones.*—The only circumstance of regularity which can be said to attend these remarkable phenomena is their position upon the sun. They are invariably confined to two moderately broad zones parallel to the solar equator, separated from it by a space several degrees in breadth. The equator itself, and this space which thus separates the macular zones, are absolutely divested of such phenomena.

Thus, for example, in the latter part of 1836 and the beginning of 1837, when a large number of spots became apparent, their position was such as is represented in *fig.* 728.,

where E Q represents the sun's equator, and *m m' n n'* the northern, and *p p' q q'* the southern macular zones.

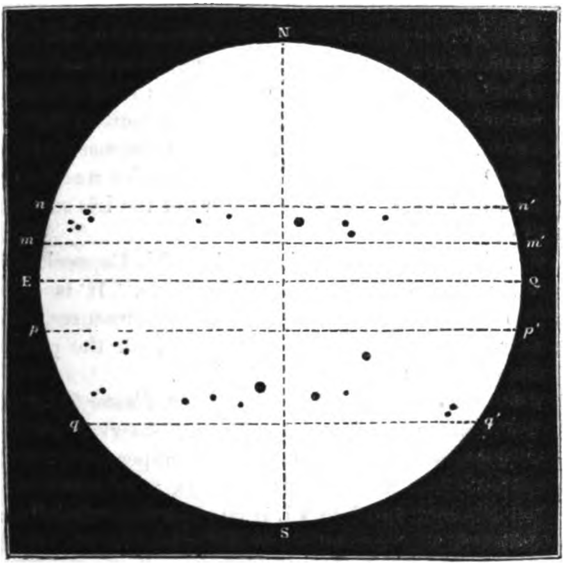


Fig. 728.

2540. *Observations and drawings of M. Capocci.*—The astronomers who have within the last quarter of a century made the most important contributions, by their observations and researches, to this subject, are M. Capocci, of Naples, Dr. Pastorff, of Frankfort (on the Oder), and Sir John Herschel.

M. Capocci made a series of observations on the spots which were developed on the sun's disk in 1826, when he recognised most of the characters above described. He observed that, during the increase of the spot from its first appearance as a dark point, the edges were sharply defined, without any indication of the gradually fading away of the fringes into the dark central spot, or into each other; a character which was again observed by Sir J. Herschel, in 1837. He found, however, that the same character was not maintained when the sides began to contract and the spots to diminish: during that process the edges were less strongly defined, being apparently covered by a sort of luminous atmosphere, which often extended

so completely across the dark nucleus as to throw a thin thread of light across it, after which the spot soon filled up and disappeared. Capocci concurs with Sir W. Herschel in regarding the internal fringes surrounding the dark nucleus as the section of the inferior stratum of the atmosphere which forms the coating of the sun; he nevertheless thinks that there are indications of solid as well as gaseous luminous matter.

Capocci also observed veins of more intensely luminous matter on the fringes converging towards the nucleus of the spot, which he compares to the structure of the iris surrounding the pupil of the eye.

The drawings of the spots observed by M. Capocci, given in Plate V., will illustrate these observations. It is to be regretted, however, that he has not given any measures, either in his memoirs or upon his drawings, by which the position or magnitude of the spots can be determined.

2541. *Observations and drawings of Dr. Pastorff, in* 1826.—Dr. Pastorff commenced his course of solar observations as early as 1819. He observed the spots which appeared in 1826, of which he published a series of drawings, from which we have selected those given in Plate VI. from observations made in September and October, contemporaneously with those of M. Capocci. Pastorff gives the position of all, and the dimensions of the principal spots. The numbers on the horizontal and vertical lines express the apparent distances of the spots severally from the limb of the sun in each direction. The actual dimensions may be estimated by observing that $1''$ measured at right angles to the visual ray represents 466 miles.

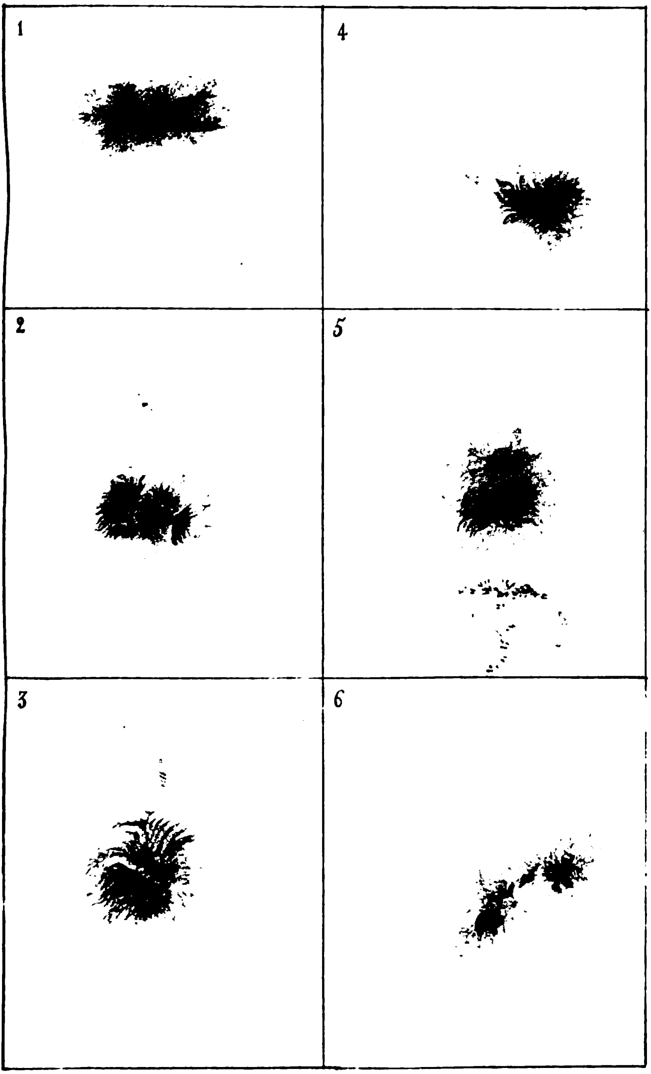
2542. *Observations and drawings of Pastorff, in* 1828.—In May and June, 1828, a profusion of spots were developed, which were observed and delineated by Pastorff with the most elaborate accuracy.

In Plate VII. *fig.* 1. represents the positions of the spots as they appeared on the disk of the sun on the 24th of May, at 10 A.M. and *figs.* 2, 3, 4, and 5, represent their forms and magnitudes. The letters A, B, C, D, in *fig.* 1. give the positions of the spots marked by the same letters in *figs.* 2, 3, 4, and 5.

The dimensions of the principal spot of the group A were stupendous; measured in a plane at right angles to the visual line, the length was $466 \times 100 = 46{,}600$ miles, and the breadth $466 \times 60 = 27{,}960$ miles.

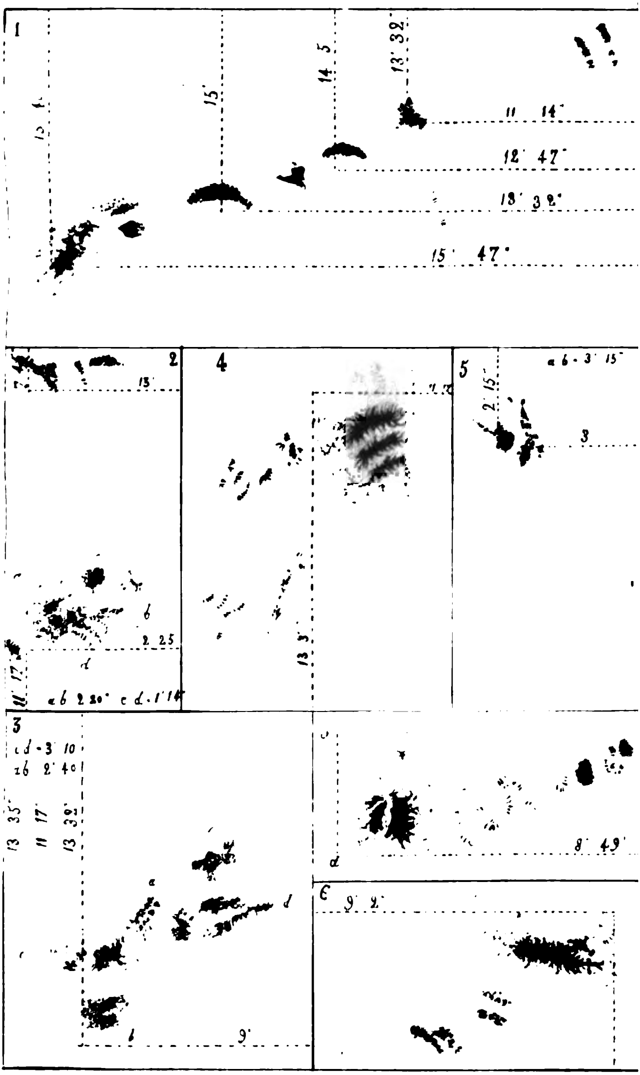
Solar Spots observed by Capocci 1826.

V.



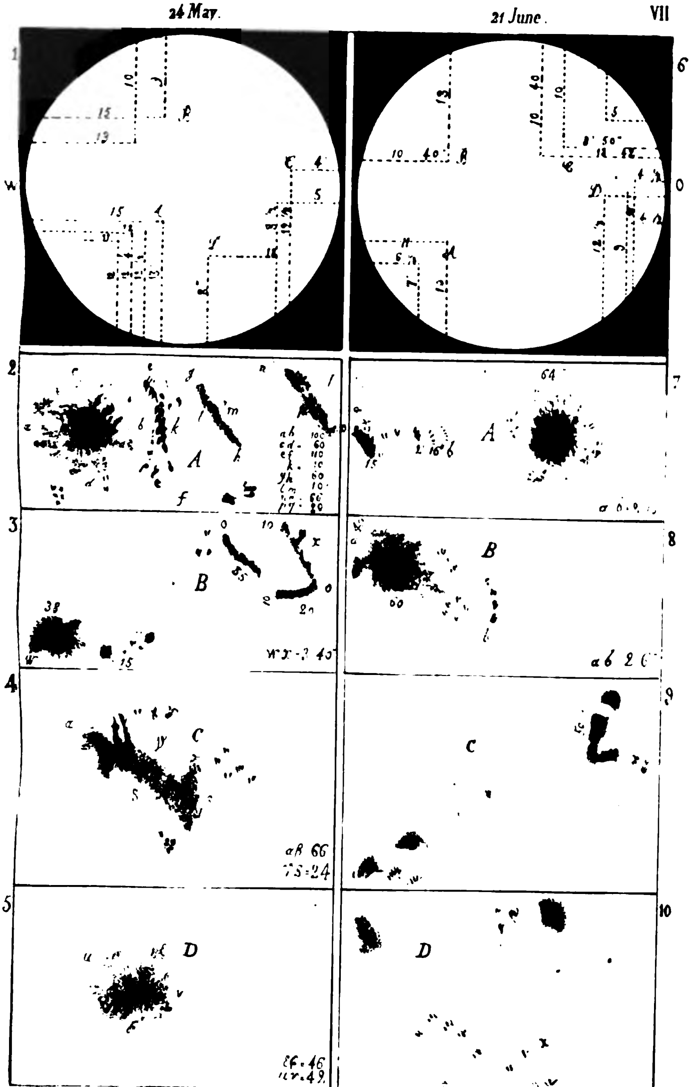
1 July 1 10^h a m. 2 Sep. 2 10^h a m. 3 Sep. 29 10^h a.m.
4 Oct 2 10^h a. m 5 Oct. 2 10^h a. m. 6 Oct. 6 10^h a. m.

Solar Spots observed by Pastorff 1826



1. Oct. 6 8h 15m a. m. 2 Oct. 12. 11h a. m. 3. Oct 23 9h 30 a. m
4. Sep. 27 9h a. m. 5 Oct. 2 8h a. m. 6. Oct. 8h a. m.

Solar Spots observed by Pastorff 1828.



L. T.

Imp. Lemercier Paris.

The apparent breadth of the black bottom of the spot was 40″, which corresponds to an actual breadth of $466 \times 40 = 18{,}640$ miles. So that the globe of the earth might pass through such a hole, leaving a distance of upwards of 5000 miles between its surface and the edges of the chasm.

The superficial dimensions of the several groups of spots observed on the sun on the 24th of May, at 10 A. M., including the shelving sides, were calculated to be as follows: —

	Square Geog. Miles.
Group A, principal spot - - -	928,000,000
Ditto, smaller spots - - - -	736,000,000
Group B - - - - -	296,000,000
Group C - - - - -	232,000,000
Group D - - - - -	304,000,000
Total area - - -	2,496,000,000

Thus it appears that the principal spot of the group A covered a space equal to little less than five times the entire surface of the earth; and the total area occupied by all the spots collectively amounted to more than twelve times that surface.

On the days succeeding the 24th of May, all the spots were observed to change their form and magnitude from day to day. The great spot of the group A, which even when so close to the limb of the sun as 5′, or a sixth of the apparent diameter, still measured 80″ by 40″, was especially rapid in its variation. Its shelving sides, as well as its dark bottom, were constantly varied, and luminous clouds were seen floating over the latter.

After the disappearance of this large spot, and several of the lesser ones of the other groups, a new spot of considerable magnitude made its appearance on the 13th of June, at the eastern edge of the disk, which gradually increased in magnitude for eight days. On the 21st of June, at half-past 9 in the morning, the disk of the sun exhibited the spots whose position is represented in *fig.* 6. Plate VII., and whose forms and magnitudes are indicated in *figs.* 7, 8, 9, and 10.

The chief spot of the group A was nearly circular, and measured 64″ in apparent diameter, the diameter of its dark base being about 30″, which, without allowing for projection, represent actual lengths of $466 \times 64 = 29{,}824$ miles, and $466 \times 30 = 13{,}980$ miles, the former being above $3\frac{1}{2}$ times, and the latter nearly $1\frac{3}{4}$

times the earth's diameter. The process of formation of this spot, surrounded by luminous clouds, was clearly seen. The shelving sides were traversed by luminous ravines or rills, converging towards the centre of the black nucleus, and exhibiting the appearance which Capocci compared to the structure of the iris.

On the same day (the 21st), another large spot, B, *fig.* 8. appeared, which measured 60″ by 40″.

Pastorff rejects the supposition that these spots were the mere reappearances of those which had been observed on the 24th of May, since they differed essentially in their form, and still more in their *entourage.*

2543. *Observations of Sir J. Herschel in* 1837.—Sir J. Herschel, at the Cape of Good Hope, in 1837, observed the spots which at that time appeared upon the sun, and has given various drawings of them in his Cape Observations. These diagrams do not differ in any respect in their general character from those of Capocci and Pastorff. Sir J. Herschel recognised on this occasion the striated or radiated appearance in the fringes already noticed by Capocci and Pastorff. He thinks that this structure is intimately connected with the physical agency by which the spots are produced.

2544. *Boundary of fringes distinctly defined.*—It is observed by Sir J. Herschel that one of the most universal and striking characters of the solar spots is, that the penumbral fringe and black spot are distinctly defined, and do not melt gradually one into the other. The spots are intensely black, and the penumbral fringe of a perfectly uniform degree of shade. In some cases there are two nuances of fringe, one lighter than the other; but in that case no intermixture or gradual fading away of one into the other is apparent. "The idea conveyed," observes Sir J. Herschel, "is more that of the successive withdrawal of veils,—the partial removal of definite films,—than the melting away of a mist or the mutual dilution of gaseous media." This absence of all graduation, this sharply marked suddenness of transition, is, as Sir J. Herschel also notices, entirely opposed to the idea of the easy miscibility of the luminous, non-luminous, and semi-luminous constituents of the solar envelope.

2545. *Solar facules and lucules.*—Independently of the dark spots just described, the luminous part of the solar disk is not uniformly bright. It presents a mottled appearance, which

may be compared to that which would be presented by the undulated and agitated surface of an ocean of liquid fire, or to a stratum of luminous clouds of varying depth and having an unequal surface, or the appearance produced by the slow subsidence of some flocculent chemical precipitates in a transparent fluid, when looked at perpendicularly from above. In the space immediately around the edges of the spots extensive spaces are observed, also covered with strongly defined curved or branching streaks, more intensely luminous than the other parts of the disk, among which spots often break out. These several varieties in the intensity of the brightness of the disk have been differently designated by the terms *facules* and *lucules*. These appearances are generally more prevalent and strongly marked near the edges of the disk.

2546. *Incandescent coating of the sun gaseous.*—Various attempts have been made to ascertain by the direct test of observation, independently of conjecture or hypothesis, the physical state of the luminous matter which coats the globe of the sun, whether it be solid, liquid, or gaseous.

That it is not solid is admitted to be proved conclusively by its extraordinary mobility, as indicated by the rapid motion of the edges of the spots in closing; and it is contended that a fluid capable of moving at the rate of 44 miles per hour cannot be supposed to be liquid, an elastic fluid alone admitting of such a motion.

2547. *Test of this proposed by Arago.*—Arago has, however, suggested a physical test, by which it appears to be proved that this luminous matter must be gaseous; in short, that the sun must be invested with an ocean of flame, since flame is nothing more than aëriform fluid in a state of incandescence (1584). This test proposed is based upon the properties of polarised light.

It has been proved that the light emitted from an incandescent body in the liquid or solid state, issuing in directions very oblique to the surface, even when the body emitting it is not smooth or polished, presents evident marks of polarisation, so that such a body, when viewed through a polariscopic telescope, will present two images in complementary colours (1290). But, on the other hand, no signs of polarisation are discoverable, however oblique may be the direction in which the rays are emitted, if the luminous matter be flame.

2548. *Its result.*—The light proceeding from the disk of the sun has been accordingly submitted to this test. The rays proceeding from its borders evidently issue in a direction as oblique as possible to the surface, and therefore, under the condition most favourable to polarisation, if the luminous matter were liquid. Nevertheless, the borders of the double image produced by the polariscope show no signs whatever of complementary colours, both being equally white even at the very edges.

This test is only applicable to the luminous matter at or near the edge of the disk, because it is from this only that the rays issue with the necessary obliquity. But since the sun revolves on its axis (2533), every part of its surface comes in succession to the edge of the disk; and thus it follows that the light emanating from every part of it is in its natural or unpolarised state, even when issuing at the greatest obliquity; and, consequently, that the luminous matter is every where gaseous.

2549. *The sun probably invested with a double gaseous coating.*—All the phenomena which have been here described, and others which our limits compel us to omit, are considered as giving a high degree of physical probability to the hypothesis of Sir W. Herschel already noticed, in which the sun is considered to be a solid, opaque, non-luminous globe invested by two concentric strata of gaseous matter, the first, or that which rests immediately on the surface, being non-luminous, and the other, which floats upon the former, being luminous gas or flame. The relation and arrangement of these two fluid strata may be illustrated by our own atmosphere, supporting upon it a stratum of clouds. If such clouds were flame, the condition of our atmosphere would represent the two strata on the sun.

The spots in this hypothesis are explained by occasional openings in the luminous stratum by which parts of the opaque and non-luminous surface of the solid globe are disclosed. These partial openings may be compared to the openings in the clouds of our sky, by which the firmament is rendered partially visible.

The apparent diameter of the sun is not, therefore, the diameter of the solid globe, but that of the globe bounded by the surface of the superior or luminous atmosphere; and this circumstance may throw some light upon the small computed mean density of the sun, since considering the high degree of rarefaction which must be supposed to characterise these atmospheric strata, and especially the superior one, the density

of the solid globe will necessarily be much more considerable than the mean density of the volume in which such rarefied matter is included.

2550. *A third gaseous atmosphere probable.* — Many circumstances supply indications of the existence of a gaseous atmosphere of great extent above the luminous matter which forms the visible surface of the sun. It is observed that the brightness of the solar disk is sensibly diminished towards its borders. This effect would be produced if it were surrounded by an imperfectly transparent atmosphere, whereas if no such gaseous medium surrounded it, the reverse of such an effect might be expected, since then the thickness of the luminous coating measured in the direction of the visual ray would be increased very rapidly in proceeding from the centre towards the edges. This gradual diminution of brightness in proceeding towards the borders of the solar disk has been noticed by many astronomers; but it was most clearly manifested in the series of observations made by Sir J. Herschel in 1837, so conclusively, indeed, as to leave no doubt whatever of its reality on the mind of that eminent observer. By projecting the image of the sun's disk on white paper by means of a good achromatic telescope, this diminution of light towards the borders was on that occasion rendered so apparent, that it appeared to him surprising that it should ever have been questioned.

2551. *Its existence indicated by solar eclipses.* — But the most conclusive proofs of the existence of such an external atmosphere are supplied by certain phenomena observed on the occasion of total eclipses of the sun, which will be fully explained in another chapter of this volume.

2552. *Sir J. Herschel's hypothesis to explain the solar spots.* — The immediate cause of the spots being proved to be occasional ruptures of continuity in the ocean of luminous fluid which forms the visible surface of the solar globe, it remains to discover what physical agency can be imagined to produce dynamical phenomena on a scale so vast as that which the changes of appearance of the spots indicate.

The regions of the spots being two zones parallel to the solar equator, manifests a connection between these phenomena and the sun's rotation. The like regions on the earth are the theatres of the trade-winds and anti-trades, and of hurricanes,

tornadoes, waterspouts, and other violent atmospheric disturbances. On the planets, the same regions are marked by belts, appearances which are traced by analogy to the same physical causes as those which produce the trades and other atmospheric perturbations prevailing in the tropical and ultra-tropical zones. Analogy, therefore, suggests the inquiry, whether any physical agencies can exist upon the sun similar to those which produce these phenomena on the earth and planets.

So far as relates to the earth it is certain, and so far as relates to the planets probable, that the immediate physical cause of these phenomena is the inequality of the exposure of the earth's surface to solar radiation, and the consequent inequality of temperature produced in different atmospheric zones, either by the direct or reflected calorific rays of the sun, combined with the earth's rotation (2528). But since the sun is itself the common fountain of heat, supplying to all, and receiving from none, no similar agency can prevail upon it. It remains, therefore, to consider whether the play of the physical principles which are in operation on the sun itself, irrespective of any other bodies of the system, can supply an explanation of such a local difference of temperature as, combined with the sun's rotation, would produce any special physical effects on the macular zones by which the phenomena of the spots might be explicable.

The heat generated by some undiscovered agency upon the sun is dispersed through the surrounding space by radiation. If, as may be assumed, the rate at which this heat is generated be the same on all parts of the sun, and if, moreover, the radiation be equally free and unobstructed from all parts of its surface, it is evident that an uniform temperature must be everywhere maintained. But if, from any local cause, the radiation be more obstructed in some regions than in others, heat will accumulate in the former, and the local temperature will be more elevated there than where the radiation is more free.

But the only obstruction to free radiation from the sun must arise from the atmosphere with which to an height so enormous it is surrounded. If, however, this atmosphere have everywhere the same height and the same density, it will present the same obstruction to radiation, and the effective radia-

tion which takes place through it, though more feeble than that which would be produced in its absence, is still uniform.

But since the sun has a motion of rotation on its axis in 25$^{d.}$ 7$^{h.}$ 48$^{m.}$, its atmosphere, like that of the earth, must participate in that motion and the effects of centrifugal force upon matter so mobile: the equatorial zone being carried round with a velocity greater than 300 miles per second, while the polar zones are moved at a rate indefinitely slower, all the effects to which the spheroidal form of the earth is due will affect this fluid with an energy proportionate to its tenuity and mobility, the consequence of which will be that it will assume the form of an oblate spheroid, whose axis will be that of the sun's rotation. It will flow from the poles to the equator, and its height over the zones contiguous to the equator will be greater than over those contiguous to the poles, in a degree proportionate to the ellipticity of the atmospheric spheroid.

Now, if this reasoning be admitted, it will follow that the obstruction to radiation produced by the solar atmosphere is greatest over the equator, and gradually decreases in proceeding towards either pole. The accumulation of heat, and consequent elevation of temperature, is, therefore, greatest at the equator, and gradually decreases towards the poles, exactly as happens on the earth from other and different physical causes.

The effects of this inequality of temperature, combined with the rotation, upon the solar atmosphere, will of course be similar in their general character, and different only in degree from the phenomena produced by the like cause on the earth. Inferior currents will, as upon the earth, prevail towards the equator, and superior counter-currents towards the poles (2528). The spots of the sun would, therefore, be assimilated to those tropical regions of the earth in which, for the moment, hurricanes and tornadoes prevail, the upper stratum which has come from the equator being temporarily carried downwards, displacing by its force the strata of luminous matter beneath it (which may be conceived as forming an habitually tranquil limit between the opposite upper and under currents), the upper of course to a greater extent than the lower, and thus wholly or partially denuding the opaque surface of the sun below. Such processes cannot be unaccompanied by vorticose motions, which, left to themselves, die away by degrees, and dissipate, with this peculiarity, that their lower portions come to rest more

speedily than their upper, by reason of the greater distance below, as well as the remoteness from the point of action, which lies in a higher region, so that their centre (as seen in our waterspouts, which are nothing but small tornadoes) appears to retreat upwards.*

Sir J. Herschel maintains that all this agrees perfectly with what is observed during the obliteration of the solar spots, which appear as if filled in by the collapse of their sides, the penumbra closing in upon the spot and disappearing afterwards.

It would have rendered this ingenious hypothesis still more satisfactory, if Sir J. Herschel had assigned a reason why the luminous and subjacent non-luminous atmosphere, both of which are assumed to be gaseous fluids, do not affect, in consequence of the rotation, the same spheroidal form which he ascribes to the superior solar atmosphere.

2553. *Calorific power of solar rays.*—It has been already shown (2217) that the intensity of heat on the sun's surface must be seven times as great as that of the vivid ignition of the fuel in the strongest blast furnace. This power of solar light is also proved by the facility with which the calorific rays pass through glass. Herschel found, by experiments made with an actinometer, that 81·6 per cent. of the calorific rays of the sun penetrate a sheet of plate-glass 0·12 inch thick, and that 85·9 per cent. of the rays which have passed through one such plate will pass through another.†

2554. *Probable physical cause of solar heat.*—One of the most difficult questions connected with the physical condition of the sun, is the discovery of the agency to which its heat is due. To the hypothesis of combustion, or any other which involves the supposition of extensive chemical change in the constituents of the surface, there are insuperable difficulties. Conjecture is all that can be offered, in the absence of all data upon which reasoning can be based. Without any chemical change, heat may be indefinitely generated either by friction or by electric currents, and each of these causes have accordingly been suggested as a possible source of solar heat and light. According to the latter hypothesis, the sun would be a great ELECTRIC LIGHT in the centre of the system.

* Herschel's Cape Observations, p. 434. † Ibid. p. 133.

CHAP. XII.

THE SOLAR SYSTEM.

2555. *Perception of the motion and position of surrounding objects depends upon the station of the observer.*—The facility, clearness, and certainty with which the motions, distances, magnitudes, and relative position and arrangement of any surrounding objects can be ascertained, depends, in a great degree, upon the station of the observer. The form and relative disposition of the buildings, streets, squares, and limits of a great city, are perceived, for example, with more clearness and certainty if the station of the observer be selected at the summit of a lofty building, than if it were at any station level with the general plane of the city itself. This advantage attending an elevated place of observation is much augmented if the objects observed are affected by various and complicated motions *inter se.* A general, who directs the evolutions of a battle, seeks an elevated position from which he can obtain, as far as it is practicable to do so, a *bird's eye* view of the field; and it was at one time proposed to employ captive balloons by which observers could be raised to a sufficient elevation above the plane of the military manœuvres.

All these difficulties, which arise from the station of the observer being in the general plane of the motions observed, are, however, infinitely aggravated when the station has itself motions of which the observer is unconscious; in such case the effects of these motions are optically transferred to surrounding objects, giving them apparent motions in directions contrary to that of the observer, and apparent velocities, which vary with their distance from the observer, increasing as that distance diminishes, and diminishing as that distance increases.

All such effects are imputed by the unconscious observer to so many real motions in the objects observed; and, being mixed up with the motions by which such objects themselves are actually affected, an inextricable confusion of changes of position, apparent and real, results, which involves the observer in

obscurity and difficulty, if his purpose be to ascertain the actual motions and relative distances and arrangement of the objects around him.

2556. *Peculiar difficulties presented by the solar system.*—All these difficulties are presented in their most aggravated form to the observer, who, being placed upon the earth, desires to ascertain the motions and positions of the bodies composing the solar system. These bodies all move nearly in one plane, and from that plane the observer never departs: he is, therefore, deprived altogether of the facilities and advantages which a bird's eye view of the system would afford. He is like the commander who can find no station from which to view the evolutions of the army against which he has to contend, except one upon a dead level with it, but with this great addition to his embarrassment, that his own station is itself subject to various changes of position, of which he is altogether unconscious, and which he can only ascertain by the apparent changes of position which they produce among the objects of his observation and inquiry.

The difficulties arising out of these circumstances obstructed for ages the progress of astronomical science. The persuasion so universally entertained of the absolute immobility of the earth, was not only a vast error itself, but the cause of numerous other errors. It misled inquirers by compelling them to ascribe motion to bodies which are stationary, and to ascribe to bodies not stationary motions altogether different from those with which they are really affected.

2557. *Two methods of exposition.*—There are two methods by which a knowledge of the motions and arrangement of the solar system may be imparted. We may first explain its apparent motions and changes as actually seen from the earth, and deduce from them, combined with our knowledge of the motions which affect the earth itself, the real motions of the other bodies of the system; or we may, on the contrary, first explain the real motions of the entire system as they are now known, and then show how they, combined with the motion of the earth, produce the apparent motions.

The former method would perhaps be more strictly logical, since it would proceed from observed facts as data to the conclusions to be deduced from them; while the other method first assumes, as known, that which we desire to ascertain, and then

shows that it is compatible with all the observed phenomena. Nevertheless, for elementary purposes, such as those to which this volume is directed, the latter method is preferable; we shall, therefore, explain the motions and relative arrangement of the bodies of the system, showing, as we proceed, how their motions cause the phenomena which are observed in the heavens.

2558. *General arrangement of bodies composing the solar system.*—The solar system is an assemblage of great bodies, globular in their form, and analogous in many respects to the earth. Like the earth, they revolve round the sun as a common centre in orbits which do not differ much from circles: all these orbits are very nearly, though not exactly, in the same plane with the annual orbit of the earth, and the orbital motions all take place in the same direction as that of the earth.

Several of these bodies are the centres of secondary systems, another order of smaller globes revolving round them respectively in the same manner and according to the same dynamical laws as govern their own motion round the sun.

2559. *Planets primary and secondary.*—This assemblage of globes which thus revolve round the sun as a common centre, of which the earth itself is one, are called PLANETS; and the secondary globes, which revolve round several of them, are called SECONDARY PLANETS, SATELLITES, or MOONS, one of them being our moon, which revolves round the earth as the earth itself revolves round the sun.

2560. *Primary carry with them the secondary round the sun.*—The primary planets which are thus attended by satellites, carry the satellites with them in their orbital course; the common orbital motion, thus shared by the primary planet with its secondaries, not preventing the harmonious motion of the secondaries round the primary as a common centre.

2561. *Planetary motions to be first regarded as circular, uniform, and in a common plane.*—It will be conducive to the more easy and clear comprehension of the phenomena to consider, in the first instance, the planets as moving round the sun as their common centre in exactly the same plane, in exactly circular orbits, and with motions exactly uniform. None of these suppositions correspond precisely with their actual motions; but they represent them so very nearly, that nothing short of very precise means of observation and measurement is capable of detecting their departure from them. The motions

of the system thus understood will form a first and very close approximation to the truth. The modifications to which the conclusions thus established must be submitted, so as to allow for the departures of the several planets from the plane of the ecliptic, of their orbits from exact circles, and of their motions from perfect uniformity, will be easily introduced and comprehended. But even these will supply only a second approximation. Further investigation will show series after series of corrections more and more minute in their quantities, and requiring longer and longer periods of time to manifest the effects to which they are directed.

2562. *This method follows the order of discovery.* — As to the rest, in following this order, proceeding from first suppositions, which are only rough approximations to the truth, to others in more exact accordance with it, we, in fact, only follow the order of discovery itself, by which the laws of nature were thus gradually, slowly, and laboriously evolved from masses of obscure and inexact hypotheses.

2563. *Inferior and superior planets.* — The concentric orbits of the planets then are included one within another, augmenting successively in their distances from the centre, so as in general to leave a great space between orbit and orbit. The third planet, proceeding from the sun outwards, is the earth. Two orbits, those of the planets called Mercury and Venus, are therefore included within the earth's orbit, which itself is included within the orbits of all the other planets.

Those planets which are included within the orbit of the earth are called INFERIOR PLANETS, and all the others are called SUPERIOR PLANETS.

2564. *Periods.* — The PERIODIC TIME of a planet is the interval between two successive returns to the same point of its orbit, or, in short, the time it takes to make a complete revolution round the sun. It is found by observation, as might be naturally expected, that the periodic time increases with the orbit, being much longer for the more distant planets; but, as will appear hereafter, this increase of the periodic time is not in the same proportion as the increase of the orbit.

2565. *Synodic motion.* — The motion of a planet considered merely in relation to that of the earth, without reference to its actual position in its orbit, is called its SYNODIC MOTION.

2566. *Geocentric and heliocentric motions.* — The position

and motion of a planet as they appear to an observer on the earth are called GEOCENTRIC*; and as they would appear if the observer were transferred to the sun, are called HELIOCENTRIC.*

2567. *Heliocentric motion deducible from geocentric.* — Although the apparent motions cannot be directly observed from the sun as a station, it is a simple problem of elementary geometry to deduce them from the geocentric motions, combined with the relative distances of the earth and planet from the sun; so that we are in a condition to state with perfect clearness, precision, and certainty, all the phenomena which the motions of the planetary system would present, if, instead of being seen from the moveable station of the earth, they were witnessed from the fixed central station of the sun.

2568. *Relation between the daily heliocentric motion and the period.* — If the mean daily heliocentric motion of a planet be expressed by a, and the periodic time in days by P, it is evident that $a \times$ P will express 360°, providing that a is expressed in degrees. Thus we shall have

$$a^\circ \times P = 360^\circ;$$

and hence it follows, that if either the period P or the daily heliocentric motion be given, the other may be computed; for we shall have

$$a^\circ = \frac{360^\circ}{P}, \quad P = \frac{360^\circ}{a^\circ}.$$

It is usual to express a, not in degrees, but in seconds. In that case it will be necessary to reduce 360° also to seconds. We shall therefore have (2292)

$$a'' = \frac{1296000}{P}, \quad P = \frac{1296000}{a''}.$$

2569. *Daily synodic motion.* — The daily synodic motion is the angle by which the planet departs from or approaches to the earth in its course round the sun. Thus if A express in degrees the angle formed by two lines drawn from the sun, one to the planet and the other to the earth, the daily synodic motion will be the daily increase or decrease of A produced by

* From the Greek words γῆ (gē) and ἥλιος (helios), signifying *the earth* and *the sun*.

the motions of the earth and planet. Now, since the earth and planet both move in the same direction round the sun with different angular motions, the increase or decrease of A will be the difference of their motions. Thus, if the planet move through 3° while the earth moves through 1° per day, it is evident that the daily increase or decrease of A will be 2°; and if, while the earth moves through 1°, the planet move through ½°, the daily increase or decrease of A will be ½°.

If we express, therefore, the daily synodic motion of a planet by σ, its daily heliocentric motion by a, and that of the earth by ε, we shall have, for an inferior planet, whose angular motion exceeds that of the earth,

$$\sigma = a - \varepsilon;$$

and for a superior planet, whose angular motion is slower,

$$\sigma = \varepsilon - a.$$

2570. ***Relation between the synodic motion and the period.***— Since the daily heliocentric motions are found by dividing 360° by the periods, we shall have for an inferior planet

$$\sigma^{\circ} = \frac{360^{\circ}}{\text{P}} - \frac{360^{\circ}}{\text{E}}, \quad \sigma'' = \frac{1296000}{\text{P}} - \frac{1296000}{\text{E}};$$

and for a superior planet

$$\sigma^{\circ} = \frac{360^{\circ}}{\text{E}} - \frac{360^{\circ}}{\text{P}}, \quad \sigma'' = \frac{1296000}{\text{E}} - \frac{1296000}{\text{P}}.$$

2571. *Elongation.*—The geocentric position of a planet in relation to the sun, or the angle formed by lines drawn from the earth to the sun and planet, is called the ELONGATION of the planet, and is EAST or WEST, according as the planet is at the one side or the other of the sun.

2572. *Conjunction.*—When the elongation of a planet is nothing, it is said to be in *conjunction*, being then in the same direction as the sun when seen from the earth.

2573. *Opposition.*—When the elongation of a planet is 180°, it is said to be in OPPOSITION, being then in the quarter of the heavens directly opposite to the sun.

It is evident that a planet which is in conjunction passes the meridian at or very near noon, and is therefore above the horizon during the day, and below it during the night.

On the other hand, a planet which is in opposition passes

the meridian at or very near midnight, and therefore is above the horizon during the night, and below it during the day.

2574. *Quadrature.*—A planet is said to be in quadrature when its elongation is 90°.

In this position it passes the meridian at about six o'clock in the morning, when it has western quadrature, and six o'clock in the evening, when it has eastern quadrature. It is, therefore, above the horizon on the eastern side of the firmament during the latter part of the night in the former case, and on the western side during the first part of the night in the latter case. It is a morning star in the one case, and an evening star in the other.

2575. *Synodic period.*—The interval which elapses between two similar elongations of a planet is called the SYNODIC PERIOD of the planet. Thus, the interval between two successive oppositions, or two successive eastern or western quadratures, is the synodic period.

2576. *Inferior and superior conjunction.*—A superior planet can never be in conjunction except when it is placed on the side of the sun opposite to the earth, so that a line drawn from the earth through the sun would, if continued beyond the sun, be directed to the planet. An inferior planet is, however, also in conjunction when it crosses the line drawn from the earth to the sun, between the earth and sun. The former is distinguished as SUPERIOR and the latter as INFERIOR conjunction.

As inferior conjunction necessarily supposes the planet to be nearer to the sun than the earth, and opposition supposes it to be more distant, it follows that inferior planets alone can be in inferior conjunction, and superior planets alone in opposition.

2577. *Relation between the periodic time and synodic period.*—Since the synodic period is the interval between two similar positions of the earth and planet, the one must gain upon the other 360° in such interval. To perceive this, let S, *fig.* 729. be the sun, P′ the earth, and P an inferior planet when in inferior conjunction, the common direction of the motions of both being indicated by the arrows. Leaving this position, the angular motion of the planet round S being the more rapid, it gains upon the earth as the minute-hand of a watch gains upon the hour-hand; and when, after making a complete revolution, the planet returns to the point P, the earth will have advanced from P′ in the direction of the arrow, so that before

the next inferior conjunction can take place the planet must pass beyond P, and overtake the earth. Let p' be the position

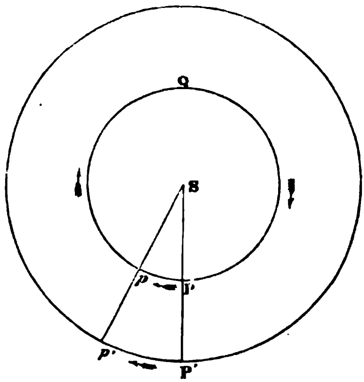


Fig. 729.

of the earth when this takes place, the planet being then at p. It is evident, therefore, that in the interval between two successive inferior conjunctions the planet describes round the sun 360°, together with the angle p S P, which the earth has described in the same interval. If this angle, described by the earth in the synodic period, be called A, the angle described by the planet in the same interval will be 360° + A.

If P represent the place of the earth, and P′ that of a superior planet in opposition, the earth leaving P, and having a more rapid angular motion round S, will get before the planet as the minute-hand gets before the hour-hand, and when it returns to P the planet will have advanced in its orbit, so that before another opposition can take place the earth must overtake it. If this happen when the planet is at p', the earth in the synodic period will have made an entire revolution, and have in addition described the angle p S P, or A, which the planet has described. Thus, while A expresses the angle which the superior planet describes in the synodic period, 360° + A expresses the angle described by the earth in the same time.

If σ, as before, express the daily synodic motion of the planet, we shall have

$$\sigma^\circ = \frac{360^\circ}{T} \qquad \sigma'' = \frac{1296000}{T},$$

and consequently

$$\mathrm{T} = \frac{360^\circ}{\sigma^\circ} = \frac{1296000}{\sigma''}.$$

Thus, when the daily synodic motion is given, the synodic time can be computed, and *vice versâ.*

But since $\sigma = a - \epsilon$, $\sigma = \epsilon - a$ (2569), according as the planet is inferior or superior, we shall have for an inferior planet

$$\frac{360^\circ}{\mathrm{T}} = \frac{360^\circ}{\mathrm{P}} - \frac{360^\circ}{\mathrm{E}},$$

and therefore

$$\frac{1}{\mathrm{T}} = \frac{1}{\mathrm{P}} - \frac{1}{\mathrm{E}}$$

for an inferior planet, and

$$\frac{360^\circ}{\mathrm{T}} = \frac{360^\circ}{\mathrm{E}} - \frac{360^\circ}{\mathrm{P}}$$

and therefore

$$\frac{1}{\mathrm{T}} = \frac{1}{\mathrm{E}} - \frac{1}{\mathrm{P}}$$

for a superior planet, showing in each case the arithmetical relation between T, P, and E.

2578. *The apparent motion of an inferior planet.*— To deduce the apparent from the real motion of an inferior planet, let E, *fig.* 730., be the place of the earth, *s* that of the sun, and *c b c′ e* the orbit of the planet; the direction of the planet's motion being shown by the arrows, the positions which it assumes successively are indicated at *c′*, *a′*, *e*, *a*, *c*, *b*, *e′*, and *b′*. Since the earth moves round the sun in the same direction as the planet, the apparent motion of the sun *s* will be from the left to the right of an observer looking from E at *s*; and since this motion is always from west to east, the planet will be west of the sun when it is any where in the semicircle *c e′ b′ c′*, and east of it when it is any where in the semicircle *c′ a′ e a c*.

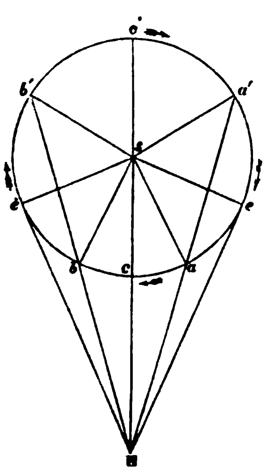


Fig. 730.

The elongation (2571) of the planet, being the angle formed by lines drawn to the sun and planet from the earth, will always be east when the planet is in the semicircle *c′ec*, and west when in the semicircle *c′e′c*.

The planet will have its greatest elongation east when the line E *e* directed to it from the earth is a tangent to its orbit, and in like manner its greatest elongation west when the line E *e′* is a tangent to the orbit.

In these positions the angle *se*E, or *se′*E, at the planet is 90°, and consequently the elongation and the angle *es*E, or *e′s*E at the sun, taken together, make up 90°.

It appears, therefore, that the greatest elongation of an inferior planet must be less than 90°.

If the earth were stationary the real orbital motion of the planet would give it an apparent motion alternately east and west of the sun, extending to a certain limited distance, resembling the oscillation of a pendulum. While the planet moves from *c′* to *e*, it will appear to depart from the sun eastward, and when it moves from *e* to *c*, it will appear to return to the sun; the elongation in the former case constantly increasing till it attain its maximum eastward, and in the latter constantly decreasing till it become nothing. It is to be observed, however, that the orbital arc *c′e* being greater than *ec*, the time of attaining the greatest eastern elongation after superior conjunction is greater than the time of returning to the sun from the greatest elongation to inferior conjunction.

After inferior conjunction, while the planet passes from *c* to *e′*, its elongation constantly increases from nothing at *c* to its maximum west at *e′*; and when the planet moves from *e′* to *c′*, it again decreases until it becomes nothing at superior conjunction. Since the orbital arcs, *ce′* and *e′c′*, are respectively equal to *ce* and *c′e*, it follows, that the interval from inferior conjunction to the greatest elongation west, is equal to the interval from the greatest elongation east to inferior conjunction. In like manner, the interval from superior conjunction to the greatest elongation east, is equal to the interval from the greatest elongation west to superior conjunction.

The oscillation of the planet alternately east and west is therefore made through the same angle—that is, the angle *e*E*e′*, included by tangents drawn to the planet's orbit from the earth; but the apparent motion from the greatest elongation west to

the greatest elongation east is slower than the apparent motion from the greatest elongation east to the greatest elongation west, in the ratio of the length of the orbitual arcs $ec'e'$ to ece'.

The planet being included within the orbit of the earth, the orbital motion of the earth will give it an apparent motion in the ecliptic in the same direction as the apparent motion of the sun; but since the apparent motion of a visible object increases as its distance decreases, and *vice versâ*, and since the planet being at a considerable distance from the centre of the earth's orbit, the distance of the earth from it is subject to variation, the apparent motion imparted to the planet by the earth's orbital motion will be subject to a proportionate variation, being greatest when the planet is in inferior conjunction, and least when in superior conjunction.

The apparent motion of the planet, as it is projected upon the firmament by the visual ray, arises from the combined effect of its own orbital motion and that of the earth. Now it is evident from what has been just explained, that the effect of the planet's own motion is to give it an apparent motion from west to east while passing from its greatest elongation west through superior conjunction to its greatest elongation east, and a contrary apparent motion from east to west while passing from its greatest elongation east to its greatest elongation west through inferior conjunction.

But since, in all positions, the effect of the orbital motion of the earth is to give the planet an apparent motion directed from west to east, both causes combine to impart to it this apparent motion while passing from its western to its eastern elongation through superior conjunction. On the other hand, the effect of the orbital motion of the planet being an apparent motion from east to west in passing from its eastern to its western elongation through inferior conjunction, while, on the contrary, the earth's motion imparts to it an apparent motion from west to east, the actual apparent motion of the planet, resulting from the difference of these effects, will be westward or eastward according as the effect of the one or the other predominates, and the planet will be stationary when these opposite effects are equal.

In leaving the greatest eastern elongation the effect of the earth's motion predominates, and the apparent motion of the planet continues to be, as before, eastward. As, in approaching inferior conjunction, the direction of the planet's motion be-

comes more and more transverse to the visual line, and the distance of the planet decreases, the effect of the planet's motion increasing becomes, at length, equal to the effect of the earth's motion, and the planet then becomes stationary. This takes place at a certain elongation east. After this, the effect of the planet's motion predominating, the apparent motion becomes westward, and this westward motion continues through inferior conjunction, until the planet acquires a certain elongation west, equal to that at which it became previously stationary. Here the effects becoming again equal, the planet is again stationary, after which, the effect of the earth's motion predominating, the apparent motion becomes eastward, and continues so to the greatest elongation west, after which, as before, both causes combine in rendering it eastward.

2579. *Direct and retrograde motion.*—When a planet appears to move in the direction in which the sun appears to move, its apparent motion is said to be DIRECT; and when it appears to move in the contrary direction, it is said to be RETROGRADE.

From what has been explained above, it appears that the apparent motion of an inferior planet is always direct, except within a certain elongation east and west of inferior conjunction, when it is retrograde.

The extent of this arc of retrogression depends on the relative distances, and consequent relative orbital velocities, of the earth and planet.

2580. *Apparent motion as projected on the ecliptic.*—From what has been here explained the apparent motion of the planet on the firmament will be easily understood. Let ABEFK, *fig.* 731., represent the ecliptic in which the planet is at present supposed to move. While passing from its western to its eastern elongation it appears to move in the same direction as the sun, from A towards B. As it approaches B its apparent motion eastward becomes gradually slower until it stops altogether at B, and becomes, for a short interval, stationary; it then moves westward, returning upon its course to C, where it again becomes stationary; after which it again moves eastward, and continues to move in that direction till it arrives at a certain point D, where it again becomes stationary; and then, returning upon its course, it again moves westward to E, where it again becomes stationary; after which it again changes its direction and moves eastward to F, where, after being stationary, it turns westward, and so on.

The middle points of the arcs BC, DE, FG, &c. of retrogression are those at which the planet is in inferior conjunction; and the

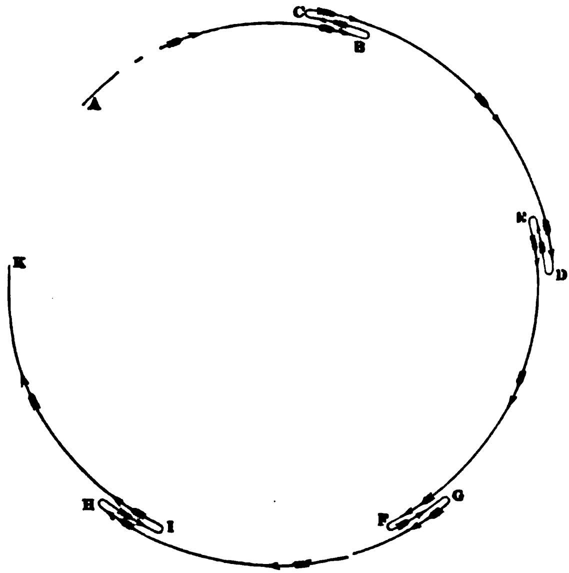


Fig. 731.

middle points of the arcs CD, EF, GH, &c. of progression are those at which the planet is in superior conjunction.

2581. *Origin of the term "planet."*—These complicated and apparently irregular movements, by which the planets are distinguished from all other celestial objects, suggested to the ancients, whose knowledge of astronomy was too imperfect to enable them to trace such motions to fixed and regular laws, the name planet, from the Greek word πλανήτης (planetes), *wanderer.*

2582. *Apparent motion of a superior planet.*—To deduce the apparent motion of a superior planet from the real orbital motions of the earth and the planet, let S, *fig.* 732., be the place of the sun, P that of the planet, and E E′ E‴ E″ the orbit of the earth included within that of the planet, the direction of the motions of the earth and planet being indicated by the arrows.

When the earth is at E‴, the sun S and planet P are in the same visual line, and the planet is consequently in conjunction.

When the earth moves to e', the elongation of the planet west of the sun is $\text{S}e'\text{P}$. This elongation increasing as the earth moves

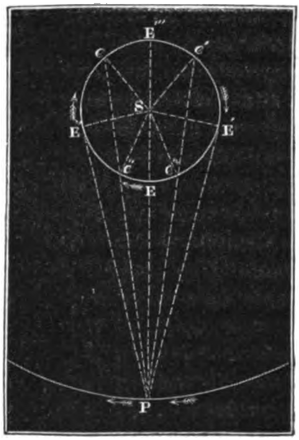


Fig. 732.

in its orbit, becomes 90° at E′, when the visual direction E′P of the planet is a tangent to the earth's orbit, and the planet is then in its western quadrature.

While the earth continues its orbital motion to e''', the elongation west of the sun continues to increase, and at length, when the earth comes to the position E, it becomes 180°, and the planet is in opposition.

After passing E, when the earth moves towards e'', the elongation of the planet is east of the sun, and is less than 180°, but greater than 90°. As the earth continues to advance in its orbit, the elongation decreasing becomes 90° when, at E″, the visual direction of the planet is a tangent to the earth's orbit. The planet is then in its eastern quadrature.

As the earth moves from E″ to E‴, the elongation, being still east, constantly decreases until it becomes nothing at E‴, where the planet is in conjunction.

2583. *Direct and retrograde motion.*—If the planet were immoveable, the effect of the earth's motion would be to give it an oscillatory motion alternately eastward and westward through the angle E′PE″, which the earth's orbit subtends at the planet. While the earth moves from E″ through E‴ to E′, the planet would appear to move *eastward* through the angle E′PE″, and

while the earth moves from E′ through E to E″, it would appear to move *westward* through the same angle.

Thus the effect of the earth's motion alone is to make the planet appear to move from east to west and from west to east alternately through a certain arc of the ecliptic, the length of which will depend on the relation between the distances of the earth and planet from the sun, the arc being in fact measured by the angle which the earth's orbit subtends at the planet, and, consequently, this angle of apparent oscillation will decrease in the same ratio as the distance of the planet increases.

The times in which the two oscillations eastward and westward would be made are not equal, the time from the western to the eastern quadrature being less than the time from the eastern to the western quadrature in the ratio of the orbital arc E′ E E″ to the arc E″ E‴ E′.

It is evident that the more distant the planet P is the less unequal these arcs, and, consequently, the less unequal the intervals between quadrature and quadrature will be.

But, meanwhile, the earth being included within the orbit of the planet, the effect of the planet's orbital motion will be to give it an apparent motion in the ecliptic always in the same direction in which the sun would move when in the same place, and therefore always eastward or direct.

This apparent motion, though always direct, is not uniform, since it increases in the same ratio as the distance of the earth from the planet decreases, and *vice versâ*. This apparent motion thus due to the planet's own orbital motion is, therefore, greater from western to eastern quadrature than from eastern to western quadrature.

From eastern to western quadrature, through conjunction, the apparent motion of the planet is direct, because both its own orbital motion and that of the earth combine to render it so. From western quadrature, as the planet approaches opposition, the effect of the earth's motion is to render the planet retrograde, while the effect of its own motion is to render it direct. On leaving quadrature the latter effect predominates, and the apparent motion is direct; but at a certain elongation, before arriving at opposition, the effect of the earth's motion increasing becomes equal to that of the planet, and, neutralising it, renders the planet stationary; after which, the effect of the earth's motion predominating, the planet becomes retrograde, and con-

tinues so until it acquires an equal elongation east, when it again becomes stationary, and is afterwards direct, and continues so.

2584. *Apparent motion projected on the ecliptic.*—Let A, *fig.* 733., represent the place of a superior planet when moving from

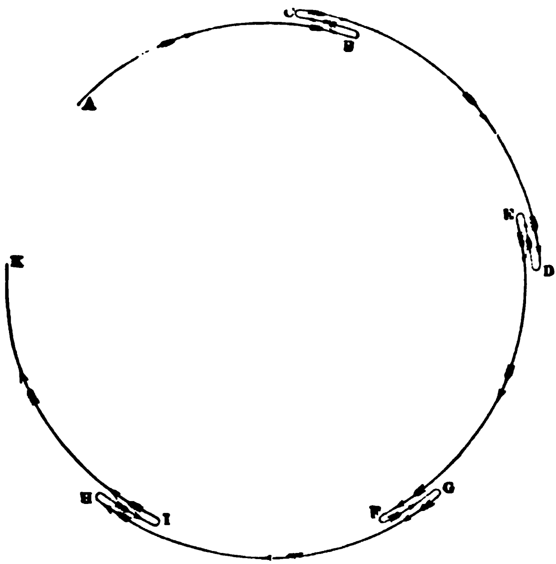


Fig. 733.

its western quadrature towards conjunction, its apparent motion being then direct. Let B be the point where it becomes stationary after its eastern quadrature; its apparent motion then becoming retrograde, it appears to return upon its course and moves westward to C, where it again becomes stationary; after which it again returns on its course and moves direct or eastward, and continues so until it arrives at a certain point D after its western quadrature, when it again becomes stationary, and then again retrogrades, moving through the arc DE, which will be equal to BC; after which it will again become direct, and so on.

The places of the planet's opposition are the middle points of the arcs of retrogression BC, DE, FG, &c.; and the places of conjunction are the middle points of the arcs of progression CD, EF, GH, &c.

It is evident, therefore, that the apparent motion of a superior planet projected on the ecliptic is, in all respects, similar to that of an inferior planet, the difference being, that in the latter the middle point of the arc of retrogression corresponds to inferior conjunction, while in the former it corresponds to opposition.

It will be apparent from what has been explained that the angle which the earth gains upon the planet in the interval between s western and eastern quadratures is the angle which the earth's orbit subtends at the planet, or twice the annual parallax (2442) of the planet.

2585. *Conditions under which a planet is visible in the absence of the sun.* — It is evident that to be visible in the absence of the sun a celestial object must be so far elongated from that luminary as to be above the horizon before the commencement of the morning twilight or after the end of the evening twilight. One or two of the planets have, nevertheless, an apparent magnitude so considerable, and a lustre so intense, that they are sometimes seen with the naked eye, even before sunset or after sunrise, and may, in some cases, be seen with a telescope when the sun has a considerable altitude. In general, however, to be visible without a telescope, a planet must have an elongation greater than 30° to 35°.

2586. *Evening and morning star.*—Since the inferior planets can never attain so great an elongation as 90°, they must always pass the meridian at an interval considerably less than six hours before or after the sun. If they have eastern elongation they pass the meridian in the afternoon, and are visible above the horizon after sunset, and are then called EVENING STARS. If they have western elongation they pass the meridian in the forenoon, and are visible above the eastern horizon before sunrise, and are then called MORNING STARS.

2587. *Appearance of superior planets at various elongations.* —A superior planet, having every degree of elongation east and west of the sun from 0° to 180°, passes the meridian during its synodic period at all hours of the day and night. Between conjunction and quadrature, its elongation east or west of the sun being less than 90°, it passes the meridian earlier than six o'clock in the afternoon in the former case, and later than six o'clock in the forenoon in the latter case, being, like an inferior planet, an evening star in the former and a morning star in the latter case.

At eastern quadrature it passes the meridian at six in the evening, and at western quadrature at six in the morning, appearing still as an evening star in the former and as a morning star in the latter case.

Between the eastern quadrature and opposition, the elongation being more than 90° east of the sun, the planet must pass the meridian between six o'clock in the evening and midnight, and is therefore visible from sunset until some hours before sunrise. Between western quadrature and opposition, the elongation being more than 90° west of the sun, the planet must pass the meridian at some time between midnight and six o'clock in the morning, and it is therefore visible from some hours after sunset until sunrise.

At opposition the planet passes the meridian at midnight, and is therefore visible from sunset to sunrise.

2588. *To find the periodic time of the planet.*—There are several solutions of this problem, which give results having different degrees of approximation to the exact value of the quantity sought.

2589. 1°. *By means of the synodic period.*—If the synodic period T be ascertained by observation, we shall have for an inferior planet (2577),

$$\frac{1}{T} = \frac{1}{P} - \frac{1}{E},$$

and consequently

$$\frac{1}{P} = \frac{1}{E} + \frac{1}{T};$$

and for a superior planet

$$\frac{1}{T} = \frac{1}{E} - \frac{1}{P},$$

and therefore

$$\frac{1}{P} = \frac{1}{E} - \frac{1}{T}.$$

In each case, therefore, P may be found, E and T being known.

This method gives a certain approximation to the value of the period; but the synodic time not being capable of very exact appreciation by observation, the method does not supply extremely accurate results.

2590. 2°. *By observing the transit through the nodes.*—The

periodic time may also be determined by observing the interval between two successive passages of the planet through the plane of the ecliptic.

It has been already stated that, although the planets move *nearly* in the plane of the ecliptic, they do not *exactly* do so. Their paths are inclined at very small angles to the ecliptic, and they consequently must pass from one side to the other of the plane of the earth's orbit twice in each revolution. If the moments of thus passing through the plane of the earth's orbit on the same side of the sun be observed twice in immediate succession, the interval will be the periodic time.

Owing to the very small inclination of the orbits in general, it is impossible to ascertain with great precision the time of the centre of the planet passing through the ecliptic, and therefore this method is only approximation.

2591. 3°. *By comparing oppositions or conjunctions having the same sidereal place.*—The periodic time of a planet being approximately found by either of the preceding methods, it may be rendered more exact by the following.

When a planet is in superior conjunction or in opposition its place in the firmament is the same, whether viewed from the earth or from the sun. Now, if two oppositions or conjunctions separated by a long interval of time be found, at which the apparent place of the planet in the firmament is the same, it may be inferred that a complete number of revolutions must have taken place in the interval. Now the periodic time being found approximately by either of the methods already explained, it will be easy to find by it how many revolutions of the planet must have taken place between the two distant oppositions. If the periodic time were known with precision, it would divide the interval in question *without a remainder;* but being only approximate it divides it with a remainder. Now the nearest multiple of the approximate period to the interval between the two oppositions will be that multiple of the true period which is exactly equal to the interval. The division of the interval by the number thus determined will give the more exact value of the period.

2592. 4°. *By the daily angular motion.*—The daily angular geocentric motion may be observed, and the heliocentric motion thence computed. If the mean heliocentric daily motion a'' can be obtained by means of a sufficient number

of observations, the period will be given by the formula (2568),

$$P = \frac{1296000}{a''}.$$

2593. *To find the distances of the planets from the sun.*— One of the most obvious methods of solving this problem is by observing the elongation of the planet, and computing, as always may be done, the angle at the sun. Two angles of the triangle formed by the earth, sun, and planet, will thus be known, and a triangle may be drawn of which the sides will be in the same proportion as those of the triangle in question. The ratio of the earth's distance from the sun to the planet's distance from the sun will thus become known (2296); and as the earth's distance has been already ascertained, the planet's distance may be immediately computed.

Other methods of determining the distances will be explained hereafter.

2594. *Phases of a planet.*—While a planet revolves, that hemisphere which is presented to the sun is illuminated, and the other dark. But since the same hemisphere is not presented generally to the earth, it follows that the visible hemisphere of the planet will consist of a part of the dark and a part of the enlightened hemisphere, and, consequently, the planet will exhibit PHASES, the varieties and limits of which will depend upon the relative directions of the lines drawn from the earth and sun to the planet. It is evident that the section of the planet at right angles to a line drawn from the sun to its centre is the base of its enlightened hemisphere, while the section at right angles to a line drawn from the earth to its centre, is the base of its visible hemisphere. The less the angle included between these lines is, the greater will be the portion of the visible hemisphere which is enlightened.

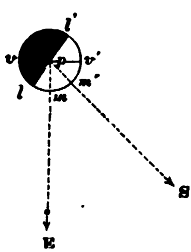


Fig. 734.

Let p, *fig.* 734., be the centre of the planet, pS the direction of a line drawn to the sun, and pE that of one drawn to the earth; ll' will then be the base of the enlightened, and vv' the base of the visible hemisphere of the planet. The point m' will be the centre of the former, and m of the latter. The visible hemisphere

will then be enlightened over the space $v'm'ml$, the part vl being dark. This dark part will be measured by the arc vl, which is evidently equal to mm', and therefore measured by the angle formed by the lines pS and pE drawn from the planet in the directions of the sun and the earth.

When this angle SpE is less than 90°, as in *fig.* 734., the breadth of the enlightened part $v'm'ml$ of the visible hemisphere is greater than 90°, and the planet appears gibbous, as the moon does when between opposition and quadratures.

When the angle SpE is greater than 90°, as in *fig.* 735., the breadth $v'l$ of the enlightened part of the visible hemisphere is less than 90°, and the planet appears as a crescent, like the moon between conjunction and quadrature.

When the angle SpE = 0, which happens when the earth is between the sun and planet, as in *fig.* 736., the centre m' of the

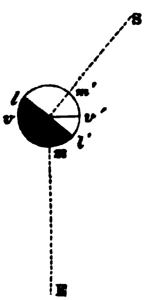


Fig. 735.

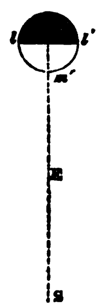


Fig. 736.

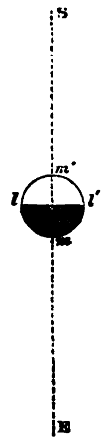


Fig. 737.

enlightened hemisphere is presented to the earth, and the planet appears with a full phase, as the moon does in opposition. This always happens when the planet is in opposition.

When the angle SpE becomes =180°, as in *fig.* 737., the centre m of the dark hemisphere is presented to the earth, and therefore the entire hemisphere turned in that direction is dark. This takes place when the planet is between the earth and sun,

which can only happen when an inferior planet is in inferior conjunction.

2595. *Phases of an inferior planet.*—It will be evident from inspecting the diagram, *fig.* 730., representing the relative positions of an inferior planet with respect to the sun and earth, that the angle formed by lines drawn from the planet to the sun and earth passes through all magnitudes from 0° to 180°, and consequently such a planet exhibits every variety of phase. Passing from *c* towards *e'*, the angle *s b* E gradually decreases from 180° to 90°, and therefore the phase, at first a thin crescent, increases in breadth until it is halved like the moon in quadrature. From *e'* to *c'* the angle *s b'* E gradually decreases from 90° to 0°, and the planet beginning by being gibbous, the breadth of the enlightened part gradually increases until it becomes full at *c'*. From *c'* to *e*, and thence to *c*, these phases are reproduced for like reasons in the opposite order.

2596. *Phases of a superior planet.*—It will be evident on inspecting *fig.* 738., that in all positions whatever of a superior planet, the lines drawn from it to the earth are inclined at an angle less than 90°; and this angle is so much the smaller the greater the orbit of the planet is comparatively with that of the earth. The angle *s a* E being nothing at *o* increases until the planet is in quadrature at *q'*, where it is greatest; and then the breadth of the enlightened part is least, and is equal to the difference between the angle *s q'* E and 180°. From *q'* to *c* the angle *s b'* E decreases, and becomes nothing at *c*. The planet is therefore full at opposition and conjunction, and is most gibbous at quadrature.

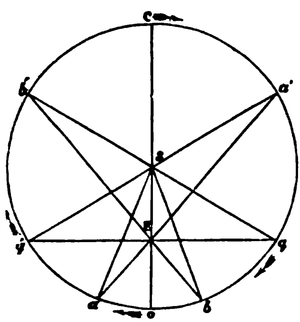


Fig. 738.

It will appear hereafter that, with one exception, all the superior planets are at distances from the sun so much greater than that of the earth, that even at quadrature the angle *s q'* E is so small, that the departure of the phase from fulness is not sensible.

2597. *The planets are subject to a central attraction.*—If a body in motion be not subject to the action of any external force,

it must move in a straight line. If, therefore, it be observed to move in any curvilinear path, it may be inferred that it is acted upon by some force or forces exterior to it, which constantly deflect it from the straight course which, in virtue of its inertia, it must follow if left to itself (220). This force must, moreover, be incessant in its operation, since, if its action were suspended for a moment, the body during such suspension would move in the direction of the straight line, which would be a tangent to the curve at the point where the action of the force was suspended.

Now, since the orbits of the planets, including the earth, are all curved, it follows that they are all under the incessant operation of some force or forces, and it becomes an important problem to determine what is the direction of these forces, whether they are one or several, and, in fine, whether they are of invariable intensity, or, if variable, what is the law and conditions of their variation.

2598. *What is the centre to which this attraction is directed?* — We are aided in this inquiry by a principle of the highest generality and the greatest simplicity, established by Newton, the demonstration of which forms the subject of the first two propositions of his celebrated work, entitled the "PRINCIPIA."

2599. *General principle of the centre of equal areas demonstrated.* — If from any point taken as fixed a straight line be drawn to a body which moves in a curvilinear path, such line is called the *radius vector* of the moving body with relation to that point as a centre of motion. As the body moves along its curvilinear path, the radius vector sweeps over a certain superficial area, greater or less, according to the velocity and direction of the motion and the length of the radius vector. This superficial space is called the "area described by the radius vector," or, sometimes, the "area described by the moving body."

Thus, for example, if C, *fig.* 739., be the point taken as the centre of motion, and B B″ be a part of the path of the moving body, CB and CB′ will be two positions of the "radius vector," and in passing from one of these positions to the other, it will sweep over or "describe" the superficial space or "area" included between the lines CB and CB′, and the body

is said shortly to "describe this area round the point C as a centre."

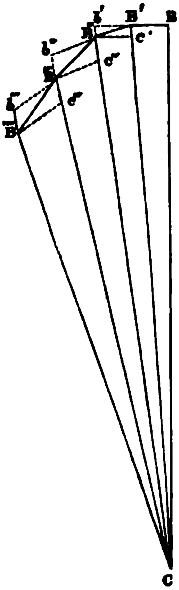


Fig. 739.

Now, according to the principle established by Newton, it appears that whenever a body moves in a curvilinear orbit, under the attraction of a force directed to a fixed centre, such a body will describe round such centre equal areas in equal times; and it is proved also conversely, that if a point can be found within the curvilinear orbit of a revolving body, round which such body describes equal areas in equal times, such body is in that case subject to the action of a single force, always directed towards that point as a centre. It follows, in short, that "*the centre of equable areas is the centre of force, and that the centre of force is the centre of equable areas.*"

As this is a principle of high generality and capital importance, and admits of demonstration by the most elementary principles of mechanics and geometry, it may be proper to explain it here.

If a body B move independently of the action of any force upon it, its motion must be in a straight line, and must be uniform. It must, therefore, move over equal spaces per second. Let its velocity be such, that in the first second it would move from B to B′. In the next second, if no force acted upon it, it would move through the equal space B′b' in the same direction. But if at B′ it receive from a force directed to C, an impulse which in a second would carry it from B′ to c', it will then be affected by two motions, one represented by B′b', and the other by B′c', and it will move in the diagonal B′B″ of the parallelogram, and at the end of the second second will be at B″.

Now, in the first second, the *radius vector* described the area BCB′, and in the next second it described the area B′CB″. It is easy to show that these areas are equal. For since BB′ = B′b', the areas BCB′ and B′Cb' are equal; and since b'B″ is parallel to B′C, the areas B′Cb' and B′CB″ are equal by the well-known

property of triangles. Therefore the areas BCB′ and B′CB″, described by the radius vector in the first and second seconds, are equal.

If the body received no impulse from the central force at B″, it would move over B″b″ = B′B″ in the third second, but receiving from the central force another impulse sufficient to carry it from B″ to c″, it again moves over the diagonal B″B‴ of the next parallelogram, and at the end of the third second is found at B‴. It is shown in the same manner that the area of the triangle B″CB‴ is equal to B′CB″ and to BCB′; so that in every succeeding second the radius vector describes round C an equal area.

In this case it has been supposed that the force, instead of acting continuously, acts by a succession of impulses at the end of each second, and the body describes, not a curve, but a polygon. If the succession of impulses were by tenths, hundredths, or thousandths, or any smaller fraction of a second, the areas would still be in the ratio of the times, but the polygon would have more numerous and smaller sides. In fine, if the intervals of the action of the force be infinitely small, the sides of the polygon would be infinitely small in magnitude and great in number. The force would, in fact, be continuous, instead of being intermitting, and the path of the body would be a curve, instead of being a polygon. The areas, however, described by the radius vector round the centre of force C, would still be proportional to the time.

The converse of the principle is easily inferred by reasoning altogether similar. If C, *fig.* 739., be the centre of equal areas, it will be the centre of attraction; for let B′b′ be taken equal to BB′. The triangular area B′Cb′ will then be equal to the area BCB′ by the common properties of triangles, and since the areas described round C in successive seconds are equal, we have the area B′CB″ = BCB′, and therefore = B′Cb′. Hence we infer that B′CB″ = B′Cb′, and therefore that the line b′B″ is parallel to B′c. The force therefore expressed by the diagonal B′B″ of the parallelogram is equivalent to the forces expressed by the sides. The body at B′ therefore, besides the projectile force BB′ or B′ b′, is urged by a central force directed to C.

2600. *Linear, angular, and areal velocity.*—In the description and analysis of the planetary motions, there are three

quantities which there is frequent occasion to express in reference to the unit of time, and to which the common name of "velocity" is consequently applied.

1°. The linear velocity of a planet is the actual space over which it moves in its orbit in the unit of time. We shall invariably express this velocity by V.

2°. The angular velocity is the angle (B C B′ in *fig.* 739.), which the radius vector from the sun to the planet moves over in the unit of time. We shall invariably express this by the Greek letter a.

3°. The areal velocity is the area (B C B′ in *fig.* 739.), which the radius vector from the sun to the planet sweeps over in the unit of time. We shall express this by A.

2601. *Relation between angular and areal velocities.*—If B″ c′, *fig.* 739., be supposed to be perpendicular to B′ C, the area of the triangle B′ C B″ will be $\frac{1}{2}$ B′ C × B″ c′. But since in this case B″ c′ may be considered as the arc of a circle, of which C is the centre and B″ C the radius, we shall have (2292),

$$\text{B}'' c' = \frac{r \times a}{206265},$$

where the distance B″ C of the planet from the sun, or the radius vector, is expressed by r. Hence we have

$$\text{A} = \tfrac{1}{2}\,\text{B}'\,\text{C} \times \text{B}''\,c' = \frac{\tfrac{1}{2}\,r^2 \times a}{206265}.$$

Hence the areal velocity is always proportional to the product of the angular velocity and the square of the radius vector or distance.

To ascertain, therefore, whether any point within the orbit of a planet be the "centre of equal areas," and therefore the centre of attraction, it is only necessary to compare the angular velocity round such point with the square of the distance; and if their product be always the same, or, in other words, if the angular velocity increase in the same ratio as the square of the distance or radius vector decreases, and *vice versâ*, then the point in question must be the centre of equal areas, and therefore the centre of attraction.

2602. *Case of the motion of the earth.*—In the case of the earth, the variation of its distance from the sun is inversely as the variation of the sun's apparent diameter, which may be accurately observed, as may also be the sun's apparent motion

in the firmament. Now, it is found that the apparent motion of the sun increases exactly in the same ratio as the square of its apparent diameter, and therefore inversely as the square of its distance; from which it follows that its centre is the centre of equal areas for the earth's motion, and therefore the centre of attraction.

2603. *Case of the planets.*—In the same manner, by calculating from observation the angular motions of the planets, and their distances from the sun, it may be shown that their angular motions are inversely as the squares of their distances, and consequently that the centre of the sun is the centre of the attraction which moves them.

2604. *Orbits of the planets ellipses.*—By comparing the variation of the distance of any planet from the sun with the change of direction of its radius vector, it may be ascertained that its orbit is an ellipse, the centre of the sun being at one of the foci, in the same manner as has been already explained in the case of the earth.

2605. *Perihelion, aphelion, mean distance.*—That point of the elliptic orbit at which a planet is nearest to the sun is called PERIHELION, and that point at which it is most remote is called APHELION.

The MEAN DISTANCE of a planet from the sun is half the sum of its greatest and least distances.

2606. *Major and minor axes, and eccentricity of the orbit.*—The *fig.* 740. represents an ellipse, of which F is the focus and C the centre. The line C F continued to P and A is the MAJOR AXIS, sometimes called the transverse axis. Of all the diameters which can be drawn through the centre C, terminating in the curve, it is the longest, while M C M′, drawn at right angles to it, called the MINOR AXIS, is the shortest. The line F M′, which is equal to P C, half the major axis, and therefore to half the sum of the greatest and least distances of the ellipse from its focus, is the MEAN DISTANCE.

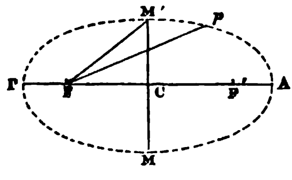


Fig. 740.

A planet is, therefore, at its mean distance from the sun

when it is at the extremities of the minor axis of its orbit.

There is another point F′ on the major axis, at a distance F′ C from the centre, equal to F C, which has also the geometric properties of the focus. It is sometimes distinguished as the EMPTY FOCUS of the planet's orbit.

Ellipses may be more or less ECCENTRIC, that is to say, more or less oval. The less eccentric they are the less they differ in form from a circle. The degree in which they have the oval form depends on the ratio which the distance F C of the focus from the centre bears to P C, the semi-axis major. Two ellipses of different magnitudes in which this ratio is the same, have a like form, and are equally eccentric. The less the ratio of C F to C P is, the more nearly does the ellipse resemble a circle. This ratio is, therefore, called the ECCENTRICITY.

The eccentricity of a planet's orbit will, therefore, be that number which expresses the distance of the sun from the centre of the ellipse, the semi-axis major of the orbit being taken as the unit.

2607. *Apsides, anomaly.*—The points of PERIHELION and APHELION, are called by the common name of APSIDES.

If an eye placed at the sun F look in the direction of P, that point will be projected upon a certain point on the firmament. This is called the PLACE OF PERIHELION.

The angle formed by a line drawn from the sun to the place *p* of a planet, and the major axis of its orbit, or, what is the same, the angular distance of the planet from its perihelion, as seen from the sun, is called its ANOMALY.

If an imaginary planet be supposed to move from perihelion to aphelion with any uniform angular motion round the sun in the same time that the real planet moves between the same points with a variable angular motion, the anomaly of this imaginary planet is called the MEAN ANOMALY of the planet.

2608. *Place of perihelion.*—The PLACE OF PERIHELION is expressed by indicating the particular fixed star at or near which the planet at P is seen from F, or, what is the same, the distance of that point from some fixed and known point in the heavens. The point selected for this purpose is the vernal equinoxial

point, or the *first point of Aries* (2435). The distance of perihelion from this point, as seen from the sun, is called the LONGITUDE OF PERIHELION, and is an important condition affecting the position of the planet's orbit in space.

2609. *Eccentricities of orbits small.*—The planets' orbits, like that of the earth, though elliptical, are very slightly so. The eccentricities are so minute, that if the form of the orbit were delineated on paper, it could not be distinguished from a circle except by very exactly measuring its breadth in different directions.

2610. *Law of attraction deduced from elliptic orbit.*—As the equable description of areas round the centre of the sun proves that point to be the centre of attraction, the elliptic form of the orbit and the position of the sun in the focus indicate the LAW according to which this attraction varies as the distance of the planet from the sun varies. Newton has demonstrated, in his PRINCIPIA, that such a motion necessarily involves the condition that the intensity of the attractive force, at different points of the orbit, varies inversely as the square of the distance, increasing as the square of the distance decreases, and *vice versâ.*

2611. *The orbit might be a parabola or hyperbola.*—Newton also proved that the converse is not necessarily true, and that a body may move in an orbit which is not elliptical round a centre of force which varies according to this law. But he showed that the orbit, if not an ellipse, must be one or other of two curves, a PARABOLA or HYPERBOLA, having a close geometric relation to the ellipse, and that in all cases the centre of force would be the focus of the curve.

These three sorts of curves, the ellipse, the parabola, and hyperbola, are those which would be produced by cutting a cone in different directions by a plane, and they are hence called the CONIC SECTIONS.

2612. *Conditions which determine the species of the orbit.*—The conditions under which the orbit of a planet might be a parabola or hyperbola, depend on the relation which the velocity of the motion of the planet, at any given point of the orbit, bears to the intensity of the attractive force at that point. It is demonstrable that, if the velocity with which a planet moves at any given point of its orbit were suddenly augmented in a certain proportion, its orbit would become a

parabola, and if it were still more augmented, it would become an hyperbola.

The ellipse is a curve which, like the circle, *returns into itself*, so that a body moving in it must necessarily retrace the same path in an endless succession of revolutions. This is not the character of the parabola or hyperbola. They are not closed curves, but consist of two branches which continue to diverge from each other without ever meeting. A planet, therefore, which would thus move, would pass near the sun once, following a curved path, but would then depart never to return.

2613. *Law of gravitation general.* — The elliptic form of the orbit of a planet indicates the law which governs the variation of the sun's attraction from point to point of such orbit; but beyond this orbit it proves nothing. It remains, therefore, to show from the planetary motions round the sun, and from the motions of the satellites round their primaries, that the same law of attraction by which the intensity decreases as the square of the distance from the centre of attraction increases, and *vice versâ*, is universal.

The attraction exerted upon any body may be measured, in general, as that of the earth on bodies near its surface is measured, by the spaces through which the attracted body would be drawn in a given time. It has been shown, that the attraction which the earth exerts at its surface, is such as to draw a body towards it through 193 inches in a second. Now if the space through which the sun would, by its attraction at any proposed distance, draw a body in one second could be found, the attraction of the sun at that distance could be exactly compared with and measured by the attraction of the earth, just as the length of any line or distance is ascertained by applying to it and comparing it with a standard yard measure.

2614. *Method of calculating the central force by the velocity and curvature.* — Now the space through which any central attraction would draw a body in a given time can be easily calculated, if the body in question moves in a circular or nearly circular orbit round such a centre, as all the planets and satellites do.

Let E, *fig.* 741., be the centre of attraction, and E m the distance or radius vector. Let $m\,m' = \text{v}$, the linear velocity. Let $m\,n$ and $m'\,n'$ be drawn at right angles to E m, and therefore

parallel to each other. The velocity $m\,m'$ may be considered as compounded of two (173), one in the direction $m\,n'$ of the tangent, and the other $m\,n$ directed towards the centre of attraction E. Now if the body were deprived of its tangential motion $m\,n'$, it would be attracted towards the centre E, through the space $m\,n$, in the unit of time. By means of this space, therefore, the force which the central attraction exerts at m can be brought into direct comparison with the force which terrestrial gravity exerts at the surface of the earth.

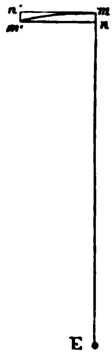


Fig.

It follows, therefore, that if f express the space through which such a body would be drawn in the unit of time, falling freely towards the centre of attraction, we shall have $f = m\,n$. But by the elementary principles of geometry,

$$m\,n \times 2\,\text{E}\,m = m\,m'^{2}.$$

Therefore,

$$f = \frac{\text{V}^2}{2r};$$

that is, the space through which a body would be drawn towards the centre of attraction, if deprived of its orbital motion, in the unit of time, is found by dividing the square of the linear orbital velocity by twice its distance from the centre of attraction.

Since $\text{V} = \dfrac{r \times a}{206265}$ (2601), we shall also have

$$f = \frac{r \times a^2}{2 \times 206265}.$$

The attractive force, or, what is the same, the space through which the revolving body would be drawn towards the centre in the unit of time, can, therefore, be always computed by these formulæ, when its distance from the centre of attraction and its linear or angular velocity are known.

Since (2568)

$$a = \frac{1296000}{\text{P}},$$

this, being substituted for a in the preceding formula, will give

$$f = 412541 \times \frac{r}{\text{P}^2};$$

by which the attractive force may always be calculated when the distance and period of the revolving body are known.

2615. *Law of gravitation shown in the case of the moon.* — The attraction exerted by the earth, at its surface, may be compared with the attraction it exerts on the moon by these formulæ.

In the case of the moon $v=0{\cdot}6356$ miles, and $r=239{,}000$ miles, and by calculation from these data, we find

$$f=0{\cdot}0000008459^{\text{miles}}=0{\cdot}0536^{\text{inch.}}$$

The attraction exerted by the earth at the moon's distance would, therefore, cause a body to fall through 536 ten-thousandths of an inch, while at the earth's surface it would fall through 193 inches (247).

The intensity of the earth's attraction on the moon is, therefore, less than its attraction on a body at the surface, in the ratio of 1,930,000 to 536, or 3600 to 1, or, what is the same, as the square of 60 to 1.

But it has been shown that the moon's distance from the earth's centre is 60 times the earth's radius. It appears, therefore, that in this case the attraction of the earth decreases as the square of the distance from the attracting centre increases; and that, consequently, the same law of gravitation prevails as in the elliptic orbit of a planet.

2616. — *Sun's attraction on planets compared — law of gravitation fulfilled.* — In the same manner, exactly, the attractions which the sun exerts at different distances may be computed by the motions and distances of the planets. The distance of a planet gives the circumference of its orbit, and this, compared with its periodic time, will give the arc through which it moves in a day, an hour, or a minute. This, represented by *m m'*, *fig.* 741., being known, the space *m n* through which the planet would fall towards the sun in the same time may be calculated, and this being done for any two planets, it will be found that these spaces are in the inverse ratio of the squares of their distances.

Thus, for example, let the earth and Jupiter be compared in this manner. If D express the distance from the sun in miles, P the period in days, A the arc of the orbit in miles described by the planet in an hour, and H the space *m n* in miles, through which the planet would fall towards the sun in an hour if the tangential force were destroyed, we shall then have

	D	P	A	H
Earth - -	95,000,000	365·26	68,091	24·4020
Jupiter - -	494,000.000	4332·62	29,850	0·9019

Now, on comparing the numbers in the last column with the squares of those in the first column, we find them in almost exact accordance. Thus,

$$(95)^2 : (494)^2 :: 24{\cdot}402 : 0{\cdot}9024.$$

The difference, small as it is, would disappear, if exact values were taken instead of round numbers.

2617. *The law of gravitation universal.*—Thus is established the great natural law, known as the law of gravitation, at least, so far as the action of the sun upon the planets, and the planets on their satellites, is concerned. But this law does not alone affect the central attracting bodies. It belongs equally to the revolving bodies themselves. Each planet attracts the sun, and each satellite attracts its primary as well as being attracted by it; and this reciprocal attraction depends on the mass of the revolving as well as on the mass of the central body and their mutual distance.

The planets moreover, as well as the satellites, attract each other, and thus modify, to some small extent, the effects of the predominant central attraction.

2618. *Its analogy to the general law of radiating influences.*—It will be observed that this law is similar to that which governs light, heat, sound, and other physical principles which are propagated by radiation; and it might thus be inferred that gravitation is an agency of which the seat is the sun, or other gravitating body, and that it emanates from it as other physical principles obeying the same law are supposed to do.

2619. *Not, however, to be identified with them.* — No such hypothesis as this, however, is either assumed or required in astronomy. The law of gravitation is taken as a general fact established by observation, without reference to any *modus operandi* of the force. The planets may be *drawn* towards the sun by an agency whose seat is established in the sun, or they may be *driven* towards it by an agency whose seat is outside and around the system, or they may be *pressed* towards it by an agency which appertains to the space in which they move. Nothing is assumed in astronomy which would be incompatible with any one of these modes of action. The law of gravitation

assumes nothing more than that the planets are subject to the agency of a force which is every where directed to the sun, and whose intensity increases as the square of the distance from the sun decreases, and *vice versâ;* and this, as has been shown, is proved as a matter of fact independent of all theory or hypothesis.

2620. *The harmonic law.* — A remarkable numerical relation thus denominated prevails between the periodic times of the planets and their mean distances, or major axes of their orbits. If the squares of the numbers expressing their periods be compared with the cubes of those which express their mean distances, they will be found to be very nearly in the same ratio. They would be exactly so if the masses or weights of the planets were absolutely insignificant compared with that of the sun. But although these masses, as will appear, are comparatively very small, they are sufficiently considerable to affect, in a slight degree, this remarkable and important law.

Omitting for the present, then, this cause of deviation, the harmonic law may be thus expressed. If P, P', P'', &c. be a series of numbers which express or are proportional to the periodic times, and r, r', r'', &c. to the mean distances of the planets, we shall have

$$\frac{r^3}{P^2} = \frac{r'^3}{P'^2} = \frac{r''^3}{P''^2}, \text{ \&c.};$$

that is, the quotients found by dividing the numbers expressing the cubes of the distances by the numbers which express the squares of the periods are equal, subject nevertheless to such deviations from the law as may be due to the cause above mentioned.

2621. *Fulfilled by the planets.—Method of computing the distance of a planet from the sun when its periodic time is known.* — To show the near approach to numerical accuracy with which this remarkable law is fulfilled by the motions of the planets composing the solar system, we have exhibited in the following table the relative approximate numerical values of their several distances and periods, and have shown that the quotients found by dividing the cubes of the distances by the squares of the periods are sensibly equal: —

	Distance. r	Period. P	Cube of Distance. r^3	Square of Period. P^2	Ratio of Cube of Distance to Sq. of Period. $\frac{r^3}{P^2}$
Mercury -	0·387	0·241	57961	580	100
Venus - -	0·723	0·615	377933	3782	100
Earth - -	1·00	1·00	1000000	10000	100
Mars - -	1·52	1·88	3525688	35344	100
Planetoids -	2·50	4·00	15625000	160000	100
Jupiter - -	5·20	11·86	140608000	1406596	100
Saturn - -	9·24	29·50	868250664	8702500	100
Uranus - -	19·18	84·00	7055792632	70560000	100
Neptune -	30·00	164·60	27000000000	270931600	100

In general, the distance of a planet from the sun can be computed by means of this law, when the distance of the earth and the periodic times of the earth and planet are known.

For this purpose find the number which expresses the periodic time P of the planet, that of the earth being expressed by 1; and let D be the number which expresses the mean distance of the planet from the sun, that of the earth being also expressed by 1. We shall then, according to the harmonic law, have

$$1^2 : P^2 :: 1^3 : D^3.$$

But since

$$1 \times 1 = 1 \qquad 1 \times 1 \times 1 = 1,$$

we shall have

$$P^2 = D^3.$$

To find the distance D, therefore, it is only necessary to find the number whose cube is the square of the number expressing the period, or, what is the same, to extract the cube root of the square of the period.

2622. *Harmonic law deduced from the law of gravitation.*— It is not difficult to show that this remarkable law is a necessary consequence of the law of gravitation.

Supposing the orbits of the planets to be circular, which for this purpose they may be taken to be, let the distance, period, and angular velocity of any one planet be expressed by r, P, and a, and those of any other by r', P', and a', and let the forces with which the sun attracts them respectively be expressed by f and f'. We shall then, according to what has been proved (2614), have

$$f = \frac{r \times a^2}{2 \times 206265} \qquad f' = \frac{r' \times a'^2}{2 \times 206265}$$

and therefore

$$f : f' :: r \times a^2 : r' \times a'^2.$$

But by the law of gravitation

$$f : f' :: r'^2 : r^2,$$

therefore

$$r'^2 : r^2 :: r \times a^2 : r' \times a'^2,$$

and consequently

$$r'^3 \times a'^2 = r^3 \times a^2.$$

But the angles described in the unit of time are found by dividing 360° by the periodic times. Therefore,

$$a = \frac{360°}{P} \qquad a' = \frac{360°}{P'},$$

and consequently

$$\frac{r^3}{P^2} = \frac{r'^3}{P'^2},$$

which is, in fact, the harmonic law.

It is easy, by pursuing this reasoning in an inverse order, to show that if the harmonic law be taken, as it may be, as an observed fact, the law of gravitation may be deduced from it.

2623. *Kepler's laws.* — The three great planetary laws explained in the preceding paragraphs — 1. The equable description of areas; 2. The elliptic form of the orbits; and 3, the harmonic law — were discovered by Kepler, whose name they bear. Kepler deduced them as matter-of-fact from the recorded observations of himself and other astronomers, but failed to show the principle by which they were connected with each other. Newton gave their interpretation, and showed their connexion as already explained.

2624. *Inclination of the orbits — nodes.* — In what has been stated the planets are regarded as moving in the plane of the earth's orbit. If this were strictly true, no planet would ever be seen on the heavens out of the ecliptic. The inferior planets, when in inferior conjunction, would *always* appear as spots on the sun; and when in superior conjunction, they, as well as the superior planets, would *always* be behind the sun's disk. This is not the case. The planets generally, superior and inferior, are seldom seen actually upon the ecliptic, although they are never far removed from it. The centre of the planet,

twice in each revolution, is observed upon the ecliptic. The points at which it is thus found upon the plane of the earth's orbit are at opposite sides of the sun, 180° asunder, as seen from that luminary. At one of them the planet passes from the south to the north of the ecliptic, and at the other from the north to the south.

2625. *Nodes, ascending and descending.* — Those points, where the centre of a planet crosses the ecliptic, are called its NODES, that at which it passes from south to north being called the ASCENDING NODE, and the other the DESCENDING NODE.

While the planet passes from the ascending to the descending node it is north of the ecliptic; and while it passes from the descending to the ascending node, it is south of it.

All these phenomena indicate that the planet does not move in the plane of the ecliptic, but in a plane inclined to it at a certain angle. This angle cannot be great, since the planet is never observed to depart far from the ecliptic. With a few exceptions, which will be noticed hereafter, the obliquity of the planets' orbits do not amount to more than 7°.

2626. *The zodiac.* — The planets, therefore, not departing more than about 8° from the ecliptic, north or south, their motions are limited to a zone of the heavens bounded by two parallels to the ecliptic at this distance, north and south of it.

2627. *Methods of determining a planet's distance from the sun.* — This problem may be solved with more or less approximation by a great variety of different methods. In all it is assumed that the earth's distance from the sun is previously ascertained, the immediate result being in every case the determination of the ratio which the planet's distance from the sun bears to the earth's distance.

2628. 1°. *By the elongation and synodic motion.* — This method has been already explained (2592).

2629. 2°. *By the greatest elongation for an inferior planet.*— When an inferior planet is at its greatest elongation, the angle P, included by lines drawn from it to the sun and earth, will be 90° (2576), and consequently the two angles at E and S taken together will be 90°. If the elongation E be observed, the angle at S will be 90°—E, and will therefore be known, and thus the distances SP and EP may be computed as in the first method.

2630. 3°. *By the greatest and least apparent magnitudes.* —

If m and m' be the apparent magnitudes of an inferior planet when at inferior and superior conjunction, and e and p be the distances of the earth and planet from the sun, $e-p$ will be the distance of the planet from the earth at inferior, and $e+p$ at superior conjunction (2577); and since the apparent magnitudes are in the inverse ratio of these distances (1118), we shall have

$$\frac{m}{m'} = \frac{e+p}{e-p},$$

and consequently

$$p = \frac{m-m'}{m+m'} \times e.$$

If $m + m'$ be the apparent magnitudes of a superior planet in opposition and conjunction, its distances at these points will be $p-e$ and $p+e$, and we shall have as before

$$\frac{m}{m'} = \frac{p+e}{p-e},$$

and therefore

$$p = \frac{m+m'}{m-m'} \times e.$$

2631. 4°. *By the harmonic law.* — This law (2614) being deduced from the law of gravitation (2616), independently of the observation and comparison of times and distances, it may be used for the determination of the distances, the times being known. Let E and P be the periodic time of the earth and planet, and e and p their distances from the sun. We shall then, by the harmonic law, have

$$\frac{p^3}{e^3} = \frac{\mathrm{P}^2}{\mathrm{E}^2},$$

and therefore

$$p^3 = \frac{\mathrm{P}^2}{\mathrm{E}^2} \times e^3,$$

and thus the distance p may be found.

2632. *To determine the real diameters and volumes of the bodies of the system.* — The apparent diameter at a known distance being observed, the real diameter may be computed by the principle explained in 2299. The linear value of 1″ at the distance being known, the real diameter will be obtained by

multiplying such value by the apparent diameter expressed in seconds.

The discs of the inferior planets not being visible at inferior conjunction when their dark hemispheres are presented to the earth, and being lost in the effulgence of the sun at superior conjunction, can only be observed between their greatest elongation and superior conjunction, when they appear gibbous. The distance of the planet from the earth is computed in this position by knowing the distances of the planet and the earth from the sun, and the angle under the lines drawn from the sun to the earth and planet, which can always be computed (2587). This distance being obtained, the linear value of 1″ at the planet will be found (2298), which, being multiplied by the greatest breadth of its gibbous disk, the real diameter will be obtained.

In the case of the superior planets, their diameters may be best obtained when in opposition, because then they appear with a full disk, and, being nearer to the earth than at any other elongation, have the greatest possible magnitude. Their distance from the earth in this position is always the difference between the distances of the earth and planet from the sun.

When the real diameters are found the volumes will be obtained, since they are as the cubes of the real diameters.

2633. *Methods of determining the masses of the bodies of the solar system.* — The work of the astronomer is but imperfectly performed when he has only mentioned the distances and magnitudes, and ascertained the motions and velocities, of the great bodies of the universe. He must not only measure, but weigh these stupendous masses.

The masses or quantities of matter in bodies upon the surface of the earth are estimated and compared by their weights—that is, by the intensity of the attraction which the earth exerts upon them. It is inferred that equal quantities of matter at equal distances from the centre of the earth are attracted by equal forces, inasmuch as all masses, great and small, fall with the same velocity (234).

The intensity of the attraction with which the earth thus acts upon a body at any given distance from its centre depends on the mass or quantity of matter composing the earth. If the mass of the earth were suddenly increased in any proposed ratio, the weights of all bodies on its surface, or at any given distance from its centre, would be increased in the same ratio,

and, in like manner, if its mass were diminished, the weights would be decreased in the same ratio. In fine, the weights of bodies at any given distance from the earth's centre would vary with, and be exactly proportional to, every variation in the mass of the earth.

This principle is general. If M and M′ be any two masses of matter, the attractions which they will exert upon any bodies, placed at equal distances from their centres of gravity, will be in the exact proportion of the quantities of ponderable matter composing them.

But it will be convenient to obtain the relation between the masses and the attractions they exert at unequal distances. For this purpose, let the attractions which they exert at equal distances be expressed by f and f', and let the common distance at which those attractions are exerted be expressed by x, and let F and F′ express the attractions which they respectively exert at any other distances, r and r', and we shall have, according to the general law of gravitation,

$$f : \mathrm{F} :: \frac{1}{x^2} : \frac{1}{r^2}$$

$$f' : \mathrm{F}' :: \frac{1}{x^2} : \frac{1}{r'^2},$$

and consequently

$$\frac{f}{\mathrm{F}} \times \frac{1}{r^2} = \frac{f'}{\mathrm{F}'} \times \frac{1}{r'^2},$$

from which it follows that

$$\frac{f}{f'} = \frac{\mathrm{F} \times r^2}{\mathrm{F}' \times r'^2}.$$

But since the masses M and M′ are proportional to the attractions f and f', we have

$$\frac{f}{f'} = \frac{\mathrm{M}}{\mathrm{M}'},$$

and therefore

$$\frac{\mathrm{M}}{\mathrm{M}'} = \frac{\mathrm{F} \times r^2}{\mathrm{F}' \times r'^2},$$

that is, the attracting masses are proportional to the products obtained, by multiplying any two forces exerted by them by the squares of the distances at which such forces are exerted.

Hence in all cases in which the attractive forces exerted by

any central masses at given distances can be measured by any known or observable motions, or other mechanical effects, the proportion of the attracting masses can be determined.

2634. *Method of estimating central masses round which bodies revolve.*—If bodies revolve round central attracting masses as the planets revolve round the sun, and the satellites round their primaries, the ratio of the attracting forces, and therefore that of the central masses, can be deduced from the periods and distances of the revolving bodies by the principles and method explained in 2614.

Thus if P and P′ be the periods of two bodies revolving round different attracting masses M and M′ at the distances r and r', we shall have

$$\frac{F}{F'} = \frac{r}{P^2} \times \frac{P'^2}{r'} = \frac{r}{r'} \times \frac{P'^2}{P^2};$$

and substituting this for $\frac{F}{F'}$, in the formula found in 2633, we have

$$\frac{M}{M'} = \frac{r^3}{r'^3} \times \frac{P'^2}{P^2},$$

By this principle the ratio of the attracting masses can always be ascertained when the periods of any bodies revolving round them at known distances are known.

2635. *Method of determining the ratio of the masses of all planets which have satellites to the mass of the sun.*—This problem is nothing more than a particular application of the principle explained above.

To solve it, it is only necessary to ascertain the period and distance of the planet and the satellite, and substitute them in the formula determined in 2634. The arithmetical operations being executed, the ratio of the masses will be determined.

2636. *To determine the ratio of the mass of the earth to that of the sun.*—Since the earth has a satellite, this problem will be solved by the method given in 2635.

If r and r' express the distances of the earth from the sun and moon, and P and P′ the periods of the sun and moon, we shall have

$$\frac{r}{r'} = 400 \qquad \frac{P'}{P} = \frac{27{\cdot}30}{365{\cdot}25} = \frac{1}{13{\cdot}38}$$

$$\frac{r^3}{r'^3} = 64000000 \quad \frac{P'^2}{P^2} = \frac{1}{179 \cdot 024},$$

which being substituted, and the operations executed, gives

$$\frac{M}{M'} = 357500.$$

2637. *To determine the masses of planets which have no satellites.* — According to what has been explained, the masses of the bodies composing the solar system are measured, and compared one with another, by ascertaining, with the necessary precision, any similar effects of their attractions, and allowing for the effects of the difference of distances. The effects which are thus taken to measure the masses and to exhibit their ratio to the mass of the sun in the case of planets attended by satellites, is the space through which a satellite would be drawn by its primary, and the space through which a planet would be drawn in the same time by the sun. These spaces indicate the actual forces of attraction of the planet upon the satellite and of the sun upon the planet, and when the effect of the difference of distance is allowed for, the ratio of the mass of the planet to the mass of the sun is found.

In the case of planets not attended by satellites, the effect of their gravitation is not manifested in this way, and there is no body smaller than themselves, and sufficiently near them to exhibit the same easily measured and very sensible effects of their attraction, and hence there is considerable difficulty, and some uncertainty, as to their exact masses.

2638. *Mass of Mars estimated by its attraction upon the earth.* — The nearest body of the system to which Mars approaches is the earth, its distance from which in opposition is nearly fifty millions of miles, or half the distance of the earth from the sun. Now, since the volume of Mars is only the eighth part of that of the earth, it may be presumed, that whatever be its density its mass must be so small, that the effect of its attraction on the earth at a distance so great must be very minute, and therefore difficult to ascertain by observation. Nevertheless, small as the effect thus produced is, it is not imperceptible, and a certain deviation from the path it would follow, if the mass of Mars were not thus present, has been observed. To infer from this deviation, the mass of Mars is, however, a problem of much greater complexity than

the determination of the mass of a planet by observing its attraction upon its satellite. The method adopted for the solution of the problem is a sort of "trial and error." A conjectural mass is first imputed to Mars, and the deviation from its course which such a mass would cause in the orbital motion of the earth is computed. If such deviation is greater or less than the actual deviation observed, another conjectural mass, greater or less than the former, is imputed to the planet, and another computation made of the consequent deviation, which will come nearer to the true deviation than the former. By repeating this approximative and tentative process a mass is at length found, which, being imputed to Mars, would produce the observed deviation; and this is accordingly assumed to be the true mass of the planet.

In this way the mass of Mars has been approximatively estimated at the seventh part of the mass of the earth.

The smallness of this mass compared with its distance from the only body on which it can exert a sensible attraction will explain the difficulty of ascertaining it, and the uncertainty which attends its value.

2639. *Masses of Venus and Mercury.*—The same causes of difficulty and uncertainty do not affect in so great a degree the planet Venus, whose mass is somewhat greater than that of the earth, and which moreover comes when in inferior conjunction within about thirty millions of miles of the earth. The effects of the attraction of the mass of this planet upon the earth's orbital motion are therefore much more decided. The deviation produced by it is not only easily observed and measured, but it affects in a sensible manner the position of the plane of the earth's orbit. By the same system of "trial and error," the mass of this planet is ascertained to be greater by a twentieth than that of the earth.

The difficulties attending the determination of the mass of Mercury are still greater than those which affect Mars, and its true value is still very uncertain. Attempts have lately been made to approximate to its value, by observing the effects of its attraction on one of the comets.

2640. *Methods of determining the mass of the moon.*—Owing to its proximity and close relation to the earth, and the many and striking phenomena connected with it, the determination of the mass of the moon becomes a problem of considerable im-

portance. There are various observable effects of its attraction by which the ratio of its mass to those of the sun or earth may be computed.

2641. 1°. *By nutation.*—It will be shown hereafter that the attractions of the masses of the sun and moon upon the protuberant matter surrounding the equator of the terrestrial spheroid produce a regular and periodic change in the direction of the axis of the earth, and consequently a corresponding change in the apparent place of the celestial pole. The share which each mass has in these effects being ascertained, their relative attractions exerted upon the redundant matter at the terrestrial equator is found, and the effect of the difference of distance being allowed for, the ratio of the attracting masses is obtained.

2642. 2°. *By the tides.*—It has been shown (Chap. X.), that, by the attractions of the masses of the sun and moon, the tides of the ocean are produced. The share which each mass has in the production of these effects being ascertained, and the effect of the difference of distance being allowed for, the ratio of the masses of the sun and moon is obtained.

2643. 3°. *By the common centre of gravity of the moon and the earth.*—It has been stated that the centre of attraction round which the moon moves in her monthly course is the centre of the earth. This is nearly, but not exactly true. By the law of gravitation the centre of attraction is not the centre of the earth, but the centre of gravity of the earth and moon, that is, a point whose distance from the centre of the earth has to its distance from the centre of the moon the same ratio as the mass of the moon has to the mass of the earth (309). Around this point, which is within the surface of the earth, both the earth and moon revolve in a month, the point in question being always between their centres. If, then, the position of this point can be found, the ratio of its distances from the centres of the earth and moon will give the ratio of their masses.

Now, the monthly motion of the earth round such a centre would necessarily produce a corresponding apparent monthly displacement of the sun. Such displacement, though small (not amounting to more than a few seconds), is nevertheless capable of observation and measurement. The exact place of the sun's centre being therefore computed on the supposition of the absence of the moon, and compared with its observed place, the

motion of the earth's centre and the position of the point round which it revolves has been determined, and the relative masses of the earth and moon thus found.

2644. 4°. *By terrestrial gravity.*—By what has been already explained, the space through which the moon would be drawn towards the earth in a given time by the earth's attraction can be determined. Let this space be expressed by s. The linear velocity v of the moon in its orbit can also be determined. Now, if r be the radius of the orbit, we shall have (2614)

$$2\,r \times s = v^2,$$

and consequently

$$r = \frac{v^2}{2\,s}.$$

We find, therefore, the radius vector of the moon's orbit by dividing the square of its linear velocity by twice the space through which it would fall towards the earth in the unit of time. But this radius vector is the distance of the moon's centre from the common centre of gravity of the earth and moon. The distance of that point, therefore, from the centre of the earth, and consequently the ratio of the masses of the earth and moon, will be thus found.

All these methods give results in very near accordance, from which it is inferred that the mass of the moon is not less than the seventy-fifth, nor greater than the eightieth, part of the mass of the earth, and it is consequently the twenty-eighth millionth part of the mass of the sun.

2645. *To determine the masses of the satellites.* — The same difficulties which attend the determination of the masses of the planets not accompanied by satellites also attend the determination of the masses of satellites themselves, and the same methods are applicable to the solution of the problem. The masses of the satellites of Jupiter and the other superior planets are ascertained in relation to those of their primaries by the disturbing effects which they produce upon the motions of each other.

2646. *To determine the densities of the bodies of the system.*— The masses and volumes being ascertained, the densities are found by dividing the masses by the volumes. Thus, if D and D′ be the densities of the earth and a planet, M and M′ their masses, and V and V′ their volumes, we shall have

$$\text{D} : \text{D}' :: \frac{\text{M}}{\text{V}} : \frac{\text{M}'}{\text{V}'}.$$

2647. *The method of determining the superficial gravity on a body.* — When it is considered how important an element in all the mechanical and physical phenomena on the surface of the earth, the intensity of gravity at the surface is, it will be easily understood that in the investigation of the superficial condition and local economy of the other bodies of the solar system, the determination of the intensities of the forces with which they attract bodies placed on or near their surfaces, is a problem of considerable interest.

If the mass of the earth be expressed by M, its semidiameter by r, and the force of gravity on its surface by g, while M′, r', and g' express the same physical quantities in relation to any other body having the form of a globe, we shall have

$$g : g' :: \frac{\text{M}}{r^2} :: \frac{\text{M}'}{r'^2},$$

because, by the general law of gravitation, the force is in the direct ratio of the masses and the inverse ratio of the square of the attracted body from their centre, and in this case the attracted body being supposed to be at their surfaces, those distances will be their semidiameters.

From the preceding proportion may be inferred the formula

$$\frac{g}{g'} = \frac{\text{M}}{\text{M}'} \times \frac{r'^2}{r^2},$$

by which the superficial gravity may always be computed when the ratios of the masses and the diameters are known.

2648. *Superficial gravity of the sun.* — The mass of the sun being 355,000 times that of the earth, while its diameter is 110 times that of the earth, we shall have

$$g : g' :: 1 : \frac{355{,}000}{12{,}100} = 28 \cdot 9.$$

It appears, therefore, that the weight of a body placed at the surface of the sun is twenty-nine times its weight on the surface of the earth.

A man, whose average weight would be 1½ cwt. on the earth, would weigh 2 tons and 1-3d if transferred to the surface of the sun. The human frame, organised as it is, would be crushed under its own weight if removed there.

Muscular force is therefore 29 times more efficacious upon the earth than it would be upon the sun.

2649. *Superficial gravity on the moon.* — The mass of the moon has been ascertained to be the 80th part of that of the earth, while the diameter of the moon is about the fourth part of that of the earth. We have, therefore, in the case of the moon

$$g : g' :: 1 : \frac{16}{80} = \frac{1}{5};$$

so that the superficial gravity on the moon is five times less than on the earth. A man weighing 1·5 cwt. on the earth would only weigh 0·3 cwt., or 33⅗ lbs., if transferred to the moon.

2650. *Classification of the planets in three groups. — First group — the terrestrial planets.* — Of the planets hitherto discovered, three which present in several respects remarkable analogies to the earth, and whose orbits are included within a circle which exceeds the earth's distance from the sun by no more than one-half, have been from these circumstances denominated TERRESTRIAL PLANETS. Two of these, MERCURY and VENUS, revolve within the orbit of the earth; and the third, MARS, revolves in an orbit outside that of the earth, its distance from the earth when in opposition being only half the earth's distance from the sun.

2651. *Second group — the planetoids.* — A chasm having a width measuring little less than four times the earth's distance, separated, for many ages after astronomy had made considerable progress, the terrestrial planets from the more remote members of the system. The labours of observers during the last half century, but chiefly during the last seven years, have filled this chasm with no less than twenty-three planets, distinguished from all the other bodies of the system by their extremely minute magnitudes, and by the circumstance of revolving in orbits very nearly equal. These bodies have been distinguished by the name of ASTEROIDS or PLANETOIDS, the latter being preferable as the most characteristic and appropriate.

2652. *Third group — the major planets.* — Outside the planetoids, and at enormous distances from the sun and from each other, revolve four planets of stupendous magnitude — named JUPITER, SATURN, URANUS, and NEPTUNE: the two former being visible to the naked eye, were known to the

ancients; the two latter are telescopic, and were discovered in modern times.

CHAP. XIII.

THE TERRESTRIAL PLANETS.

I. Mercury.

2653. *Period.* — The nearest of the planets to the sun, and that which completes its revolution in the shortest time, is Mercury.

The synodic period of this planet, determined by immediate observation, is 115·88 days. Hence we shall have by the formula (2589).

$$\frac{1}{P} = \frac{1}{115{\cdot}88} + \frac{1}{365{\cdot}25} = \frac{1}{87{\cdot}98}.$$

The period of Mercury is, therefore, 87·98, or very nearly 88 days.

By methods of calculation susceptible of still greater precision, the period is found to be 87·97 days.

If the earth's period be expressed by 1, that of Mercury will, therefore, be 0·2408.

2654. *Heliocentric and synodic motions.* — The mean daily heliocentric motion is, therefore, (2568),

$$a = \frac{1296000}{87{\cdot}97} = 14732''{\cdot}5 = 245'{\cdot}5 = 4^{\circ}.092.$$

The mean daily synodic motion is (2569)

$$\sigma = a - \varepsilon = 14732''{\cdot}5 - 3548''{\cdot}2 = 11184''{\cdot}3 = 186'{\cdot}4 = 3^{\circ}{\cdot}11.$$

2655. *Distance determined by greatest elongation.* — Owing to the ellipticity of the planet's orbit, its greatest elongation is subject to some variation. Its mean amount is, however, about 22°·5. If the radius r of the planet's orbit, drawn from the sun to the planet at the point of its greatest elongation, were the arc of a circle, having the earth's distance from the sun as radius, we should have (2294)

$$r = \frac{95{,}000{,}000}{57{\cdot}3} \times 22^{\circ}{\cdot}5 = 37{,}303{,}000 \text{ miles.}$$

But the radius r being, in fact, the sine of 22°·5, and not the arc itself, the value of r is a little less, being about $36\frac{3}{4}$ millions of miles.

2656. *By the harmonic law.* — If r express the distance of the planet, that of the earth being 1, we shall have (2621)

$$r^3 = 0{\cdot}2408^2 = 0{\cdot}387^3.$$

The distance of the planet is, therefore, 0·387. But the mean distance of the earth being 95 millions of miles, we shall have for the mean distance of the planet

$$r' = 95{,}000{,}000 \times 0{\cdot}387 = 36{,}770{,}000 \text{ miles.}$$

2657. *Mean and extreme distances from the earth.* — The eccentricity of the orbit of Mercury is much more considerable than those of the planets generally, being a little more than 0·2, expressed in parts of the mean distance. The distance of the planet from the sun is, therefore, subject to a variation, amounting to so much as a fifth part of its mean value. The greatest and least distances from the sun are, therefore,

$$36\tfrac{3}{4} + 7\tfrac{1}{3} = 43\tfrac{5}{6} \text{ millions of miles in aphelion}$$
$$36\tfrac{3}{4} - 7\tfrac{1}{3} = 29\tfrac{2}{3} \quad \text{,,} \quad \text{,,} \quad \text{perihelion.}$$

The distance is, therefore, subject to a variation in the ratio of 5 to 7 very nearly.

The mean distances of the planet from the earth are, therefore,

$$95 - 36\tfrac{3}{4} = 59\tfrac{1}{4} \text{ mill. of miles at inf. conj.}$$
$$95 + 36\tfrac{3}{4} = 131\tfrac{3}{4} \quad \text{,,} \quad \text{,,} \quad \text{sup. conj.}$$

These distances are subject to an increase and diminution of seven and one-third millions of miles due to the eccentricity of the orbit of the planet, and one million and a-half of miles due to the eccentricity of the orbit of the earth.

2658. *Scale of the orbit relatively to that of the earth.*—The orbit of Mercury and a part of that of the earth are exhibited on their proper scale in *fig.* 742., where SE is the earth's distance from the sun, and $mm''m$ the orbit of the planet. The lines Em'' drawn from the earth touching the orbit of the planet determine the positions of the planet when its elongation is greatest east and west of the sun. The points m are the positions of the planet at inferior and superior conjunction.

2659. *Apparent motion of the planet.* — The effects of the combination of the orbital motions of the planet and the earth

upon the apparent place of the planet will now be easily comprehended.

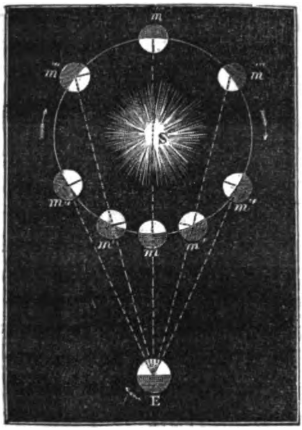


Fig. 742.

Since the mean value of the greatest elongation m'' ES$=22\frac{1}{2}°$, the arc $mm''=67\frac{1}{2}°$ and therefore $m''\,m\,m''=67\frac{1}{2}°\times 2=135°$. The times of the greatest elongations east and west therefore divide the whole synodic period into two unequal parts, in one of which, that from the greatest elongation east through inferior conjunction to the greatest elongation west, the planet gains upon the earth 135°; and in the other, that from the greatest elongation west, through superior conjunction to the greatest elongation east, it gains $360°-135°=225°$. Since the parts into which the synodic period is thus divided are proportional to these angles, they will be (taking the synodic period in round numbers as 116 days),

$$\frac{135}{360}\times 116 = 43\tfrac{1}{2}\text{ days};$$

$$\frac{225}{360}\times 116 = 72\tfrac{1}{2}\text{ days}.$$

And since the former interval is divided equally by the epoch of inferior, and the latter by the epoch of superior, conjunction, it follows, that the intervals between inferior conjunction and greatest elongation are $21\frac{3}{4}$ days, and the intervals between superior conjunction and greatest elongation are $36\frac{1}{4}$ days.

The interval between the times at which the planet is stationary, before and after inferior conjunction, is subject to some variation, owing to the eccentricities of the orbits both of the planet and the earth, but chiefly to that of the planet's orbit, which is considerable. If its mean value be taken at 22 days, the angle gained by the planet on the earth in that interval being

$$3°{\cdot}11\times 22 = 68°{\cdot}4,$$

the angular distances of the points at which the planet is stationary from inferior conjunction as seen from the sun would be 34°·2, which would correspond to an elongation of about 21°, as seen from the earth. This result, however, is subject to very great variation, owing to the eccentricity of the planet's orbit and other causes.

2660. *Conditions which favour the observation of an inferior planet.* — These conditions are threefold; 1. The magnitude of that portion of the enlightened hemisphere which is presented to the earth. 2. The elongation. 3. The proximity of the planet to the earth.

Since it happens that the positions which render some of these conditions most favourable render others less so, the determination of the position of greatest apparent brightness is somewhat complicated. When the planet is nearest to the earth its dark hemisphere is presented towards us (2595); besides which, being in inferior conjunction, it rises and sets with the sun, and is only present in the day time. At small elongations in the inferior part of the orbit its distance from the earth is not much augmented, but it is still overpowered by the sun's light, and would only appear as a thin crescent when it would be possible to see it. At the greatest elongation, when it is halved, it is most removed from the interference of the sun, but is brightest at a less elongation, even though it moves to a greater distance from the earth, since it gains more by the increase of its phase than it loses by increased distance and diminished elongation.

Owing to the very limited elongation of Mercury, that planet, even when its apparent distance from the sun is greatest, sets in the evening long before the end of twilight; and when it rises before the sun, the latter luminary rises so soon after it that it is never free from the presence of so much solar light as to render it extremely difficult to see the planet with the naked eye.

In these latitudes Mercury is therefore rarely seen with the naked eye. It is said that Copernicus himself never saw this planet, a circumstance which, however, may have been owing, in a great degree, to the unfavourable climate in which he resided. In lower latitudes, where the diurnal parallels are more nearly vertical and the atmosphere less clouded, it is more

frequently visible, and there it is more conspicuous, owing to the short duration of twilight.

2661. *Apparent diameter,—its mean and extreme values.*—Owing to the variation of the planet's distance from the earth, its apparent diameter is subject to a corresponding change. At its greatest distance its apparent diameter is $4\frac{1}{2}''$, and at its least distance $11\frac{1}{2}''$, its value at the mean distance being $6\frac{1}{2}''$.

The apparent diameter of the moon being familiar to every eye supplies a convenient and instructive comparison by which the apparent magnitudes of other objects may be indicated, and we shall refer to it frequently for that purpose. The disk of the full moon subtends an angle of $1800''$ to the eye. It follows, therefore, that the apparent diameter of Mercury when it appears as a thin crescent near inferior conjunction is about the 150th part, near the greatest elongation it is the 280th part, and near superior conjunction the 400th part, of the apparent diameter of the moon. With a magnifying power of 140, it would therefore, at its greatest elongation, appear with a disk half the apparent diameter of the moon.

2662. *Real diameter.* —The distance of Mercury in inferior conjunction being $36\frac{3}{4}$ millions of miles, the linear value of $1''$ at it then is (2298)

$$\frac{59{,}250{,}000}{206{,}265} = 287{\cdot}2 \text{ miles.}$$

At this distance its apparent diameter is $11\frac{1}{4}''$; and if D' express its real diameter we shall have

$$D' = 287{\cdot}2 + 11{\cdot}25 = 3231 \text{ miles.}$$

Other observations make the diameter somewhat less, and fix it at 2950 miles.

2663. *Volume.*—If V' express the volume, that of the earth being V, we shall have

$$\frac{V'}{V} = \frac{D'^3}{D^3} = \left(\frac{3231}{7912}\right)^3 = \frac{1}{14{\cdot}6}$$

The volume is therefore less than the 14th part of that of the earth. If the lesser estimate of the diameter of Mercury be adopted, it will follow that its volume is about the 17th part of that of the earth. The relative volumes are represented by M and E, *fig.* 743.

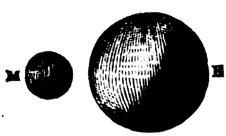


Fig. 743.

2664. *Mass and density.*—Some uncertainty has hitherto attended the calculation of the density and mass of this planet, owing to the absence of a satellite. The disturbances produced by it upon the motion of Encke's comet (a body which will be described in another chapter) have, however, supplied the means of a closer approximation to it. By this means it has been found that if M′ express the mass of the planet and M that of the earth, we shall have

$$\frac{M'}{M} = \frac{100}{1225};$$

so that the mass is 12¼ times less than that of the earth.

If d' and d express the densities of the planet and the earth, we shall therefore have

$$\frac{d'}{d} = \frac{M'}{M} \times \frac{V}{V'} = \frac{100}{1225} \times \frac{147}{10} = \frac{1470}{1225} = 1{\cdot}20.$$

Other estimates make it 1·12. So that it may be inferred that the density of Mercury exceeds that of the earth by an eighth to a fifth.

2665. *Superficial gravity.*—If g' express the force of gravity on the surface of Mercury, g being the force on the surface of the earth, we shall have (2639)

$$\frac{g'}{g} = \frac{M'}{M} \times \frac{D'^2}{D^2} = 0{\cdot}5.$$

The superficial gravity is therefore only half the same force on the earth. Muscular and other forces not depending on weight are therefore twice as efficacious. The height through which a body would fall in a second would be 96½ inches, or a little more than eight feet.

2666. *Solar light and heat.*—The apparent magnitude of the sun is greater than upon the earth, in the same ratio as the distance is less; and owing to the considerable ellipticity of Mercury's orbit, it has apparent magnitudes sensibly different in different parts of Mercury's year. The apparent diameter of the sun as seen from the earth being 30′, its apparent diameter seen from Mercury will be

in perihelion $\quad 30' \times \frac{1000}{307} = 92'{\cdot}5,$

in aphelion $\quad 30' \times \frac{1000}{467} = 64'{\cdot}2,$

$$\text{at mean distance } 30' \times \frac{1000}{387} = 78'.$$

Thus the apparent diameter when least is twice, and when greatest three times, that under which the sun appears from the earth.

In *fig.* 744, 5., the relative apparent magnitudes of the sun, as

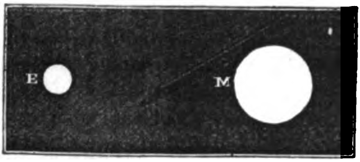


Fig. 744.

Fig. 745.

seen from the earth and from Mercury, at the mean distance and extreme distances, are represented at E, M, M′, and M″. If E be supposed to represent the apparent disk of the sun as seen from the earth, M will represent it as it appears to Mercury at the mean distance, M′ at aphelion and M″ at perihelion.

Since the illuminating and heating power of the sun's rays, whatever be the physical condition of the surface of the planet, must vary in the same proportion as the apparent area of the sun's disk, it follows, that the light and warmth produced by the sun on the surface of the planet will be greater in perihelion than in aphelion, in the ratio of 9 to 4, and, consequently, there must be a succession of seasons on this planet, depending exclusively on the ellipticity of the orbit, and having no relation to the direction of its axis of rotation or the position of the plane of its equator with relation to that of its orbit. The passage of the planet through its perihelion must produce a summer, and its passage through aphelion a winter, the mean temperature of the former *ceteris paribus* being above twice that of the latter.

If the axis of the planet be inclined to the plane of its orbit, another succession of seasons will be produced, dependent on such inclination and the position of the equinoctial points. If these points coincide with the apsides of the orbit, the summers and winters arising from both causes will either respectively coincide, or the summer from each cause will coincide with the winter from the other. In the former case the intensities of the seasons and their extreme temperatures will be augmented

by the coincidence, and in the latter they will be mitigated, the summer heat from each cause tempering the winter cold from the other.

If, on the other hand, the line of apsides be at right angles to the direction of the equinoxes, the summer and winter from each cause will correspond with the spring and autumn from the other, and a curious and complicated succession of seasons must ensue, depending on the degree of obliquity of the axis of the planet, compared with the effects of the eccentricity of its orbit.

In comparing the calorific influence of the sun on Mercury and the earth, it must be remembered that the actual temperature produced by the solar rays depends on the density of the atmosphere through which they pass, by which the heat is collected and diffused. The density of the sun's rays above the snow-line in the tropics is as great as at the level of the sea, but the temperatures of the air and surrounding objects are extremely different. Notwithstanding, therefore, the greater density of the solar rays, the atmospheric conditions of the planet may be such that the superficial temperature may not be different from that of the earth.

The intensity of the solar light must be greater than at the earth in the ratio of four to one when the planet is in aphelion, and nine to one when in perihelion. Its effects on vision, however, may be rendered the same by the mere adaptation of the contractile power of the pupil of the eye. (1129.)

2667. *Method of ascertaining the diurnal rotation of the planets.* — One of the most interesting objects of telescopic inquiry regarding the condition of the planets is, the question as to their diurnal rotation. In general, the manner in which we should seek to ascertain this fact would be, by examining with powerful telescopes the marks observable upon the disk of the planet. If the planet revolve upon an axis, these marks, being carried round with it, would appear to move across the disk from one side to the other; they would disappear on one side, and, remaining for a certain time invisible, would reappear on the other, passing, as before, across the visible disk. Let any one stand at a distance from a common terrestrial globe, and let it be made to revolve upon its axis: the spectator will see the geographical marks delineated on it pass across the hemisphere which is turned towards him. They will successively disappear and reappear. The same effects must, of

course, be expected to be seen upon the several planets, if they have a motion of rotation resembling the diurnal motion of our globe.

2568. *Difficulty of this question in the case of Mercury.* — This is a species of observation which has not yet been successfully made in the case of Mercury. Sir John Herschel, who has enjoyed more than common advantages for telescopic observation under different climates, affirms, that little more can be certainly affirmed of Mercury than that it is globular in form, and exhibits phases, and that it is too small and too much lost in the constant and close effulgence of the sun to allow the further discovery of its physical condition. Other observers, however, claim the discovery of indications not only of rotation but other physical characters. Schröter says, that by examining daily the appearance of the cusps of the crescent he ascertained that it has a motion of rotation in $24^{h.}\ 5^{m.}\ 28^{s.}$

2669. *Alleged discovery of mountains.* — The same observer claims the discovery of mountains on Mercury, and even assigns their height, estimating one at 2132 yards, and another 18,978 yards.

These observations, not having been confirmed, must be considered apochryphal.

II. Venus.

2670. *Period.* — The next planet proceeding outwards from the sun is Venus, which revolves in an orbit within that of the earth, and which, after the sun and moon, is the most splendid object in the firmament.

The synodic period, ascertained by observation, is 584 days. Her period deduced from this (2589) is, therefore,

$$\frac{1}{P} = \frac{1}{365{\cdot}25} - \frac{1}{584} = \frac{1}{225}$$

By the other methods it is more exactly determined to be 224·7 days.

If the earth's period be taken as the unit, that of Venus will, therefore, be 0·61.

2671. *Heliocentric and synodic motions.* — The mean daily heliocentric motion of Venus is, therefore (2568),

$$a = \frac{1296000}{224{\cdot}7} = 5768'' = 96'{\cdot}13 = 1^{\circ}{\cdot}6,$$

and the mean daily synodic motion is (2569)

$$\sigma = a - \epsilon = 5768'' - 3548'' = 2220'' = 37',$$

2672. *Distance, by greatest elongation.*—The mean amount of the greatest elongation of Venus being found by observation to be about 45° or 46°, it follows that in that position lines drawn to the earth and sun from the planet would form the sides of a square, of which the earth's distance from the sun is the diagonal. If, therefore, the earth's distance be expressed by 1·0000, that of Venus would be 0·7071.

2673. *By the harmonic law.*—If r express the mean distance of the planet from the sun (2621), we have

$$r^3 = 0{\cdot}61^2 = 0{\cdot}719^3.$$

Therefore $r = 0{\cdot}719$; and since the mean distance of the earth is 95 millions of miles, we shall have

$$r' = 95{,}000{,}000 \times 0{\cdot}719 = 68{,}300{,}000.$$

By more exact methods the distance is found to be $68\frac{3}{4}$ millions of miles.

2674. *Mean and extreme distances from the earth.*—Its distances from the earth at inferior conjunction, greatest elongation, and superior conjunction, are therefore

26,250,000 miles at inf. con.
65,000,000 miles at greatest elon.
163,750,000 miles at super. con.

The eccentricity of the orbit of Venus being less than 0·007, these distances are subject to very little variation from that cause. The extreme distance of the planet from the sun will be

$68\frac{3}{4} - 0\frac{1}{2} = 68\frac{1}{4}$ millions of miles in perihelion,
$68\frac{3}{4} + 0\frac{1}{2} = 69\frac{1}{4}$ „ „ aphelion.

These distances of the planet from the earth are subject therefore to an increase and diminution, amounting to half a million of miles, due to the eccentricity of the planet's orbit, and one and a half million of miles due to that of the earth's orbit.

2675. *Scale of the orbit relative to that of the earth.*—The relation of the orbit of Venus to the earth is represented in *fig.* 746., where SE represents the earth's distance from the sun, and vSv the mean diameter of the planet's orbit on the same scale. The angles SEv'' represent the greatest elongation of

the planet, which is about 46°. The lesser elongations v''' E S are those at which the planet appears with less than a full disk, or gibbous, as at v''', or as a crescent, as at v'. (2595.)

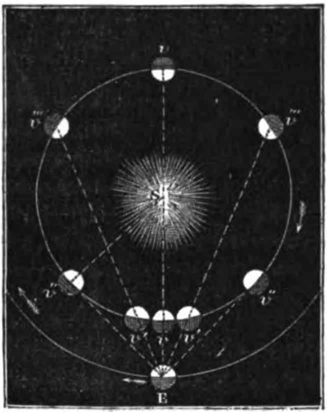


Fig. 746.

2676. *Apparent motion.* — Since the mean value of the greatest elongation is ascertained to be 46°, the angle at the sun, v'' S E = 44°, and consequently the angle v'' S v'', included between the greatest elongations east and west, is 88.° Since the time taken by the planet to gain this angle upon the earth bears the same ratio to the synodic period as this angle bears to 360°, the intervals into which the synodic period is divided by the epochs of greatest elongation, are

$$\frac{88}{360} \times 584 = 142{\cdot}8 \text{ days.}$$

$$\frac{272}{360} \times 584 = 441{\cdot}2 \text{ days.}$$

The intervals between inferior conjunction and greatest elongation are therefore $71\frac{1}{2}$ days, and the intervals between superior conjunction and greatest elongation are $220\frac{1}{2}$ days.

2677. *Stations and retrogression.* — From a comparison of the orbital motions and distances of the earth and planet, it is found that the epochs at which it is stationary are about twenty days before and after inferior conjunction. Now, since the planet gains 0°·61 per day upon the earth, this interval corresponds to an angle of

$$20 \times 0^\circ{\cdot}61 = 12^\circ{\cdot}2,$$

at the sun, which corresponds to an elongation of 25°.

The arc of retrogression is little less than a degree.

2678. *Conditions which favour the observation of Venus.*— This planet presents itself to the observer under conditions in many respects more favourable for telescopic examination than

Mercury. The actual diameter of Venus is more than twice that of Mercury. It approaches nearer to the earth in the inferior part of its orbit in the ratio of 13 to 30. It elongates itself from the sun to the distance of 46°, while the elongation of Mercury is limited to 22½°. The latter is never seen, except in strong twilight. Venus, especially in the lower latitudes, is seen at a considerable elevation long after the cessation of evening and before the commencement of morning twilight, and when she has a gibbous or a crescent phase. The planet appears brightest when its elongation is about 40° in the superior part of her orbit.

2679. *Evening and morning star.—Lucifer and Hesperus.*— This planet for these reasons is, next to the sun and moon, the most conspicuous and beautiful object in the firmament. When it has western elongation, it rises before the sun, and is called the MORNING STAR. When it has eastern elongation it sets after the sun, and is called the EVENING STAR.

The ancients gave it, in the former position, the name LUCIFER (the harbinger of day), and in the latter HESPERUS.

2680. *Apparent diameter.* — Owing to the great difference between its distance from the earth at inferior and superior conjunctions, the apparent diameter of this planet varies in magnitude within wide limits. At superior conjunction it is only 10″, from which to inferior conjunction it gradually enlarges until it becomes 62″, and in some positions even so much as 76″. At its greatest elongation its apparent diameter is about 25″, and at its mean distance 16½″.

Thus, when the planet appears as a thin crescent immediately before or after inferior conjunction, the magnitude is such that the line joining the cusps is the 30th part of the line joining the cusps of the crescent moon, and a telescope having a magnifying power no greater than 30 will show it with an apparent size equal to that of the crescent moon to the naked eye.

At or near the greatest elongation it requires a magnifying power of 70, and near superior conjunction one of 180, to produce a like effect.

2681. *Difficulties attending the telescopic observations of Venus.* — Notwithstanding this the greatest difficulties have attended the telescopic observation of this planet. Its intense lustre dazzles the eye, and aggravates all the optical imperfections of the instrument.

The low altitudes at which the observations are generally made constitute another difficulty, the irregular effects of refraction interfering materially with the appearance. Some observers have consequently contended that the best position for observations upon it is near superior conjunction, when its phase is full, and when by proper expedients it may be observed at midday within a few degrees of the sun's disk.

2682. *Real diameter.*—The linear value of 1″ at Venus, when she appears as a thin crescent near her inferior conjunction, is

$$\frac{26{,}250{,}000}{206{,}265} = 127{\cdot}2 \text{ miles.}$$

At this distance her apparent diameter is 61″; and if D′ express her real diameter, we shall have

$$D' = 127{\cdot}2 \times 61 = 7760 \text{ miles.}$$

The magnitude of Venus is, therefore, nearly equal to that of the earth.

2683. *Mass and density.*—By the methods explained in (2639), it has been ascertained that the mass of Venus is greater than that of the earth in the ratio of 113 to 100; and as the volumes are nearly equal their densities are also nearly equal.

2684. *Superficial gravity.* — All the conditions which affect the gravity of bodies on the surface of Venus being the same, or nearly so, as those which affect bodies on the earth, the superficial gravity is nearly the same.

2685. *Solar light and heat.*—The density of the solar rays is greater than upon the earth in the inverse ratio of the squares of the numbers 7 and 10, which express their distances from the sun. The intensity is, therefore, greater at Venus in the ratio of 2 to 1.

The relative apparent magnitudes of the sun's disk at Venus and the earth are represented at V and E, *fig.* 747. Owing to the very small eccentricity of the orbit this magnitude is not subject to any very sensible variation.

Fig. 747.

2686. *Rotation—probable mountains.* — Although there is very little doubt of the fact that this planet has a diurnal rotation analogous to that of the earth, the observations which might have been expected to demonstrate it in a satisfactory

manner have been obstructed by the causes already noticed (2681). Nevertheless Cassini, in the 17th century, and Schröter towards the close of the 18th, with instruments very inferior to the telescopes of the present day, deduced from the phases a period of rotation in complete accordance with the results of the most recent observations.

These astronomers found that the points of the horns of the crescent observed between inferior conjunction and greatest elongation appeared at certain moments to lose their sharpness, and to become as it were blunted. This appearance was, however, of very short duration, the horn after some minutes always recovering its sharpness. Such an effect would obviously be produced by a local irregularity of surface on the planet, such as a lofty mountain, which would throw a long shadow over that part of the surface which would form the point of the horn. Now, admitting this to be the cause of the phenomenon, it ought to be reproduced by the same mountain at equal intervals, this interval being the time of rotation of the planet. Such a periodical recurrence was accordingly ascertained.

2687. *Observations of Cassini, Herschel, and Schröter.*— From such observations the elder Cassini, so early as 1667, inferred the time of rotation of the planet to be $23^{h}. 16^{m}.$, a period not very different from that of the earth. Soon after this, Bianchini, an Italian astronomer, published a series of observations tending to call in doubt the result obtained by Cassini, and showing a period of 576 hours. Sir William Herschel resumed the subject, aided by his powerful telescopes, in 1780, but without arriving at any satisfactory result, except the fact that the planet is invested with a very dense atmosphere. He found the cusps (contrary to the observations of Cassini, and, as we shall see, of more recent astronomers) always sharp, and free from irregularities. Schröter made a series of most elaborate observations on this planet, with a view to the determination of its rotation. He considered not only that he saw periodical changes in the form of the points of the horns, but also spots, which had sufficient permanency to supply satisfactory indications of rotation. From such observations he inferred the time of rotation to be $23^{h}. 21^{m}. 7{\cdot}98^{s}.$ From observations upon the horns, he inferred also that the southern hemisphere of the planet was more mountainous than the northern; and he attempted, from observations on the bluntness periodically pro-

duced on the southern point of the crescent, to estimate the height of some of the mountains, which he inferred to amount to the almost incredible altitude of twenty-two miles.

2688. *Observations of MM. Beer and Mädler — time of rotation.* — Although the estimate of the planet's rotation resulting from the observations of Schröter, corroborating those of Cassini, has been generally accepted by the scientific world, the question was not regarded as definitively settled; and a series of observations was made by MM. Beer and Mädler, between 1833 and 1836, which went far to confirm the conclusions of Cassini and Schröter; and the still more recent observations of De Vico at Rome may be considered as removing all doubt that the period of the planet's rotation does not vary much from $23\frac{1}{4}^{h}$.

2689. *Beer and Mädler's diagrams of Venus.* — In *fig.* 748.,

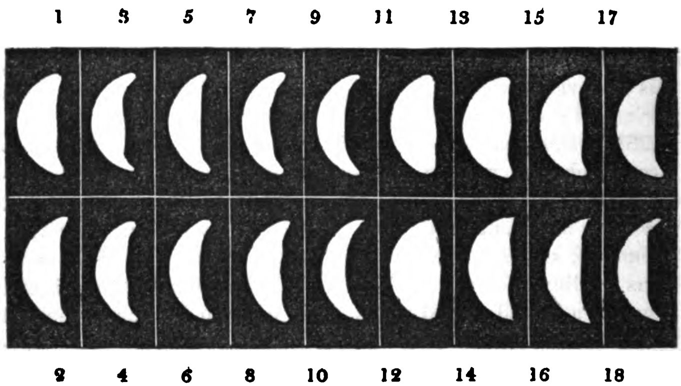


Fig. 748.

are represented a series of eighteen diagrams of the planet, selected from a much greater number made by MM. Beer and Mädler at the dates indicated above. These drawings were when taken the planet was approaching inferior conjunction, the planet being observed either before sunset or during twilight.

If the surface of the planet were exempt from considerable inequalities, the concave edge of the crescent would be a sensible ellipse, subject to no other deficiency of perfect regularity and sharpness, save such as might be explained by the gradual faintness of illumination due to the atmosphere of Venus. The mere inspection of the diagrams is enough to show that such is

not the appearance of the disc. Irregularities of curvature and of the forms of the cusps are apparent, which can only arise from corresponding irregularities of the surface of the planet. If the want of sharpness in the horns of the crescent arose from any effect produced by the terrestrial atmosphere on the optical image of the disc it would equally affect both cusps. Several of the diagrams, for example, *figs.* 1, 2, 3, 7, 8, 15, 17, are at variance with such an hypothesis, the cusps being obviously different in form.

In corroboration of the observations of Schröter, it was ascertained that the southern cusp was subject to greater and more frequent changes of form than the northern, from which it was inferred that the southern hemisphere of the planet is the more mountainous. It is remarkable that the same character is found to prevail on the moon.

It was not only observed that the irregularities of the concave edge of the crescent were subject to a change visible from 5^m. to 5^m., but that the same forms were reproduced after an interval of $23\frac{1}{4}^h$., subject to an error not exceeding from 5 to 10 minutes.

2690. *More recent observations of De Vico.* — In fine, De Vico, observing at a still later date at Rome, favoured by the clear sky of Italy, made several thousand measurements of the planet in its phases, the general result of which is in such complete accordance with those of MM. Beer and Mädler, that the fact of the planet's rotation may be now regarded as satisfactorily demonstrated, and that its period does not differ much from 23^h. 15^m.

2691. *Direction of the axis of rotation unascertained.* — If such difficulties have attended the mere determination of the rotation, it will be easily conceived that those which have attended the attempts to ascertain the direction of the axis of rotation have been much more insurmountable. The observations above described, by which the rotation has been established, supply no ground by which the direction of the axis could be ascertained. No spot has been seen the direction of whose motion could indicate that of the axis. It was conjectured, with little probability, by some observers, that the axis was inclined to the orbit at the angle of 75°. This conjecture, however, has not been confirmed.

2692. *Twilight on Venus and Mercury.*—The existence of

an extensive twilight in these planets has been well ascertained. By observing the concave edge of the crescent which corresponds to the boundary of the illuminated and dark hemispheres, it is found that the enlightened portion does not terminate suddenly, but there is a gradual fading away of the light into the darkness, produced by the band of atmosphere illuminated by the sun which overhangs a part of the dark hemisphere, and produces upon it the phenomena of twilight.

Some observers have seen on the dark hemisphere of the planet *Venus* a faint reddish and grayish light, visible on parts too distant from the illuminated hemisphere to be produced by the light of the sun. It was conjectured that these effects are indications of the play of some atmospheric phenomena in this planet similar to the *aurora borealis.*

In fine, it may be stated generally, that so far as relates to the physical condition of the inferior planets, the whole extent of our certain knowledge of them is, that they are globes like the earth, illuminated and warmed by the sun; that they are invested with atmospheres probably more dense than that of the earth; and since observations render probable the existence of vast masses of clouds on Venus, if not on Mercury, analogy justifies the inference that liquids exist on these planets.

2693. *Spheroidal form unascertained—suspected satellite.* — One of the phenomena from which the rotation, as well as the direction of the axis, might be inferred, is the spheroidal form of the planet. To ascertain this by observations of the disk, it would be necessary to see the planet with a full phase. But when the inferior planets have that phase, they are near superior conjunction, and therefore lost in the solar light. It has been nevertheless contended, that when Venus is most remote from her node, she is sufficiently removed from the plane of the ecliptic to be observed with a good telescope at noon when in superior conjunction. No observation, however, of this kind has ever yet been made, and the spheroidal form of the planet is unascertained.

Several observers of the last two centuries concurred in maintaining that they had seen a satellite of Venus. Cassini, the elder, imagined he saw such a body near the planet on the 25th of January, 1672, and again on 27th of August, 1686; Short, the well-known optical instrument maker, on 3d November, 1740; Montaigne, the French astronomer, in May, 1761; several ob-

servers in March, 1764, all agree in reporting observations of such a body. In each case the phase was similar to that of Venus, and the apparent diameter about a fourth of that of the planet. By collecting these observations, Lambert computed the orbit of the supposed satellite.

In opposition to all this, it may be stated that notwithstanding the immense improvement in optical instruments, and especially in the construction of telescopes of power far surpassing any of which the observers before the present century were in possession, no trace of such a body has been detected, although observers have increased in number, activity, and vigilance, in a proportion greater still than that of the improvement of telescopes. It must, therefore, be concluded, at least for the present, that the supposed appearances recorded by former observers were illusive.

III. Mars.

2694. *Position in the system.*—Proceeding outwards from the sun, the third planet in the order of distance is the Earth. The fourth in order, whose orbit circumscribes that of the earth, is the planet Mars.

2695. *Period.*—The synodic period of Mars is found by observation to be 780 days. It follows from this that if P express the periodic time of the planet in days, we shall have

$$\frac{1}{P} = \frac{1}{365} - \frac{1}{780} = \frac{1}{687}.$$

The periodic time of Mars is therefore 687 days, or, as appears by more exact methods of calculation and observation, 686·979 days.

The earth's period being taken as the unit, the period of Mars will therefore be 1·881.

2696. *Distance.*—To compute by the Harmonic Law the mean distance of Mars from the sun, we have therefore

$$1{\cdot}88^2 = 1{\cdot}5246^3.$$

The mean distance is therefore 1·5246, that of the earth being the unit, and the mean distance in miles is

$$95{,}000{,}000 \times 1{\cdot}524 = 144{,}780{,}000,$$

or about 144¾ millions of miles.

2697. *Eccentricity—mean and extreme distances from the*

earth.—The eccentricity of the orbit of Mars being about 0·09, the distance is subject to a variation, the extreme amount of which is less than one-tenth of its mean value. The extreme distances are

$144\frac{3}{4} + 13 = 157\frac{3}{4}$ million miles in aphelion.
$144\frac{3}{4} - 13 = 131\frac{3}{4}$ million miles in perihelion.

It appears, therefore, that the mean distances of the planet from the earth are

In Opposition	-	-	$144\frac{3}{4} - 95 = 49\frac{3}{4}$ million miles.
In Conjunction	-	-	$144\frac{3}{4} + 95 = 239\frac{3}{4}$ million miles.
In Quadrature	-	-	$= 109$ million miles.

These distances are subject to variation, whose extreme limit is about 15 millions of miles, owing to the combined effects of the eccentricities of the two orbits. Although the mean distance of the planet in opposition from the earth is about half the distance of the sun, it may in certain positions of the orbit come within a distance of 35 hundredths of the sun's distance. In the opposition which took place in September, 1830, the distance of the planet was only 38th hundredths of the sun's mean distance.

2698. *Heliocentric and synodic motions.*—The mean daily Heliocentric motion of Mars is (2568)

$$a = \frac{1296000}{687} = 18''86 = 31'{\cdot}1.$$

The mean synodic motion is therefore (2569)

$$\sigma = \epsilon - a = 3548 - 1886 = 16''62 = 27'{\cdot}7.$$

2699. *Scale of orbit relatively to that of the earth.*—If S, *fig.* 749., represent the position of the sun, and SM the distance of Mars, the orbit of the earth will be represented by EE″E‴E′.

2700. *Division of the synodic period.*—The earth is at E‴ when Mars is in conjunction, at E′ when in quadrature west of the sun, at E when in opposition, and at E″ when in quadrature east of the sun.

The angle of elongation SE′M being 90°, and the mean value of SM being 1·52, that of SE′ being expressed by 1, it follows that the angle E′SM will be about 48°, and therefore E′SE‴ = 180° — 48° = 132°.

Since the synodic period is 780 days, the mean time between quadrature and opposition will be

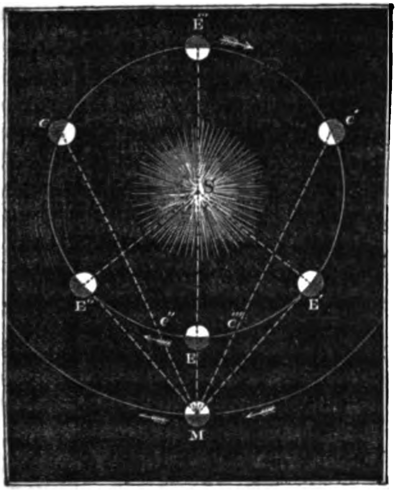


Fig. 749.

$$\frac{48}{360} \times 780 = 104 \text{ days};$$

and the mean time between quadrature and conjunction will be

$$\frac{132}{360} \times 780 = 286 \text{ days}.$$

2701. *Apparent motion.* — The various changes of the apparent positions of the planet and sun during the synodic period may, therefore, be easily explained. At conjunction the earth being at E''', the planet and sun pass the meridian together. In this case, the planet being above the horizon only during the day, is not visible. After conjunction, the planet passes the meridian in the forenoon, and is therefore visible above the eastern horizon before sunrise. Before conjunction it passes the meridian in the afternoon, and is therefore visible above the western horizon after sunset.

At the time of the western quadrature, the earth being at E', the planet passes the meridian about 6 A. M., and at the time of western quadrature the earth being at E'' it passes the meridian about 6 P.M. The planet has these positions about 286 days, more or less, after and before its conjunction.

At the time of opposition, the earth being at E, the planet passes the meridian at midnight; and is therefore above the horizon from sunset till sunrise. Before opposition it passes the meridian before midnight, and is above the horizon chiefly during the later part of the night, and after opposition it passes the meridian after midnight, and is therefore above the horizon chiefly during the earlier part of the night.

The interval during which it is visible more or less in the absence of the sun, being that during which it passes from western to eastern quadrature through opposition is, in the case of Mars, $104 \times 2 = 208$ days.

2702. *Stations and retrogression.*—The elongations at which Mars is stationary, and the lengths of his arc of retrogression, vary to some extent with the distances of the planet from the sun and earth, which distances depend on the ellipticity of the two orbits, and the direction of their major axes. In 1854 Mars will be in opposition on 1st March, and will be stationary on the 17th January and 10th April. The right ascension on these days will be,

17 Jan. - R. A. - -	11^h	19^m	32^s
10 April - R. A. - -	10	4	56
		14	36

It follows, therefore, that the extent of retrogression in right ascension will then be $14^m\ 36^s$, which reduced to angular magnitude is

$$(14^m\ 36^s) \times 15 = 219' = 3^\circ - 39'.$$

2703. *Phases.*—At opposition and conjunction the same hemisphere being turned to the earth and sun, the planet appears with a full phase. In all other positions the lines drawn from the planet to the earth and sun, making with each other an acute angle of greater or less magnitude, the phase will be deficient of complete fullness, and the planet will be gibbous, more so the nearer it is to its quadrature, in which position the lines drawn to the earth and sun make the greatest possible angle, which being the complement of E′ S M, will be $90^\circ - 48^\circ = 42^\circ$. Of the entire hemisphere presented to the earth 138° will therefore be enlightened and 42° dark. The corresponding form of the disk, as can easily be deduced from the common principles of projection, will be that which is represented in *fig.* 750., the dark part being indicated by the dotted line.

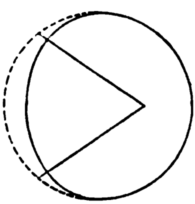


Fig. 750.

The gibbosity will be less the nearer the planet approaches to opposition or conjunction.

2704. *Apparent and real diameter.*—The apparent diameter of Mars in opposition varies between rather wide limits, in con-

sequence of the variation of its distance from the earth in that position, arising from the causes explained above. When at its mean distance at opposition the apparent magnitude does not exceed 16″, and at conjunction it is reduced to 3″·7.

In 1830, soon after opposition, when its distance from the earth was $38\frac{4}{10}$ million of miles, it exhibited a diameter of 22″; the linear value of 1″ at that distance being

$$\frac{38400000}{206265} = 185 \cdot 7 \text{ miles},$$

the diameter D′ of the planet must be

$$D' = 185 \cdot 7 \times 22 = 4085 \text{ miles}.$$

2705. *Volume.*—If V′ be the volume, that of the earth being V, we have

$$\frac{V'}{V} = \left\{\frac{4085}{7900}\right\}^3 = \frac{1}{7 \cdot 5} = 0 \cdot 133.$$

The volume is therefore less than the seventh part of that of the earth. The relative volumes of Mars and the earth are represented at M and E, *fig.* 746.

2706. *Mass and density.*—By the methods explained it has been ascertained that the mass of Mars is 145, that of the earth being 1000.

We shall have for the density therefore,

$$\frac{d'}{d} = \frac{145}{133} = 1 \cdot 09.$$

The density is very nearly equal, therefore, to that of the earth.

2707. *Superficial gravity.*—The superficial gravity being determined by the formula (2639), we shall have

$$\frac{g'}{g} = 0 \cdot 54.$$

It appears, therefore, that the force of gravity on the surface of Mars is a little more than half its intensity on the surface of the earth.

2708. *Solar light and heat.*—The mean distance of the earth from the sun being less than that of Mars in the ratio of 10 to 15, the apparent diameter of the sun as seen from Mars will be less than its diameter as seen from the earth in the same

ratio. If E, *fig.* 747., represent the apparent disk of the sun as seen from the earth, M will represent its apparent disk as seen from Mars.

Fig. 751.

Since the density of the solar radiation decreases as the square of the distance increases, its density at Mars will be less than at the earth in the ratio of 4 to 9.

So far as the illuminating and heating powers of the solar rays depend on their density, they will, therefore, be less in the same proportion.

2709. *Rotation.*—There is no body of the solar system, the moon alone excepted, which has been submitted to so rigorous and successful telescopic examination as Mars. Its proximity to the earth in opposition, when it is seen on the meridian at midnight with a full phase, affords great facility for this kind of observation.

By observing the permanent lineaments of light and shade exhibited by the disk, its rotation on its axis can be distinctly seen, and has been ascertained to take place in 24^h 37^m 10^s, the axis on which it revolves appearing to be inclined to the plane of the planet's orbit at an angle of 28° 27′. The exact direction of the axis is, however, still subject to some uncertainty.

2710. *Days and nights.*—It thus appears that the days and nights in Mars are nearly the same as on the earth, that the year is diversified by seasons, and the surface of the planet by zones and climates not very different from those which prevail on our globe. The tropics, instead of being 23° 28′, are 28° 27′ from the equator, and the polar circles are in the same proportion more extended.

2711. *Seasons and climates.*—The year consists of 668 Martial days and 16 hours, the Martial being longer than the terrestrial day in the ratio of 100 to 97.

Owing to the eccentricity of the planet's orbit, the summer on the northern hemisphere is shorter than on the southern in the ratio of 100 to 79, but owing to the greater proximity of the sun, the intensity of its light and heat during the shorter

northern summer is greater than during the longer southern summer in the ratio of 145 to 100. From the same causes, the longer northern winter is less inclement than the shorter southern winter in the same proportion.

There is thus a complete compensation in both seasons in the two hemispheres.

The duration of the seasons in Martial days in the northern hemisphere is as follows: — spring 192, summer 180, autumn 150, winter 147.

2712. *Observations and researches of Messrs. Beer and Mädler.*—It is mainly to the persevering labours of these eminent observers that we are indebted for all the physical information we possess respecting the condition of the surface of this planet. Their observations, commenced at an early epoch, were regularly organised at the time of the opposition of 1830, with a view to ascertain with certainty and precision the time of rotation of the planet, the position of its axis, and so far as might be practicable a survey of its surface. These observations have been continued during every succeeding opposition, in which the planet having northern declination rose to a sufficient altitude, and was made visible by a telescope by Fraunhoffer of four and a half feet focal length, parallactically mounted, and moved by clockwork, so as to keep the planet in the field of view notwithstanding the diurnal motion of the earth. With this instrument they were enabled to use a magnifying power of 300, and as the disk of the planet subtended in 1830 a visual angle of 22″, it was, when thus magnified, viewed under an angle of 6600″ or 110′, being nearly four times the apparent diameter of the moon.

2713. *Areographic character.*—That many of the lineaments observed are areographic, and not atmospheric, is established beyond all contestation by their permanency. They are not always visible, and when visible not always equally distinct; but are observed to retain the same forms, no matter how distant may be the intervals at which they may be submitted to examination. The elaborate researches and observations of MM. Beer and Mädler, which commenced with the opposition of 1830, were continued with unwearied assiduity in every succeeding opposition of the planet for twelve years, so far as the varying declination and the state of the weather at the epochs of the oppositions permitted. The same spots, cha-

racterised by the same forms, and the same varieties of light and shade, were seen again and again in each succeeding opposition. Changes of appearance were manifest, but through those changes the permanent features of the planet were always discerned; just as the seas and continents of the earth may be imagined to be distinguishable through the occasional openings in the clouds of our atmosphere by a telescopic observer of Mars.

2714. *Telescopic views of Mars — areographic charts of the two hemispheres.*—A large collection of drawings of the various hemispheres of Mars presented to the observer has been made by MM. Beer and Mädler. Thirty-five were made during the opposition of 1830, upwards of thirty during that of 1837, and forty during that of 1841, from a comparison of which charts were made, showing the permanent areographic lineaments of the northern and southern hemisphere.

In Plate VII. we have given six views, selected from those of Beer and Mädler, with the dates subjoined. In Plate VIII. are given the areographic charts of the two hemispheres. It will be observed, that as each spot approaches the edge of the disk its apparent form is modified by the effect of foreshortening, owing to the obliquity of the surface of the planet to the visual ray.

2715. *Polar snow observed.*— All the lineaments exhibited in these drawings were found to be permanent, except the remarkable white spots which cover the polar regions. These circular areas presented the appearance of a dazzling whiteness, and one of them was so exactly defined and so sharply terminated, that it seemed like the full disk of a small and very brilliant planet projected upon the disk, and near the edge of a larger and darker one. The appearance, position, and changes of these white polar spots have suggested to all the observers who have witnessed them, the supposition that they proceed from the polar snows accumulated during the long winter, and which, during the equally protracted summer by exposure to the solar rays, more full by 7° degrees than at the poles of the earth, are partially dissolved, so that the diameter of the snow circle is diminished.

The increase and diminution of this white circle takes place at epochs and in positions of the axis of the planet, such as are in complete accordance with this supposition.

MARS

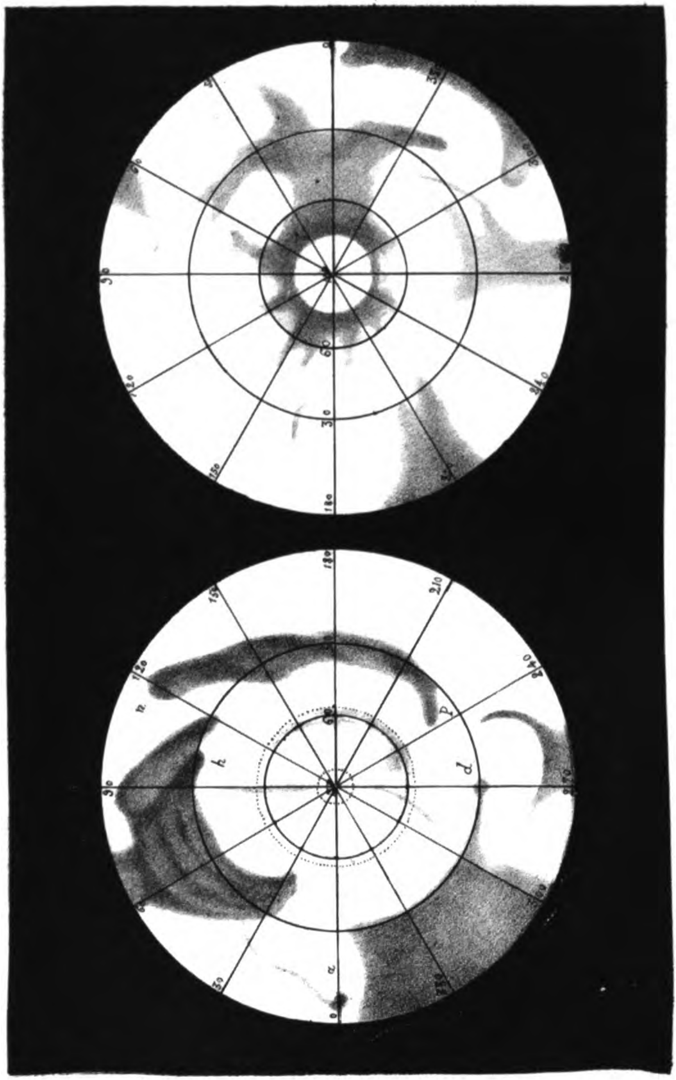
From telescopic drawings by MM. Beer & Mädler

VIII

1 Sep 14 10h 30m 2 Sep. 14 13h 3 Sep. 13 10h 6m
4 Oct. 14 7h 37m 5. Oct. 19 8h 13m 6 Oct 20 10h 20m

MARS

Telescopic projections of the two Hemispheres by Mädler



2716. *Position of areographic meridians determined.*—The leg and foot-shaped spot marked *p n* in the southern hemisphere, was distinctly seen and delineated in all the oppositions. This was one of the spots from the apparent motion of which the time of rotation was deduced.

The spot *a* in the southern hemisphere connected with a large adjacent spot by a sinuous line, was also one of those whose position was most satisfactorily established. This spot was selected as the observatory of Greenwich has been upon the earth, to mark the meridian from which longitudes are reckoned.

The spot *efh*, chiefly situate in the southern, but projecting into the northern hemisphere between the 90th and 105th degrees of longitude, was also well observed on repeated occasions.

According to Mädler, the reddish parts of the disk are chiefly those which correspond to 40° long. and 15° lat. S.

The two concentric dotted circles marked round the south pole indicate the limits of the white polar spot as seen on different occasions in 1830 and 1837. The redness of this planet is much more remarkable to the naked eye than when viewed with the telescope. In some cases, during the observations of MM. Beer and Mädler, no redness was discoverable, and when it was perceived it was so faint that different observers at the same moment were not agreed as to its existence. It was found that the prevailing colour of the spots was generally yellow rather than red.

Independently of any effect which could be ascribed to projection or foreshortening, it was found that the lineaments were always seen with much greater distinctness near the centre of the disk than towards its borders. This is precisely the effect which might be expected from a dense atmosphere surrounding the planet.

2717. *Possible satellite of Mars.*—Analogy naturally suggests the probability that the planet Mars might have a moon. These attendants appear to be supplied to the planets in augmented numbers as they recede from the sun; and if this analogy were complete, it would justify the inference that Mars must at least have one, being more remote from the sun than the earth, which is supplied with a satellite. No moon has ever been discovered in connection with Mars. It has, however, been contended that we are not therefore to conclude that the

planet is destitute of such an appendage; for as all secondary planets are much less than their primaries, and as Mars is by far the smallest of the superior planets, its satellite, if such existed, must be extremely small. The second satellite of Jupiter is only the forty-third part of the diameter of the planet; and a satellite which would only be the forty-third part of the diameter of Mars, would be under one hundred miles in diameter. Such an object could scarcely be discovered even by powerful telescopes, especially if it do not recede far from the disk of the planet.

The fact that one of the satellites of Saturn has been discovered only within the last few years, renders it not altogether improbable that a satellite of Mars may yet be discovered.

CHAP. XIV.

THE PLANETOIDS.

2718. *A vacant place in the planetary series.* — At a very early epoch in the progress of astronomy it was observed that the progression of the distances of the planets from the sun was characterised by a remarkable numerical harmony in which nevertheless a breach of continuity existed between Mars and Jupiter. This arithmetical progression was first loosely noticed by Kepler, but it was not until towards the close of the last century that the more exact conditions of the law and the close degree of approximation with which it was fulfilled, with the exception just noticed, was fully explained.

This numerical relation prevails between the distances of the successive orbits of the other planets measured from that of the first planet Mercury. It was observed that such distances formed very nearly a series in duple progression, so that each distance is twice the preceding one, with the sole exception already mentioned. Although this law is not fulfilled like those of Kepler, with numerical precision, there is nevertheless so striking an approximation to it as to produce a strong impression that it must be founded upon some physical cause and not merely accidental. To show the near approximation to its

exact fulfilment we have placed in the following table the succession of calculated distances from Mercury's orbit, which will exactly fulfil it in juxtaposition with the actual distances of the planets, the earth's distance from the sun being the unit.

	Calculated Distance from Mercury.	Actual Distance from Mercury.
Venus - - -	0·3362	0·3362
Earth - - -	0·6724	0·6129
Mars - - -	1·3448	1·1366
Absent planet - -	2·6896	
Jupiter - - -	5·3792	4·8157
Saturn - - -	10·7584	9·1517
Uranus - - -	21·5168	18·7953

By comparing these numbers it will be apparent that although the succession of distances does not correspond precisely with a numerical series in duple progression, there is nevertheless a certain approach to such a series, and at all events a glaring breach of continuity between Mars and Jupiter.

Towards the close of the last century, professor Bode of Berlin revived this question of a deficient planet, and gave the numerical progression which indicated its absence in the form in which it has just been stated; and an association of astronomers was formed under the auspices of the celebrated Baron de Zach of Gotha, for the express purpose of organising and prosecuting a course of observation, with the special purpose of searching for the supposed undiscovered member of the solar system. The very remarkable results which have followed this measure, the consequences of which have not even yet been fully developed, will presently be apparent.

2719. *Discovery of Ceres.*—On the first day of the present century, Professor Piazzi observing in the fine serene sky of Palermo, noticed a small star of about the 7th or 8th magnitude which was not registered in the catalogues. On the night of the 2nd again observing it, he found that its position relative to the surrounding stars was sensibly changed. The object appearing to be invested with a nebulous haze he took it at first for a comet, and announced it as such to the scientific world. Its orbit being however computed by Professor Gauss, of Göttingen, it was found to have a period of 1652 days, and a mean distance from the sun expressed by 2·735, that of the earth being 1.

By comparing this distance with that given in the preceding table at which a planet was presumed to be absent, it will be

seen that the object thus discovered filled the place with striking arithmetical precision.

Piazzi gave to this new member of the system the name CERES.

2720. *Discovery of Pallas.* — Soon after the discovery of Ceres the planet passing into conjunction ceased to be visible. In searching for it after emerging from the sun's rays in March 1802, Dr. Olbers noticed on the 28th a small star in the constellation of Virgo, at a place which he had examined in the two preceding months, and where he knew that no such object was *then* apparent. It appeared as a star of the seventh magnitude, the smallest which is visible without a telescope. In the course of a few hours he found its position visibly changed in relation to the surrounding stars. In fine the object proved to be another planet bearing a striking analogy to Ceres, and what was then totally unprecedented in the system, moving in an orbit at very nearly the same mean distance from the sun, and having therefore nearly the same period.

Dr. Olbers called this planet PALLAS.

2721. *Olbers' hypothesis of a fractured planet.* — This circumstance, combined with the exceptional minuteness of these two planets, suggested to Olbers the startling, and then, as it must have appeared, extravagantly improbable hypothesis, that a single planet of the ordinary magnitude existed formerly at the distance indicated by Bode's analogy, — that it was broken into small fragments either by internal explosion from some cause analogous to volcanic action, or by collision with a comet, — that Ceres and Pallas were two of its fragments, and in fine, that it was very likely that many other fragments, smaller still, were revolving in similar orbits, many of which might reward the labour of future observers who might direct their attention to these regions of the firmament.

In support of this curious conjecture it was urged that in the case of such a catastrophe as was involved in the supposition the fragments, according to the established laws of physics, would necessarily continue to revolve in orbits not differing much in their mean distances from that of the original planet; that the obliquities of the orbits to each other and to that of the original planet might be subject to a wider limit; that the eccentricities might also have exceptional magnitudes; and, finally, that such bodies might be expected to have magnitudes

so indefinitely minute as to be out of all analogy or comparison, not only with the other primary planets, but even with the smallest of the secondary ones.

Ceres and Pallas both were so small as to elude all attempts to estimate their diameters, real or apparent. They appeared like stellar points with no appreciable disk, but surrounded with a nebulous haziness, which would have rendered very uncertain any measurement of an object so minute. Sir W. Herschel thought that Pallas did not exceed 75 miles in diameter. Others have admitted that it might measure a few hundred miles. Ceres is still smaller.

The obliquity of the orbit of Ceres to the plane of the ecliptic is above $10\frac{1}{2}°$ and that of Pallas more than $34\frac{1}{2}°$. Both planets therefore when most remote from the ecliptic pass far beyond the limits of the zodiac, and differ in obliquity from each other by a quantity far exceeding the entire inclination of any of the older planets.

It was further observed by Dr. Olbers, that at a point near the descending node of Pallas the orbits of the two planets very nearly coincided.

Thus it appeared that all the conditions which rendered these bodies exceptional, and in which they differed from the other members of the solar system, were precisely those which were consistent with the hypothesis of their origin advanced by Dr. Olbers.

2722. *Discovery of Juno.*—A year and a half elapsed before any further discovery was produced to favour this hypothesis. Meanwhile observers did not relax their zeal and their labours, and on Sept. 1. 1804, at ten o'clock P. M., Professor Harding, of Lilienthal, discovered another minute planet, which observation soon proved to agree in all its essential conditions with the hypothesis of Olbers, having a mean distance very nearly equal to those of Ceres and Pallas, an exceptional obliquity of 13°, and a considerable eccentricity.

This planet was named JUNO.

Juno has the appearance of a star of the 8th magnitude, and a reddish colour. It was discovered with a very ordinary telescope of 30 inches focal length and 2 inches aperture.

2723. *Discovery of Vesta.* — On the 29th of March, 1807, Dr. Olbers discovered another planet under circumstances precisely similar to those already related in the cases of the former

discoveries. The name VESTA was given to this planet, which, in its minute magnitude and the character of its orbit, was analogous to Ceres, Pallas, and Juno.

Vesta is the brightest and apparently the largest of all this group of planets, and when in opposition may be sometimes distinguished by good and practised eyes without a telescope. Observers differ in their impressions of the colour of this planet. Harding and other German observers consider her to be reddish; others contend that she is perfectly white. Mr. Hind says that he has repeatedly examined her under various powers, and always received the impression of a pale yellowish cast in her light.

2724. *Discovery of the other Planetoids.*—The labours of the observers of the beginning of the century having been now prosecuted for some years without further results were discontinued, and it is probable that but for the admirable charts of the stars which have been since published, no other members of this remarkable group of planets would have been discovered. These, however, containing all the stars up to the 9th or 10th magnitude, included within a zone of the firmament 30° in width, extending to 15° on each side of the celestial equator, supplied so important and obvious an instrument of research, that the subject was again resumed with a better prospect of successful results. It was only necessary for the observer, map in hand, to examine, degree by degree, the zone within which such bodies are known to move, and to compare star by star the heavens with the map. When a star is observed which is not marked on the map, it is watched from hour to hour, and from night to night. If it do not change its position it must be inferred that it has been omitted in the construction of the map, and it is marked upon it in its proper place. If it change its position it must be inferred to be a planet, and its orbit is soon calculated from its observed changes of position.

By these means M. Henke, an amateur observer of Dreissen in Prussia, discovered on the 8th December, 1845, another of the small planets, which has been named Astræa.

Since that time the progress of planetary discovery in the same region has advanced with extraordinary rapidity. Three planets were discovered in 1847, one in 1848, one in 1849, three in 1850, two in 1851, and in fine, not less than eight in 1852.

In their exceptional minuteness of volume, their mean distances from the sun, and the very variable obliquities and eccentricities of their orbits, they all resemble the first four discovered in the beginning of the century, and are therefore in complete accordance with the conditions mentioned in the curious hypothesis of Olber's above stated.

In the following table is given a complete list of the planetoids discovered up to the close of the last year (1852), with the dates of their discovery, and the names of their discoverers.

The planet discovered by M. Gasparis, on the 17th of March, 1852, was observed by that astronomer at the Naples Observatory, on the 17th, 19th, and 20th March. It appeared as a star of the 10th or 11th magnitude. The observations were published in the "Comptes Rendus" of the Academy of Sciences, Paris, tome xxxiv. p. 532.

The planet discovered by M. Luther was observed by that astronomer at Bilk near Dusseldorf, on the 17th April, and again by M. Argelander, on the 22d April, at Bonn. The observations were published in the "Comptes Rendus" of the Paris Academy, tome xxxiv. p. 647.

2725. *Table showing the number of Planetoids discovered before* 1*st January*, 1853, *the names conferred upon them, their discoverers, and the dates of their discovery.*

	Name.	Discoverer.	When discovered.	Place of Observation.
1	Ceres.	Piazzi.	Jan. 1. 1801.	Palermo.
2	Pallas.	Olbers.	March 28. 1802.	Bremen.
3	Juno.	Harding.	Sept. 1. 1804	Lilienthal.
4	Vesta.	Olbers.	March 29. 1807.	Bremen.
5	Astræa.	Henke.	Dec. 8. 1845.	Dreissen (Prussia).
6	Hebe.	Henke.	July 1. 1847.	Dreissen.
7	Iris.	Hind.	Aug. 13. 1847.	London.
8	Flora.	Hind.	Oct. 18. 1847.	London.
9	Metis.	Graham.	April 25. 1848.	Markree (Ireland).
10	Hygeia.	De Gasparis.	April 12. 1849.	Naples.
11	Parthenope.	De Gasparis.	May 11. 1850.	Naples.
12	Victoria (called Clio by American astronomers).	Hind.	Sept. 13. 1850.	London.
13	Egeria.	De Gasparis.	Nov. 2. 1850.	Naples.
14	Irene *	Hind.	May 19. 1851.	London.
15	Eunomia.	De Gasparis.	July 29. 1851.	Naples.
16	Pysche.	De Gasparis.	March 17. 1852.	Naples.
17	Thetis.	Luther.	April 17. 1852.	Bilk (Dusseldorf).
18	Melpomene.	Hind.	June 24. 1852.	London.
19	Fortuna.	Hind.	Aug. 22. 1852.	London.
20	Massalia.	Chacornac.	Sept. 20. 1852.	Marseilles.
21	Lutetia.	Goldschmit.	Nov. 15. 1852.	Paris.
22	Calliope.	Hind.	Nov. 16. 1852.	London.
23	Thalia.	Hind.	Dec. 15. 1852.	London.

* This planet was discovered by M. de Gasparis four days later, at Naples, before that astronomer had received the information of the discovery of Mr. Hind.

2726. *The discovery of these mainly due to amateur astronomers.*—Dr. Olbers was a practitioner in medicine, Messrs. Henke, Luther, and Goldschmit amateur observers, Mr. Hind has been engaged in the private observatory of Mr. Bishop, in the Regent's Park, and Mr. Graham in that of Mr. Cooper, at Markree, in the county of Sligo, in Ireland. It appears, therefore, that of these twenty-three members of the solar system the scientific world owes no less than fourteen to amateur astronomers, and observatories erected and maintained by private individuals, totally unconnected with any national or public establishments, and receiving no aid or support from the state. Mr. Hind has obtained for himself the honourable distinction which must attach to the discoverer of eight of these bodies. Five are due to M. de Gasparis, assistant astronomer at the Royal Observatory at Naples.

M. Hermann Goldschmit is an historical painter, a native of Francfort on the Maine, but resident for the last eighteen years in Paris. He discovered the planet with a small ordinary telescope, placed in the balcony of his apartment, No. 12. rue de Seine, in the Faubourg St. Germain.

2727. *Their remarkable accordance with Dr. Olbers' hypothesis.*—The orbits of several of those observed in 1852 have not yet been calculated, but all those which have been computed are comprised between the mean distances 2·2 and 3·2, that to the earth being 1·0. The magnitudes of all of these bodies, with one or two exceptions, are too minute to be ascertained by any means of measurement hitherto discovered, and may be inferred with great probability not to exceed 100 miles in diameter. The largest of the group is probably less than 500 miles in diameter. It cannot fail, therefore, to be observed in how remarkable a manner they conform to the conditions involved in the hypothesis of Dr. Olbers.

2728. *Force of gravity on the planetoids.*—From the minuteness of their masses, the force of gravity on the surfaces of these bodies must be very inconsiderable, and this would account for a much greater altitude of their atmospheres than is observed on the larger planets, since the same volume of air feebly attracted would dilate into a volume comparatively enormous. Muscular power would be more efficacious on them in the same proportion. Thus a man might spring upwards sixty or eighty perpendicular feet, and return to the ground sustaining no

greater shock than would be felt upon the earth in descending from the height of two or three feet. "On such planets," observes Herschel, "giants might exist, and those enormous animals which on earth require the buoyant power of water to counteract their weight."

CHAP. XV.

THE MAJOR PLANETS.

I. Jupiter.

2729. *Jovian system.*—Passing across the wide space which lies beyond the range of the three planets which, with the earth, revolve as it were under the wing of the sun,—a space which was regarded as an anomalous desert in the planetary regions until contemporary explorers found there what seem to be the ruins of a shattered world,—we arrive at the theatre of other and more stupendous cosmical phenomena. The succession of planets, broken by the absence of one in the place occupied by the planetoids, is resumed, and four orbs are found constructed upon a comparatively Titanic scale, each attended by a splendid system of moons presenting a miniature of the solar system itself, and revolving round the common centre of light, heat, and attraction, at distances which almost confound the imagination.

2730. *Period.*—The synodic period of Jupiter is ascertained by observation to be 398 days. Hence to obtain its periodic time P, we have (2589)

$$\frac{1}{P} = \frac{1}{365 \cdot 25} - \frac{1}{398} = \frac{1}{4332 \cdot 6}$$

The period is therefore 4332·6 days, or 11·86 years.

2731. *Heliocentric and Synodic motions.*—The daily angular heliocentric motion of Jupiter is therefore

$$\frac{360^\circ}{4333} = 0^\circ \cdot 083 = 5'.$$

The mean angle gained daily by the earth or sun upon Jupiter, is therefore

$$0^\circ \cdot 9856 - 0^\circ \cdot 083 = 0^\circ \cdot 9026 = 54' \cdot 156.$$

2732. *Distance.*—The distance of Jupiter from the sun may be computed by means of the Harmonic Law (2621), the period being known. This method gives

$$(11{\cdot}83)^2 = (5{\cdot}2028)^3.$$

The mean distance of Jupiter from the sun is therefore $5\frac{1}{4}$ times that of the earth, and since the earth's mean distance is 95 millions of miles, that of Jupiter must be 494 millions of miles.

The eccentricity of Jupiter's orbit being 0·048, this distance is liable to variation, being augmented in aphelion and diminished in perihelion by 24 millions of miles. The greatest distance of the planet from the sun is therefore 518, and the least 470, millions of miles.

The small eccentricity of the orbit of this planet, combined with its small inclination to the plane of the ecliptic, is of great importance in its effect in limiting the disturbances consequent upon its mass, which, as will hereafter appear, is greater than the aggregate of the masses of all the other planets primary and secondary taken together. If the orbit of Jupiter had an eccentricity and inclination as considerable as those of the planet Juno, the perturbations produced by his mass upon the motions of the other bodies of the system, would be twenty-seven times greater than they are with its present small eccentricity and inclination.

2733. *Relative scale of the orbits of Jupiter and the Earth.*—The relative magnitudes of the distances of Jupiter and the earth from the sun, and the apparent magnitude of the orbit of the earth as seen from Jupiter, are represented in *fig.* 752., where the planet is at J, the sun at S, and the orbit of the earth E E′E‴E″.

The direction of the orbital motions being represented by the arrows, it will be evident that when the earth is at E the planet is in opposition, at E‴ in conjunction, at E′ in quadrature west, and at E″ in quadrature east of the sun.

2734. *Annual parallax of Jupiter.*—To determine the angle SJE′, which the semi-diameter of the earth's orbit subtends at Jupiter, or the annual parallax of the planet, it may be assumed without material inexactness that SE′ is nearly equal to an arc described with J as centre, and SJ as radius, and consequently (2294)

$$\text{SJE}' = \frac{57^\circ{\cdot}3}{5{\cdot}2} = 11^\circ.$$

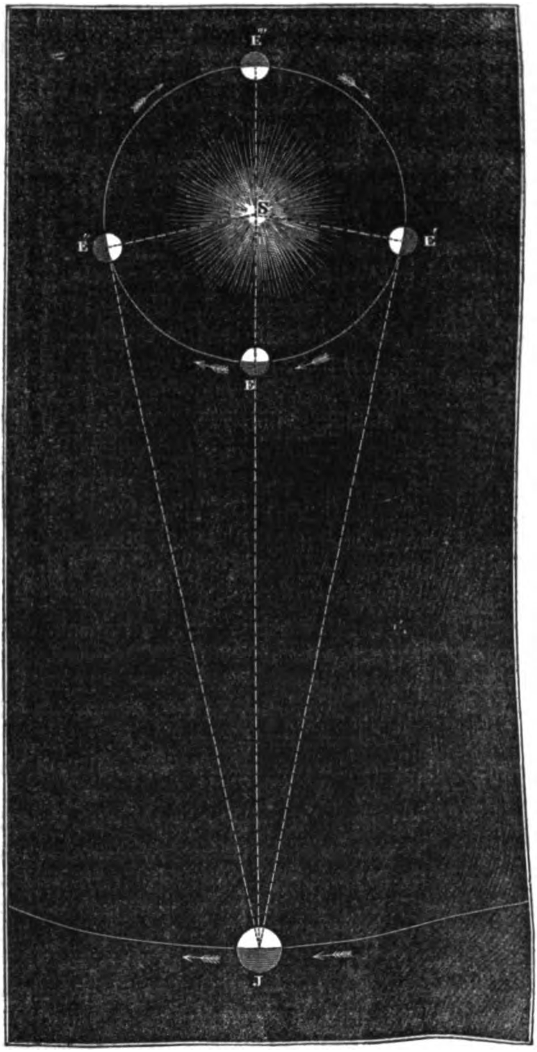


Fig. 752.

The annual parallax of Jupiter is therefore 11°, and consequently the orbit of the earth subtends at the planet an angle of 22°.

2735. *Variation of distance from the earth.*—Since the greatest and least distances JE''' and JE of Jupiter from the earth are the sum and difference of the distances of the planet and earth from the sun, we shall have

$$JE''' = 494 + 95 = 589 \text{ millions of miles.}$$
$$JE = 494 - 95 = 399 \text{ millions of miles.}$$
$$JE' = \sqrt{(494^2 - 95^2)} = 485 \text{ millions of miles.}$$

The extreme distances of the planet are therefore in the ratio of 6 to 5 nearly.

By the ellipticity of the earth's orbit, the distances at opposition and conjunction may be increased or diminished by $1\frac{1}{2}$ million of miles, and by that of the planet's orbit by 24 millions of miles. From both causes combined they may vary from their mean values more or less by $25\frac{1}{2}$ millions of miles.

2736. *Its prodigious orbital velocity.*—The velocities with which the planets move through space in their circumsolar courses are on the same prodigious scale as their distances and magnitudes. It is impossible, by the mere numerical expression of these enormous magnitudes and motions, to acquire any tolerably clear or distinct notion of them. A cannon ball moving at the rate of 500 miles an hour, would take nearly a century to come from Jupiter to the earth, even when the planet is nearest to us, and a steam-engine moving on a railway at 50 miles an hour would take nine centuries to perform the same trip.

Taking the diameter of Jupiter's orbit at 1000 millions of miles, its circumference is above 3000 millions of miles, which it moves over in 4333 days. The distance it travels is, therefore, 700,000 miles per day, 30,000 per hour, 500 per minute, and $8\frac{1}{3}$ per second,—a speed sixty times greater than that of a cannon ball.

2737. *Intervals between opposition, conjunction, and quadrature.*—If the distance of the planet from the sun bore an indefinitely great ratio to that of the earth, the quadratures would divide the semi-synodic period into parts precisely equal, for in that case JE' and JE'' would be practically parallel, and the bent line E'SE'' would become straight, and would be a

diameter of the earth's orbit. Although this is not the case, the angle formed by S E′ and S E″ being less than 180° by the magnitude of the angle E′ S E″ only, the intervals into which the semi-synodic period is divided are not very unequal.

We shall have the angle E′ S E″ $= 180° - 22° = 158°$, and it is evident that the time of gaining this angle will bear the same proportion to the synodic period which the angle itself bears to 360°. Hence, it follows, that if t express the interval from the quadrature west to the quadrature east, and t' the interval from the quadrature east to the quadrature west, we shall have

$$t = \frac{158}{360} \times 398 = 174\tfrac{2}{3} \text{ days}$$

$$t' = \frac{202}{360} \times 398 = 223\tfrac{1}{3} \text{ days.}$$

It follows, therefore, that the interval between opposition and quadrature is $87\frac{1}{3}$ days, and the interval between conjunction and quadrature is $111\frac{2}{3}$ days.

These are mean values of the intervals which are subject to variation owing to the eccentricities of the orbits of the earth and planet.

2738. *Jupiter has no sensible phases.*—The mere inspection of the diagram, *fig.* 748., will show that this planet cannot be sensibly gibbous in any position. The position in which the enlightened hemisphere is in view most obliquely is when the earth is at E′ or E″, and the planet consequently in quadrature, and even then the centre of the visible hemisphere is only 11° distant from the centre of the enlightened hemisphere (2734).

2739. *Appearance in the firmament at night.*— Since between quadrature and opposition the planet is above the horizon during the greater part of the night, and appears with a full phase, it is thus favourably placed for observation during 6 months in 13 months.

2740. *Stations and retrogression.* —From a comparison of the orbital motions and distance of Jupiter and the earth it appears that the planet is stationary at about two months before and two months after opposition; and since the earth gains upon the planet at the daily rate of 0°·907, the angle it gains in two months must be

$$0°{\cdot}907 \times 61 = 54°{\cdot}43.$$

The angular distance of the points of station from opposition, as seen from the sun, is therefore about 54°, which corresponds to an elongation of 114°.

The planet is therefore stationary at about 66° on each side of its opposition.

Its arc of retrogression is a little less than 10°, and the time of describing it varies from 117 to 123 days.

2741. *Apparent and real diameters.*—The apparent diameter of Jupiter when in opposition varies from 42″ to 48″, according to the relative positions of the planet and the earth in their elliptic orbits. At its mean opposition distance from the earth its apparent magnitude is 45″. In conjunction the mean apparent diameter is 30″, its value at the mean distance from the earth being 37½″.

At the distance of 399 millions of miles the linear value of 1″ is

$$\frac{399000000}{206265} = 1934 \text{ miles},$$

and consequently, the planet's diameter D′ will be

$$D' = 1934 \times 45 = 87030 \text{ miles}.$$

According to more accurate methods, the mean diameter is ascertained to be 88640 miles. The diameter of Jupiter is therefore 11·18 times that of the earth.

2742. *Jupiter a conspicuous object in the firmament—relative splendour of Jupiter and Mars.*—Although the apparent magnitude of Jupiter is less than that of Venus, the former is a more conspicuous and more easily observable object, inasmuch as when in opposition it is in the meridian at midnight, and when its opposition takes place in winter it passes the meridian at an altitude nearly equal to that which the sun has at the summer solstice. By reason, therefore, of this circumstance, and the complete absence of all solar light, the splendour of the planet is very great, whereas Venus, even at the greatest elongation, descends near the horizon before the entire cessation of twilight.

The apparent splendour of a planet depends conjointly on the apparent area of its disk, and the intensity of the illumination of its surface. The area of the disk is proportional to the square of its apparent diameter, and the illumination of the surface depends conjointly on the intensity of the sun's light at

the planet, and the reflecting power of the surface. On comparing Mars with Jupiter, we find the apparent splendour of the latter planet much greater than it ought to be, as compared with the former, if the reflecting power of these surfaces were the same, and are consequently compelled to conclude that the surface of Mars is endowed with some physical quality, in virtue of which it absorbs much more of the solar light incident upon it than that of Jupiter does. When the apparent diameter of the latter is twice that of the former, its apparent area is fourfold that of the former. But the intensity of the solar light at Jupiter is at the same time about thirteen times less than at Mars; and if the reflective power of the surfaces were equal, the apparent splendour of Mars would be more than three times that of Jupiter. The reflective power must, therefore, be less in a sufficient proportion to explain the inferior splendour of Mars, unless, indeed, the very improbable supposition be admitted that there may be a source of light in Jupiter independent of solar illumination.

2743. *Surface and volume.*—If S and V be the surface and volume of the earth, S′ and V′ being those of Jupiter, we shall have

$$S' = 125 \times S \qquad V' = 1397{\cdot}4 \times V.$$

The surface of Jupiter is therefore above 125 times, and its volume about 1400 times, those of the earth.

To produce a globe such as that of Jupiter it would be necessary to mould into a single globe 1400 globes like that of the earth.

The relative magnitudes of the globes of Jupiter and the earth are represented in *fig.* 753. by J and E.

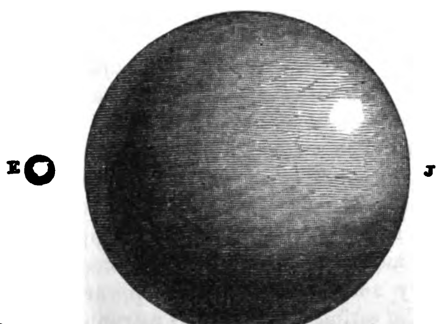


Fig. 753.

2744. *Solar light and heat.*—The mean distance of Jupiter from the sun being 5·2 times that of the earth, the apparent diameter of the sun to the inhabitants of that planet will be less than its apparent diameter at the earth in the proportion of 5·2 to 1. The relative apparent magnitudes of the disk of the sun at Jupiter and at the earth are represented in *fig.* 754. at E and J.

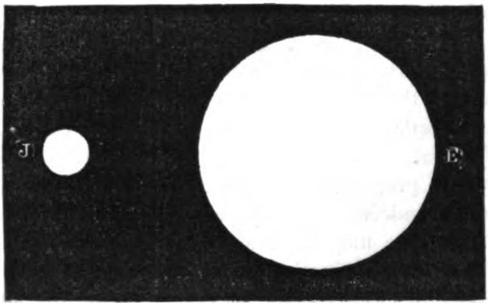


Fig. 754.

The density of solar radiation being in the exact proportion of the apparent superficial magnitudes of the disks, the illuminating and heating powers of the sun will, *ceteris paribus*, be less in the same proportion at Jupiter than at the earth.

As has been already observed, however, this diminished power as well of illumination as of warmth, may be compensated by other physical provisions.

2745. *Rotation and direction of the axis.*—Although the lineaments of light and shade on Jupiter's disk are generally subject to variations, which prove them to be, for the most part, atmospheric, nevertheless permanent marks have been occasionally seen, by means of which the diurnal rotation and the direction of the axis have been ascertained within very minute limits of error. The earlier observers, whose instruments were imperfect, and observations consequently inaccurate comparatively with those of more recent date, ascertained nevertheless the period of rotation with a degree of approximation to the results of the most elaborate observations of the present day, which is truly surprising, as may appear by the following statement of the estimates of various astronomers:—

	H.	M.	S.
Cassini (1665) - - - - -	9	56	
Silvabelle - - - - -	9	56	
Schröter (1786) - - - -	9	55	33
Airy - - - - - -	9	55	24·6
Mädler (1835) - - - - -	9	55	26·56

The estimate of Professor Airy is based upon a set of observations made at the Cambridge Observatory. That of Mädler is founded upon a series of observations, commencing on the 3rd of November, 1834, and continued upon every clear night until April, 1835, during which interval the planet made 400 revolutions. These observations were favoured by the presence of two remarkable spots near the equator of the planet, which retained their position unaltered for several months. The period was determined by observing the moments at which the centres of the spots arrived at the middle of the disk.

The direction of the apparent motion of the spots gave the position of the equator, and consequently of the axis, which is inclined to the plane of the planet's orbit at an angle of 3° 6′.

The length of the Jovian day is therefore less than that of the terrestrial day in the ratio of 596 to 1440, or 1 to 2·42.

2746. *Jovian years.* — Since the period of Jupiter is 4332·6 terrestrial days, it will consist of

$$4332{\cdot}6 = 10484{\cdot}9$$

Jovian days.*

2747. *Seasons.* — At the Jovian equinoxes the length of the day in terrestrial time must be $4^{h\cdot}\ 57^{m\cdot}\ 43{\cdot}5^{s\cdot}$ Owing to the very small obliquity of the plane of the planet's equator to that of its orbit, not much exceeding the eighth part of the obliquity of the earth's equator, the difference of the extreme length of the days at midsummer and midwinter, even at high latitudes, must necessarily be small. Thus at

	H.	M.	S.
Lat. 40°. — Longest day - - -	5	6	26
Shortest day - - -	4	49	14
Difference - -	0	17	12
Lat. 60°. — Longest day - - -	5	15	47
Shortest day - - -	4	39	53
Difference - -	0	35	54

* The day here computed is the sidereal day, which, in the case of the superior planets, differs from the mean solar day by a quantity so insignificant that it may be neglected in such illustrations as these.

The diurnal phenomena at midwinter and midsummer on the earth in latitudes higher than $66\frac{1}{2}°$ are only exhibited on Jupiter within a small circle circumscribing the pole at a distance of 3° 6′.

The extremes of temperature, so far as they depend on the varying distance of the planet from the sun, being in the proportion of the squares of the aphelion and perihelion distances, are as

$$518^2 : 470^2 :: 5 : 6 \text{ nearly.}$$

It appears, therefore, that except in the near neighbourhood of the poles the vicissitudes of temperature and season to which the surface of this planet is exposed, whether arising from the obliquity of its axis or the eccentricity of its orbit, are confined within extremely narrow limits.

2748. *Telescopic appearance of Jupiter.* — Of all the bodies of the system, the moon perhaps alone excepted, Jupiter presents to the telescopic observer the most magnificent spectacle. Notwithstanding its vast distance, such is its stupendous magnitude that it is seen under a visual angle nearly twice that of Mars. A telescope of a given power, therefore, shows it with an apparent disk four times greater. It has, consequently, been submitted to examination by the most eminent observers, and its appearances described with great minuteness of detail. The apparent diameter in opposition (when it is on the meridian at midnight) is about the fortieth part of that of the moon, and, therefore, a telescope with the very moderate magnifying power of forty, presents it to the observer with a disk equal to that with which the full moon is seen with the naked eye.

2749. *Magnifying powers necessary to show the features of the disk.* — A power of four or five is sufficient to enable the observer to see the planet with a sensible disk; a power of thirty shows the more prominent belts and the oval form of the disk produced by the oblateness of the spheroid; a power of forty shows it with a disk as large as that which the full moon presents to the naked eye; but to be enabled to observe the finer streaks which prevail at greater distances from the planet's equator, it is not only necessary to see the planet under favourable circumstances of position and atmosphere, but to be aided by a well-defining telescope with magnifying powers varying from 200 to 300.

2750. *Belts — their arrangement and appearance.* — The planet, when thus viewed, appears to exhibit a disk, the ground of which is a light yellowish colour, brightest near its equator, and melting gradually into a leaden-coloured gray towards the poles, still retaining, nevertheless, somewhat of its yellowish hue. Upon this ground are seen a series of brownish-gray streaks, resembling in their form and arrangement the streaks of clouds which are often observed in the sky on a fine calm evening after sunset. The general direction of these streaks is parallel to the equator of the planet, though sometimes a departure from strict parallelism is observable. They are not all equally conspicuous or distinctly defined. Two are generally strikingly observable, being extended north and south of the planet's equator, separated by a bright yellow zone, being a part of the general ground of the disk. These principal streaks commonly extend around the globe of the planet, being visible without much change of form during an entire revolution of Jupiter. This, however, is not always the case, for it has happened, though rarely, that one of these streaks, at a certain point, was broken sharply off so as to present to the observer, an extremity so well defined and unvarying for a considerable time as to supply the means of ascertaining, with a very close approximation, the time of the planet's rotation. The borders of these principal streaks are sometimes sharp and even, but, sometimes (those especially which are further from the equator), rugged and uneven, throwing out arms and offshoots.

2751. *Those near the poles more faint.* — On the parts of the disk more remote from the equator, the streaks are much more faint, narrower, and less regular in their parallelism, and can seldom be distinctly seen, except by practised observers, with good telescopes. With these, however, what appears near the poles, in instruments of inferior power, as a dim shading of a yellowish gray hue, is resolved into a system of fine parallel streaks in close juxtaposition, which becoming closer in approaching the pole, finally coalesce.

2752. *Disappear near the limb.*— In general, all the streaks become less and less distinct towards either the eastern or western limb, disappearing altogether at the limb itself.

2753. *Belts not zenographical features, but atmospheric.* — Although these streaks have infinitely greater permanency than the arrangements of the clouds of our atmosphere, and are,

as we have seen, even more permanent than is necessary for the exact determination of the planet's rotation, they are nevertheless entirely destitute of that permanence which would characterise Zenographic features, such as are observed, for example, on Mars. The streaks, on the contrary, are subject to slow but evident variations, so that after the lapse of some months the appearance of the disk is totally changed.

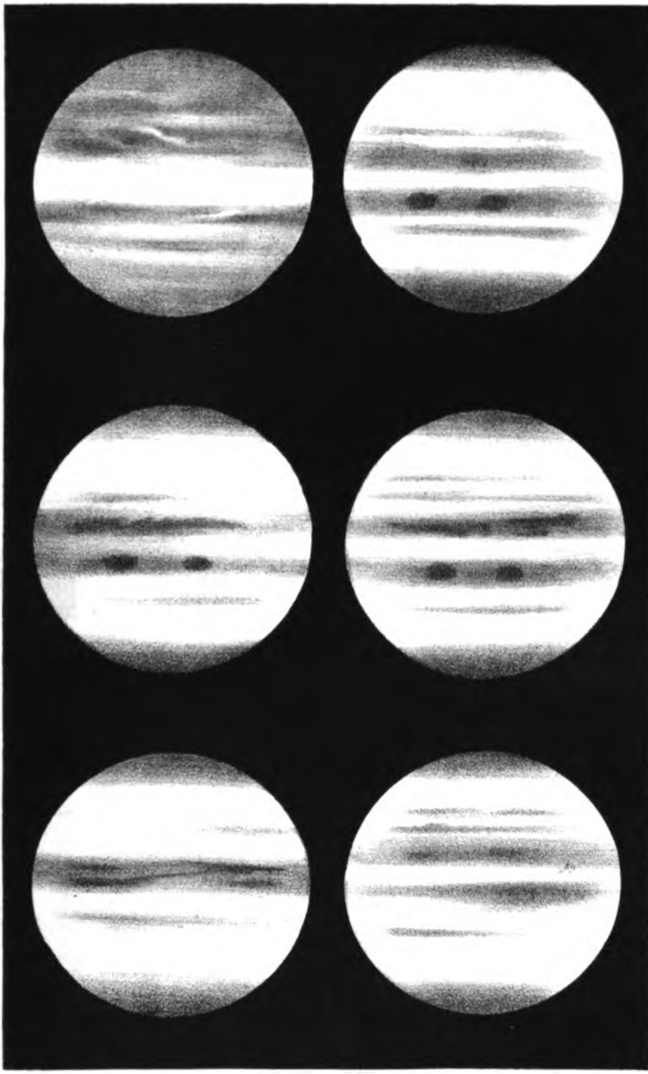
2754. *Telescopic drawings of Jupiter by Mädler and Herschel.* — These general observations on the appearance of Jupiter's disk will be rendered more clearly intelligible by reference to the telescopic drawings of the planet given in plate X. In *fig.* 1. is given a telescopic view of the disk by Sir John Herschel, as it appeared in the 20-feet reflector at Slough on the 23rd Sept. 1832. The other views were made by M. Mädler from observations taken in 1835, and 1836, at the dates indicated on the plate.

2755. *Observations and conclusions of Mädler.* — The two black spots represented in *figs.* 2, 3, and 4, were those by which the time of rotation was determined (2745.). They were first observed by Mädler, on the 3rd of Nov. 1834. The effect of the rotation on these spots was so apparent that their change of position with relation to the centre of the disk, in the short interval of five minutes, was quite perceivable. A third spot, much more faint than these, was visible at the same time, the distances separating the spots being about 24° of the planet's surface. It was estimated that the diameter of each of the two spots represented in the diagrams was 3680 miles, and the distance between them was sometimes observed to increase at the rate of half a degree, or 330 miles, in a month. The two spots continued to be distinctly visible from the 3rd of November, 1834, when they were first observed, until the 18th of April, 1835; but during this interval the streak on which they were placed, had entirely disappeared. It became gradually fainter in January (see *fig.* 4.), and entirely vanished in February; the spots, however, retaining all their distinctness. The planet, after April passing towards conjunction, was lost in the light of the sun; and when it reappeared in August, after conjunction, the spots had altogether vanished.

The observations being continued, the drawings, *figs.* 5. and 6., were made from observations, on the 16th and 17th of January, 1836, when the entire aspect of the disk was changed. The two

JUPITER
From telescopic drawings by Madler & Herschel

X



1 2 3 4 5 6

1 Sep 25 1832 2 Dec 23 1834 3 Dec 23 1834
4 Jan 2 1836 5 Jan 16 1836 6 Jan 17 1836

figures 5. and 6. represent opposite hemispheres of the planet. The former presents a striking resemblance to the principal belts in the drawing of Sir J. Herschel, *fig.* 1.

It was remarked that the two spots, when carried round by the rotation, became invisible at 55° to 57° from the centre of the disk. This is an effect which would be produced if the spots were openings in the mass of clouds floating in the atmosphere of the planet, and would be explicable in the same manner as is the disappearance of spots on the sun in approaching the edges of the disk. A proper motion with a slow velocity, and in a direction contrary to the rotation of the planet, was observed to affect the spots, and this motion continued with greater uniformity in March and April, after the disappearance of the belt.

It was calculated that the velocity of their proper motion over the surface of the planet, was at the rate of from three to four miles an hour.

Although the two black spots were not observed by Mädler until the first days of November, they had been previously seen and examined by Schwabe, who observed them to undergo several curious changes, in one of which one of them disappeared for a certain interval, its place being occupied by a mass of fine dots. It soon, however, reappeared as before.

From all these circumstances, and many others developed in the course of his extensive and long-continued observations, Mädler considers it highly probable, if not absolutely certain, that the atmosphere of Jupiter is continually charged with vast masses of clouds which completely conceal his surface; that these clouds have a permanence of form, position, and arrangement to which there is nothing analogous in the atmosphere of the earth, and that such permanence may in some degree be explained by the great length and very small variation of the seasons. He thinks it probable that the inhabitants of places in latitudes above 40° never behold the firmament, and those in lower latitudes only on rare occasions.

To these inferences it may be added that the probable cause assigned for the distribution of the masses of clouds in streaks parallel to the equator, is the prevalence of atmospheric currents analogous to the trades, and arising from a like cause, but marked by a constancy, intensity, and regularity exceeding those which prevail on the earth, inasmuch as the diurnal

motion of the surface of Jupiter is more rapid than that of the earth in the combined proportion of the velocity of the diurnal rotation and the magnitude of the circumference, that is, as 27 to 1 nearly.

It is also probable that the bright yellowish general ground of Jupiter's disk consists of clouds, which reflect light much more strongly than the most dense masses which are seen illuminated by the sun in our atmosphere; and that the darker streaks and spots observed upon the disk are portions of the atmosphere, either free from clouds and through which the surface of the planet is visible more or less distinctly, or clouds of less density and less reflecting power than those which float over the general atmosphere and form the ground on which the belts and spots are seen.

That the atmosphere has not any very extraordinary height above the surface of the planet, is proved by the sharply defined edge of the disk. If its height bore any considerable proportion to the diameter of the planet, the light towards the edges of the disk would become gradually fainter, and the edges would be nebulous and ill-defined. The reverse is the case.

2756. *Spheroidal form of the planet.*—The disk of Jupiter, seen with magnifying powers as low as 30, is evidently oval, the lesser axis of the ellipse coinciding with the axis of rotation, and being perpendicular to the general direction of the belts. This fact supplies a striking confirmation of the results attained in the measurement of the curvature of the earth; and, as in the case of the earth, the degree of oblateness of Jupiter is found to be that which would be produced upon a globe of the same magnitude, having a rotation such as the planet is observed to have.

At the mean distance from the earth, the apparent diameters of the disk are ascertained by exact micrometric measures to be—

	Miles.
Equatorial Diameter - - - -	38·4″ = 92,080
Polar Diameter - - - -	35·6″ = 85,210
Mean Diameter - -	= 88,645

The polar diameter is therefore less than the equatorial, in the ratio of 356 to 384 or 100 to 108 nearly. Other estimates give the ratio as 100 to 106.

2757. *Jupiter's satellites.*—When Galileo directed the first telescope to the examination of Jupiter, he observed four

minute stars, which appeared in the line of the equator of the planet. He took these at first to be fixed stars, but was soon undeceived. He saw them alternately approach to, and recede from the planet, observed them pass behind it and before it; and oscillate, as it were, to the right and the left of it, to certain limited and equal distances. He soon arrived at the obvious conclusion that these objects were not fixed stars, but that they were bodies which revolved round Jupiter in orbits, at limited distances, and that each successive body included the orbit of the others within it; in short, that they formed a miniature of the solar system, in which, however, Jupiter himself played the part of the sun. As the telescope improved, it became apparent that these bodies were small globes, related to Jupiter in the same manner exactly as the moon is related to the earth; that, in fine, they were a system of four moons, accompanying Jupiter round the sun.

2758. *Rapid change and great variety of phases.*—But connected with these appendages there is perhaps nothing more remarkable than the period of their revolutions. That moon which is nearest to Jupiter, completes its revolution in forty-two hours. In that brief space of time it goes through all its various phases; it is a thin crescent, halved, gibbous, and full. It must be remembered, however, that the day of Jupiter, instead of being twenty-four hours, is less than ten hours. This moon, therefore, has a month equal to a little more than four Jovian days. In each day it passes through one complete quarter; thus, on the first day of the month it passes from the thinnest crescent to the half moon; on the second, from the half moon to the full moon; on the third, from the full moon to the last quarter; and on the fourth returns to conjunction with the sun. So rapid are these changes that they must be actually visible as they proceed.

The apparent motion of this satellite in the firmament of Jupiter is at the rate of more than 8° per hour, and is the same as if our moon were to move over a space equal to her own apparent diameter, in rather less than four minutes. Such an object would serve the purpose of the hand of a stupendous celestial clock.

The second satellite completes its revolution in about eighty-five terrestrial hours, or about eight and a half Jovian days. It passes, therefore, from quarter to quarter in twenty-one hours,

or about two Jovian days, its apparent motion in the firmament being at the rate of about 4·25° per hour, which is as if our moon were to move over a space equal to nine times its own diameter per hour, or over its own diameter in less than seven minutes.

The movements and changes of phase of the other two moons are not so rapid. The third passes through its phases in about 170 hours, or seventeen Jovian days, and its apparent motion is at the rate of about 1° per hour. The fourth and last completes its changes in 400 hours, or forty Jovian days, and its apparent motion is at the rate of little less than 1° per hour, being double the apparent motion of our moon.

Thus the inhabitants of Jupiter have four different months of four, eight, seventeen, and forty Jovian days, respectively.

2759. *Elongation of the satellites.*— The appearance which the satellites of Jupiter present when viewed with a telescope of moderate power, is that of minute stars ranged in the direction of a line drawn through the centre of the planet's disk nearly parallel to the direction of the belts, and therefore coinciding with that of the planet's equator. The distances to which they depart on the one side or the other of the planet, are so limited that the whole system is included within the field of any telescope whose magnifying power is not considerable; and their elongations from the centre of the planet can therefore be measured with great precision by means of the wire micrometers.

When the apparent diameter of the planet in opposition is 45″, the greatest elongations of the satellites from the centre of the planet's disc are as follow: —

I. - - -	135″
II. - - -	215″
III. - - -	346″
IV. - - -	585″

It follows, therefore, that the entire system is comprised within a visual area of about 1200″ in extent, being two-thirds of the apparent diameter of the moon. If, therefore, we conceive the moon's disk to be centrically superposed on that of Jupiter, not only would all the satellites be covered by it, but that which elongates itself most from the planet would not approach nearer to the moon's edge than one-sixth of its apparent diameter.

If all the satellites were at the same time at their greatest

elongations, they would, relatively to the apparent diameter of the planet, present the appearance represented in *fig.* 755.

Fig. 755.

2760. *Distances from Jupiter.*—The actual distances of the satellites from the centre of the planet may be immediately inferred, from a comparison of their greatest elongations with the apparent semi-diameter of the planet. Since, in the case above supposed, the apparent semi-diameter of the planet is 22·5″, the distances will be found expressed with reference to the semi-diameter as the unit, by dividing the greatest elongations expressed in seconds by 22·5. This gives for the distances:—

I. - - -	$\frac{135}{22 \cdot 5} = 6 \cdot 0.$
II. - - -	$\frac{215}{22 \cdot 5} = 9 \cdot 6.$
III. - - -	$\frac{346}{22 \cdot 5} = 15 \cdot 4.$
IV. - - -	$\frac{585}{22 \cdot 5} = 26.$

Relatively to the magnitude of the planet, therefore, the satellites revolve much closer to it than the moon does to the earth. The distance of the moon is nearly 60 semi-diameters of the earth, while the distance of the most remote of Jupiter's moons is not more than 26 semi-diameters, and that of the nearest only six, from his centre.

Owing, however, to the greater dimensions of Jupiter, the actual distances of the satellites, expressed in miles, are (except that of the first) greater than the distance of the moon from the earth.

2761. *Harmonic law observed in the Jovian system.*—That the same law of gravitation which reigns throughout the material universe, prevails in this system, is rendered manifest by the accordance of the motions and distances of the satellites with the harmonic law. In the following table numerical relations establishing this are exhibited:—

	D	P	D^3	P^2	$\frac{P^2}{D^3}$
I.	6·0	43	216	1849	8·6
II.	9·6	85	885	7225	8·2
III.	15·4	172	3652	29,584	8·1
IV.	27·0	400	19,683	160,000	8·1

The want of exact equality in the numbers in the last column, by which the ratio of the squares of the periods to the cubes of the distances are expressed, is to be ascribed partly to using round numbers only, and partly to the effects of the mutual disturbances produced by the satellites upon each other and by the spheroidal form of the planet itself.

2762. *Singular relation between the motions of the first three satellites.*— On comparing the periods of the first three satellites, it is evident that they are in the ratio of the numbers 1, 2, and 4. For we have—

$$43 : 86 : 171 :: 1 : 2 : 4.$$

Since the mean angular velocities, or, what is the same, the mean apparent motions as seen from Jupiter, are found by dividing 360° by the periodic times, it follows that these motions for the three satellites are in the inverse ratio of 1, 2, and 4, that is, as 1, $\frac{1}{2}$, and $\frac{1}{4}$; and, therefore, that the mean apparent motion of the second satellite is half, and that of the third one-fourth of the mean apparent motion of the first.

It follows, also, that if twice the mean motion of the third be added to the mean motion of the first, the sum will be three times the mean motion of the second. This will be rendered evident by expressing these motions by general symbols. Let m', m'', and m''' express the mean hourly apparent motions. We shall have—

$$m = \tfrac{1}{2} m' \qquad\qquad m''' = \tfrac{1}{4} m';$$

and consequently

$$m' + 2\,m''' = m' + \tfrac{1}{2}\,m' = \tfrac{3}{2}\,m' = 3\,m''.$$

2763. *Corresponding relation between their mean longitudes.* —The longitudes of satellites are referred to their primaries as visual centres. Thus the mean longitudes of Jupiter's satellites are their mean angular distances from the first point of Aries as seen from Jupiter. Now, it follows that the relation which has been shown to prevail between the mean motions of

the first three satellites, also prevails between their mean longitudes. Let these longitudes at any proposed time be l', l'', l'''; and after a given interval, during which all the satellites will have augmented their longitudes, let them be L', L'', L'''. The angles or arcs moved through in the interval will be $L'-l'$, $L''-l''$, $L'''-l'''$; and since these will represent and be proportional to the mean apparent motions we shall have—

$$(L'-l')+2\,(L'''-l''')=3\,(L''-l'');$$

from which is inferred—

$$3\,L''-(L'+2\,L''')=3\,l''-(l'+2\,l''').$$

It appears, therefore, that the difference between three times the longitude of the second and the sum of the longitude of the first and twice that of the third is invariable; but what this invariable difference is does not appear from the mere relation of the periods. A single observation of the positions of the three satellites at any proposed moment, is sufficient to ascertain this difference; since whatever it may be at any one moment, it must always continue to be. Now, it may be thus easily ascertained by observations made at any proposed time, that this difference is exactly 180°. We shall thus have, as a permanent relation between the positions of these three satellites—

$$3\,L''-(L'+2\,L''')=180°;$$

so that, whenever the positions of any two of them are given, the position of the other can be found.

It follows from this relation, that the three satellites can never have at the same time the same phase; for if they had, they must necessarily have the same visual direction, and consequently the same longitude, which would be incompatible with the preceding relation. If two of them have nearly the same phase, the third must have a phase differing from it by 180°, 90°, or 60°, according to the satellites which agree in their phase.

If the second and third have nearly the same phase, we shall have $L''=L'''$; and therefore—

$$3L''-L'-2L''=L''-L'=180°.$$

The first will have a position, and therefore a phase, in direct opposition to the common phase of the second and third. If one be new, the other will be full, and *vice versâ*.

If the first and second have a common phase, we shall have $L' = L''$; and therefore—

$$3L' - L' - 2L''' = 2(L' - L''') = 180°.$$
$$L' - L''' = 90°.$$

The third satellite will therefore be 90° from the common direction of the other two, and will therefore have a phase different from theirs by 90°. If one be full or new, the other will be in the quarters, and *vice versâ*.

In fine, if the first and third have a common phase, we shall have $L' = L'''$; and, consequently—

$$3L'' - 3L' = 180°, \qquad L'' - L' = 60°.$$

The second will therefore have a position 60° different from the common direction of the other two, and its phase will differ in the same degree from their common phase. If one be full, the other will be gibbous; and if one be new, the other will be a crescent; the breadth of the gibbous phase being 120°, and that of the crescent 60°.

The student will find no difficulty in tracing the effects of this relation in all other phases.

An attempt has been made to trace the remarkable relation between the periods here noticed to the effects of the mutual gravitation of the satellites; and Laplace has shown that, if such a relation prevailed *nearly* at any one epoch, the mutual gravitation of the satellites would render it in process of time *exact*. There would seem, therefore, to be a tendency to such a relation, as a consequence of the general law of gravitation.

2764. *Orbits of satellites.* — The orbits of the satellites are ellipses of very small ellipticity, inclined to the plane of Jupiter's orbit at very small angles, as is made apparent by their motions being always very nearly coincident with the plane of the planet's equator, which is inclined to that of its orbit at the small angle of 3° 5′ 30″.

2765. *Apparent and real magnitudes.* — The satellites, although reduced by distance to mere lucid points in ordinary telescopes, not only exhibit perceptible disks when observed by instruments of sufficient power, but admit of pretty accurate measurement. At opposition, when the apparent diameter of the planet is 45″, all the satellites subtend angles exceeding 1″,

and the third and fourth appear under angles of $1\frac{3}{4}''$ and $1\frac{1}{2}''$. By observing these apparent diameters with all practicable precision, and multiplying them by the linear value of $1''$, as already determined (2741.), their real diameters may be ascertained as follows:—

		Miles.
I.	- - -	$1''\cdot194 \times 1934 = 2309$.
II.	- - -	$1''\cdot070 \times 1934 = 2069$.
III.	- - -	$1''\cdot747 \times 1934 = 3378$.
IV.	- - -	$1''\cdot495 \times 1934 = 2891$.

It appears, therefore, that with the exception of the second, which is exactly equal in magnitude to the earth's moon, all the others are on a much larger scale; and one of them, the third, is greater than the planet Mercury, while the fourth is very nearly equal to it.

2766. *Apparent magnitudes as seen from Jupiter.*—By comparing their real diameters with their distances, the apparent diameters of the several satellites, as seen from Jupiter, may be easily ascertained. By dividing the actual distances of the satellites from Jupiter by 206,265, we obtain the linear value of $1''$ at such distance; and by dividing the actual diameters of the satellites respectively by this value, we obtain, in seconds, their apparent diameters as seen from Jupiter.

In making this calculation, however, it is necessary to take into account the magnitude of the semi-diameter of the planet; since it is from the surface, and not from the centre, that the satellite is viewed.

It follows, from a calculation made on these principles, that the apparent magnitudes of the four satellites, seen from any part of the surface not far removed from the equator of the planet, are, for the first $35' \, 30''$, for the second $19' \, 30''$, for the third $18' \, 16''$, and for the fourth $8' \, 58''$.

The first satellite, therefore, has an apparent diameter equal to that of the moon; the second and third are nearly equal and about half that diameter; and the apparent diameter of the other satellite is about the fourth part of that of the moon.

It may be easily imagined what various and interesting nocturnal phenomena are witnessed by the inhabitants of Jupiter, when the various magnitudes of these four moons are combined

with the quick succession of their phases, and the rapid apparent motions of the first and second.

By the relation (2763) between the mean motions of the first three satellites, they never can be at the same time on the same side of Jupiter; so that whenever any one of them is absent from the firmament of the planet at night, one at least of the others must be present. The Jovian nights are, therefore, always moonlit, except during eclipses (which take place at every revolution), and often enlightened at once by three moons of different apparent magnitudes and seen under different phases.

2767. *Parallax of the satellites.* — Owing to the small proportion which the distances of the satellites bear to the semidiameter of the planet, the effects of their parallax as observed from the surface of Jupiter, are out of all analogy with any phenomena of a like kind upon the earth. The nearest body in the universe to the earth, the moon, is at the distance of sixty semidiameters, and its horizontal parallax is consequently less than 1°; while the most remote of Jupiter's satellites is only twenty-seven, and the nearest only six semidiameters from his centre.

By the method explained in 2327, the horizontal parallaxes, π, π', π'', π''', of the four satellites, may be determined, and are —

$$\pi = \frac{57{\cdot}3^\circ}{6} = 9{\cdot}5^\circ. \qquad \pi' = \frac{57{\cdot}3^\circ}{9{\cdot}6} = 6^\circ. \qquad \pi'' = \frac{57{\cdot}3^\circ}{15{\cdot}4} = 3{\cdot}6^\circ.$$

$$\pi''' = \frac{57{\cdot}3^\circ}{27} = 2{\cdot}1^\circ.$$

2768. *Apparent magnitudes of Jupiter seen from the satellites.* — Since the apparent diameter of the planet seen from a satellite, is twice its horizontal parallax (2327), it follows that the apparent diameter of Jupiter seen from the first satellite is 19°, from the second 12°, from the third 7°, and from the fourth 4·25°. The disk of Jupiter, therefore, appears to the first with a diameter eighteen times greater, and a surface 320 times greater than that of the full moon.

2769. *Satellites invisible from a circumpolar region of the planet.* — It is easy to demonstrate in general that an object cannot be seen from any part of the surface of a planet, which is at a distance from its pole less than the horizontal parallax

of the object. Let N P S, *fig.* 756., be a meridian of the planet, N S its axis, O an object at a distance, O C, from its centre. Suppose a line O P drawn from O, touching the meridian at P, the angle P O C will be the horizontal parallax of O; and since the angle O P C = 90°, the angles P C O and P O C taken together are 90°. But since the angle N C O is also 90°, it follows that the angle N C P, and, therefore, the arc N P which it measures, is equal to the horizontal parallax P O C.

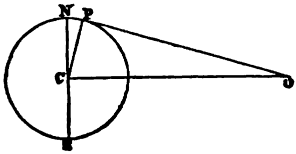


Fig. 756.

Now, it is evident, that O is not visible from any part of the meridian between P and N. If, therefore, a parallel of latitude be supposed to be described round the pole at a distance from it equal to the horizontal parallax of any object, such object cannot be seen from any part of circumpolar region included within such parallel.

It follows from this, from the values of the horizontal parallaxes of the satellites found above (2732), and in fine from the fact the satellites move nearly in the plane of the planet's equator, that the first satellite is invisible at all parts within a parallel described round the pole at a distance of 9·5°, the second at 6°, the third at 3·6°, and the fourth at 2·1°.

2770. *Rotation on their axes.*— One of the peculiarities in the motion of our moon which distinguishes it in a remarkable manner from the planets, is its revolution upon its axis. It will be remembered, that the planets generally rotate on their axes in times somewhat analogous to that of the earth. Now, on the contrary, the moon revolves on its axis in the same time that it takes to revolve round the earth; in consequence of which adjustment of its motions, it turns the same hemisphere continually towards the earth.

Some observations of Sir William Herschel have rendered it probable, that the Jovian moons also revolve on their axes once, in the time of their respective revolutions round the planet. These observations cannot be repeated without the aid of telescopes as powerful as those of the elder Herschel, and it may be expected that those of Lord Rosse and others may supply further evidence on this question.

2771. *Mass of Jupiter.*—The ratio of the mass of Jupiter to that of the sun, can be deduced from the motion of any of the satellites, by the method explained in (2635).

If r and P express the distance and period of the planet, r' and P' those of the satellite, and M and M' the masses of the sun and planet, we shall have—

$$\frac{M}{M'} = \left(\frac{r}{r'}\right)^3 \times \left(\frac{P'}{P}\right)^2$$

By substituting in this formula the distances and periods for each of the four satellites, we shall find the following values of $\frac{M}{M'}$,

I. $\frac{M}{M'} = 1130$. II. $\frac{M}{M'} = 1123$. III. $\frac{M}{M'} = 1121$. IV. $\frac{M}{M'} = 1095$.

The small discrepancy between these values is due chiefly to the causes, already explained, for the departure of the harmonic law from absolute precision.

The following are the estimates of the mass of the planet obtained by processes susceptible of greater precision:—

Laplace - - - -	1070.
Nicolai - - - -	1054.
Airy - - - -	1048·69.
Santini - - - -	1050.
Bessel - - - -	1046.

The last three computations were conducted on principles such as to secure the greatest attainable precision, and these estimates are confirmed by observations on the perturbations produced by Jupiter on the smaller planets.

Since the mass of the sun is about 355,000 times that of the earth, while it is only 1050 times that of Jupiter, it follows, that the mass of Jupiter exceeds that of the earth in the ratio of 3550 to 10·50, or 338 to 1.

The comparatively great mass of Jupiter explains the very short periods of his satellites compared with that of the moon.

At greater distances from Jupiter than that of the moon from the earth, they nevertheless revolve in periods much shorter than that of the moon, and are affected by centrifugal forces, which exceed that of the moon in a ratio which may be determined by the periods and distances, and which must be resisted

by the attraction of a central mass proportionally greater than that of the earth. It would be easy to show that, if the earth were attended by a similar system of moons, at like distances from its centre, their periods would be about eighteen times greater than those of Jupiter's satellites.

2772. *Their mutual perturbations.*—The mutual attractions of the masses of the satellites, and the inequality of the attraction of the sun upon them, produce an extremely complicated system of disturbing actions on their motions, which has nevertheless been brought with great success under the dominion of analysis by Laplace and Lagrange. This is especially the case with the three inner satellites, whose motions, but for this cause, would be sensibly uniform. The effect of these disturbing forces is nevertheless mitigated and limited by the very small excentricities and inclinations of the orbits of the satellites.

2773. *Density.*—The volume of Jupiter being greater than that of the earth in the ratio of 1400 to 1, while its mass is greater in the inferior ratio of 338 to 1 nearly, it follows, that the density of the matter composing the planet, is less than the mean density of the earth in the ratio of the above numbers. We have, therefore—

$$\frac{d'}{d} = \frac{338}{1400} = 0{\cdot}2415.$$

Its mean density is, therefore, less than one-fourth of that of the earth; and since the mean density of the earth is 5·67 times that of water, the density of Jupiter is 1·37 times that of water.

2774. *Masses and densities of the satellites.*—The masses of the satellites are determined by their mutual disturbances, by means of the general principle explained in (2637), and the densities are deduced as usual from a comparison of these masses with their volumes. In the following table are given the masses as compared with the primary and with the earth, and their densities as compared with the earth and with water.

	Mass, that of Jupiter = 1.	Mass, that of Earth = 1.	Density, that of Earth = 1.	Density, that of Water = 1.
I.	0·0000173	0·00576	0·02016	0·1143
II.	0·0000232	0·00773	0·03015	0·1710
III.	0·0000885	0·02947	0·06984	0·3970
IV.	0·0000427	0·01422	0·03925	0·2225

Thus it appears that the density of the matter composing these satellites, is much smaller than those of any other bodies of the system, whose densities are known.

It follows, therefore, that the first satellite must be composed of matter which is twice as light as cork, the density of which is 0·240; and that of the third, which consists of the heaviest matter, is not more dense than the lightest sort of wood, such, for example, as the common poplar, whose density is 0·383 (787).

It is remarkable that this extremely small degree of density is not found in the earth's satellite, the density of which, though less than that of the earth, is still more than twice the density of water.

The planets Mercury and Mars, which are so nearly of the same magnitudes as the third and fourth satellites, show, in a striking manner, the difference of the matter composing them by the great difference of their densities. The mean specific weight of the materials composing these planets, is nearly the same as that of those which compose the earth, while the materials of the third satellite are thirteen times, and that of the fourth twenty-five times lighter.

2775. *Superficial gravity on Jupiter.*—The gravity by which bodies placed on the surface of this planet are affected, omitting the consideration of the modifying effects of its spheroidal form and its rotation, may be computed by means of its mass, and its mean semi-diameter by the method already explained.

Let m' = Jupiter's mass, that of the earth being = 1;
r' = Jupiter's mean semi-diameter, that of the earth being = 1;
g' = superficial gravity, that of the earth being = 1;
we shall then have (2647), (2771)—

$$g' = \frac{m'}{r'^2} = \frac{338}{11{\cdot}45^2} = 2{\cdot}6.$$

2776. *Centrifugal force at Jupiter's equator.* — In the case of Jupiter, owing to the great degree of its oblateness and its rapid rotation, this force of superficial gravitation is subject to much greater variation than on the earth. To determine this variation, it will be necessary to compute the centrifugal force by which bodies placed on the equator of the planet are affected.

Let C = centrifugal force related to the terrestrial gravity as the unit,

g = 16·08 feet,

V = the velocity of Jupiter's equator in feet per second due to his rotation,

we shall then have (313)—

$$C = \frac{V^2}{2r' \times g}$$

The value of V deduced from the equatorial diameter of the planet (2756) and the time of rotation (2747) is 42760; and it follows, therefore, that C = 0·234. Deducting this from the superficial gravity undiminished by rotation, already computed (2775), we shall find the effective equatorial superficial gravity

$$2{\cdot}616 - 0{\cdot}234 = 2{\cdot}382.$$

2777. *Variation of superficial gravity from equator to pole.*—The well-known theorem of Clairault, already quoted (2384), by which the oblateness, the variation of superficial gravity, and the centrifugal force are connected, supplies the means of determining this.

Let e and w, as in 2384, express respectively the fraction of its whole length by which the equatorial exceeds the polar diameter, and the fraction of its whole weight by which the weight of a body and the pole exceeds the weight of the same body at the equator, and in fine, let c express the equatorial centrifugal force as a fraction of the effective equatorial superficial gravity. By the theorem of Clairault, these three quantities are related in the manner expressed in the following formula:—

$$e + w = 2{\cdot}5\, c.$$

But from what has been already explained $e = 0{\cdot}08$,

$$c = \frac{C}{g'} = \frac{0{\cdot}234}{2{\cdot}382} = 0{\cdot}096;$$

and consequently $w = 0{\cdot}16$.

From whence it follows that the weights of bodies are increased by 16 per cent. when transferred from the pole to the equator.

A mass of matter, therefore, which upon the earth's surface would weigh 1000 pounds, would weigh, if placed upon Jupiter's equator, 2382 pounds, and if placed at his pole, would weigh 2763 pounds.

The height through which a body would fall in a second would be 16·08 × 2·382 = 38·3 feet at its equator, and 44·4 at the pole.

The length of a seconds pendulum varies in the exact ratio of the forces of gravity which produce its vibration (542); and if the length of the seconds pendulum on the surface of the earth be taken in round numbers as 39 inches, that of a seconds pendulum at Jupiter's equator would be 39 × 2·382 = 92·91 inches, and at the poles 107·77 inches.

2778. *Density must increase from the surface to the centre.*— It is easy to show that the oblateness of Jupiter is incompatible with the supposition of his uniform density. It was demonstrated by Newton that, if the earth's density were uniform, its oblateness would be $\frac{1}{230}$; and the same would be true of any spheriod of uniform density, revolving on its axis in the same time. But the oblateness will be increased in the same ratio as the square of the time of rotation and as the density are diminished. If, therefore, R express the time of rotation of Jupiter, that of the earth being 1, and d the mean density of Jupiter, that of the earth being 1, the oblateness which the planet would have if its density were uniform would be—

$$e = \frac{1}{230} \times \frac{1}{R^2} \times \frac{1}{d}.$$

But $R = \frac{1}{2\,406}$ and $d = 0·228$; therefore

$$e = \frac{2·406^2}{230 \times 0·228} = 0·1104.$$

But the oblateness deduced from observation being only 0·08, it follows that the density cannot be uniform.

It is easy to perceive that, if the density augmented from the centre to the surface, the effect of the centrifugal force upon the component parts of the mass would have a tendency to render the oblateness still greater than it would be with the same mass having an uniform density. Since, therefore, the actual oblateness is incompatible either with an uniform density, or with a density decreasing from the surface to the centre, it follows that the density must increase from the surface to the centre.

The mean density of the planet being 0·228, it follows, therefore, that the mean density of the superficial stratum must be

less than this, though in what proportion cannot be determined by these data.

If the mean superficial density of Jupiter bear the same proportion to the mean density of its entire mass, as the mean superficial density of the earth bears to the mean density of its entire mass, it will follow, that the mean superficial density of Jupiter will be half its mean density, and will, consequently, be 0·114; and since the mean density of the earth related to that of water as the unit, is 5·67, it would follow upon this supposition, that the actual mean density of the superficial stratum of Jupiter would be $0{\cdot}114 = 5{\cdot}67 = 0{\cdot}645$.

Water, which discharges so many important functions in the physical economy of the earth, has a specific gravity 2·8 times less than the mean specific gravity of the superficial stratum of the globe. If a like fluid on Jupiter, serving like purposes, be similarly related to the mean density of its surface, its specific would therefore be—

$$\frac{0{\cdot}645}{2{\cdot}8} = 0{\cdot}23,$$

which would be more than three times lighter than sulphuric ether, the lightest known liquid, and nearly equal in levity to cork.

2779. *Utility of the Jovian system as an illustration of the solar system.*—It is not merely as a model on a small scale of the solar system, so far as relates to the analogy presented by the motions of the satellites round Jupiter to the motion of the planets round the sun, and the striking confirmation of the theory of gravitation afforded by the exhibition of the play of Kepler's Laws, that the Jovian system is to be regarded with interest by the physical astronomer. All the effects of the reciprocal gravitation of the planets one upon another, which mathematicians have succeeded in explaining upon the principles of the theory of gravitation, all the perturbations and inequalities, many of which, in the case of the planets, will take thousands of centuries to complete their periods and re-commence their course, all these are exhibited on a greatly reduced scale in the Jovian system. As the central mass is reduced in a thousand-fold proportion, and the distances of the bodies revolving round it in a still greater ratio, the cycles of the perturbations and inequalities are similarly reduced. Millions of years are reduced to thousands, centuries to months, months to days, days to hours. Phenomena, the periods of

which would far surpass, not the life of man only, but the whole extent of time embraced within human records and traditions, are reproduced and completed in this miniature system, within such moderate limits of time as to bring them within the scope of actual observation. The analyst is thus enabled to see practically verified, those conditions of equilibrium and stability, which it would take countless ages to develope in the solar system.

II. Saturn.

2780. *Saturnian system.* — Beyond the orbit of Jupiter a space but little less in width than that which separates that planet from the sun is unoccupied. At its limit we encounter the most extraordinary object in the system, — a stupendous globe, nearly nine hundred times greater in volume than the earth, surrounded by two, at least, and probably by several thin flat rings of solid matter, outside which revolve a group of eight moons; this entire system moving with a common motion so exactly maintained, that no one part falls upon, overtakes, or is overtaken by another, in their course around the sun.

Such is the Saturnian system, the central body of which was known as a planet to the ancients, the annular appendages and satellites being the discovery of modern times.

2781. *Period.* — By the usual methods the period of Saturn has been ascertained to be 10759·22 days, or 29·48 years.

2782. *Heliocentric motion.* — The mean heliocentric motion is therefore

$$\frac{360^\circ}{29{\cdot}48} = 12{\cdot}23^\circ \text{ annually.}$$
$$= 1{\cdot}018^\circ \text{ monthly.}$$
$$= 0{\cdot}033^\circ = 2' \text{ daily.}$$

2783. *Synodic motion.* — The apparent mean daily motion of the sun being 0·9856°, the mean daily synodic motion, or its mean daily increment of elongation, is

$$0{\cdot}9856^\circ - 0{\cdot}033^\circ = 0{\cdot}9526^\circ:$$

and the synodic period is therefore

$$\frac{360}{0{\cdot}9526} = 377{\cdot}9 \text{ days.}$$

The interval between the successive oppositions of the planet is therefore a year and thirteen days.

2784. *Distance.* — The mean distance from the sun may be determined by the harmonic law. We have

$$(24 \cdot 48)^2 = (9 \cdot 54)^3.$$

Fig. 757.

The distance is therefore 9·54; or more exactly 9·5387861, that of the earth being = 1.

Taking the earth's mean distance as 95 millions of miles, that of Saturn will then be 906 millions of miles.

The eccentricity of Saturn's orbit being 0·056, this distance is liable to variation, being augmented in aphelion, and diminished in perihelion, by a twentieth of its whole amount. The greatest distance of the planet from the sun is therefore 950, and the least is 850, millions of miles.

2785. *Relative scale of orbit and distance from the earth.* — The relative proportion of the orbits of Saturn and the earth are represented in *fig.* 757., where E E′ E″ is the earth's orbit, and S S′ Saturn's distance from the sun. The four positions of the earth indicated are,

E when the planet is in opposition.
E‴ when the planet is in conjunction.
E′ in quadrature west of the sun.
E″ in quadrature east of the sun.

2786. *Annual parallax of Saturn.* — Since S S′ is 9·54 times S E′, we shall have for the angle S′ S E′,

$$S S' E' = \frac{57 \cdot 30}{9 \cdot 54} = 6°.$$

The semi-diameter of the earth's orbit therefore subtends at Saturn an angle of only 6°. The apparent diameter of a globe, which would fill the entire orbit of the earth seen from Saturn, would, therefore, be no more than 12°, or twenty-four times the apparent diameter of the sun as seen from the earth.

2787. *Great scale of the orbital motion.* — The distance of Saturn from the sun is therefore so enormous, that if the whole earth's orbit, measuring nearly 200 millions of miles in diameter, were filled with a sun, that sun seen from Saturn would be only about twenty-four times greater in its apparent diameter than is the actual sun seen from the earth. A cannon ball moving at 500 miles an hour would take 91,000 years, and a railway train moving 50 miles an hour would take 910,000 years to move from Saturn to the sun. Light, which moves at the rate of nearly 200,000 miles per second, takes 5 days, 18 hours, and 2 minutes to move over the same distance. Yet to this distance solar gravitation transmits its mandates, and is obeyed with the utmost promptitude and the most unerring precision.

Taking the diameter of Saturn's orbit at 1800 millions of miles, its circumference is 5650 millions of miles, over which it moves in 10,759 days. Its daily motion is therefore 525,140 miles, and its hourly 21,880 miles.

2788. *Division of synodic period.* — Since the angle E′S′S = 6°, the angle ESE′ = 84° and E′SE‴ = 96°. Since the synodic period is 378 days, the intervals between

$$\text{opposition and quadrature} = \frac{84}{360} \times 378 = 88{\cdot}2.$$

$$\text{conjunction and quadrature} = \frac{96}{360} \times 378 = 100{\cdot}8.$$

It appears, therefore, that in 88 days after its opposition the planet is in its eastern quadrature, and passes the meridian about 6 in the afternoon. After a further interval of 101 days it arrives at conjunction; after which it acquires western elongation, passing the meridian in the forenoon; and at 101 days from conjunction it attains its western quadrature, passing the meridian at about 6 a.m. After another interval of 88 days it returns to opposition.

2789. *No phases.* — It is evident from what has been ex-

plained in relation to Jupiter (2738), that neither Saturn nor any more distant planet can have sensible phases.

2790. *Variation of the planet's distance from the earth.*— The distances of Saturn from the earth are therefore

$$\begin{aligned} \text{S}'\text{E} &= 906-95=811 \quad \text{millions of miles in opposition.} \\ \text{S}'\text{E}''' &= 906+95=1001 \quad \text{millions of miles in conjunction.} \\ \text{S}'\text{E}' &= \ . \ . \ . \ = 900{\cdot}6 \quad \text{millions of miles in quadrature.} \end{aligned}$$

These distances are subject to some variation, owing to the eccentricities of the orbits of Saturn and the earth. The amount of this variation, arising from the eccentricity of Saturn's orbit, is, as has been shown, 100 millions of miles. The variation due to the earth's orbit is comparatively small, being under two millions of miles.

2791. *Stations and retrogression.*—From a comparison of the orbital motion and varying distance between the earth and Saturn, it appears that the stations of the planet take place at about 65 days before and after opposition. Since the earth gains upon the planet at the mean rate of 0·9526° per day, the angle at the sun corresponding to 65 days will be

$$0{\cdot}9526° \times 65 = 61{\cdot}92°;$$

which corresponds to an elongation of 113°. The planet is therefore stationary at elongation 67° east and west of opposition.

Its arc of retrogression varies from 6°41′ to 6°55′.

2792. *Apparent and real diameter.*—This planet appears as a star of the first magnitude, with a faint reddish light. Its apparent brightness, compared with that of Mars, is greater than that which is due to their apparent magnitudes and distances, a circumstance which is explained, as in the case of Jupiter, by the more feebly reflective power of the surface of Mars.

The disk is visibly oval, and traversed like that of Jupiter by streaks of light and shade parallel to its greater axis; but these belts are much more faint and less pronounced than those of Jupiter. One principal grey belt, which lies along the greater axis of the disk, is almost unchangeable.

Sir William Herschel imagined that the disk had the form of an oblong rectangle, rounded at the corners, the length being in the direction of the belts. More recent observations and

micrometrical measurements made at Konigsburg, by Professor Bessel, and at Greenwich by Mr. Main, have shown, however, the true form to be an ellipse. According to these measures the apparent magnitude of the greater axis of the disk is 17·053″, and that of the lesser axis 15·394″. The observations of Professor Struve, made with the Dorpat instruments, give 17·991 for the greater axis; the difference of the two estimates 0·938 being less than a second.

At the mean distance of Saturn the linear value of a second is

$$\frac{906000000}{2 \cdot 06265} = 4392 \cdot 5.$$

The actual magnitude of the greater axis would therefore be

$$4392 \cdot 5 \times 17 \cdot 053 = 74900 \text{ Bessel.}$$
$$4392 \cdot 5 \times 17 \cdot 991 = 79160 \text{ Struve.}$$

The oblateness expressed as a fraction of the greater axis is 0·097, or a little less than a tenth.

The lesser axis of the planet therefore, according to Struve, measures 71,100 miles, and the mean diameter 75,000 miles.

2793. *Surface and volume.* — Taking the mean diameter of the planet as 9·46, that of the earth being 1, the surface will be $9 \cdot 46^2 = 89 \cdot 5$, and the volume $9 \cdot 46^3 = 847$ times greater than those of the earth.

The relative volumes of Saturn and the earth are represented in *fig.* 758.

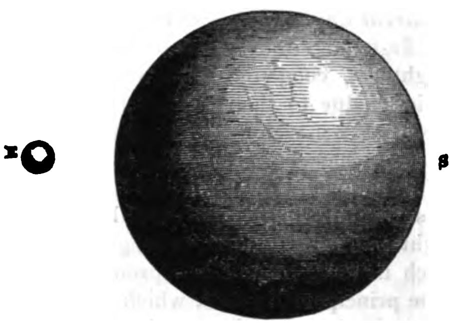


Fig. 758.

2794. *Diurnal rotation.* — From observations on the apparent motion of spots on the disk of the planet it has been ascer-

tained to have a motion of rotation upon the shorter axis of the ellipse formed by its disk in $10^{h.}\ 29^{m.}\ 17^{s.}$ A terrestrial day is therefore equal to 2·3 Saturnian days.

2795. *Inclination of the axis to the orbit.* — The general direction of the motion of rotation has been ascertained to be such, that the inclination of the equator of the planet to the plane of the orbit is 26° 48′ 40″, and its inclination to the plane of the ecliptic is 28° 10′ 47·7″.

The axis, like that of the earth, and those of the other planets, whose rotation has been ascertained, is carried parallel to itself in the orbital motion of the planet.

The consequence of this arrangement is that the year of Saturn is varied by the same succession of seasons subject to the same range of temperature as those which prevail on our globe.

2796. *Saturnian days and nights. Year.* — The alternation of light and darkness is therefore nearly the same as upon Jupiter. This rapid return of day, after an interval of five hours night, seems to assume the character of a *law* among the major planets, as the interval of twelve hours certainly does among the minor planets.

The year of Saturn is equal in duration to 10,759 terrestrial days, or to 258,192 hours. But since a terrestrial day is equal to 2·3 Saturnian days, the number of Saturnian days in the Saturnian year must be 247,457.

2797. *Belts and atmosphere.* — Streaks of light and shade, parallel in their general direction, to the planet's equator have been observed on Saturn similar, in all respects, to the belts of Jupiter, and affording like evidence of an atmosphere surrounding the planet, attended with the like system of currents analogous to the trades. Such an inference involves, as in the former case, the admission of liquid producing vapour to form clouds and other meteorological phenomena.

2798. *Solar light and heat.* — The apparent diameter of the sun as seen from Saturn is 9·54 times less than as seen from the earth; and since its mean apparent diameter, as seen from the earth, is 1923″, its apparent diameter, as seen from Saturn, must be

$$\frac{1923''}{9{\cdot}54} = 201''{\cdot}57 = 3'21''{\cdot}57.$$

The comparative apparent magnitudes are represented in

fig. 759., where E represents the disk of the sun as seen from the earth, and S as seen from Saturn.

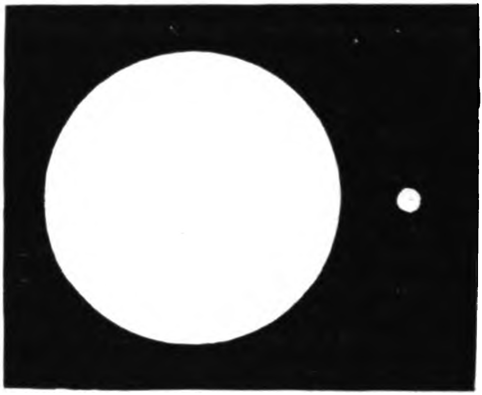
E  S

Fig. 759.

The intensity of solar light is less in the ratio of 1 to $9{\cdot}54^2 = 91$; and its optical and calorific influences with this reduced intensity are subject to the observations already made in the case of Jupiter (2744).

2799. *Rings.*—The invention of the telescope having invested astronomers with the power of approaching, for optical purposes, hundreds of times closer to the objects of their observation, one of the earliest results of the exercise of this improved sense was the discovery that the disk of Saturn differed in a remarkable manner from those of the other planets in not being circular. It seemed at first to be a flattened oblong oval, approaching to the form of an elongated rectangle, rounded off at the corners. As the optical powers of the telescope were improved, it assumed the appearance of a great central disk, with two smaller disks, one at each side of it. These lateral disks, in fine, took the appearance of handles or ears, like the handles of a vase or jar, and they were accordingly called the ansæ of the disk, a name which they still retain. At length, in 1659, Huygens explained the true cause of this phenomenon, and showed that the planet is surrounded by a ring of opaque solid matter, in the centre of which it is suspended, and that what appear as ansæ are those parts of the ring which lie beyond the disk of the planet at either side, which by projection are reduced to the form of the

parts of an ellipse near the extremities of its greater axis, and that the open parts of the ansæ are produced by the dark sky visible through the space between the ring and the planet.

The improved telescopes and greatly multiplied number, and increased zeal and activity of observers, have supplied much more definite information as to the form, dimensions, structure, and position of this most extraordinary and unexampled appendage.

It has been ascertained, that it consists of an annular plate of matter, the thickness of which is very inconsiderable compared with the superficies. It is nearly, but not precisely concentric with the planet and in the plane of its equator. This is proved by the coincidence of the plane of the ring with the general direction of the belts, and with that of the apparent motion of the spots by which the diurnal rotation of the planet has been ascertained.

When telescopes of adequate power are directed to the ring presented under a favourable aspect, dark streaks are seen upon its surface similar to the belts of the planet. One of these having been observed to have a permanence which seemed incompatible with the admission of the same atmospheric cause as that which has been assigned to the belts, it was conjectured that it arose from a real separation or division of the ring into two concentric rings placed one within the other. This conjecture was converted into certainty by the discovery, that the same dark streak is seen in the same position on both sides of the ring. It has even been affirmed by some observers that stars have been seen in the space between the rings; but this requires confirmation. It is, however, considered as proved, that the system consists of two concentric rings of unequal breadth, one placed outside the other without any mutual contact.

The plane of the rings, being always at right angles to the axis of the planet, is, like the axis, carried by the orbital motion of the planet parallel to itself, so that during the year of Saturn, it undergoes changes of position in relation to the radius vector of the planet, or to a line drawn from the sun analogous to those which the earth's equator undergoes. Since the plane of the rings coincides with that of the Saturnian equator, therefore, it will be directed to the sun at the epochs of the Saturnian equinoxes; and, in general, the angle which the radius vector from

the sun makes with the plane of the ring, will be the sun's declination as seen from Saturn. This angle, therefore, at the Saturnian solstices will be equal to the obliquity of Saturn's equator to his orbit, that is, to 26° 48′ 40″ (2795), and at the Saturnian equinoxes will be 0°.

2800. *Position of nodes of ring and inclination to ecliptic.*— The investigation of the position of the plane of the ring in space was undertaken and conducted with great ability and success by Prof. Bessel, by means of an elaborate comparison of all the recorded observations on the phases of the ring from 1701 to 1832. The result proved that the line of intersection of the plane of the ring, and, therefore, that of the equator of the planet, with the plane of the ecliptic, is parallel to that diameter of the celestial sphere, which connects the two opposite points whose longitudes are 166° 53′ 8·9″ and 346° 53′ 8·9″, the former being the longitude of the point at which the rings pass from the south to the north of the ecliptic, and which is, therefore, the ascending node of the rings. It also resulted from this investigation that the angle formed by the plane of the rings, and, therefore, of the Saturnian equator with the plane of the ecliptic, is 28° 10′ 44·7″.

These longitudes and obliquity were those which corresponded to the 1st of January, 1800. It was shown that the nodes of the ring have a retrograde motion on the ecliptic at the mean rate of 46·462″ per annum.

It resulted from the observations of Professor Struve, made with the great Dorpat refractor, that the obliquity of the plane of the ring to that of the ecliptic is 28° 5′ 54″, subject to a possible error of 6′ 24″.

The observations and measurements of these two eminent astronomers are, therefore, in as perfect accordance as the degree of perfection to which the instruments of observation have been brought admits.

2801. *Obliquity of ring to the planet's orbit.* — The position of the plane of the ring in relation to the ecliptic being thus determined, its position in relation to that of Saturn's orbit can be ascertained; and this is the more necessary to be done, inasmuch as considerable discrepancy prevails between the statements of different authorities respecting this element.

Let A, *fig.* 760., be the ascending node of Saturn's orbit and A′ the ascending node of the ring, A A′ being consequently an arc

of the ecliptic. Let $r r'$ be the direction of the plane of the ring, $n n'$ its intersection with the orbit of the planet, of which, there-

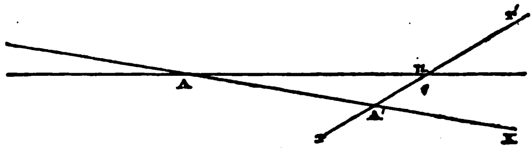


Fig. 760.

fore, An is an arc, and n the point where the plane of the ring intersects that of the orbit. It has been found that the longitude of A is 111° 56′ 37·4″; and since that of A′ is 166° 53′ 8·9″, we have AA′ = 54° 56′ 31·5″. The angle nAA′, which is the obliquity of Saturn's orbit, being 2° 29′ 35·7″, and the angle nA′X, the obliquity of the ring to the orbit being 28° 10′ 44·7″, it follows, by formulæ of spherical trigonometry, that AnA′ or the obliquity of the ring to the orbit of the planet, is 26° 48′ 40″; that An, or the distance of the intersection of the plane of the ring with the plane of the planet's orbit from the ascending node of the planet, is 58° 57′ 30″; and, in fine, the distance nA of the same point from the ascending node of the ring is 4° 32′ 30″.

2802. *Conditions which determine the phases of the ring.* — The relation between the phases of the ring and the position of the planet is easily ascertained. Let a be the semidiameter of the rings as seen undiminished by projection; let b be the lesser semi-axis of the ellipse produced by the projection of the ring; and let D be the angle which the visual ray makes with the plane of the ring. We shall then have, by the common principles of projection,

$$b = a \times \sin. D.$$

But since the visual ray is the line drawn from the planet to the earth, it is evident that the angle D will be the declination of the earth as seen from the planet. Now, if L express the arc of the ecliptic between the earth and the ascending node of the ring as seen from the planet, and O the obliquity of the plane of the ring to the ecliptic, we shall have, by the common principles of trigonometry,

$$\text{Sin D} = \sin. O \times \sin. L;$$

and consequently

$$\frac{b}{a} = \text{sin. O} \times \text{sin L.}$$

But since the distance of the earth from the ascending node as seen from the planet is equal to 180°, diminished by the distance of Saturn from the same point as seen from the earth, L may be taken in the preceding formula to express the latter distance, the sine being the same.

If the position of the planet with relation to the ascending node of the ring be known, the preceding formula will therefore serve to deduce the obliquity from the phase of the ring, or *vice versâ.*

Since O = 28° 10′ 44·7″, we shall have Sin. O = 0·4722. We shall therefore have

$$\frac{b}{a} = 0{\cdot}4722 \times \text{sin. L},$$

by which the phases of the ring for any given distance from its node may be computed.

In the following table the ratio $\frac{b}{a}$ of the semi-axis of the ellipse formed by the projection of the ring, and the ratio $\frac{b}{r}$ of the lesser semi-axis to the semi-diameter of the planet are given for every 10° from the node.

L	10°	20°	30°	40°	50°	60°	70°	80°	90°
$\frac{b}{a}$	0·082	0·161	0·236	0·303	0·362	0·409	0·444	0·465	0·472
$\frac{b}{r}$	0·183	0·360	0·526	0·676	0·806	0·911	0·989	1·04	1·05

From this table it appears that the lesser semi-axis increases as the planet moves from the ascending node of the ring until it is 90° from that point, at which its ratio to the major semi-axis is that of 472 to 1000, being a little less than half the major semi-axis. From the numbers given in the third line of the table it appears that the lesser semi-axis becomes equal to the equatorial semidiameter of the planet at about 71° from the ascending node of the ring, and exceeds it by a twentieth of its

length at 90°. But as the polar diameter of the planet is less by a tenth than the equatorial, it follows, that at 90° from the node of the ring the lesser semi-axis exceeds the polar semi-diameter of the planet by an eighteenth of its length, it follows, therefore, that a corresponding breadth of the ring will, in this case, be visible above the disk of the planet.

Since the entire breadth of the rings is three-fourths of the semi-diameter of the planet, and since they are reduced one-half by foreshortening their apparent breadth measured in the direction of the lesser axis of the ellipse at 90° from the node is three-eighths of the semidiameter of the planet, it follows, therefore, that a seventh part of the entire breadth of the rings is visible above the disk of the planet at 90° from the node, the entire upper segment of the disk being projected upon the ring.

From 90° to the descending node of the ring the like phases are presented in a contrary order; and while the planet moves from the descending to the ascending node a similar series of phases are presented; the position of the plane of the ring with relation to the poles of the planet, however, being reversed, that which is in one case interposed between the observer and the northern hemisphere of the planet will be interposed in the other cases between him and the southern hemisphere.

In the diagrams *figs.* 761—765., the phases of the rings indicated in the preceding table are exhibited.

If the thickness of the ring were uniform and sufficiently great to subtend a sensible visual angle at Saturn's distance, the phase presented to the planet at the nodes of the ring would be such as that represented in *fig.* 761. The phases at 1, 2, 4, and 6–7 years, after passing the nodes, are roughly sketched in *figs.* 761—765.

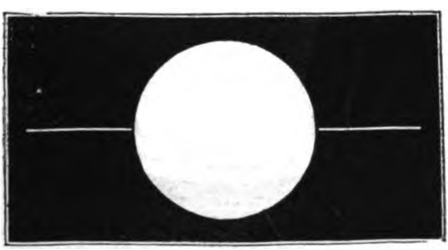


Fig. 761.

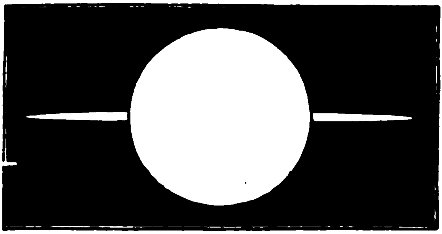


Fig. 762.

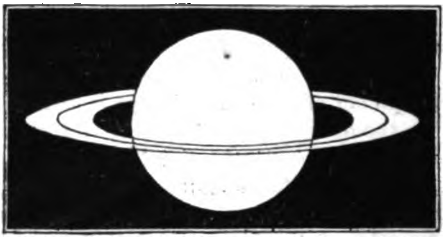


Fig. 763.

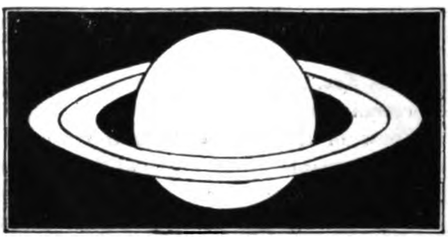


Fig. 764.

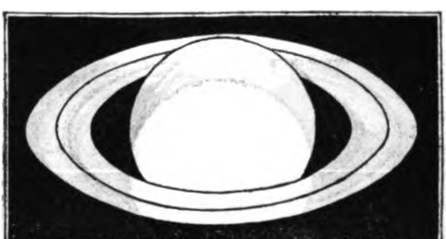


Fig. 765.

2803. *Apparent and real dimensions of the rings.* — The breadth of the rings as well as of the intervals which separate

them from each other and from the planet, have been submitted to very precise micrometric observations; and the results obtained by different observers do not differ from each other by a fortieth part of the whole quantity measured. In the following table are given the results of the micrometric observations of Professor Struve, reduced to the mean distance.

		Apparent Magnitude at mean Distance.	In Semi-diameters of the Planet.	Miles.
Semi-diameter of the planet - - -	r	8″·995	1·000	39,580
Exterior semi-diameter of exterior ring - -	a	20·047	2·229	88,209
Interior do. do. - -	a'	17·644	1·961	77,636
Breadth of exterior ring - - - -	$a-a'$	2·403	0·268	10,573
Exterior semi-diameter of interior ring - -	b	17·237	1·916	75,845
Interior do. do. - -	b'	13·334	1·482	58,669
Breadth of interior ring - - - -	$b-b'$	3.903	0·434	17,176
Width of interval between the rings - -	$a'-b$	0·407	0·045	1,791
Width of interval between planet and interior ring	$b'-r$	4·339	0·482	19,089
Breadth of the double ring, including interval -	$a-b'$	6·713	0·747	29,540

The relative dimensions of the two rings, and of the planet within them, are represented in *fig.* 766., projected upon the

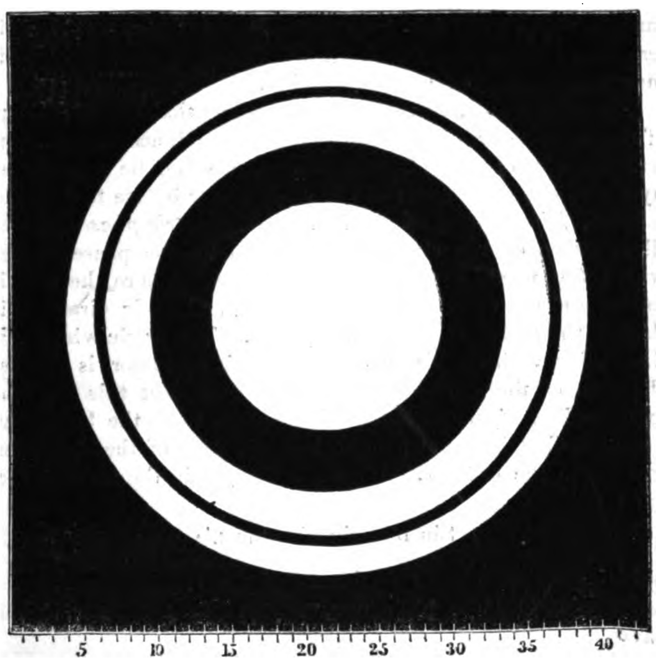


Fig. 766.

common plane of the rings and the planet's equator. Each division of the subjoined scale represents 5,000 miles.

The visual angle subtended at the earth by the extreme diameter of the external ring, when the planet is in opposition, is 48″, which is about one thirty-seventh part of the moon's apparent diameter.

2804. *Thickness of the rings.*—The thickness of the rings is so extremely minute, that the nicest micrometric observations have hitherto failed to supply the data necessary to determine it with any degree of precision or certainty. It is so inconsiderable, that when the plane of the ring is directed to the earth, and, consequently, the edge alone is presented to the eye, it is invisible even with telescopes of great power, or, if seen, it is so imperfectly defined as to elude all micrometric observation. When it was in this position in 1833, Sir J. Herschel observed it with a telescope, which would certainly have rendered distinctly visible a line of light one twentieth of a second in breadth. Since the linear value of 1″ at Saturn's mean distance is about 4400 miles, it would follow that the thickness is less than 220 miles. Sir J. Herschel admits, however, that it may possibly be so great as 250 miles.

The thickness is, therefore, certainly less than the 100th part of the extreme breadth of the two rings, and, according to the scale on which the *fig.* 752. is drawn, it would be represented by the thickness of a leaf of the volume now before the reader.

2805. *Illumination of the ring.*—*Heliocentric phases.*—The illumination of the rings is determined by the phases under which they would be viewed from the sun. There the illuminating and the visual rays are identical, and their direction is that of the radius vector of the planet. The angle which this line makes with the plane of the Saturnian equator, is the declination of the sun as seen from Saturn. Let this angle be expressed by D', the distance of the same from the Saturnian vernal equinoxial point L', and the inclination of the Saturnian equator to the orbit O'. We shall then, as in the former case, have

$$\text{Sin D}' = \sin \text{O}' \times \text{Sin L}';$$

and if b' express lesser semi-axes of the ellipse to which the major is reduced by projection, as seen from the sun, we shall have

$$\frac{b}{a'} = \text{Sin. } o' \times \text{sin. } \text{L}'.$$

By this formula the ratios of B′ to A and r may be determined for all values of L′, as in the former case. In the following table these are given as they have been computed, for $o' = 26° \ 48' \ 40''$.

L′	10°	20°	30°	40°	50°	60°	70°	80°	90°
$\frac{b'}{a}$	0·078	0·154	0·226	0·290	0·346	0·390	0·424	0·444	0·451
$\frac{b'}{r}$	0·175	0·344	0·503	0·646	0·770	0·870	0·945	0·990	1·005

2806. *Shadow projected on the planet by the rings.*—Since the lines by which these phases are determined are those of the solar rays, it is evident that the parts of the rings intercepted by the planet, and those of the planet intercepted by the rings, are exactly those which reciprocally bound the shadows projected on them.

The preceding table of Heliocentric phases will therefore serve for the determination of the limits of the shadows.

At the equinoxes, the edge of the rings being presented to the sun, the shadow projected on the equator of the planet will depend altogether on the thickness of the rings, and the apparent diameter of the sun, as seen from the planet. The latter being only 3½, the solar rays which touch the edge of the thickness may be considered as nearly parallel, and the breadth of the shadow will be nearly equal to the thickness of the ring. The shadow will therefore, in this case, be in a thin dark line, extending along the equator of the planet, *fig.* 767., covering a

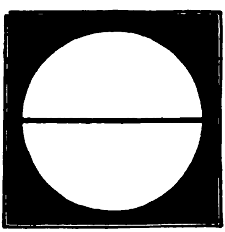


Fig. 767.

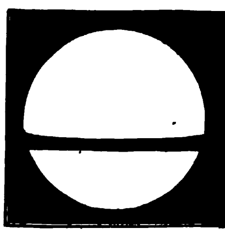


Fig. 768.

zone of the firmament whose breadth must be about fifteen

times greater than the apparent diameter of the sun. A total solar eclipse at the equator of the planet would, therefore, be produced by the shadow of the ring, and would continue until the sun would gain or lose 44′ declination.

When the planet presents the phase represented in *fig.* 763., its enlightened hemisphere would be traversed by the shadow represented in *fig.* 768., and in like manner the shadows pro-

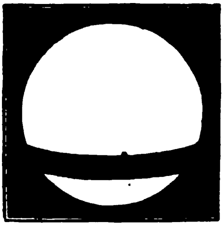


Fig. 769.

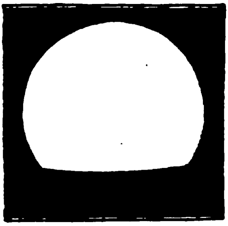


Fig. 770.

duced under the phases *figs.* 764, 765, are represented in *figs.* 769, 770.

2807. *Shadow projected by the planet on the ring.*—The shadow projected on the surface of the ring by the globe of the planet will vary with the sun's declination, as seen from the planet, because the angle at which the plane of the ring is inclined to the axis of the shadow of the planet is equal to the declination.

At the equinoxes, the declination being 0°, the plane of the ring passes through the axis of the shadow, and the breadth of the arc of the ring on which the shadow falls is nearly equal to the diameter of the planet, the angle of the cone of the shadow not exceeding 3′, since it is very nearly equal to the apparent diameter of the sun as seen from Saturn.

If a express the arc of the external edge of the ring on which the shadow falls, it is evident that

$$\text{Sin. } \tfrac{1}{2}\, a = \frac{r}{a},$$

r expressing, as before, the equatorial semi-diameter of the planet, and a the semi-diameter of the outer edge of the ring. The values already determined being given to these, we have

$$\text{Sin. } \tfrac{1}{2}\, a = \frac{10{,}000}{22{,}287} = 0{\cdot}4486.$$

Hence it appears that $a = 53° 18' 48''$; and in like manner it may be shown that the shadow covers 84° 52′ of the inner edge of the inner ring.

The lateral edges of the shadow in this case are rectilinear and sensibly parallel, a consequence due to the angle of the cone being insignificant, *fig.* 771.

As the declination of the sun increases the section of the cone of the shadow by the plane of the ring becomes elliptical, and the edges of the shadow on the ring are curved, *fig.* 772., while its breadth is more contracted. When the sun comes to the Satur-

Fig. 771.

Fig. 772.

Fig. 773.

nian solstice, and its declination is 26° 48′ 40″, the vertex of the elliptic section of the cone falls upon the outer edge of the ring, and the shadow has the form of an elliptic segment at the extremity of the major axis of the ellipse, *fig.* 773.

2808. *The shadows partially visible from the earth.*—It is

evident that, the visual ray of an observer placed on the sun coinciding with the luminous ray, none of the shadows produced by the projection of the ring on the planet or of the planet on the ring, could be visible to him, since the object producing the shadow would be interposed with geometrical precision between his eye and the shadow. But if the observer be transferred to the earth, the visual ray will form an angle with the luminous ray, which, though it cannot in any case exceed 6°, is sufficient, under certain circumstances of relative position, to remove the observer from the direction of the luminous ray to such an extent as to disclose to him a part, though a small part, of the shadow which to an observer at the sun is wholly intercepted.

In this manner, when the planet is in quadrature, a small part of the shadow it projects upon the ring is visible on the east or on the west side of the disk, according as the sun is west or east of the planet; and in like manner, the difference between the angles which the visual and luminous rays form with the plane of the ring discloses, in certain cases, a small breadth of the shadow projected on the planet's disk by the ring, which is accordingly seen as a thin dark streak crossing the disk of the planet in contact with the ring.

These phenomena prove that both the planet and the ring consist of matter having no light of their own, and deriving their entire illumination from the sun.

2809. *Conditions under which the ring becomes invisible from the earth.*—The rings of Saturn viewed from the earth may become invisible, either because the parts presented to the eye are not illuminated by the sun, or, being illuminated, have dimensions too small to subtend a sensible visual angle.

It has been explained that, in every position assumed by the planet in its orbital motion, one side or the other of the rings is illuminated with more or less intensity, except at the Saturnian equinoxes, when, the plane of the ring passing through the sun, its edge alone is illuminated. Owing to the extreme thinness of the plate of matter composing the rings, they cease in this case to be visible, except by feeble and uncertain indications observed with high magnifying powers, which will be noticed hereafter. It has been inferred by Sir John Herschel, from observations made with telescopes of great power, that the major limit of their possible thickness is 250 miles. The visual

angle which this thickness would subtend at the distance of Saturn in opposition, is (2790)

$$\frac{250}{800,000,000} \times 206,265'' = 0{\cdot}064''.$$

The visual angle would, therefore, be less than the fifteenth part of a second.

The rings, therefore, disappear from this cause at Saturn's equinoxes, which occur at intervals of 14¾ years.

When the dark side of the rings is exposed to the earth it is evident that the sun and earth must be on opposite sides of the plane of the rings, and therefore that plane must have such a position that its direction would pass between the sun and the earth. This can only happen within a certain limited distance of the planet's equinoxes.

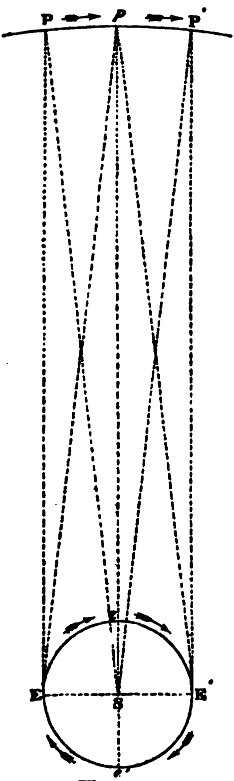


Fig. 774.

Let S, *fig.* 774., represent the place of the sun, EE′ the orbit of the earth, and *p* the place of Saturn at the time of either of his equinoxes. From what has been already explained, it appears that in this position the edge of the ring is presented to S. Since the ring is carried parallel to itself in the orbital motion of the planet, the edge of the ring, before arriving at the point *p*, must have been directed successively to the points of the earth's orbit between E and the diameter *ee′*, and, after passing *p*, must be directed successively to the points between the diameter *ee′* and E′. If lines E P and E′ P′ be drawn from E and E′ parallel to S*p*, the edge of the ring will be directed across some part of the earth's orbit, so long as the planet is passing between P and P′. At P it will be directed to E, and at P′ to E′. Before arriving at P, the edge of the ring will be directed to the left of the earth's orbit, and after passing P′, to the right of it.

It is evident, therefore, that before the

planet arrives at P, whatever be the position of the earth in its orbit, the earth, as well as the sun, must be to the right of the direction of the edge of the rings, and consequently on their illuminated side; and after passing P′, the earth, as well as the sun, must be to the left of the edge of the rings, and therefore still on the illuminated side. The rings, therefore, must everywhere be visible, except when the planet is at or between the points P and P′.

When the planet is at either of the points P or P′, it *may* happen that at the same moment the earth is at the corresponding point E or E′. In that contingency the edge of the ring, being the only part exposed to the observer, would be invisible because of its minuteness, in the same manner as when the planet is at *p*. In that case the disappearance of the planet would be of short duration, because the orbital motion of the earth would soon bring it on the enlightened side of the ring. Thus, if when the planet is at P the earth is at E, the latter, moving much faster than the planet, advances before it, and being then on the same side of the ring with the sun, the illuminated side is exposed to the earth, and therefore visible; and if when the planet is at P′ the earth is at E′, the latter moving towards *e′* while the planet advances to the right of P′, the earth and planet are both on the left of the edge of the rings, and, as before, the enlightened side of the rings is exposed to the earth, and they are therefore visible.

If, while the planet is between P and *p*, the earth, moving from *e′* towards E, pass through the point to which the edge of the rings is directed, the rings will after such passage cease to be visible, because the earth will then be on their dark side. It is possible that after this, and before the planet arrives at *p*, the earth, moving from E towards *e*, overtaking the direction of the edge, may again pass through the point to which it is directed. If this happen the rings will again become visible, because the earth will thus pass from their dark to their illuminated side.

If in this case, while the earth moves towards E′ and *e′*, the planet pass through *p*, the rings will again become invisible, because, their edge passing from one side of the sun to the other, the side presented towards E′ will be dark, and that towards E illuminated. If in this case the earth, moving from E′ towards *e′*, again pass through the point to which the edge of the rings

is directed, it will again pass from the dark to the enlightened side, and the rings will again become visible.

The angle E p S, being the annual parallax of Saturn (2744), is 6°, and consequently E p E′ or P S P′ is 12°. But since the annual heliocentric motion of Saturn is 12°·22, the time of moving from P to P′ is

$$\frac{12}{12{\cdot}22} \times 365 = 358\tfrac{1}{2} \text{ days},$$

or about $6\frac{1}{2}$ days less than the time the earth takes to make a complete revolution, so that while the planet moves from P to P′ the earth moves through about 354° of its orbit.

It appears, therefore, that the interval during which the planet is within such a distance of its equinoctial point as to render the disappearance of the rings from one or other of these several causes possible, the earth makes very nearly a complete revolution, and is therefore at one time or other in a position to meet the direction of the edges at least once, and the relative position of it and the planet may be such as to cause several disappearances of the rings within six months before and after the Saturnian equinox.

All these various phenomena were witnessed at the last Saturnian equinox in 1848. The northern surface of the ring had then been visible for nearly fifteen years. The motions of the planet and the earth brought the plane of the ring to that position on the 22nd of April in which, its edge being presented to the earth, it became invisible, the sun being still north of the plane. On the 3rd of September the sun, passing through the plane of the ring, illuminated its southern surface, and, the earth being on the same side, the ring was visible. On the 12th, the earth again passing through the plane of the ring, its northern surface was exposed to the observer, which was invisible, the sun being on the southern side. The ring continued thus to be invisible until the 18th of January, 1849, when, the earth once more passing through the plane of the ring, the southern surface illuminated by the sun came into view. This side of the ring will continue to be exposed to both the earth and the sun until 1861–2, the epoch of the next equinox, when a like succession of appearances and disappearances will take place,—the sun and earth eventually passing to the northern side, on which they will continue for a like interval.

2810. *Schmidt's observations and drawings of Saturn with the ring seen edgeways.*—At the last Saturnian equinox, which took place in 1848, a series of observations was made at Bonn, the results of which have demonstrated the existence of great inequalities of surface on the rings, having the character of mountains of considerable elevation. The observations were made and published, accompanied by seventeen drawings of the appearance of the planet, its belts, and ring, by M. Julius Schmidt, of the Bonn Observatory.*

We have selected from these drawings four, which are given in Plate XI.

On the 26th of June, the planet presented an appearance, *fig.* 1., closely resembling that of Jupiter, except that a dark streak was seen along its equator, produced by the shadow of the ring, the earth being then a little above the common plane of the ring and the sun. A few feeble streaks, of a greyish colour, were visible on each hemisphere, which however disappeared towards the poles. A very feeble star was seen at the western extremity of the ring, which was supposed to be one of the nearer satellites. The ring exhibited the appearance of a broken line of light projecting from each side of the planet's disk.

After this day the shadow across the planet disappeared, but was again faintly seen on the 25th of July.

The ring continued to be invisible until the 3rd of September, when a very slight indication of it was seen, but on the next night it became distinctly visible with an interruption in two places, as represented in *fig.* 2. The bright equatorial belt was divided into two unequal parts by the ring, the northern portion being the narrower. Three small satellites were seen in the prolongation of the direction of the ring.

On the 5th, the ring was symmetrically broken on both sides, *fig.* 3.

On the 7th, the western side was divided into three parts.

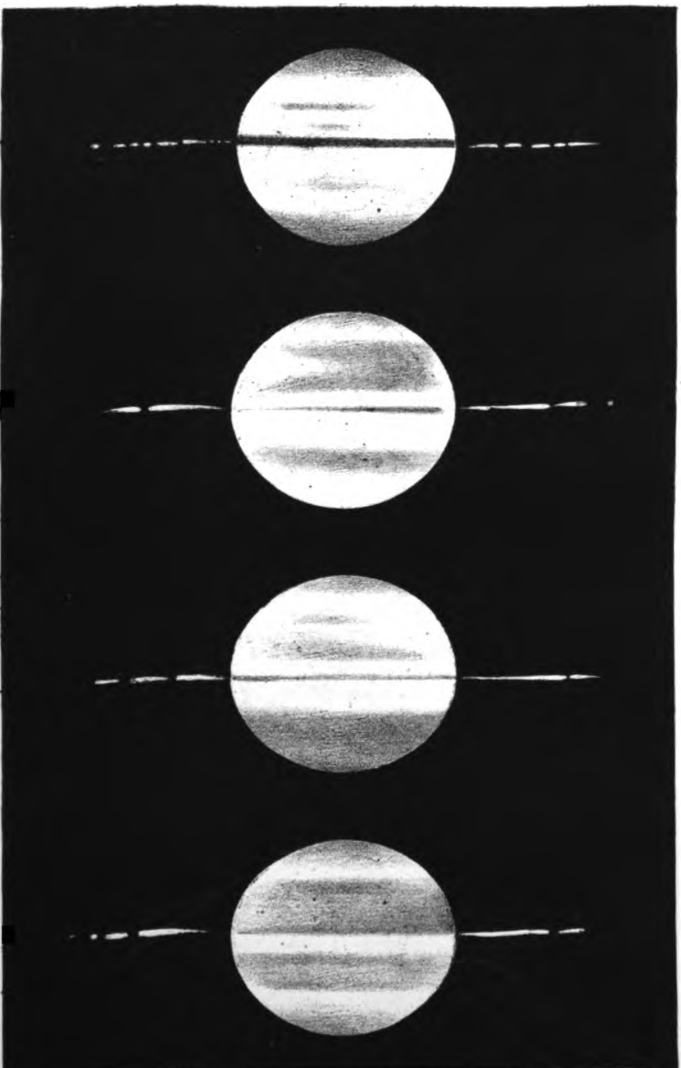
On the 11th, the ring and planet presented the appearance represented in *fig.* 4.

The broken and changing appearances of the ring on this occasion, can only be explained by the admission of great inequalities of surface rendering some parts of the ring so thick

* Astron. Nachr. Schumacher, Vol. xxviii. No. 650.

SATURN

As seen at his equinox in 1848 by Schmidt.



1

2

3

4

1 June 26. 2 Sep. 4 3 Sep 5 4 Sep 11.

as to be visible, and others so thin as to be invisible, when presented edgeways to the observer.

2811. *Observations of Herschel.*—These observations of Schmidt are corroborative of those made at a much earlier epoch by Sir W. Herschel, who discovered the existence of appearances on the surface of the rings indicating mountainous nequalities.

2812. *Supposed multiplicity of rings.*—Some observations made at Rome and elsewhere gave grounds for the conjecture, that the outer ring, instead of being double, is quintuple, and that instead of having a single division, there are four. It was even affirmed with some confidence, that the ring was septuple, and consisted of seven concentric rings suspended in the same plane. These conjectures were founded upon the supposed permanence of the black circular and concentric streaks which are observed upon the surface of the rings, and which are quite analogous to the belts of the planet. This assumed permanence has not, however, been re-observed, although the planet has been examined by numerous observers, with telescopes of very superior power to those with which the observations were made which formed the ground of the conjecture.

The passage of Saturn diametrically across any fixed star of sufficient magnitude, at the epoch of the Saturnian solstice, when the plane of the ring is inclined at the greatest angle to the visual line, would supply the most eligible means of testing the multiple structure of the rings; for in that case the light of the star would be seen with the telescope to flash through each successive opening between ring and ring, provided that the width of such opening were sufficient to allow the visual ray to clear the thickness of the rings.

2813. *Ring probably triple—observations of Messrs. Lassell and Dawes.*—Nevertheless, there are well ascertained appearances on the surface of the outer ring, which have been thought to indicate a second division, and that the ring is triple. So early as 1838 Professor Encké noticed an appearance which indicated a division, and even made drawings in which such a division is indicated. (See Berlin Trans. 1838.) On the 7th September, 1843, Messrs. Lassell and Dawes, unaware apparently of Encké's observations, with a nine-feet Newtonian reflector constructed by Mr. Lassell, saw what they considered to be a division of the outer ring. The observation was made

under a magnifying power of 450, which gave a sharply defined disk to the planet, and exhibited the principal division of the rings as a continuous, distinctly seen, black streak, extending all round the surface of the ring. A dark line on the outer ring, near the extremities of the ellipse, was not only distinctly seen, but an estimate of its breadth compared with that of the principal division was made by both these observers from which it appeared that this breadth was about one-third of the space which separates the two principal rings. Its place upon the outer ring was a little less than half the entire width of the ring from the outer edge, and it was equally visible at both ends of the ellipse. No appearances could be discovered of any other divisions, although the shading of the belts on the inner ring was distinguished.

2814. *Researches of Bessel corroborate these conjectures.* — Bessel compared all the observations made on the rings from 1700 to 1833, with the view of determining with more precision the nodes of the ring, and found that the ring has frequently been seen when it ought to have been invisible, if the several concentric rings of which it consists were all in the same plane and had a uniform surface. He found that the appearances and disappearances had no certain or regular epochs, and did not correspond with each other, even to the same observer, using the same instrument. Thus Schwabe, at Dessau, saw the line of light formed by the rings near their equinox, resolve itself into a series of points. Schmidt, as has been stated, saw it become a broken line, changing from night to night its form. Other observers saw the ring disappear on one side of the disk, while it was apparent on the other. From all these phenomena, it is inferred, that probably the rings are in planes slightly different; that their edge is not regularly circular, but notched and dinged; and, in fine, that their surfaces are characterised by considerable mountainous undulations.

2815. *Discovery of an inner ring imperfectly reflective and partially transparent.* — But the most surprising result of recent telescopic observations of this planet has been the discovery of a ring, composed, as it would appear, of matter reflecting light much more imperfectly than the planet or the rings already described; and what is still more extraordinary, transparent to such a degree, that the body of the planet can be seen through it.

In 1838, Dr. Galle, of the Berlin observatory, noticed a phenomenon, which he described as a gradual shading off of the inner ring towards the surface of the planet, as if the solid matter of the ring were continued beyond the limit of its illuminated surface, this continuation of the surface being rendered visible by a very feeble illumination such as would attend a penumbra upon it; and measures of this obscure surface were published by him in the "Berlin Transactions" of that year.

The subject, however, attracted very little attention until towards the close of 1850, when Professor Bond, of Boston, and Mr. Dawes in England, not only recognised the phenomenon noticed by Dr. Galle, but ascertained its character and features with great precision. The observations of Professor Bond were not known in England until the 4th of December; but the phenomenon was very fully and satisfactorily seen and described by Mr. Dawes, on the 29th of November. That astronomer, on the 3rd December, called the attention of Mr. Lassell to it, who also witnessed it on that evening at the observatory of Mr. Dawes; and both immediately published their observations and descriptions of it, which appeared in Europe simultaneously with those of Professor Bond.

It was not, however, until 1852 that the transparency was fully ascertained. From some observations made in September, Mr. Dawes strongly suspected its existence, and about the same time it was clearly seen at Madras by Captain Jacob, and in October by Mr. Lassell at Malta, whither he had removed his observatory to obtain the advantages of a lower latitude and more serene sky. The result of these observations has been the conclusive proof of the unique phenomenon of a semi-transparent annular appendage to this planet.

2816. *Drawing of the planet and rings as seen by Mr. Dawes.*—The planet surrounded by this compound system of rings is represented in Plate XII. The drawing is reduced from the original sketch, made by Mr. Dawes, of the planet as seen with his refractor of 6⅓ inch aperture, at Wateringbury, in November 1852. Another representation of the planet as seen by Mr. Lassell at Malta, in December 1852, has been lithographed, and is almost identical with that of Mr. Dawes. In both, the form and appearance of the obscure ring and its partial transparency are rendered quite manifest. The princi-

pal division of the bright rings is visible throughout its entire circumference. The black line, supposed to be a division of the outer ring, is visible in the drawing of Mr. Dawes; but was not at all seen by Mr. Lassell.

A remarkably bright thin line, at the inner edge of the inner bright ring, which appears in the Plate XII., was distinctly seen by Mr. Dawes in 1851 and 1852.

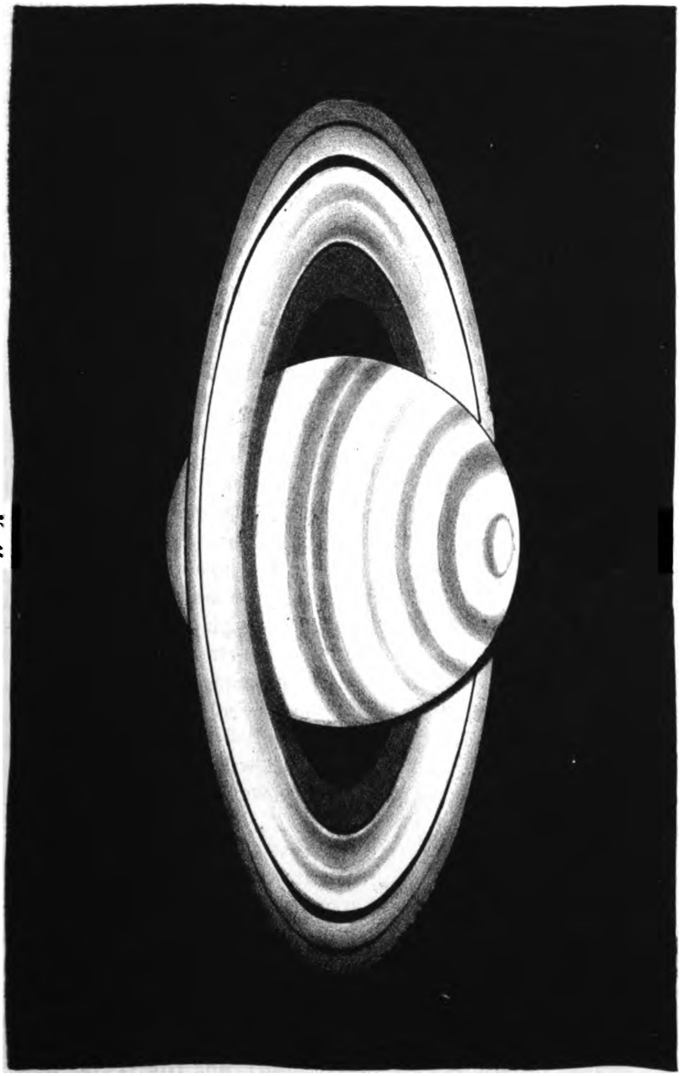
The inner bright ring is always a little brighter than the planet. It is not, however, uniformly bright. Its illumination is most intense at the outer edge, and grows gradually fainter towards the inner edge, where it is so feeble as to render it somewhat difficult to ascertain its exact limit. It would seem as if the imperfectly reflective quality there approaches to that of the obscure ring recently discovered. The open space between the ring and the planet has the same colour as the surrounding sky.

2817. *Bessel's calculation of the mass of the rings.*—Bessel has attempted to determine the mass of the system of rings by the perturbation they produce upon the orbit of the sixth satellite. He estimates it at 1-118th part of the mass of the planet. The thickness of the rings being too minute for measurement, no estimate of the density of the matter composing them can be hence obtained; but if the density be assumed to be equal to that of the planet (which will be explained hereafter), it would follow that the thickness of the rings would be about 138 miles, which is not far from the estimate of their thickness made by observers. If this thickness be admitted, the edge of the rings would subtend an angle of the 1-32nd part of a second at Saturn's mean distance. Hence it will be understood that the ring must disappear, even in powerful telescopes, when presented edgeways.

2818. *Stability of the rings.* — One of the circumstances attending this planet, which has excited most general astonishment, is the fact that the globe of the planet, and two, not to say more, stupendous rings, carried round the sun with a velocity of 22,000 miles an hour, subject to a periodical variation not inconsiderable, due to the varying distance of the planet from the sun, should nevertheless maintain their relative position for countless ages undisturbed, the globe of the planet remaining still poised in the middle of the rings, and the rings, two or several as the case may be, remaining one within the other

SATURN.

As seen in November 1852 with a refractor of 6 ⅓ inch aperture at Wateringbury near Maidstone. by W. R. Dawes. XII.



without material connection or apparent contact, no one of the parts of this most marvellous combination having ever gained or lost ground upon the other, and no apparent approach to collision having taken place, notwithstanding innumerable disturbing actions of bodies external to them.

2819. *Cause assigned for this stability.*—The happy thought of bringing the rings under the common law of gravitation, which gives stability to satellites, has supplied a striking and beautiful solution for this question. The manner in which the attraction of gravitation, combined with centrifugal force, causes the moon to keep revolving round the earth without falling down upon it by its gravity on the one hand, or receding indefinitely from it by the centrifugal force on the other, is well understood. In virtue of the equality of these forces, the moon keeps continually at the same mean distance from the earth while it accompanies the earth round the sun. Now it would be easy to suppose another moon revolving by the same law of attraction at the same distance from the earth. It would revolve in the same time, and with the same velocity, as the first. We may extend the supposition with equal facility to three, four, or a hundred moons, at the same distance. Nay, we may suppose as many moons placed at the same distance round the earth as would complete the circle, so as to form a ring of moons touching each other. They would still move in the same manner and with the same velocity as the single moon.

If such a ring of moons were beaten out into the thin broad flat rings which actually surround Saturn, the circumstances would be somewhat changed, inasmuch as the periods of each concentric zone would vary in a certain ratio, depending on its distance from the centre of Saturn, so that each such zone would have to revolve more rapidly than those within it, and less rapidly than those outside it. But if the entire mass were coherent, as the component parts of a solid body are, the complete ring might revolve in a periodic time less than that due to its exterior and longer than that due to its interior parts. In fact, the period of its revolution would be the period due to a certain zone lying near the middle of its breadth, exactly as the time of oscillation of a compound pendulum is that which is proper to the centre of oscillation (543). Indeed, the case of the oscillation of a pendulum and the conditions which deter-

mine the centre of oscillation afford a very striking illustration of the physical phenomena here contemplated.

2820. *Rotation of the rings.* — Now the observations of Sir William Herschel on certain appearances upon the surface of the rings led to the discovery that they actually have a revolution round their common centre and in their own plane, and that the time of such revolution is very nearly equal to the periodic time of a satellite whose distance from the centre of the planet would be equal to that of the middle point of the breadth of the rings.

But if the principles above explained be admitted, it would follow that each of the concentric zones into which the ring is divided would have a different time of revolution, just as satellites at different distances have different periodic times; and it is extremely probable that such may be the case, because no observations hitherto made afford results sufficiently exact and conclusive as to either establish or overturn such an hypothesis.

It appears, therefore, in fine, that the stability of the rings is explicable upon the same principle as the stability of a satellite.

2821. *Eccentricity of the rings.* — The fact that the system of rings is not concentrical with the planet resulted from some observations made by Messrs. Harding and Schwabe; after which the subject was taken up by Professor Struve, who, by delicate micrometric observations and measurements executed with the great Dorpat instrument, fully established the fact, that the centre of the rings moves in a small orbit round the centre of the planet, being carried round by the rotation of the rings.

2822. *Arguments for the stability founded on the eccentricity.* — Sir John Herschel has indicated, in this deviation of the centre of the rings from the centre of the planet, another source of the stability of the Saturnian system. If the rings were "mathematically perfect in their circular form, and exactly concentric with the planet, it is demonstrable that they would form (in spite of their centrifugal force) a system in a state of *unstable equilibrium*, which the slightest external power would subvert — not by causing a rupture in the substance of the rings — but by precipitating them, *unbroken*, on the surface of the planet. For the attraction of such a ring or rings on a

point or sphere eccentrically situate within them is not the same in all directions, but tends to draw the point or sphere toward the nearest part of the ring, or away from the centre. Hence, supposing the body to become, from any cause, ever so little eccentric to the ring, the tendency of their mutual gravity is, not to correct but to increase this eccentricity, and to bring the nearest parts of them together. Now, external powers, capable of producing such eccentricity, exist in the attractions of the satellites; and in order that the system may be *stable*, and possess within itself a power of resisting the first inroads of such a tendency, while yet nascent and feeble, and opposing them by an opposite or maintaining power, it has been shown that it is sufficient to admit the rings to be *loaded* in some part of their circumference, either by some minute inequality of thickness, or by some portions being denser than others. Such a load would give to the whole ring to which it was attached somewhat of the character of a heavy and sluggish satellite, maintaining itself in an orbit with a certain energy sufficient to overcome minute causes of disturbance, and establish an average bearing on its centre. But even without supposing the existence of any such load — of which, after all, we have no proof — and granting, therefore, in its full extent, the general instability of the equilibrium, we think we perceive, in the periodicity of all the causes of disturbance, a sufficient guarantee of its preservation. However homely be the illustration, we can conceive nothing more apt in every way to give a general conception of this maintenance of equilibrium, under a constant tendency to subversion, than the mode in which a practised hand will sustain a long pole in a perpendicular position resting on the finger, by a continual and almost imperceptible variation of the point of support. Be that, however, as it may, the observed oscillation of the centres of the rings about that of the planet is in itself the evidence of a perpetual contest between conservative and destructive powers — both extremely feeble, but so antagonising one another as to prevent the latter from ever acquiring an uncontrollable ascendancy, and rushing to a catastrophe."

Sir J. Herschel further observes, that since "the least difference of velocity between the planet and the rings must infallibly precipitate the one upon the other, never more to separate (for, once in contact, they would attain a position of

stable equilibrium, and be held together ever after by an immense force), it follows either that their motions in their common orbit round the sun must have been adjusted to each other by an external power with the minutest precision, or that the rings must have been formed about the planet while subject to their common orbital motion, and under the full and free influence of all the acting forces."

2823. *Satellites.* — Saturn is attended by eight satellites, seven of which move in orbits whose planes coincide very nearly with that of the equator of the planet, and therefore with the plane of the rings. The orbit of the remaining satellite, which is the most distant, is inclined to the equator of the planet at an angle of about 12° 14′, and to the plane of the planet's orbit at nearly the same angle.

2824. *Their nomenclature.* — In the designations of the satellites, much confusion has arisen from the disagreement of astronomers as to the principle upon which the numerical order of the satellites should be determined. Some name them first, second, third, &c., in the order of their discovery; while others designate them in the order of their distances from Saturn. It has been proposed to remove all confusion, by giving them names, taken, like those of the planets, from the heathen divinities. The following metrical arrangement of these names, in the order of their distances, proceeding from the most distant inwards, has been proposed, as affording an artificial aid to the memory: —

Iapetus, Titan; Rhea, Dione, Tethys *;
Enceladus, Mimas ———.

2825. *Order of their discovery.* — Since this was suggested, the eighth satellite situate between Iapetus and Titan has been discovered, and called Hyperion.

The order of their discovery was as follows: —

Name.	Discoverers.	When discovered.
Iapetus.	Huygens.	1655.
Titan.	D. Cassini.	1672.
Rhea.	Do.	1672.
Dione.	Do.	1684.
Enceladus.	Sir W. Herschel.	1789.
Mimas.	Do.	1789.
Hyperion.	Messrs. Lassell and Bond.	1848.

* Pronounced Tĕthys.

Hyperion was discovered on the same night, 19th Sept. 1848, by Mr. Lassell of Liverpool, and Professor Bond of the University of Cambridge in the United States.

2826. *Their distances and periods.* — The periodic times and mean distances of these bodies from the centre of Saturn, ascertained by the same kind of observations as already explained in the case of the satellites of Jupiter, are as follows:—

Order.	Name.	Period.					Distance.
		D.	H.	M.	S.	Saturnian Days.	Semidiameters of Saturn.
1	Mimas - - -	0	22	37	22 9	2·16	3·3607
2	Enceladus - -	1	8	53	6·7	3·14	4·3125
3	Tethys - - -	1	21	18	25·7	4·32	5·3396
4	Dione - - -	2	17	41	8·9	6·26	6·8398
5	Rhea - - -	4	12	25	10·8	10·60	9·5528
6	Titan - - -	15	22	41	25·2	35·48	22·1450
7	Hyperion - - -	22	12	?	?	61· ?	28· ?
8	Iapetus - - -	79	7	53	40·4	185·	64·3590

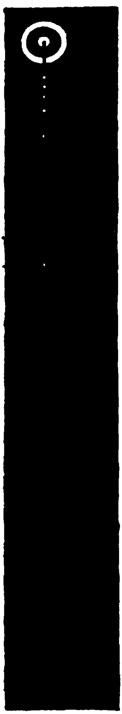


Fig. 775.

2827. *Harmonic law observed.* — By comparing the numbers expressing the ratio of the periods and distances, it will be found that the numbers expressing these fulfil the harmonic law, subject to such deviations from its rigorous observance as are due to the influence of small disturbing causes already noticed. Thus, if D express the mean distance of any satellite from the centre of the planet, and P its period, it will be found that the relation

$$\frac{D^3}{P^2} = 74$$

will be very nearly true for all the satellites.

2828. *Elongations and relative distances.* — The greatest elongations of the satellites from the primary, and the scale of their distances in relation to the diameters of the planet and its rings, are represented in *fig.* 775.

It appears, therefore, that the orbit of the most remote of the satellites subtends a visual angle of only 1286″ at the earth, being about two-thirds of the apparent diameter of the sun or moon, and within this small visual space all the vast physical machinery and phenomena which we have here noticed are in operation, and within such a space have these extraordinary discoveries been made.

The apparent diameter of the external edge of the rings is only 44″, or the fortieth part of the apparent diameter of the sun or moon; yet within that small circle have been observed and measured the planet, its belts, atmosphere, and rotation, and the two rings, their magnitude, rotation, and the lineaments of their surface.

2829. *Various phases and appearances of the satellites to observers on the planet.* — All that has been said of the phases and appearances of the moons of Jupiter, as presented to the inhabitants of that planet, is equally applicable to the satellites of Saturn, with this difference, that instead of four there are eight moons continually revolving round the planet, and exhibiting all the monthly changes to which we are accustomed in the case of the solitary satellite of the earth.

The periods of Saturn's moons, like those of Jupiter, are short, with the exception of those most remote from the primary. The nearest passes through all its phases in 22½ hours, and the fourth, counting outwards, in less than 66 hours. The next three have months varying from 4 to 22 terrestrial days.

These seven moons move in orbits whose planes are nearly coincident with the plane of the rings. The consequence of this arrangement is, that they are always visible by the inhabitants of both hemispheres when they are not eclipsed by the shadow of the planet.

The two inner satellites are seen making their rapid course along the external edge of the ring, within a very small apparent distance of it. The motion of the nearest is so rapid as to be perceivable, like that of the hour-hand of a colossal timepiece. It describes 360° in 22½ hours, being at the rate of 16° per hour, or 16′ per minute, so that in two minutes it moves over a space equal to the apparent diameter of the moon.

The eighth, or most remote satellite, is in many respects exceptional, and different from all the others. Unlike these, it moves in an orbit inclined at a considerable angle to the plane of the rings.

It is exceptional also in its distance from the primary, being removed to the distance of 64 semidiameters of Saturn. The only case analogous to this presented in the solar system is that of the earth's moon, the distance of which is 60 semidiameters of the primary.

2830. *Magnitudes of the satellites.* — Owing to the great

distance of Saturn, the dimensions of the satellites have not been ascertained. The sixth in order, proceeding outwards, is, however, known to be the largest, and it appears certain that its volume is little less than that of the planet Mars. The three satellites immediately within this, Rhea, Dione, and Tethys, are smaller bodies, and can only be seen with telescopes of great power. The other two, Mimas and Enceladus, require instruments of the very highest power and perfection, and atmospheric conditions of the most favourable nature, to be observable at all. Sir J. Herschel says, that at the time they were discovered by his father "they were seen to thread, like beads, the almost infinitely thin fibre of light to which the ring, there seen edgeways, was reduced, and for a short time to advance off it at either end, speedily to return, and hastening to their habitual concealment behind the body."

2831. *Apparent magnitudes as seen from Saturn.* — The real magnitudes of the satellites, the eighth excepted, being unascertained, nothing can be inferred with any certainty respecting their apparent magnitudes as seen from the surface of Saturn, except what may be reasonably conjectured upon analogies to other like bodies of the system. The satellites of Jupiter being all greater than the moon, while one of them exceeds Mercury in magnitude, and another is but little inferior in volume to that planet, it may be assumed with great probability of truth that the satellites of Saturn are at least severally greater in their actual dimensions than our moon.

If this be admitted, their probable apparent magnitudes as seen from Saturn may be inferred from their distances. The distance of the first, Mimas, from the nearest part of the surface of the planet, is only 94,000 miles, or about 2½ times less than the distance of the moon; the distance of the second is about half that of the moon; that of the third about two-thirds, and that of the fourth about five-sixths, of the moon's distance. If these bodies, therefore, exceed the moon in their actual dimensions, their apparent magnitudes as seen from Saturn will exceed the apparent magnitude of the moon in a still greater ratio than that in which the distance of the moon from the earth exceeds their several distances from the surface of Saturn. Of the remaining satellites, little is as yet known of the seventh, Hyperion, which has only been recently discovered; and the great magnitude of the sixth, Titan, renders it probable that,

notwithstanding its great distance from Saturn, it may still appear with a disk not very much less than that of the moon.

2832. *Horizontal parallax of the satellites.* — The horizontal parallax is determined in the same manner as for the satellites of Jupiter (2767). Let the values of it, for the eight satellites proceeding outwards, be expressed by π_1, π_2, π_3, &c., and we shall have

$$\pi_1 = \frac{57^\circ{\cdot}3}{3{\cdot}36} = 17^\circ \qquad \pi_2 = \frac{57^\circ{\cdot}3}{4{\cdot}3} = 13^\circ{\cdot}3 \qquad \pi_3 = \frac{57^\circ{\cdot}3}{5{\cdot}34} = 10^\circ{\cdot}7 \qquad \pi_4 = \frac{57^\circ{\cdot}3}{6{\cdot}84} = 8^\circ{\cdot}4$$

$$\pi_5 = \frac{57^\circ{\cdot}3}{9{\cdot}55} = 6^\circ \qquad \pi_6 = \frac{57^\circ{\cdot}3}{22{\cdot}14} = 2^\circ{\cdot}6 \qquad \pi_7 = \frac{57^\circ{\cdot}3}{28} = 2^\circ \qquad \pi_8 = \frac{57^\circ{\cdot}3}{64{\cdot}36} = 0^\circ{\cdot}9.$$

2833. *Apparent magnitudes of Saturn seen from the satellites.* — It follows, therefore, that the disk of Saturn, seen from the satellites respectively, subtends visual angles varying from 34° subtended at the nearest to 2° at the most remote.

2834. *Satellites not visible in the circumpolar regions of the planet.* — From what has been explained in (2769), combined with the observed fact that all the satellites except Iapetus move in the plane of the equator of the planet, it follows that they are severally invisible within distances of the poles of the planet expressed by their horizontal parallaxes. Thus, the first cannot be seen at latitudes higher than 73°; the second, 76°·4; the third, 79°·3, and so on.

2835. *Remarkable relation between the periods.* — The periods of the four satellites nearest to the planet have a very remarkable numerical relation. If they are expressed by P, P', P'', and P''', we shall find that

$$P'' = 2\,P, \qquad P''' = 2\,P';$$

that is, the periods of the third and fourth are respectively double those of the first and second.

2836. *Rotation on their axes.* — The case of the moon, and the observations made on the satellites of Jupiter, raise the presumption that it is a general law of secondary planets to revolve on their axes in the times in which they revolve round their primary. The great distance of Saturn has deprived observers hitherto of the power of testing this law by the Saturnian system. Certain appearances, however, which have been observed in the case of the great satellite Titan indicate, at least with regard to it, such a rotation. The variation of its apparent brightness in different parts of its orbit is very conspicuous, and

the changes have a fixed relation to its elongation, the same degree of brightness always corresponding to the same position of the satellite in relation to its primary. Now this is an effect which would be explicable on the supposition that different sides of the satellite reflect light with different degrees of intensity, and that it revolves on its axis in the same time that it revolves round its primary. It has been observed that, when the satellite has eastern elongation, it has ceased to be visible, from which it has been inferred that the hemisphere then turned to the earth has so feeble a reflective power that the light proceeding from it is insufficient to affect the eye in a sensible degree. The improvement of telescopes has enabled observers to follow it at present through the entire extent of its orbit, but the diminution of its lustre on the eastern side of the planet is still so great, that it is only seen with the greatest difficulty.

2837. *Mass of Saturn.* — The mass of Saturn is ascertained by the motion of his satellites by the method already explained (2635).

If r express the distance of the planet from the sun in semi-diameters of the orbit of a satellite, P the period of the planet taking that of the satellite for the unit, and M the mass of the sun taking that of the planet for the unit, we shall have

$$M = \frac{r^3}{P^2}.$$

By substituting for these symbols the numbers which they represent in the case of Saturn and his satellites, values will be found for M, a mean of which is about 3500, showing that the mass of Saturn is the 3500th part of the mass of the sun.

Since the earth is the 355,000th part of the mass of the sun, it follows that the mass of Saturn is 100 times that of the earth.

2838. *Density.* — Since the mass of Saturn is only 100 times that of the earth, while his volume is about 1000 times greater, it follows that this planet is composed of matter whose mean density is about ten times less than that of the earth; and since the density of the earth is five and a half times greater than that of water, it follows that the density of Saturn is a little more than half that of water. This is the density of the light sorts of wood, such as cedar and poplar, and is about twice the density of cork (787).

2839. *Superficial gravity.* — The gravity by which bodies on the surface of Saturn are attracted, omitting for the moment all consideration of the effects of the spheroidal form and rotation, may be computed by the principles already explained by its mass and mean diameter.

Let m' = the mass, that of the earth being 1;
r' = the mean semidiameter, that of the earth being 1;
g' = superficial gravity, that of the earth being 1.

We shall then have

$$g' = \frac{m}{r'^2} = \frac{101 \cdot 6}{9 \cdot 48^2} = 1 \cdot 13.$$

Superficial gravity, therefore, on Saturn exceeds that upon the earth by thirteen per cent., omitting the effects of form and rotation.

2840. *Centrifugal force at Saturn's equator.* — The force of gravity on Saturn, as on Jupiter, is subject to considerable variation, arising from the counteracting effects of centrifugal force.

Let c = centrifugal force at the planet's equator related to terrestrial gravity as the unit;
g = 16·08 feet;
v = velocity of Saturn's equator in feet per second.

We shall then have (313)

$$c = \frac{v^2}{2 l' \times g}.$$

The value of v deduced from the equatorial diameter of the planet and the time of its rotation is 34,775, and it follows, therefore, that c = 0·1799.

Deducting this from the superficial gravity, undiminished by rotation already computed (2839), we shall have

$$g' - c = 1 \cdot 13 - 0 \cdot 1799 = 0 \cdot 9501\,;$$

which is, therefore, the effective gravity at Saturn's equator related to terrestrial gravity as the unit.

2841. *Variation of gravity from equator to pole.* — Let e, w, and c retain their former significations (2777) (2384), and by the theorem of Clairaut, already noticed, we shall have

$$e + w = 2 \cdot 5c.$$

But, by what has been proved, we have

$$e=0\cdot097 \qquad c=\frac{c}{g'-c}=\frac{0\cdot1799}{0\cdot9501}=0\cdot189\,;$$

from which it follows that

$$w=2\cdot5\times0\cdot189-0\cdot097=0\cdot3755.$$

It follows, therefore, that a body which weighs 10,000 lbs. at Saturn's equator would weigh 13,755 lbs. if transported to his pole; and a body which weighs 100 lbs. placed upon the earth would weigh 95 lbs. on Saturn's equator.

The height through which a body would fall in a second upon Saturn will be

$16\cdot08\times0\cdot95=15\cdot28$ feet at the equator;
$15\cdot28\times1\cdot375=21\cdot01$ feet at the pole.

The relative heights through which bodies would fall, and the lengths of pendulums, may be determined in the same manner as already explained (2777).

2842. *Prevailing errors respecting the uranography of Saturn.* — The rings must obviously form a most remarkable object in the firmament of observers stationed upon Saturn, and must play an important part in their uranography. The problem to determine their apparent magnitude, form, and position, in relation to the fixed stars, the sun, and Saturnian moons, has, therefore, been regarded as a question of interesting speculation, if not of great scientific importance; and has, accordingly, more or less engaged the attention of astronomers. It is nevertheless a singular fact, that, although the subject has been discussed and examined by various authorities for three-quarters of a century, the conclusions at which they have arrived, and the views which have been generally expressed and adopted respecting it, are completely erroneous.

In the Berlin *Jahrbuch* for 1786, Professor Bode published an essay on this subject, which, subject to the imperfect knowledge of the dimensions of the rings which had then resulted from the observations made upon them, does not seem to differ materially in principle from the views adopted by the most eminent astronomers of the present day.

2843. *Views of Sir J. Herschel.* — Sir John Herschel, in his Outlines of Astronomy, edit. 1849, states that the rings as

seen from Saturn appear as vast arches spanning the sky from horizon to horizon, holding an almost invariable situation among the stars; and that, in the hemisphere of the planet which is on their dark side, a solar eclipse of fifteen years' duration takes place.

This statement, which has been reproduced by almost all writers both in England and on the Continent, is incorrect in both the particulars stated. *First*, the rings do not hold an almost invariable position among the stars. On the contrary, their position with relation to the fixed stars is subject to a change so rapid that it must be sensible to observers on the planet, the stars seen on one side of the rings passing to the other side from hour to hour. *Secondly*, no such phenomenon as a solar eclipse of fifteen years' duration, or any phenomenon bearing the least analogy to it, can take place on any part of the globe of Saturn.

2844. *Theory of Mädler.*—Among the continental astronomers who have recently reviewed this question, the most eminent is Dr. Mädler, to whose observations and researches science is so largely indebted for the information we possess respecting the physical character of the surface of the Moon and Mars.

This astronomer maintains, like Herschel, that the rings hold a fixed position in the firmament, their edges being projected on parallels of declination, and that, consequently, all celestial objects are carried by the diurnal motion in circles parallel to them, so that in the same latitude of Saturn the same stars are always covered by the rings, and the same stars are always seen at the same distance from them.

This is also incorrect. The zones of the firmament covered by the rings are not bounded by parallels of declination, but by curves which intersect these parallels at various angles.

Dr. Mädler enters into elaborate calculations of the solar eclipses which take place during the winter half of the Saturnian year. According to him, at a certain epoch after the autumnal equinox in each latitude, the sun passes under the outer ring and is eclipsed by it, and continues to be thus eclipsed until, by its increasing declination, it emerges from the lower edge of that ring and passes into the opening between the rings, where it continues to be visible for an interval greater or less according to the latitude of the observer,

until the further increase of its declination causes it to pass under the edge of the inner ring, where it is again eclipsed. The further increase of its declination in certain latitudes would, according to this astronomer, carry it beyond the lower edge of the inner ring, after which it would be seen below the ring uneclipsed. After the solstice in such latitudes, when the sun returns towards the celestial equator, its decreasing declination would carry it successively first under the inner and then under the outer ring. There would thus be, according to Mädler, in such latitudes two solar eclipses of long duration, one by each ring before the winter solstice, and two others of like duration, but in a contrary order, after the winter solstice. In certain latitudes, however, the declination of the lower edge of the inner ring being greater than the obliquity of the orbit to the Saturnian equator, the sun would not emerge from the inner ring, and in this case there would be only one eclipse by the inner ring, and that at mid-winter; but, as before, two, one before and the other after the solstice, by the outer ring, separated from the former by the time during which the sun passes across the interval between the rings.

Dr. Mädler computes the duration of these various eclipses in the different latitudes of Saturn, and gives a table, by which it would appear that the solar eclipses which take place behind the inner ring vary in length from three months to several years, that the duration of the eclipses produced by the outer ring is still greater, and that the duration of the appearance of the sun in the interval between the rings varies in different latitudes from ten days to seven and eight months.*

2845. *Correction of the preceding views.* — These various conclusions and computations of Dr. Mädler, and the reasoning on which they are based, are altogether erroneous; and the solar phenomena which he describes have no correspondence with, nor any resemblance to, the actual uranographical phenomena.

We shall now explain, so far as the necessary limits of the present volume will admit, what the actual phenomena are which would be witnessed by an observer stationed at different parts of the surface of Saturn. It will not, however, be possible to enter into the details of the reasoning upon which the conclusions are based. For this we must refer to a memoir by the

* See Populäre Astronomie, von Dr. J. H. Mädler. Berlin, 1852.

author of this volume, read before the Royal Astronomical Society of London, and which will be seen in their Transactions.

2846. *Phenomena presented to an observer stationed at Saturn's equator — Zone of the firmament covered by the ring.* — The station of the observer in this case being in the plane of the ring, and the heavens having the character of a right sphere, the ring will cover a zone of the firmament coinciding with the prime vertical, which, in this case, is also the celestial equator. It will therefore pass through the zenith of the observer at right angles to his meridian, descending to the horizon at the east and west points. The only part of the system of rings exposed to view is the inner edge of the inner ring. This edge is illuminated at night by the sun at all times, except at the equinox, when the sun, being in the plane of the ring, one semicircle of the ring throws its shadow on the other; and excepting, also, that arc of the ring on which the shadow of the planet falls. In the day-time the edge of the ring is rather strongly illuminated by light reflected from the extensive and not very remote surface of the planet.

The thickness of the ring not being exactly ascertained, the apparent width of the zone of the firmament which it covers cannot be determined with precision.

If, however, the major limit of 250 miles, assigned to the thickness by Sir J. Herschel, be adopted, the corresponding limit of the apparent width, obtained by the usual method of calculation, by comparing this thickness with its distance from the observer, will give 45′ as the apparent breadth of the zone occupied by the ring at the zenith; and since the observer, being stationed at a point O (*fig.* 776.) considerably removed from the centre C of the ring, is at a greater distance O P″, O *p*″ from those points of the ring which meet the horizon than from that which is at the zenith, the apparent breadth of the ring will gradually decrease from the zenith Z to the horizon in the same proportion as the distance of successive points P, P′ &c. of the ring from the station of the observer increases. From the known values of the diameters of the planet and the ring already given, it is easy to

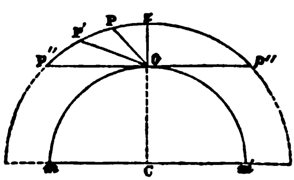


Fig. 776.

show that the apparent breadth of the ring at the horizon will be less than its apparent breadth at the zenith in the ratio of 9 to 4 very nearly, that being the inverse ratio of the distance of the two points from the observer. If, therefore, the apparent breadth of the ring at the zenith z be 45′, its apparent breadth at the horizon P″ p″ will be 20′.

It appears, therefore, that a zone of the firmament is in this case covered by the ring extending 22′½ N. and S. of the equator at the zenith, and 10′ N. and S. of it at the horizon, and gradually decreasing in width from the one point to the other. It follows that two parallels of declination at 22′½ N. and S. touch this zone at the points where it intersects the meridian, and lie elsewhere altogether clear of it; and two other parallels, whose declination N. and S. is 10′, meet it at the horizon, and lie elsewhere altogether within it. The intermediate parallels intersect the edges of the zone at certain points between the zenith and the horizon, and pass outside it, below and under it, above those points. It follows from this, that all objects situate in such parallels rise clear of the ring, pass under it at a certain altitude, and culminate occulted by it. All parallels of declination, whose distance from the equator is less than 10′, are entirely covered by the ring from the horizon to the zenith; and all objects placed on such parellels are, consequently, occulted by the ring through the whole period of their diurnal motion.

2847. *Solar eclipses at Saturn's equator.*—These observations are applicable, of course, not only to the sun, but to all objects of changeable declination. As to the sun, its apparent diameter at Saturn being 3′·3, its disk will be in external contact with the ring at the zenith, when the declination of its centre is 24′½, and in internal contact with it when its declination is 21′.

From a calculation of the rate at which the sun changes its declination at and near Saturn's equinox, derived from the ascertained obliquity of his equator to his orbit, it may be shown that it will have the declination 24′½ at twenty-two days before the equinox, and the declination 21′ at nineteen days before it. At twenty-two days before the equinox, therefore, a partial eclipse of the sun by the ring at the zenith will commence, which will become total at nineteen days before it.

But though total at the zenith it will still be only partial at

lower altitudes, and the sun's disk will be clear of the ring altogether when still nearer to the horizon.

When the sun's declination still decreasing becomes 11′·65, its disk will be in external contact with the ring at the point where it rises and sets; and when its declination becomes 8′·35 it will be in internal contact, and therefore totally eclipsed. The sun will have the former declination at ten days and a half, and the latter at seven days and a half, before the equinox.

It follows, therefore, that the sun will be totally eclipsed from rising to setting seven days and a half before, and seven days and a half after, the equinox, to an observer stationed on the equator of the planet.

2848. *Eclipses of the satellites.*—The orbits of the six inner satellites being exactly or nearly in the plane of the ring, they will be permanently eclipsed by the ring to an observer stationed on the planet's equator, unless, indeed, the apparent magnitudes of their disks exceed the apparent width of the ring. The real magnitude of the satellites being unascertained, it is impossible to determine their apparent magnitudes; from analogy it would appear improbable that they should be so great as the apparent width of the ring even at the horizon, and unless they depart to some extent, by reason of small obliquities in their orbits, from the plane of the ring, all view of them must be permanently intercepted from an observer thus stationed.

The eighth satellite, however, whose orbit is inclined to the plane of the ring at an angle of about 13°, departs N. and S. of the ring to this extent, and is subject to eclipses similar to those of the sun already described.

2849. *Phenomena presented to an observer at other Saturnian latitudes.* — If an observer be stationed at any point on one of the meridians of the planet, on the same side of the ring as the sun, the ring will present to him the appearance of an arch in the heavens, bearing some resemblance in its form to a rainbow, the surface, however, having an appearance resembling that of the moon.

The vertex or highest point of this arch will be upon his meridian, and the two portions into which it will be divided by the meridian will be equal and similar, and will descend to the horizon at points equally distant from the meridian. The apparent breadth of this illuminated bow will be greatest upon the

meridian, and it will decrease in descending on either side towards the horizon, where it will be least. The division between the two rings will be apparent, and, except at places within a very short distance of the equator, the firmament will be visible through it.

2850. *Edges of the rings seen at variable distances from celestial equator.*—The distance of the edge of the bow from the celestial equator will not be every where the same, as it has been erroneously assumed to be. That part of the bow which is upon the meridian will be most remote from the celestial equator; and in descending from the meridian on either side towards the horizon, the declination of its edge will gradually decrease, so that those points which rest upon the horizon will be nearer to the equator than the other points.

2851. *Parallels of declination, therefore, intersect them.*—It follows from this that the parallel of declination which passes through the points where the upper edge of the bow meets the horizon will lie every where above, and the parallel which passes through the point where the upper edge crosses the meridian will lie every where below, that edge. This necessarily follows from the fact, that the declination of the points where the edge meets the horizon is less, and that of the point where it meets the meridian greater, than that of any other point upon it.

It appears from this, that all parallels whose declinations are greater than those of the points where the edge meets the horizon, and less than that of the point where it crosses the meridian, must intersect the edge between the horizon and the meridian, and must therefore lie below it under the point of intersection, and above it over the point of intersection.

These conditions are equally applicable to each of the edges of each of the rings, and will serve to determine in all cases those parallels which will intersect them respectively.

2852. *Edges placed symmetrically in relation to meridian and horizon.*—From the symmetrical position of each of the edges, with relation to the meridian and horizon, it will be apparent that the points at which each parallel intersects them will be similarly placed on each side of the meridian, having equal altitudes and equal azimuths E. and W.

2853. *Form of the projection at different latitudes.*—The general form and position of the bow formed by the projection of the rings upon the firmament in each latitude may be

easily determined by elementary geometrical principles. Let P E (*fig.* 777.) be the quadrant of the meridian of the planet passing through the station of the observer O, and let R R′ r represent one of the edges of the rings reduced by perspective to an oval,

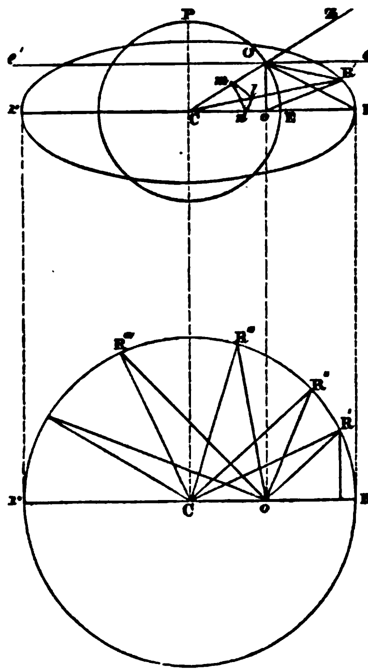


Fig. 777.

but shown as a circle in the lower figure. It will be easily perceived, that the visual ray passing through O to R carried round the circle R, R′ R″, &c., will describe the surface of an oblique cone, whose base is the plane of the ring, and whose axis O C is inclined to the base at an angle O C R, equal to the latitude of the station. The projection of the edge of the bow upon the firmament will then be the intersection of the surface of this oblique cone with the celestial sphere; and it can be demonstrated, that the point of this which is most remote from the celestial equator is the projection of the point R, where the plane of the meridian intersects the plane of the ring, that

the other points of the projection approach nearer to the celestial equator as their distance from the meridian increases, and that they have equal declinations at equal distances east and west of the meridian.

To illustrate this still more fully, let N C E, *fig.* 778., be a

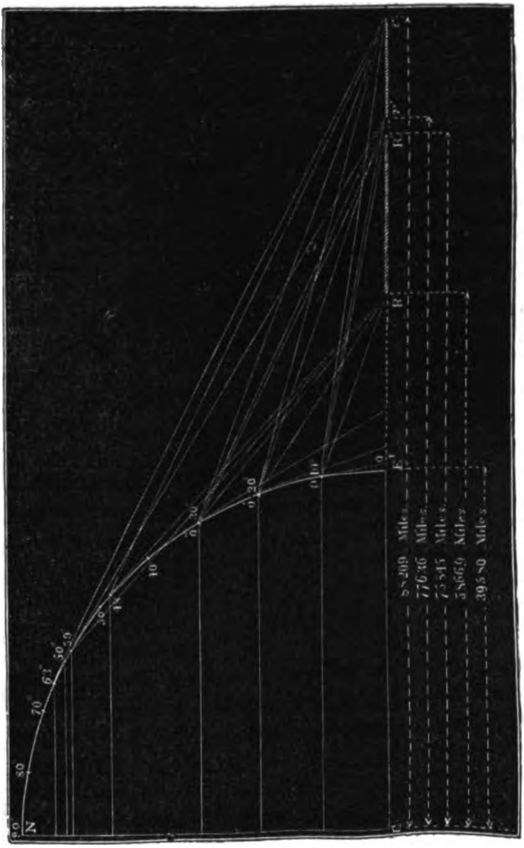


Fig. 778.

quadrant of a meridian of the planet, C being its centre and C E its semidiameter. Let R R′ be a section of the inner ring, made by the plane of the meridian continued through the ring, and let *r r′* be a like section of the exterior ring.

2854. *Rings invisible above lat.* 63° 20′ 38″.—If we suppose an observer to travel upon the meridian from the pole N, towards the equator, it is evident that at first all view of the rings will be intercepted by the convexity of the planet. If a line be drawn from *r*, the external point of the external ring, touching the planet, it can be proved that the point of contact will be at the latitude of 63° 20′ 38″; and it is evident, that when the observer has descended to this latitude the point *r* will be in his horizon, and consequently at all higher latitudes it will be below his horizon, and therefore invisible. It appears, therefore, that no part of the outer ring is visible from any latitude above 63° 20′ 38″, and that at this latitude a single point of the exterior ring is visible just touching the southern point of the horizon.

2855. *Appearance at lat.* 59° 20′ 25″.—If the observer now descend to lower latitudes, the exterior ring will begin to rise above his horizon at the southern point; and it can be shown that when he has descended to the latitude 59° 20′ 25″, his horizon will just touch the inner edge *r′* of the exterior ring, cutting off a segment of that ring, which will be seen above the horizon.

The position of the ring thus visible above the horizon, will have the appearance of a lunar segment.

2856. *Appearance at lat.* 58° 32′ 20″.—If the observer continue to descend to a lower latitude, the ring will continue to rise to a greater elevation, and the interval between the rings will become visible. When he has descended to the lat. 58° 32′ 20″ his horizon will just touch the outer edge of the inner ring, and a segment of the interval between the rings will be visible under the arch of the outer ring, which will appear projected upon the southern firmament.

The outer ring, therefore, is presented to the observer in this case as a lunar bow spanning the southern firmanent. It must be remembered that the declinations of the points at which each of the edges intersect the meridian being greater, and the declination of the points where they meet the horizon less, than that of any intermediate points, all parallels having declinations between these will intersect the edges severally, while all parallels whose declination is beyond these limits will pass altogether above or altogether below them, respectively, as the case may be.

2857. *Appearance at lat.* 47° 33′ 51″. — When the observer has descended to this latitude, the horizon will just touch the inner edge of the inner ring, cutting off a segment of it, of which the horizon is the base, the outer ring appearing as a bow above it, as represented in *fig.* 779. The same observation as to the

Fig. 779.

varying declination of the edges of both rings will be applicable in this as in the former case, and certain parallels will accordingly intersect each of the edges of the rings, and others will be entirely covered by them.

2858. *Appearance at lower latitudes.* — In descending to still lower latitudes, the elevation of the bow formed by the projection of the rings increases; and the lower ring, which hitherto has presented itself as a mere segment, having the horizon as a base, now assumes the form of a bow, inclosing below it a plane segment of the firmament, as represented in *fig.* 780.

Fig. 780.

As the latitude decreases, the amplitude and elevation of this bow also increase, but its apparent breadth diminishes, as in *figs.* 781, 782, 783. The obliquity of its several edges to the direction of the parallels of declination still continues, and it is consequently still intersected by them at every point, so that the

parallels which interlace it will lie alternately above and below its edges as already described, some parallels, however, lying

Fig. 781.

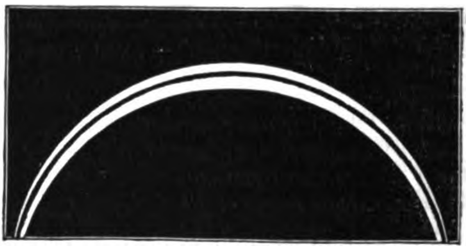


Fig. 782.

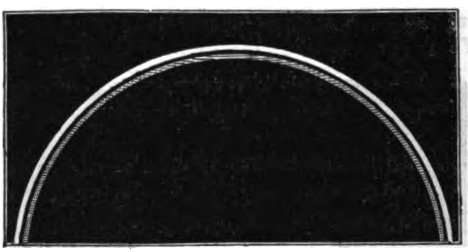


Fig. 783.

entirely above and others entirely below it, as represented in *fig.* 784. It will be easy to perceive by this, without rendering the diagrams complicated by multiplying upon them the arcs of the parallels, that they will be curiously interlaced by these parallels, which will pass alternately above and below the rings, being at one place covered by them, and at another uncovered; at one point crossing the interval between the rings, and at another

lying above or below it; some parallels lying entirely above, others entirely below, either ring, while some may be alto-

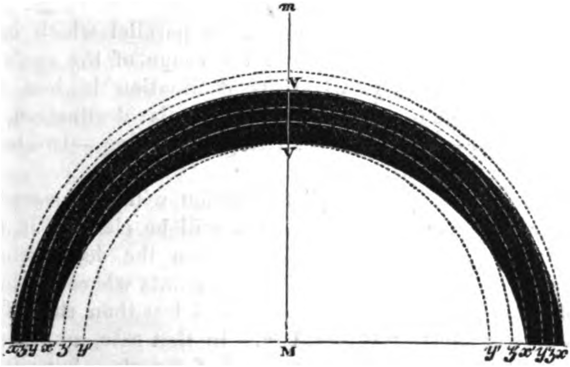


Fig 784.

gether covered by the one ring or the other. I have ascertained, by an investigation of the dimensions and form of the bow in different latitudes, that the only latitudes in which a parallel lies altogether within the interval between the rings are those included between the latitudes 58° and 56°. At all other latitudes of the planet from which the interval between the rings is visible, the parallels which pass between the ring will lie partly within the interval, and partly above or below it.

2859. *Occultations of celestial objects by the rings.* — It follows, from all this, that, in general, celestial objects are carried by their diurnal motion alternately above and below each of the edges of the rings; and if a parallel intersects all the edges, which happens in many cases, then the object in such a parallel will rise below the rings, will pass successively under them: will be occulted by each of them, will be visible in crossing the interval between them, and will culminate above them, and this will take place in precisely the same manner, and at precisely the same altitudes and azimuths E. and W. of the meridian, so that such an object will be occulted four times by the rings between rising and setting, that is to say, twice between the horizon and meridian east and west of the latter. In rising, it will be first occulted by the inner ring, then passing across the interval will be occulted by the outer ring, after which it will culminate clear of the rings; and in descending on the west towards the horizon,

it will be successively occulted by the outer and inner rings in the same manner and at the same altitudes as it was occulted before culmination, and will finally set clear of the rings.

When, as happens in some cases, the parallel which comes under these conditions falls within the range of the sun's declination, that is to say, when its declination is less than 26° 48′ 40″, the sun, attaining this particular declination, will suffer four such eclipses between rising and setting,—two before and two after culmination.

In some cases a parallel of declination will be covered by the lower ring at the meridian, but will be clear of it near the horizon. This will take place when the declination of the parallel is greater than that of the points where the lower edge of the ring meets the horizon, and less than that of the point where it meets the meridian. In that case, an object in such a parallel will rise and set clear of the ring, but will be occulted by it at culmination. Such an object, therefore, will be occulted only once between rising and setting.

An object may in like manner be occulted at or a little above the horizon by the inner ring, and may culminate in the interval, or it may, after being occulted by the inner ring, pass under the outer ring and culminate occulted by it.

In fine, all these various phenomena, and many others too numerous and complicated to be explained here, are manifested in the Saturnian firmament, and the sun itself is subject to most of them. It happens, in some cases, that a certain number of parallels of declination are entirely covered by the outer and others by the inner ring, and when the sun is found on any one of these parallels it will be eclipsed constantly from rising to setting by one or the other ring.

2860. *Zone visible between the rings.* — The zone of the heavens visible between the rings is found by calculating the visual angle subtended at the station of the observer by lines drawn to the inner edge of the outer and to the outer edge of the inner ring, supposing that the thickness of the outer ring bears an inconsiderable proportion to the width of the space which separates them; and it is evident that the magnitude of this visual angle will gradually and indefinitely decrease as the observer approaches the equator, inasmuch as the obliquity of the visual ray to the plane of the rings indefinitely increases.

2861. *Effect of the thickness of the rings on this zone.* — If,

however, the thickness of the outer ring be supposed to bear any considerable ratio to the width of the space between the rings, it will intercept a portion of the visual rays included within the angle formed by the rays drawn to the edges of the two rings, and the effective opening will be found by subtracting the visual angle subtended by the thickness of the outer ring from the visual angle subtended by the space between the rings; and since the obliquity of the visual rays bounding the former gradually diminishes in approaching the equator, while the obliquity of the visual rays bounding the latter gradually increases, it is evident that the visual angle subtended by the thickness of the outer ring continually increasing will, at some certain latitude, become equal to the visual angle subtended by the space between the rings, and at that latitude accordingly, as well as at all inferior latitudes, the thickness of the outer ring will altogether intercept the opening, and no zone of the heavens will be visible through it. I have found by calculation that if 250 miles be admitted as the major limit of the rings, all view of the heavens through the opening will be intercepted at latitudes below 8°; and if the probable minor limit of 150 miles be assumed, all view will be intercepted at and below the latitude of 5°.

2862. *Solar eclipses by the rings.* — The principles upon which solar eclipses by the rings in each latitude are calculated are, therefore, easily understood. By comparing the parallel of declination of the sun at any time with the parallels of declination of the points where each of the edges of each of the rings meets the horizon and the meridian, the conditions under which it will intersect the edges severally will be determined, and hence it will appear that a most curious and interesting body of solar phenomena not anticipated in any of the works in which the uranography of Saturn has been investigated are brought to light. In the lower latitudes the sun undergoes at certain epochs four eclipses per day, two in the forenoon and two in the afternoon, and between each pair of eclipses is seen shining through the space between the rings. In higher latitudes, at certain seasons, it does not emerge from one or other of the rings, and suffers only three eclipses, two by one ring and one by the other. At other latitudes, at certain times, it is only eclipsed at rising and setting, and at others only in culminating.

Our limits, however, preclude us from giving these details,

and others not less curious, for which the reader is referred to the paper already mentioned.

2863. *Eclipses of the satellites.* — The inner satellites being in the plane of the rings, they will necessarily be projected on a zone of the heavens outside the exterior ring, and can never be intercepted or eclipsed by the rings (*fig.* 784.).

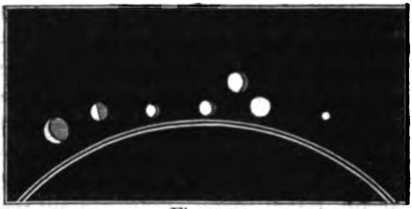


Fig. 784.

The eighth satellite, however, whose orbit has an obliquity of 12° or 13° to the plane of the rings, will be eclipsed at those latitudes at which the edges of the rings have declinations less than those which it attains. These eclipses are calculated and explained on the same principles exactly as those of the sun, *mutatis mutandis.*

2864. *Saturnian seasons.*—It has been shown (2795) that the axis of the planet is inclined to the plane of its orbit at an angle of 26° 48′ 40″, and is, like the axis of the earth, carried parallel to itself round the sun. The obliquity therefore which, so far as the sun is concerned, determines the extremes of the Saturnian seasons, differs by no more than 3° from that of the ecliptic. The tropics are parallels of latitude 26° 48′ 40″ north and south of the equator. The parallels within which the sun remains in winter below the horizon during one or more revolutions of the planet are at the latitude 63° 11′ 20″. These circles, therefore, affect the Saturnian climatology in the same manner as the tropics and polar circles affect that of the earth. The slow motion of the sun in longitude, and its rapid diurnal motion, however, must produce important differences in its effects as compared with those manifested on the earth. While the sun, as seen from the earth, changes its longitude at the mean rate of very nearly 1° per day, its change of longitude for Saturn is little more than 2′ per day; and while, as seen in the terrestrial firmament, it is carried by the diurnal motion over 1° in four minutes, a Saturnian observer sees it move over the same space in less than two minutes. If the heating and illuminating power of the sun be diminished in a high ratio by the greater distance of that luminary, some compensation may perhaps arise from the rapid alternations of light and darkness.

III. Uranus.

2865. *Discovery.*— While occupied in one of his surveys of the heavens on the night of the 13th March, 1781, the attention of Sir William Herschel was attracted by an object which he did not find registered in the catalogue of stars, and which presented in the telescope an appearance obviously different from that of a fixed star. On viewing it with increased magnifying powers, it presented a sensible disk; and after the lapse of some days, its place among the fixed stars was changed. This object must, therefore, be either a comet or a planet; and Sir W. Herschel, in the first instance, announced it as the former. When, however, submitted to further and more continued observation, it was found to move in an orbit nearly circular, inclined at a small angle to the plane of the ecliptic, and to have a disk sensibly circular.

It appeared, therefore, to have the characters, not of a comet, but a planet revolving outside the orbit of Saturn. It was named the "Georgium Sidus" by Sir W. Herschel, in compliment to his friend and patron George III. This name not being accepted by foreign astronomers, that of "Herschel" was proposed by Laplace, and to some extent for a time adopted. Definitively, however, the scientific world has agreed upon the name "Uranus," by which this member of the system is now universally designated.

2866. *Period, by synodic motion.*—Owing to the great length of the period of this planet, those methods of determination which require the observation of one or more complete revolutions could not be applied to it. The synodic period, however, or the interval between two successive oppositions, being only 369·4 days, supplied a means of obtaining a first approximation. This gives

$$\frac{1}{P}=\frac{1}{365{\cdot}25}-\frac{1}{369{\cdot}40}=\frac{1}{30643},$$

which gives a period of 30,643 days.

2867. *By the apparent motion in quadrature.*—When a planet is in quadrature, its visual direction being a tangent to the earth's orbit, its apparent place is not affected by the earth's orbital motion. In the quadrature which precedes opposition, the earth moves directly *towards* the planet; and in the quadrature which follows opposition, it moves directly *from* the

planet. In neither case, therefore, would its motion produce any apparent change of place in the planet. It follows, therefore, that when a planet is in quadrature, its apparent motion is due exclusively to its own motion, and not at all to that of the earth. The daily motion of the planet as then observed is, therefore, the actual daily increment of its geocentric longitude.

But in the case of a planet such as Uranus, or even Saturn, whose distance from the sun bears a large ratio to the earth's distance, the geocentric motion of the planet will not differ sensibly from the heliocentric motion; and, therefore, the geocentric daily increment of the planet's longitude observed when in quadrature may, to obtain an approximative value of the period, be taken as the daily increment of the heliocentric longitude. If this increment be expressed by l, we shall have

$$\text{P} = \frac{360^\circ}{l}.$$

Now, it is found that the apparent daily increment of the planet's longitude when in quadrature is $42''\cdot23$. If 360° be reduced to seconds, we shall then have

$$\text{P} = \frac{1296000}{42\cdot23} = 30689.$$

By more accurate calculation, the periodic time has been determined at 30,686·82 days, or 84 years.

2868. *Heliocentric motions.* — The mean heliocentric motion of the planet is therefore, more exactly,

$$\frac{360^\circ}{84} = 4^\circ\ 17'\ 8''\cdot5 \text{ yearly};$$

$$\frac{360^\circ}{84 \times 12} = 21'\ 25''\cdot7 \text{ monthly};$$

$$\frac{1296000}{30687} = 42\cdot''233 \text{ daily}.$$

2869. *Synodic motion.* — The mean daily apparent motion of the sun being $0^\circ\cdot9856$, or $3548''16$, the mean daily synodic motion or increment of elongation of Uranus will be

$$3548\cdot16 - 42''\cdot23 = 3505''\cdot93 = 58'\cdot43 = 0^\circ\cdot975.$$

The synodic period is therefore, more exactly,

$$\frac{360}{0\cdot975} = 369\cdot23 \text{ days}.$$

The earth, therefore, in $4\frac{1}{4}$ days overtakes the planet, after completing each sidereal revolution.

2870. *Distance.* — The mean distance r of the planet from the sun, determined by the harmonic law, is therefore

$$r^3 = 84^2 = 7056,$$
$$r = 19{\cdot}18.$$

The mean distance is therefore 19·18 times that of the earth, and, consequently, the actual distance is

$$95{,}000{,}000 \times 19{\cdot}18 = 1{,}822{,}100{,}000 \text{ miles.}$$

The distance of Uranus from the sun is therefore 1822 millions of miles, and its distance from the earth, when in opposition, is therefore 1727 millions of miles.

The eccentricity of the orbit of Uranus being 0·046, these distances are liable to only a very small variation. The distance from the sun is increased in aphelion, and diminished in perihelion by less than a twentieth of its entire amount. The plane of the orbit coincides very nearly with that of the ecliptic.

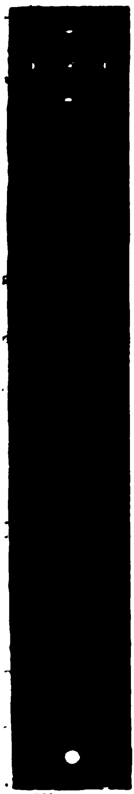


Fig. 785.

2871. *Relative orbit and distance from the earth.* — The relative proportion of the orbits of Uranus and the earth are represented in *fig.* 785., where E E′ E″ is the orbit of the earth, and S U the distance of Uranus from the sun. The four positions of the earth, corresponding to the opposition, conjunction, and quadratures of the planet, are represented as in the former cases.

2872. *Annual parallax.* — Since S U is 19·18 times S E, we shall have for the angle

$$\text{S U E}' = \frac{57^\circ{\cdot}30}{19{\cdot}18} = 3^\circ.$$

The diameter of the earth's orbit, measuring as it does nearly 200 millions of miles, therefore subtends at Uranus a visual angle of only 6°; and a globe which would fill it, seen from the planet, would have an apparent diameter only twelve times greater than that of the moon.

2873. *Vast scale of the orbital motion.* — The distance of Uranus from the sun being above nineteen times that of the earth, and the earth being at such a distance that light, moving at the rate of nearly 200,000 miles per second, takes about eight

minutes to come from the sun to the earth; it follows that it will take $19 \times 8 = 152$ minutes, or two hours and a half, to move from the sun to Uranus. Sunrise and sunset are, therefore, not perceived by the inhabitants of that planet for two hours and a half after they really take place, for the sun does not appear to rise or set until the light moving from it, at the moment it touches the plane of the horizon, reaches the eye of the observer.

The diameter of the orbit of Uranus measuring, in round numbers, 3600 millions of miles, its circumference measures 11,300 millions of miles, over which the planet moves in 30,687 days. Its mean daily motion is therefore 368,000 miles, and its hourly motion, consequently, about 15,300 miles.

2874. *Apparent and real diameters.*—The apparent diameter of Uranus in opposition exceeds 4″ by a small fraction. At the distance of the planet from the earth, in that position, the linear value of 1″ is

$$\frac{1757000000}{206265} = 8421 \text{ miles.}$$

The actual diameter of the planet is therefore

$$8421 \times 4{\cdot}1 = 34{,}526 \text{ miles,}$$

being about half that of Saturn, and a little more than $4\frac{1}{2}$ times that of the earth.

2875. *Surface and volume.*—The surface of Uranus is therefore 19 times, and its volume 82 times, that of the earth.

2876. *Diurnal rotation and physical character of surface unascertained.*—The vast distance of this planet, and its consequent small apparent magnitude and faint illumination, have rendered it hitherto impracticable to discover any indications of its diurnal rotation, the existence of an atmosphere, or any of the other physical characters which the telescope has disclosed in the case of the nearer of the great planets.

2877. *Solar light and heat.*—The apparent diameter of the sun, as seen from Uranus, is less than as seen from the earth in the ratio of 1 to 19. The magnitude of the sun's disk at the earth being supposed to be represented by E, *fig.* 786., its magnitude seen from Uranus would be U.

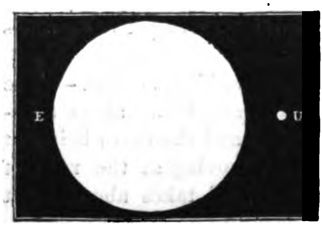


Fig. 786.

The illuminating and warming

powers of the solar rays, *under the same physical conditions*, are therefore $19^2 = 361$ times less at Uranus than at the earth.

2878. *Suspected rings.* — It was at one time suspected by Sir W. Herschel that this planet was surrounded by two systems of rings with planes at right angles to each other. Subsequent observation has not realised this conjecture.

2879. *Satellites.* — It has been ascertained that Uranus, like the other major planets, is attended by a system of satellites, the number of which is not yet certainly determined, and which, from the great remoteness of the Uranian system, cannot be seen at all except by the aid of the most perfect and powerful telescopes.

Sir W. Herschel, soon after discovering this planet, announced the existence of a system of six satellites attending it, having the periods and distances expressed in the following Table: —

Order.	Period.				Distances in semi-diameters of Uranus.
	D.	H.	M.	S.	
1	4	?			?
2	8	16	56	31·3	17·0
3	10	23	?		19 8 ?
4	13	11	7	12·6	22·8
5	38	2	?		45·5 ?
6	107	12	?		91·0 ?

Subsequent observations have confirmed this discovery so far only as relates to the four inner satellites. The fifth and sixth not having been re-observed, notwithstanding the vast improvement which has taken place in the construction of telescopes, and the greatly multiplied number and increased activity and zeal of observers, must be considered, to say the least, as problematical.

Of the four which have been re-observed, the second and fourth are by far the most conspicuous, and their distances and periods have been ascertained with all desirable accuracy and certainty. The first was re-observed by Mr. Lassell at Liverpool, and by M. Otto Struve at Dorpat, in 1847. The fourth was observed about the same time by Mr. Lassell.

2880. *Anomalous inclination of their orbits.* — Contrary to the law which prevails without any other exception in the motions of the bodies of the solar system, the orbits of the satellites of Uranus are inclined to the plane of the orbit of the planet, and therefore to that of the ecliptic, at an angle of 78° 58′, being little less than a right angle, and their motions

in these orbits are retrograde, that is to say, their longitudes as seen from Uranus continually *decrease*.

When the earth has such a position that the visual direction is at right angles to the line of nodes, the angle under the plane of the orbit and the visual line will be 78° 58′; and in certain positions of the planet they will be seen, as it were, *in plan*. Being nearly circular, the satellites will in such a position be visible revolving round the primary throughout their entire orbits, the projections not sensibly differing from circles.

2881. *Apparent motion and phases as seen from Uranus.* — The diurnal rotation and the direction of the axis of the planet being unascertained, the inclination of the orbits to the planet's equator is consequently unknown. It appears, however, that all the orbits have the same line of nodes, and are in a common plane or nearly so. Twice in each revolution of the planet this plane passes through the sun, when the satellites exhibit the same succession of phases to the planet as the moon presents to the earth, except so far as they are modified by the effects of the diurnal parallax, which are considerable, especially in the case of the nearer satellites.

Twice in each revolution of the planet, at epochs exactly intermediate between the former, the line of nodes being at right angles to the line joining the sun and planet, the plane of the satellites' orbits is nearly perpendicular to the same line. In this case the satellites, during their entire revolution, suffer no other change of phase than what may be produced by the diurnal parallax, and appear continually with the same phases as that which the moon presents at the quarters.

In the intermediate position of the planet a complicated variety of phases will be presented, which may be traced and analysed by giving due attention to the change of direction of the line of nodes of the satellites' orbits to the line joining the planet with the sun.

2882. *Mass and density of Uranus.* — Some uncertainty still attends the determination of the elements of these more distant and recently discovered planets. The mass and density of Uranus are only provisionally determined. The mass is assumed to be the 24,900th part of that of the sun, and the density the sixth of that of the earth.

IV. Neptune.

2883. *Discovery of Neptune.* — The discovery of this planet constitutes one of the most signal triumphs of mathematical science, and marks an era which must be for ever memorable in the history of physical investigation.

If the planets were subject only to the attraction of the sun, they would revolve in exact ellipses, of which the sun would be the common focus; but being also subject to the attraction of each other, which, though incomparably more feeble than that of the presiding central mass, produces sensible and measurable effects, consequent deviations from these elliptic paths, called PERTURBATIONS, take place, which will be more fully explained in a subsequent chapter. The masses and relative motions of the planets being known, these disturbances can be ascertained with such accuracy that the position of any known planet at any epoch, past or future, can be determined with the most surprising degree of precision.

If, therefore, it should be found, that the motion which a planet is observed to have is not in accordance with that which it ought to have, subject to the central attraction of the sun, and the disturbing actions of the surrounding planets, it must be inferred that some other disturbing attraction acts upon it, proceeding from an undiscovered cause, and, in this case, a problem novel in its form and data, and beset with difficulties which might well appear insuperable, is presented to the physical astronomer. If the solution of the problem, to determine the disturbances produced upon the orbit of a planet by another planet, whose mass and motions are known, be regarded as a stupendous achievement in physical and mathematical science, how much more formidable must not the converse question be regarded, in which the disturbances are given to find the planet!

Such was, nevertheless, the problem of which the discovery of Neptune has been the astonishing solution.

Although no exposition of the actual process by which this great intellectual achievement has been effected, could be comprehended without the possession of an amount of mathematical knowledge far exceeding that which is expected from the readers of treatises much less elementary than the present volume, we may not be altogether unsuccessful in attempting

to illustrate the principle on which an investigation, attended with so surprising a result, has been based, and even the method upon which it has been conducted, so as to strip the proceeding of much of that incomprehensible character which, in the view of the great mass of those who consider it without being able to follow the steps of the actual investigation, is generally attached to it, and to show at least the spirit of the reasoning by which the solution of the problem has been accomplished.

For this purpose, it will be necessary, *first*, to explain the nature and character of those disturbances which were observed and which could not be ascribed to the attraction of any of the known planets; and, *secondly*, to show in what manner an undiscovered planet revolving outside the known limits of the solar system could produce such effects.

2884. *Unexplained disturbances observed in the motion of Uranus.* — The planet Uranus, revolving at the extreme limits of the solar system, was the object in which were observed those disturbances which, not being the effects of the action of any of the known planets, raised the question of the possible existence of another planet exterior to it, which might produce them.

After the discovery of the planet by Sir W. Herschel, in 1781, its motions, being regularly observed, supplied the data by which its elliptic orbit was calculated, and the disturbances produced upon it by the masses of Jupiter and Saturn ascertained, the other planets of the system, by reason of their remoteness, and the comparative minuteness of their masses, not producing any sensible effects. Tables founded on these results were computed, and ephemerides constructed, in which the places at which the planet ought to be found from day to day for the future were duly registered.

The same kind of calculations which enabled the astronomer thus to predict the future places of the planet, would, as is evident, equally enable him to ascertain the places which had been occupied by the planet in times past. By thus examining, retrospectively, the apparent course of the planet over the firmament, and comparing its computed places at particular epochs with those of stars which had been observed, and which had subsequently disappeared, it was ascertained that several of these stars had in fact been Uranus itself, whose planetary character had not been recognised from its appearance, owing

to the imperfection of the telescopes then in use, nor from its apparent motion, owing to the observations not having been sufficiently continuous and multiplied.

In this way it was ascertained, that Uranus had been observed, and its position recorded as a fixed star, six times by Flamstead; viz., once in 1690, once in 1712, and four times in 1715; — once by Bradley in 1753, once by Mayer in 1756, and twelve times by Lemonnier between 1750 and 1771.

Now, although the observed positions of these objects, combined with their subsequent disappearance, left no doubt whatever of their identity with the planet, their observed places deviated sensibly from the places which the planet ought to have had according to the computations founded upon its motions after its discovery in 1781. If these deviations could have been shown to be irregular and governed by no law, they would be ascribed to errors of observation. If, on the other hand, they were found to follow a regular course of increase and decrease in determinate directions, they would be ascribed to the agency of some undiscovered disturbing cause, whose action at the epochs of the ancient observations was different from its action at more recent periods.

The ancient observations were, however, too limited in number and too discontinuous to demonstrate in a satisfactory manner the irregularity or the regularity of the deviation. Nevertheless, the circumstance raised much doubt and misgiving in the mind of Bouvard, by whom the tables of Uranus, based upon the modern observations, were constructed; and he stated that he would leave to futurity the decision of the question whether these deviations were due to errors of observation, or to an undiscovered disturbing agent. We shall presently be enabled to appreciate the sagacity of this reserve.

The motions of the planet continued to be assiduously observed, and were found to be in accordance with the tables for about fourteen years from the date of the discovery of the planet. About the year 1795, a slight discordance between the tabular and observed places began to be manifested, the latter being a little in advance of the former, so that the observed longitude L of the planet was greater than the tabular longitude L′. After this, from year to year, the advance of the observed upon the tabular place increased, so that the excess L—L′ of the observed above the tabular longitude was continually

augmented. This increase of L—L′ continued until 1822, when it became stationary, and afterwards began to decrease. This decrease continued until about 1830–31, when the deviation L—L′ disappeared, and the tabular and observed longitudes again agreed. This accordance, however, did not long prevail. The planet soon began to fall behind its tabular place, so that its observed longitude L, which before 1831 was greater than the tabular longitude L′, was now less; and the distance L′—L of the observed behind the tabular place increased from year to year, and still increases.

It appears, therefore, that in the deviations of the planet from its computed place, there was nothing irregular and nothing compatible with the supposition of any cause depending on the accidental errors of observation. The deviation, on the contrary, increased gradually in a certain direction to a certain point; and having attained a maximum, then began to decrease, which decrease still continues.

The phenomena must, therefore, be ascribed to the regular agency of some undiscovered disturbing cause.

2885. *A planet exterior to Uranus would produce a like effect.* — It is not difficult to demonstrate that deviations from its computed place, such as those described above, would be produced by a planet revolving in an orbit having the same or nearly the same plane as that of Uranus, which would be in heliocentric conjunction with that planet at the epoch at which its advance beyond its computed place attained its maximum.

Let A B C D E F, *fig.* 787., represent the arc of the orbit of Uranus described by the planet during the manifestation of the perturbations. Let N N′ represent the orbit of the supposed undiscovered planet in the same plane with the orbit of Uranus. Let *a*, *b*, *c*, *d*, *e*, and *f* be the positions of the latter when Uranus is at the points A, B, C, D, E, and F. It is, therefore, supposed that Uranus when at D is in heliocentric conjunction with the supposed planet, the latter being then at *d*.

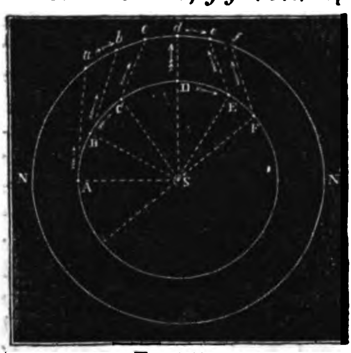


Fig. 787.

The directions of the orbital motions of the two planets are

indicated by the arrows beside their paths; and the directions of the disturbing forces* exercised by the supposed planet on Uranus are indicated by the arrows beside the lines joining that planet with Uranus.

Now, it will be quite evident that the attraction exerted by the supposed planet at *a* on Uranus at A tends to accelerate the latter. In like manner, the forces exerted by the supposed planet at *b* and *c* upon Uranus at B and C tend to accelerate it. But as Uranus approaches to D the direction of the disturbing force, being less and less inclined to that of the orbital motion, has a less and less accelerating influence, and on arriving at D, the disturbing force being in the direction D *d* at right angles to the orbital motion, all accelerating influence ceases.

After passing D the disturbing force is inclined *against* the motion, and instead of accelerating retards it; and as Uranus takes successively the positions E, F, &c. it is more and more inclined, and its retarding influence more and more increased, as will be evident if the directions of the retarding force and the orbital motion, as indicated by the arrows, be observed.

It is then apparent, that from A to D the disturbing force, accelerating the orbital motion, will transfer Uranus to a position in advance of that which it would otherwise have occupied; and after passing D, the disturbing force retarding the planet's motion will continually reduce this advance, until it bring back the planet to the place it would have occupied had no disturbing force acted; after which, the retardation being still continued, the planet will fall behind the place it would have had if no disturbing force had acted upon it.

Now it is evident that these are precisely the *kind* of disturbing forces which act upon Uranus; and it may, therefore, be inferred that the deviations of that planet from its computed place are the physical indications of the presence of a planet exterior to it, moving in an orbit whose plane either coincides with that of its own orbit or is inclined to it at a very small angle, and whose mass and distance are such as to give to its attraction the degree of intensity necessary to produce the alternate acceleration and retardation which have been observed.

* To simplify the explanation, the effect of the attraction of Uranus on the sun is omitted in this illustration. In the chapter on Perturbations the method of determining the exact direction of the disturbing force will be explained.

Since, however, the intensity of the disturbing force depends conjointly on the quantity of the disturbing mass and its distance, it is easy to perceive that the same disturbance may arise from different masses, provided that their distances are so varied as to compensate for their different weights or quantities of matter. A double mass at a fourfold distance will exert precisely the same attraction. The question, therefore, under this point of view, belongs to the class of indeterminate problems, and admits of an infinite number of solutions. In other words, an unlimited variety of different planets may be assigned exterior to the system which would cause disturbances observed in the motion of Uranus, so nearly similar to those observed as to be distinguishable from them only by observations more extended and elaborate than any to which that planet could possibly have been submitted since its discovery.

2886. *Researches of Messrs. Le Verrier and Adams.*—The idea of taking these departures of the observed from the computed place of Uranus as the data for the solution of the problem to ascertain the position and motion of the planet which could cause such deviations, occurred, nearly at the same time, to two astronomers, neither of whom at that time had attained either the age or the scientific standing which would have raised the expectations of achieving the most astonishing discovery of modern times.

M. Le Verrier, in Paris, and Mr. J. C. Adams, Fellow and Assistant Tutor of St. John's College, Cambridge, engaged in the investigation, each without the knowledge of what the other was doing, and believing that he stood alone in his adventurous and, as would then have appeared, hopeless attempt. Nevertheless, both not only solved the problem, but did so with a completeness that filled the world with astonishment and admiration, in which none more ardently shared than those who, from their own attainments, were best qualified to appreciate the difficulties of the question.

The question, as has been observed, belonged to the class of indeterminate problems. An infinite number of different planets might be assigned which would be equally capable of producing the observed disturbances. The solution, therefore, might be theoretically correct, but practically unsuccessful. To strip the question as far as possible of this character, certain conditions were assumed, the existence of which might be regarded as in the highest degree probable. Thus, it was assumed that the dis-

turbing planet's orbit was in or nearly in the plane of that of Uranus, and therefore in that of the ecliptic; that its motion in this orbit was in the same direction as that of all the other planets of the system, that is, according to the order of the signs; that the orbit was an ellipse of very small eccentricity; and, in fine, that its mean distance from the sun was, in accordance with the general progression of distances noticed by Bode, nearly double the mean distance of Uranus. This last condition, combined with the harmonic law, gave the inquirer the advantage of the knowledge of the period, and therefore of the mean heliocentric motion.

Assuming all these conditions as provisional data, the problem was reduced to the determination, at least as a first approximation, of the mass of the planet and its place in its orbit at a given epoch, such as would be capable of producing the observed alternate acceleration and retardation of Uranus.

The determination of the heliocentric place of the planet at a given epoch would have been materially facilitated if the exact time at which the amount of the advance ($L - L'$) of the observed upon the tabular place of the planet had attained its maximum were known; but this, unfortunately, did not admit of being ascertained with the necessary precision. When a varying quantity attains its maximum state, and, after increasing, begins to diminish, it is stationary for a short interval; and it is always a matter of difficulty, and often of much uncertainty, to determine the exact moment at which the increase ceases and the decrease commences. Although, therefore, the heliocentric place of the disturbing planet could be nearly assigned about 1822, it could not be determined with the desired precision.

Assuming, however, as nearly as was practicable, the longitude of Uranus at the moment of heliocentric conjunction with the disturbing planet, this, combined with the mean motion of the sought planet, inferred from its period, would give a rough approximation to its place for any given time.

2887. *Elements of the sought planet assigned by these geometers.*—Rough approximations were not, however, what MM. Le Verrier and Adams sought. They aimed at more exact results; and, after investigations involving all the resources and exhausting all the vast powers of analysis, these eminent geometers arrived at the following elements of the undiscovered planet:—

	Le Verrier.	Adams.
Epoch of the elements	1 Jan. 1847.	1 Jan. 1846.
Mean longitude at the epoch	318° 47′·4	[illegible]° 2′
Mean distance of planet from sun	36·1539	37·2474
Eccentricity of the orbit	0·107610	0·120615
Longitude of perihelion	284° 45′·8	299° 11′
Mass (sun = 1)	0·00010727	0·00015003

2888. *Its actual discovery by Dr. Galle of Berlin.*—On the 23rd of September, 1846, Dr. Galle, one of the astronomers of the Royal Observatory at Berlin, received a letter from M. Le Verrier, announcing to him the principal results of his calculations, informing him that the longitude of the sought planet must then be 326°, and requesting him to look for it. Dr. Galle, assisted by Professor Encké, accordingly did "look for it," and found it that very night. It appeared as a star of the 8th magnitude, having the longitude of 326° 52′, and consequently only 52′ from the place assigned by M. Le Verrier. The calculations of Mr. Adams, reduced to the same date, gave for its place 329° 19′, being 2° 27′ from the place where it was actually found.

2889. *Its predicted and observed places in near proximity.*—To illustrate the relative proximity of these remarkable predictions to the actual observed place, let the arc of the ecliptic, from long. 323° to long. 330°, be represented in *fig.* 788. The place assigned by M. Le Verrier for the sought planet is indicated by the small circle at L, that assigned by Mr. Adams by the small circle at A, and the place at which it was actually found by the dot at N. The distances of L and A from N may be appreciated by the circle which is described around the dot N, and which represents the apparent disk of the moon.

Fig. 788.

The distance of the observed place of the planet from the place predicted by M. Le Verrier was less than two diameters, and from that predicted by Mr. Adams less than five diameters, of the lunar disk.

2890. *Corrected elements of the planet's orbit.*—In obtaining the elements given above, Mr. Adams based his calculations on the observations of Uranus made up to 1840, while the calcu-

lations of M. Le Verrier were founded on observations continued to 1845. On subsequently taking into computation the five years ending 1845, Mr. Adams concluded that the mean distance of the sought planet would be more exactly taken at 33·33.

After the planet had been actually discovered, and observations of sufficient continuance were made upon it, the following proved to be its more exact elements: —

	Greenwich.
Epoch of the elements	1 Jan. 1847, M. Noon.
Mean longitude of epoch	328° 32′ 44″·2.
Mean distance from sun	30·0367.
Eccentricity of orbit	0·00871946.
Longitude of perihelion	47° 12′ 6″·50.
——— ascending node	130° 4′ 20″·81.
Inclination of orbit	1° 46′ 58″·97.
Periodic time	164·6181 years.
Mean annual motion	2°·18688.

2891. ***Discrepancies between the actual and predicted elements explained.*** — Now it will not fail to strike every one who devotes the least attention to this interesting question, that considerable discrepancies exist, not only between the elements presented in the two proposed solutions of this problem, but between the actual elements of the discovered planet and both of these solutions. There were not wanting some who, viewing these discordances, did not hesitate to declare that the discovery of the planet was the result of chance, and not, as was claimed, of mathematical reasoning, since, in fact, the planet discovered was not identical with either of the two planets predicted.

To draw such a conclusion from such premises, however, betrays a total misapprehension of the nature and conditions of the problem. If the problem had been determinate, and, consequently, one which admits of but one solution, then it must have been inferred, either that some error had been committed in the calculations which caused the discordance between the observed and computed elements, or that the discovered planet was not that which was sought, and which was the physical cause of the observed disturbances of Uranus. But the problem, as has been already explained, being more or less indeterminate, admits of more than one, — nay, of an indefinite number of different solutions, so that many different planets might be assigned which would equally produce the disturb-

ances which had been observed; and this being so, the discordance between the two sets of predicted elements, and between both of them and the actual elements, are nothing more than might have been anticipated, and which, except by a chance against which the probabilities were millions to one, were, in fact, inevitable.

So far as depended on reasoning, the prediction was verified; so far as depended on chance, it failed. Two planets were assigned, both of which lay within the limits which fulfilled the conditions of the problem. Both, however, differed from the true planet in particulars which did not affect the conditions of the problem. All three were circumscribed within those limits, and subject to such conditions as would make them produce those deviations or disturbances which were observed in the motions of Uranus, and which formed the immediate subject of the problem.

2892. *Comparison of the effects of the real and predicted planets.*—It may be satisfactory to render this still more clear,

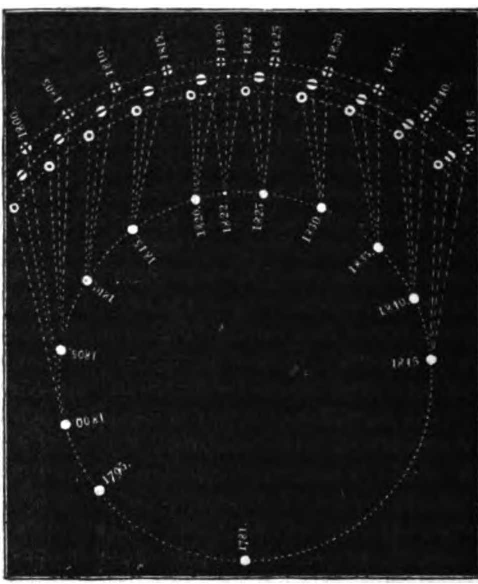


Fig. 789.

by exhibiting in immediate juxtaposition the motions of the hypothetical planets of MM. Le Verrier and Adams and the planet actually discovered, so as to make it apparent that any one of the three, under the supposed conditions, would produce the observed disturbances. We have accordingly attempted this in *fig.* 789., where the orbits of Uranus, of Neptune, and of the planets assigned by MM. Le Verrier and Adams are laid down, with the positions of the planets respectively in them for every fifth year, from 1800 to 1845 inclusively. This plan is, of course, only roughly made; but it is sufficiently exact for the purposes of the present illustration. The places of Uranus are marked by ○, those of Neptune by ⊙, those of M. Le Verrier's planet by ⊖, and those of Mr. Adams's planet by ⊕.

It will be observed that the distances of the two planets assigned by MM. Le Verrier and Adams, as laid down in the diagram, differ less from the distance of the planet Neptune than the mean distances given in their elements differ from the mean distance of Neptune. This is explained by the eccentricities of the orbit, which, in the elements of both astronomers, are considerable, being nearly an eighth in one and a ninth in the other, and by the positions of the supposed planets in their respective orbits.

If the masses of the three planets were equal, it is clear that the attraction with which Le Verrier's planet would act upon Uranus, would be less than that of the true planet, and that of Adams's planet still more so, each being less in the same ratio as the square of its distance from Uranus is greater than that of Neptune. But if the planets are so adjusted that what is lost by distance is gained by the greater masses, this will be equalised, and the supposed planet will exert the same disturbing force as the actual planet, so far as relates to the effects of variation of distance. It is true that, throughout the arcs of the orbits over which the observations extend, the distances of the three planets in simultaneous positions are not every where in exactly the same ratio, while their masses must necessarily be so; and, therefore, the relative masses, which would produce perfect compensation in one position, would not do so in others. This cause of discrepancy would operate, however, under the actual conditions of the problem, in a degree altogether inconsiderable, if not insensible.

But another cause of difference in the disturbing action of the

real and supposed planets would arise from the fact that the directions of the disturbing forces of all the three planets are different, as will be apparent on inspecting the figure, in which the degree of divergence of these forces at each position of the planets is indicated; but it will be also apparent that this divergence is so very inconsiderable that its effect must be quite insensible in all positions in which Uranus can be seriously affected. Thus, from 1800 to 1815, the divergence is very small. It increases from 1815 to 1835; but it is precisely here, near the epoch of heliocentric conjunction, which took place in 1822, that all the three planets cease to have any direct effect in accelerating the motion of Uranus. When the latter planet passes this point sufficiently to be sensibly retarded by the disturbing action, as is the case after 1835, the divergence again becomes inconsiderable.

From these considerations it will therefore be understood, that the disturbances of the motion of Uranus, so far as these were ascertained by observation, would be produced without sensible difference, either by the actual planet which has been discovered, or by either of the planets assigned by MM. Le Verrier and Adams, or even by an indefinite number of others which might be assigned, either within the path of Neptune, or between it and that of Adams's planet, or, in fine, beyond this — within certain assignable limits.

2893. *No part of the merit of this discovery ascribable to chance.* — That the planets assigned by MM. Le Verrier and Adams are not identical with the planet to the discovery of which their researches have conducted practical observers is, therefore, true; but it is also true that, if they or either of them had been identical with it, such excessive amount of agreement would have been purely accidental, and not at all the result of the sagacity of the mathematician. All that human sagacity could do with the data presented by observation was done. Among an indefinite number of *possible* planets capable of producing the disturbing action, two were assigned, both of which were, for all the purposes of the inquiry, so nearly coincident with the real planet as inevitably and immediately to lead to its discovery.

2894. *Period.* — After a complete revolution of the earth, Neptune is found to advance in its course no more than 2°·187, and consequently its period is—

$$P = \frac{360}{2{\cdot}187} = 164{\cdot}6 \text{ years,}$$

or, more exactly, 164·618 years.

2895. *Distance.* — Its mean distance R, therefore, may be determined by the harmonic law:

$$R^3 = (164{\cdot}6)^2 = 27093 = (30{\cdot}04)^3.$$

2896. *Relative orbits and distances of Neptune and the earth.* — It appears, then, in fine, that the system possesses another member still more remote from the common centre of light, heat, and attraction. In *fig.* 790. the earth's orbit is represented at EE‴; and a part of that of Neptune, on the same scale, is represented at N. The actual distance of N from S is thirty times that of E from S.

Fig. 790.

The mean distance of Neptune from the sun is, therefore,

2,850,000,000 miles.

2897. *Apparent and real diameter.* — The apparent diameter of the planet, seen when in opposition, is about 2″8. Its distance from the earth being, then,

$$2850 - 95 = 2755 \text{ mill. miles,}$$

and the linear value of 1″ at this distance being

$$\frac{2755000000}{206265} = 13{,}313 \text{ miles,}$$

the actual diameter of the planet will be

$$13313 \times 2{\cdot}8 = 37276 \text{ miles.}$$

The diameter of the planet is, therefore, a little greater than that of Uranus, about half that of Saturn, and about four and a half times that of the earth.

According to Mr. Hind, the apparent diameter is only 2·6, and the real diameter 31,000 miles, numbers which, he says, are deduced from careful measurements with some of the most powerful European telescopes.

2898. *Satellite of Neptune.*—A satellite of this planet was discovered by Mr. Lassell in October, 1846, and was afterwards observed by other astronomers both in Europe and the United States. The first observations then made raised some suspicions as to the presence of another satellite as well as of a ring analogous to that of Saturn. Notwithstanding the numerous observers, and the powerful instruments which have been directed to the planet since the date of these observations, nothing has been detected which has had any tendency to confirm these suspicions.

The existence of the satellite first seen by Mr. Lassell has, however, not only been fully established, but its motion, and the elements of its orbit, have been ascertained, first by the observations of M. O. Struve in Sept. and Dec. 1847, and later and more fully by those of his late relative M. Auguste Struve, in 1848-9.

From these observations it appears that the distance of the satellite from the planet at its greatest elongation subtends an angle of 18″ at the sun; and since the diameter of the planet subtends an angle of 2·8 at the same distance, it follows, therefore, that the distance of the satellite from the centre of the planet is equal to fourteen semidiameters of the latter.

The mean daily angular motion of the satellite round the centre of the planet is, according to the observations of Struve, 61°·2625, and consequently the period of the satellite is

$$\frac{360}{61{\cdot}2625} = 5{\cdot}8763 \text{ days},$$

or $5^{d.}\ 21^{h.}\ 1{\cdot}8^{m.}$, a result which is subject to an error not exceeding 5 minutes.

If the semidiameter of the planet be 18,750 miles, the actual distance of the satellite is

$$18{,}750 \times 12 = 225{,}000 \text{ miles},$$

being a little less than the distance of the moon from the earth's centre.

2899. *Mass and density.*—This discovery of a satellite has supplied the means of determining the mass, and therefore the density, of the planet. M. Struve, calculating by the principles already explained, has found that the mass of Neptune is the 14,446th part of the mass of the sun; and since its diameter is

about the 20th, and its volume the 8000th, part of that of the sun, its density will be about five-ninths that of the sun, and about the seventh part of the density of the earth.

Other estimates make the mass less. According to Professor Bond it is the 19,400th, and according to Mr. Hind the 17,900th, of the mass of the sun.

2900. *Apparent magnitude of the sun at Neptune.*—The apparent diameter of the sun, as seen from Neptune, being 30 times less than from the earth, is,

$$\frac{1800''}{30} = 60''.$$

The sun, therefore, appears of the same magnitude as Venus seen as a morning or evening star.

The relative apparent magnitudes are exhibited in *fig.* 791. at E and N.

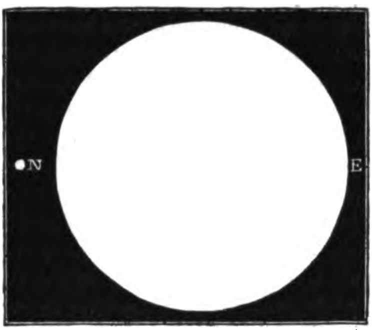


Fig. 791.

It would, however, be a great mistake to infer that the light of the sun at Neptune approaches in any degree to the faintness of that of Venus at the earth. If Venus, when that planet appears as a morning or evening star, with the apparent diameter of 60″, had a full disk (instead of one halved or nearly so, like the moon at the quarters), and if the actual intensity of light on its surface were equal to that on the surface of the sun, the light of the planet would be exactly that of the sun at Neptune. But the intensity of the light which falls on Venus is less than the intensity of the light on the sun's surface in the ratio of the square of Venus' distance to that of the sun's semi-

diameter, upon the supposition that the light is propagated according to the same law as if it issued from the sun's centre; that is, as the square of 37 millions to the square of half a million nearly, or as $37^2 : \frac{1}{4}$, that is, as 5476 to 1. If, therefore, the surface of Venus reflected (which it does not) all the light incident upon it, its apparent light at the earth (considering that little more than half its illuminated surface is seen) is about 11,000 times less than the light of the sun at Neptune.

Small, therefore, as is the apparent magnitude of the sun at Neptune, the intensity of its daylight is probably not less than that which would be produced by about 20,000 stars shining at once in the firmament, each being equal in splendour to Venus when that planet is brightest.

In addition to these considerations, it must not be forgotten that all such estimates of the comparative efficiency of the illuminating and heating power of the sun is based upon the supposition that his light is received under like physical conditions; and that many conceivable modifications in the physical state of the body or medium on or into which the light falls, and in the structure of the visual organs which it affects, may render light of an extremely feeble intensity as efficient as much stronger light is found to be under other conditions.

2901. *Suspected ring of Neptune.* — Messrs. Lassell and Challis have at times imagined that indications of some such appendage as a ring, seen nearly edgewise, were perceptible upon the disk of Neptune. These conjectures have not yet received any confirmation. When the declination of the planet will have so far increased as to present the ring, if such an appendage be really attached to the planet, at a less oblique angle to the visual ray, the question will probably be decided.

CHAP. XVI.

ECLIPSES, TRANSITS, AND OCCULTATIONS.

2902. *Interposition of celestial objects.*—The objects which in such countless numbers are scattered over the firmament being at distances and in positions infinitely various, and many of them being in motion, so that the directions of lines drawn from one to another are constantly varying, it must occasionally happen that three will come into the same line, or nearly so. Such a contingency produces a class of occasional astronomical phenomena which are invested with a high popular as well as a profound scientific interest. The rareness with which some of them are presented, their sudden and, to the vulgar mass, unexpected appearance, and the singular phenomena which often attend them, strike the popular mind with awe and terror. To the astronomer, geographer, and navigator, they subserve important uses, among which the determination of terrestrial longitudes, the more exact estimation of the sun's distance from the earth (which is the standard and modulus of all distances in the celestial spaces), and, in fine, the discovery of the mobility of light, and the measure of its velocity, hold foremost places.

When one of the extremes of the series of the three bodies which thus assume a common direction is the sun, the intermediate body deprives the other extreme body, either wholly or partially, of the illumination which it habitually receives. When one of the extremes is the earth, the intermediate body intercepts, wholly or partially, the other extreme body from the view of observers situate at places on the earth which are in the common line of direction, and the intermediate body is seen to pass across the other extreme body as it enters upon and leaves the common line of direction. The phenomena resulting from such contingencies of position and direction are variously denominated ECLIPSES, TRANSITS, and OCCULTATIONS, according to the relative apparent magnitudes of the interposing and obscured bodies, and according to the circumstances which attend them.

2903. *General conditions which determine the phenomena of interposition when one of the extreme objects is the earth.*—If the interposing and intercepted objects have disks of sensible magnitude, the effects attending their interposition will depend on the magnitude of the diameters of their disks and the apparent distance between their centres.

Let D express the apparent distance between the centres of the two disks. Let r be the semi-diameter of the nearer, and r' that of the more distant disk.

2904. *Condition of no interposition.*—If D be greater than $r+r'$, as represented at A, *fig.* 792., the disks must be entirely outside each other, and consequently no interposition can take place. The nearest points of the edges of the disks are, in this case, at a distance equal to the difference between D. and $r+r'$, that is, D$-(r+r')$.

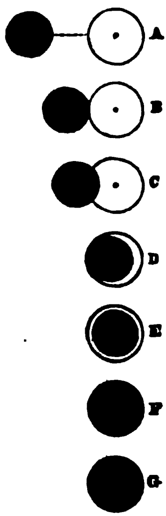


Fig. 792.

External contact.—If D$=r+r'$, as at B, the disks will touch without interposition. This is called the position of EXTERNAL CONTACT.

2905. *Partial Interposition.*—If D be less that $r+r'$, the nearer disk will be partially interposed, as at C. In this case, the greatest breadth of the obscured part of the more remote disk is $(r+r')-$D. It is evident that the less the distance D is, the greater will be this breadth, and the greater the part obscured.

2906. *Internal contact of interposing disk.*—If the interposing disk be less than the more distant, it will reduce the latter to a crescent, the points of the horns of which meet, as represented at D, when D$=r'-r$, that is, when the distance between the centres is equal to the difference of the apparent semi-diameters.

2907. *Centrical interposition of lesser disk.*—If in the same case the centres coincide, as at E, the nearer disk, covering all the central portion of the more distant, will leave uncovered around it a regular ring or annulus of visible surface, the breadth of which will be the difference $r-r'$ of the semi-diameters.

2908. *Complete interposition.*—If the nearer disk be greater

than the more remote, and the distance D between the centres be not greater than $r-r'$, the difference of the semi-diameters, the more remote disk will be completely covered, and will continue so until the centres separate to a greater distance than $r-r'$, as represented at F and G.

I. Solar Eclipses.

2909. The case of the sun and moon presents all these various appearances. The disks, though nearly equal, are each subject to a variation of magnitude confined within certain narrow limits, as has been already explained; and, in consequence, the disk of the moon is sometimes a little greater, and sometimes a little less, than that of the sun. Their centres move, as has been explained, in two apparent circles on the firmament; that of the sun in the ecliptic, and that of the moon in a circle inclined to the ecliptic at a small angle of about 5°. These circles intersect at two opposite points of the firmament, called the moon's nodes (2473). In consequence of the very small obliquity of the moon's orbit to the ecliptic, the distance between these paths, even at a considerable distance at either side of the node, is necessarily small. Now, since the centres of the disks of the sun and moon must each of them pass once in each revolution through each node, it will necessarily happen from time to time that they will be both at the same moment either at the node itself, or at some points of their respective paths so near it, that their apparent distance asunder will be less than the sum of their apparent semi-diameters, and either total or partial interposition must take place, according to the relative magnitudes of their disks, and to the distance between the points of their respective paths at which their centres are simultaneously found.

2910. *Partial solar eclipse.*—If the apparent distance D between their centres be less than the sum $(r+r')$, but greater than the difference $(r-r')$ of their apparent semi-diameters, a partial interposition will take place (2905). The greatest breadth of the obscured parts of the solar disk will in this case be equal to the difference between the sum of the apparent semi-diameters and the distance between the centres of the two disks, that is, $(r+r') - $D.

2911. *Magnitude of eclipses expressed by digits.*—If the

apparent diameter of the obscured object be supposed to be divided into twelve equal parts, each of these parts in reference to eclipses is called a DIGIT, and the magnitude of an eclipse is expressed by the number of digits contained in the greatest breadth of the obscured part of the disk. Thus, the magnitude of the eclipse will be found by dividing $r + r' - D$ by $\frac{2r}{12}$ or by $\frac{r}{6}$.

2912. *Total solar eclipse.*—To produce a total solar eclipse, it is necessary, 1st, that the apparent diameter of the moon should be equal to or greater than that of the sun, and, 2dly, that the apparent places of their centres should approach each other within a distance not greater than $r' - r$, the difference of their apparent semi-diameters. When these conditions are fulfilled, and so long as they continue to be fulfilled, the eclipse will be total (2908).

The greatest value of the apparent semi-diameter of the moon being 1006″, and the least value of that of the sun being 945″, we shall have

$$r' - r = 61''.$$

The greatest possible duration, therefore, of a total solar eclipse will be the time necessary for the centre of the moon to gain upon that of the sun $61'' \times 2 = 122''$. But since the mean synodic motion of the moon is at the rate of 30″ per minute, it follows that the duration of a total solar eclipse can never exceed four minutes.

2913. *Annular eclipses.*—When the apparent diameter of the moon is less than that of the sun, its disk will not cover that of the sun, even when concentrical with it. In this case, a ring of light would be apparent round the dark disk of the moon, the breadth of which would be equal to the difference of the apparent semi-diameters, as represented at E, *fig.* 792. When the disks are not absolutely concentrical, the distance between their centres being, however, less than the difference of their apparent semi-diameters, the dark disk of the moon will still be within that of the sun, and will appear surrounded by a luminous annulus, but in this case the ring will vary in breadth, the thinnest part being at the point nearest to the moon's centre; and when the distance between the centres is reduced to exact equality with the difference of the apparent semi-diameters, the

ring becomes a very thin crescent, the points of the horns of which unite, as represented at D, *fig.* 792.

The greatest breadth of the crescent will be in this case equal to the difference of the apparent diameters of the sun and moon.

The greatest apparent semi-diameter of the sun being 16′ 18″, and the least apparent semi-diameter of the moon being 14′ 44″, the greatest possible breadth of the annulus when the eclipse is centrical will be

$$r - r' = 16' 18'' - 14' 44'' = 1' 34'' = 94'',$$

which is about the 20th part of the mean apparent diameter of the sun.

The greatest interval during which the eclipse can continue annular is the time necessary for the centre of the moon to move synodically over $94'' \times 2 = 188''$, and, since the mean synodic motion is at the rate of 30″ per minute, this interval will be about

$$\frac{188}{30} = 6{\cdot}26 \text{ minutes},$$

or about six minutes and a quarter.

2914. *Solar eclipses can only occur at or near the epoch of new moons.*—This is evident, because the condition which limits the apparent distance between the centres of the disks to the sum of the apparent semi-diameters, involves the consequence that this distance cannot much exceed 30′, and as the difference of longitudes must be still less than this, it follows that the eclipse can only take place within less than half a degree in apparent distance, and within less than two hours of the epoch of conjunction.

2915. *Effects of parallax.*—Since the visual directions of the centres of the disks of the sun and moon vary more or less with the position of the observer upon the earth's surface, the conditions which determine the occurrence of an eclipse, and if it occur, those which determine its character and magnitude, are necessarily different in different parts of the earth. While in some places none of the conditions are fulfilled, and no eclipse occurs, in others an eclipse is witnessed which varies from one place to another in its magnitude, and in some may be total, while it is partial in others.

If the change of position of the observer upon the earth's

surface affected the visual directions of the centres of the two disks equally, which would be the case if they were equally distant, or nearly so, no change in the apparent distance between them would be produced, and in that case, the eclipse would have the same appearance exactly to all observers in every part of the earth. But the sun being about 400 times more distant than the moon, the visual direction of the centre of its disk is affected by any difference of position of the observers, to an extent 400 times less than that of the moon's centre.

Let S, E, and M, *fig.* 793., represent sections of the sun, earth, and moon, made by the plane which passes through their centres. Let a line P *m s* be drawn, touching the sun and moon, but so that they shall lie on opposite sides of it. It is evident that to an observer at P, the dark disk of the moon would touch that of the sun externally, for the apparent distance between the centres would be measured by the angle S P M, which is equal to the sum S P *s*, the apparent semi-diameter of the sun, and M *p m* that of the moon.

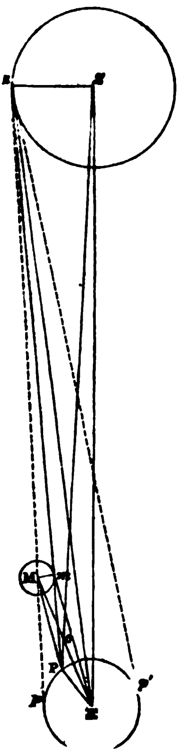


Fig. 793.

From the point *s* let lines be supposed to be drawn, touching the earth at *p* and *p′*. It is evident that, to an observer situate between P and *p′*, the apparent distance of the centres of the moon and sun would be greater than the sum of their apparent semi-diameters, and they would therefore be separated at the nearest points of their disks by a space equal to the excess of this distance above the sum of the apparent semi-diameters.

Adopting the signs already used, let *r* express the apparent semi-diameter of the sun, *r′* that of the moon, and D the apparent distance between their centres, we shall have D greater than $r + r'$ for every point from P to *p′*, and the excess will increase continually from P to *p′*.

On the other hand, for every point between P and *p*, D will be less than $r + r'$, and the sun will be eclipsed, the magnitude of the eclipse augmenting gradually from P to *p*.

The phenomena varying therefore indefinitely with the position of the observer upon the earth, it is necessary, in order to render their prediction practicable, to select a fixed position for which they may be calculated, formulæ being established, and tables prepared, by which the difference between the appearances there and at any proposed place may be computed. The fixed point selected for this purpose is the centre E of the earth.

The angular distance between the centres of the disks of the sun and moon, as seen from any place, such as P for example, is called their *apparent* distance at that place, and their angular distance, as seen from the centre E of the earth, is called their *true* distance. Thus, S P M is the apparent distance between the centres at P, and S E M is their true distance. It will be easy to show the relation which exists between these two distances.

By the principles of elementary geometry we have

$$\mathrm{SEM} = \mathrm{S}o\mathrm{M} - \mathrm{PSE}, \qquad \mathrm{S}o\mathrm{M} = \mathrm{P}o\mathrm{E} = \mathrm{MPS} + \mathrm{PME},$$

and consequently

$$\mathrm{SEM} = \mathrm{MPS} + \mathrm{PME} - \mathrm{PSE}.$$

But the angle P M E is the diurnal parallax of the moon, and P S E that of the sun, estimated in the plane of the figure. If these be expressed by ω' and ω respectively, and the apparent and true distances between the centres by D and D′ respectively, the above relation will be

$$\mathrm{D}' = \mathrm{D} + \omega' - \omega,$$

and consequently

$$\mathrm{D}' - \mathrm{D} = \omega' - \omega;$$

that is to say, the true distance exceeds the apparent by as much as the parallax of the moon exceeds that of the sun.

At the place P, from which the disks appear in external contact,

$$\mathrm{D} = r + r',$$

and therefore

$$\mathrm{D}' - (r + r') = \omega' - \omega';$$

consequently, when external contact takes place, we have

$$\mathrm{D}' = r + r' + \omega' - \omega$$

that is, the true distance between the centres is equal to the sum of the apparent diameters added to the difference of the parallaxes.

To simplify the explanation, we have here supposed the place of observation to be in the plane which passes through the centres of the sun, moon and earth, or, what is the same, the centres of the disks of the sun and moon, to be in the same vertical at the time of the observation. In the actual calculations necessary to supply an exact prediction of the beginning, middle, the end, and the magnitude of a solar eclipse, many particulars must be taken into account, which are not adapted to a work such as the present, but which present no other difficulty than such as attends elaborate arithmetical computation.

2916. *Shadow produced by an opaque globe.*—Connected with the phenomena of eclipses and transits are certain properties of shadows.

When a luminous body, radiating light in all directions around it, throws these rays upon an opaque body, that body prevents a portion of the rays from penetrating into the space behind it. That part of the space from which the light is thus excluded by the interposition of the opaque body, is called in astronomy the SHADOW of that body.

The shape, magnitude, and extent, of the shadow of an opaque body will depend partly on the shape and magnitude of the opaque body itself, and partly on that of the body from which the light proceeds.

2917. *Method of determining the form and dimensions of the shadow.*—In the cases which are actually presented in astronomy, the luminous body being the sun, and the opaque body a planet or satellite, both are globes, and the former of much greater dimensions than the latter. It is easy to show that in such case the shadow will be a cone, projected to a certain distance behind the opaque body. The length of this cone, and the angle formed at its vertex, may be computed, when the real diameters of the sun and the body which forms the shadow, and the distance of the one from the other, are known.

Let *b b′* and *a a′*, *fig.* 794., represent a section of the sun and the opaque body. Suppose the lines *b a* and *b′ a′* drawn touching these. Let them be continued until they meet at *f*. If similar lines be supposed to be drawn through all points surrounding both globes, they will include a cone the diameter of whose base is *b b′*, whose sides are *b f* and *b′ f*, and whose vertex is *f*. It will be evident that the sun's rays will be excluded from all that part of the cone which is between *a a′* and the vertex *f*. This part of

the cone, therefore, having the section of the opaque body at $a a'$ for its base, and the point f for its vertex, is the shadow.

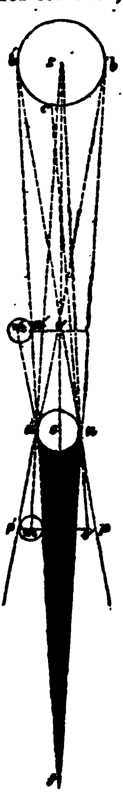


Fig. 794.

To ascertain the length l of the shadow, let r and r' express the semi-diameters of the sun, and the body $a a'$ respectively, and let d express the distance $b\ a$ between them. We shall then have, by the principles of elementary geometry,

$$r : r' :: l + d : l,$$

and consequently,

$$r \times l = r' \times l + r' \times d,$$

and therefore,

$$l \times (r - r') = r' \times d, \quad l = \frac{r' \times d}{r - r'},$$

that is, the length of the shadow is found by multiplying the distance from the sun by the semi-diameter of the body which forms the shadow, and dividing the product by the difference between the semi-diameters.

To determine the semi-angle $a f e$ of the cone we have

$$a f e = 206265'' \times \frac{r'}{l}.$$

2918. ***Method of determining the limits of the penumbra.***—If tangents be drawn transversely, such as $b\ a'$ and $b'\ a$, and be continued beyond the points a and a', the sun's rays will be partially excluded from the space included between $p\ a$ and $f a$. Any point on the line $a\ p$ will receive light from all points of the sun's disk. If the point thus illuminated be moved gradually from p towards o, it will receive less and less of the sun's light, since the globe $a\ a'$ will be more and more interposed between it and the sun. Thus, a point placed at o' receives light only from those points of the sun which lie between c and b, the rays proceeding from all points between b' and c being intercepted by $a\ a'$. As the point o' is moved towards o, the corresponding point c moves towards b, so that the portion of the sun from which it receives light constantly decreases until it arrives at the boundary $a f$ of the shadow, where all the rays are intercepted.

The light being thus partially intercepted from the space

bounded by the lines *a p* and *a f*, this space is called the PENUMBRA.

The angle *p a f*, which measures the penumbra, is equal to the visual angle *b a b*, subtended by the sun at the object which forms the shadow.

2919. *Total and partial solar eclipses explained by the lunar shadow.*—The moon projects behind it a conical shadow, the dimensions of which can be ascertained by the methods explained above. If when the moon comes between the sun and the earth, which it must do near conjunction, if it be not far removed from the node of its orbit, this shadow will be projected on a part of the hemisphere of the earth which is turned to the sun, provided its length be greater than the moon's distance, as represented in *fig.* 795. In this case, the shadow will move over certain points of the surface of the earth lying around the point to which its axis is directed. The light of the sun being

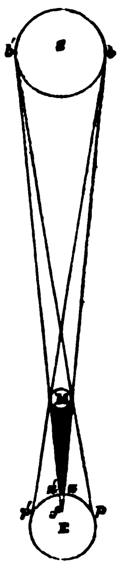


Fig. 795.

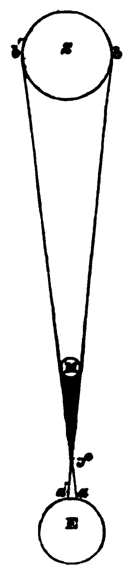


Fig. 796.

altogether intercepted within the limits of the shadow, a total eclipse will take place, the duration of which will be determined

by the limits and movement of the shadow thus projected, which is in effect the intersection of the conical shadow of the moon and the earth's surface.

To those parts of the earth which are outside the limits $a a'$ of the shadow, but within those $p p'$ of the penumbra, a partial eclipse will be exhibited, the magnitude of which will be so much the greater the nearer the place is to the axis of the shadow. All such parts will be more faintly illuminated in proportion to the extent of the sun's disk which is obscured.

2920. *Annular eclipses explained by shadow.*—If the length of the shadow be less than the moon's distance from the earth, the vertex not reaching to the earth, no part of the earth's surface can be immersed in the shadow. In that case, a centrical annular eclipse will be exhibited at those points of the earth's surface to which the axis of the shadow is directed. This case is represented in *fig.* 796., where f represents the vertex of the moon's shadow. At all places within the circle upon the earth, of which $a a'$ is the diameter, there will be a annular eclipse, and at the centre of the circle the eclipse will be centrical, the annulus being of uniform breadth. Outside this circle, so far as the penumbra extends, the eclipse will be partial, its magnitude decreasing as the distance of the place from the centre of the circle increases, until at the limit of the penumbra the phenomenon ceases to be exhibited.

2921. *Possibility of annular eclipses proved.*—To establish the possibility of an annular eclipse, and to show the relative positions of the earth and moon, in their respective orbits, when such a phenomenon takes place, it must be considered that it is necessary that the length of the moon's shadow be less than the moon's distance from the earth. By substituting for r and r', in the general formula for the value of l (2917) the actual values of the semi-diameters of the sun and moon, we find

$$\frac{r'}{r-r'}=\frac{1076}{441000}=\frac{1}{440}.$$

The exteme limits of the sun's distance d being

$$d''=96,600000,$$
$$d'=93,400000,$$

the greatest and least values of l, the length of the moon's shadow will be

$$l'' = 96{,}600000 \times \frac{1}{440} = 219545 \text{ Miles},$$

$$l' = 93{,}400000 \times \frac{1}{440} = 212272.$$

But the extreme distances of the moon from the centre of the earth are,

		Miles.
greatest distance	- -	251760,
least distance	- - -	221290,

and, therefore, the extreme distances from the surface are

$$\text{greatest distance} = 251760 \text{ Miles} - 3963 \text{ Miles} = 247797 \text{ Miles},$$
$$\text{least distance} = 221290 - 3963 = 217327.$$

Since, therefore, the length of the moon's shadow when greatest, exceeds the moon's distance from the surface of the earth when least, the surface may intersect the shadow at a point within its vertex; and since the length of the shadow when least, is less than the moon's distance from the surface of the earth when greatest, the vertex of the shadow may pass between the moon and the earth without touching the surface. In the former case, there will be a total solar eclipse at all places within the section of the shadow made by the earth's surface, and in the latter, there will be an annular eclipse at all places within the section of the cone afa', *fig.* 796., formed by the continuation of the cone of the shadow beyond its vertex f.

The extreme distance of the vertex of the cone f, *fig.* 795., within the surface of the earth, in the case of a total eclipse, will evidently be

$$219545 - 217327 = 2218 \text{ Miles},$$

and the extreme distance of the vertex f, *fig.* 794., of the cone from the surface, in case of an annular eclipse, is

$$247797 - 212272 = 35525 \text{ Miles}.$$

Thus, the vertex of the shadow, when directed to the earth's centre, ranges from 2218 miles below the surface to 35,525 miles above it.

Since the surface of the earth cuts off about the 100th part of the length of the shadow, the diameter of the circular

shadow projected upon the earth, which is the section of the conical shadow, is about the 100th part of the moon's diameter, or about 20 miles, which, however, is increased by the effect of the curvature of the earth, and considerably so by the obliquity of its surface to the axis of the shadow.

In the same manner it may be shown, that the diameter of the circle over which the eclipse may be annular, measured at right angles to the axis of the shadow, is about the sixth part of the diameter of the moon, which in like manner is augmented by curvature and obliquity.

2922. *Solar ecliptic limits.*—The moon's orbit being inclined to the ecliptic, at an angle of 5°, and, consequently, the distance of the moon's centre from the ecliptic varying in each month from 0° to 5°, while the interposition of the moon between any place on the earth and the sun, requires that the apparent distance of their centres should not exceed the sum of their apparent semi-diameters, which never much exceeds half a degree, it is clear that an eclipse can never happen except when, at the time of conjunction, the apparent distance of the moon's centre from the ecliptic is within that limit, a condition which can only be fulfilled within certain small distances of the moon's nodes.

There is a certain distance from the moon's node, *beyond* which a solar eclipse is *impossible*, and a certain lesser distance, *within* which that phenomenon is *inevitable*. These distances are called the solar ECLIPTIC LIMITS.

The mere inspection of *fig.* 794. will show that no solar eclipse can take place unless some part of the globe of the moon pass within the lines $b\,a$ and $b'\,a'$, which touch externally the globes of the sun and earth. It follows, therefore, that the major limit of the distance of the moon's centre from the ecliptic, or its latitude at the time of conjunction, which is compatible with the occurrence of an eclipse, is $m'\,o'$, or what is the same, the angle $m'\,a'\,o'$. Let this angle be expressed by L, and we have

$$\text{L} = m'\,a'\,n' + n'\,a'\,s + s\,a'\,o'.$$

But by the principles of geometry,

$$s\,a'\,o' = a'\,o'\,e - a'\,s\,e,$$

and therefore,

$$\text{L} = m'\,a'\,n' + n'\,a'\,s + a'\,o'\,e - a'\,s\,e.$$

But $m'a'n'$ and $n'a's$ are the apparent semi-diameters of the sun and moon, and $a'o'e$ and $a'se$ are their horizontal parallaxes, respectively. If the former be expressed by s and s', and the latter by h and h', we shall have

$$L = s + s' + h' - h.$$

It follows, therefore, that a solar eclipse cannot take place, unless the latitude of the moon at conjunction be less than the sum of the apparent semi-diameters of the sun and moon, added to the difference of their horizontal parallaxes.

But since all these quantities vary between a certain major and a certain minor limit, an eclipse will be possible or certain, according as the moon's latitude at conjunction is within the one limit or the other. If the latitude be within the major limit, a solar eclipse *may* take place; and if the several quantities have such values as fulfil the above condition, it *will* take place. If the latitude be within the minor limit, an eclipse *must* take place; because, whatever be their values, they must fulfil the condition.

2923. *Extreme and mean values of semi-diameters and horizontal parallaxes of sun and moon.*—The extreme and mean values of these quantities, which are very important in the theory of eclipses, and other parts of practical astronomy, are given in the following table:—

	Greatest.	Least.	Mean.
Apparent semi-diameter of Sun - -	16′ 18″	15′ 45″	16′ 1·5″
Apparent semi-diameter of Moon - -	16 46	14 44	15 45
Horizontal parallax of Sun - -	0 8·75	0 8·45	0 8·60
Horizontal parallax of Moon - -	61 18	53 58	57 38

The major limit of L will therefore be

$$L' = 1° 34' 14'',$$

and the minor limit

$$L'' = 1° 24' 19''.$$

To determine the distances from the node, which correspond to these limits of the moon's latitude at conjunction, let NMM′, *fig.* 797., represent a part of the moon's path, NSS′ a part of the ecliptic, MS the major, and M′S′ the minor limit of the value of L. The triangles MNS and M′NS′ are spherical triangles, and the

Fig. 797.

distances NS and NS′, if rigorously computed, would require the application of the rules and formulæ of spherical trigonometry. No error of importance, for the present illustration, will, however, be entailed upon the results, by treating them as plane triangles, and applying to them the principles explained in (2294.). We shall then have

$$MNS = 5° \, 8' \, 48'' \quad MS = 1° \, 34' \, 14'' \quad M'S' = 1° \, 24' \, 19'',$$

$$NS = 1° \, 34' \, 14'' \times \frac{57{\cdot}30}{5{\cdot}15} = 17° \, 27' \, 48'',$$

$$NS' = 1° \, 24' \, 19'' \times \frac{57{\cdot}30}{5{\cdot}15} = 15° \, 37' \, 36''.$$

Thus it appears, that when the distance from the node at opposition is greater than 17° 27′ 48″, an eclipse *cannot*, and when less than 16° 6′ 16″, *must*, take place. Between these limits it may or may not occur, according to the magnitude of the parallaxes and apparent diameters.

Since the sun takes more than a month to move through 32° of the ecliptic, it follows that at least one conjunction must take place within 16° of each node, and that one solar eclipse, at least, must occur near each node, and therefore two, at least, annually. But it *may* happen that two solar eclipses shall occur at the same node, and this *will* take place if the moon be in conjunction at more than 14½°, and less than 15° 37′ 36″, from the node, for in that case, it will be again in conjunction in 29½ days, in which time the sun will move through 29°, and will therefore be at 14½° on the other side of the node, and therefore within the ecliptic limit.

Thus, it is *possible* that two solar eclipses may take place at each node, and, therefore, four within the year. But even more is possible, for, as will hereafter appear, the nodes of the moon's orbit have a retrograde motion on the ecliptic, the consequence of which is, that the sun arrives at each node in less than a year after it has last passed through it, and consequently another solar eclipse *may* happen, before the lapse of a solar year, at the same node at which the first occurred.

2924. *Limits for total and annular eclipses.*—It is evident that no total or annular eclipse can be witnessed, unless the globe of the moon be fully within the tangents *b a* and *b′ a′*, *fig.* 794. That this may take place, the apparent distance of the

moon's centre m' from o' must be less than $n' o'$ by the moon's semi-diameter $m' n'$, so that we shall have

$$L = n' a' o' - n' a' m' = s a' b' + s a' o' - n' a' m';$$

and, from what has been explained above, this becomes

$$L = s - s' + h' - h.$$

By assigning to these quantities the values which render L greatest and least, we shall find that when greatest,

$$L = 62' \, 44'' = 1° \, 2' \, 44'',$$

and when least,

$$L = 52' \, 49'';$$

and according to the method previously applied, we find the corresponding distance from the nodes

$$N S = 11° \, 37' \, 36''$$
$$N' S' = 9° \, 47' \, 19''.$$

A total or annular eclipse is therefore *possible*, if conjunction takes place within 11° 37′ 36″, and *certain*, if it takes place within 9° 47′ 19″ of the node. It will be total or annular, according as the apparent diameter of the moon is greater or less than that of the sun.

2925. *Appearances attending total solar eclipses.*—A natural consequence of the diffusion of knowledge is, that while it lessens the vague sense of wonder, with which singular phenomena in nature are beheld, it increases the feeling of admiration at the harmonious laws, the development of which renders easily intelligible effects apparently strange and unaccountable. It may be imagined what a sense of astonishment, and even terror, the temporary disappearance of an object like the sun or moon must have produced in an age when the causes of eclipses were known only to the learned. Such phenomena were regarded as precursors of divine vengeance. History informs us that in ancient times armies have been destroyed by the effects of the consternation spread among them by the sudden occurrence of an eclipse of the sun. Commanders who happened to possess some scientific knowledge, have taken advantage of it to work upon the credulity of those around them by menacing them with prodigies, the near approach of which they were well

aware of, illustrating thus, in a singular and perverted manner, the maxim that knowledge is power.*

The spectacle presented during a total eclipse is always most imposing. The darkness is sometimes so intense as to render the brighter stars and planets visible. A sudden fall of temperature is sensible in the air. Vegetables and animals comport themselves as they are wont to do after sunset. Flowers close, and birds go to roost. Nevertheless, the darkness is different from the natural nocturnal darkness, and is attended with a certain indescribable unearthly light, which throws upon surrounding objects a faint hue, sometimes reddish, and sometimes cadaverously green.

Many interesting narratives have been published by scientific observers, who have been so fortunate as to witness these phenomena.

2926. *Baily's beads.*—When the disk of the moon, advancing over that of the sun, has reduced the latter to a thin crescent, it was observed by Mr. Francis Baily, that immediately before the beginning, or after the end of complete obscuration, the crescent appeared as a band of brilliant points separated by dark spaces, so as to give to it the appearance of a string of brilliant "beads." The phenomenon, which has since been frequently re-observed, thence acquired the name of "Baily's beads."

Further observation showed, that before the formation of the "beads" the horns of the crescent were sometimes interrupted and broken by black streaks thrown across them.

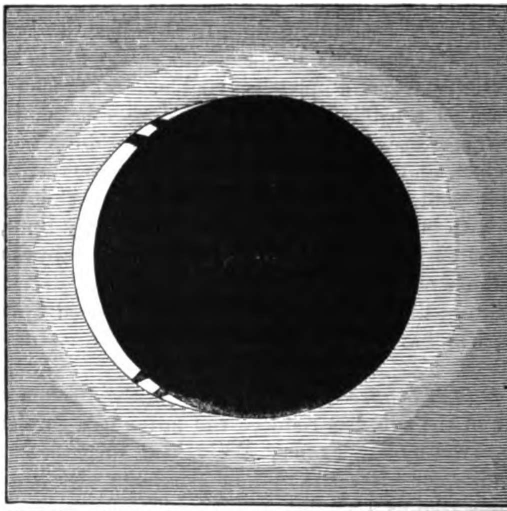
These phenomena are roughly sketched in *figs.* 798., 799.

Figs. 800. to 803. are taken from the original sketches of Mr. Baily, representing the progressive disappearance of the beads after the termination of the complete obscuration.

2927. *Produced by lunar mountains, projected on the sun's disk.*—These phenomena arise from the projection of the edge of the moon's disk, serrated by numerous inequalities of the surface, approaching so close to the external edge of the sun's disk, that the points of the projections extend to the latter, while the intermediate spaces remain uncovered. This may be very

* Columbus is said to have availed himself of his acquaintance with practical astronomy to predict a solar eclipse, and used the prediction as a means of establishing his authority over the crews of his vessels who showed indications of mutinous disobedience.

Fig. 798.



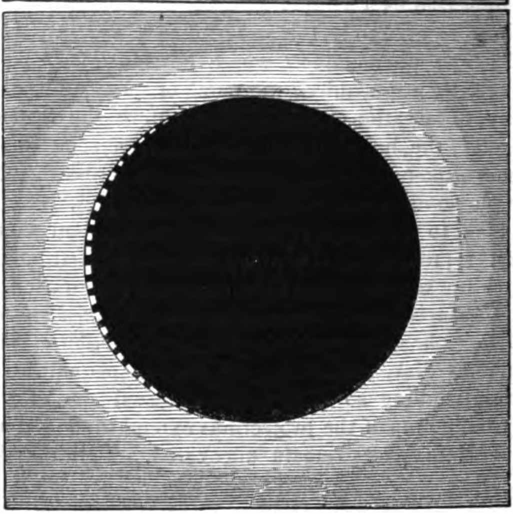
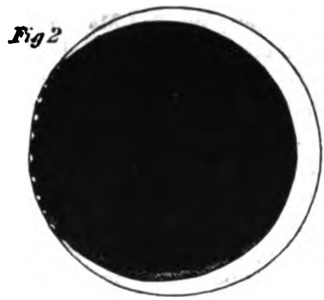


Fig. 799.

appropriately illustrated by laying the blade of a circular saw, having finely cut teeth, over a white circle of nearly equal

Fig. 800.

Fig. 801.



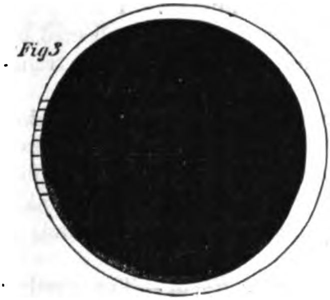


Fig. 802.

Fig. 803.

diameter upon a black ground. The white parts between the teeth will appear like a necklace of white pearls.

The fact, that in some cases the beads have not been seen, or if seen, appeared in a less conspicuous manner, may be explained by the greater or less prevalence of mountainous masses, on that part of the moon's surface which forms the edge of its disk at different times.

The beads, in general, disappear suddenly, at the moment of the commencement of total obscuration, and reappear on the other side of the lunar disk, with a somewhat startling, instantaneous effect, at the moment the total obscuration ceases.

2928. *Flame-like protuberances.*—Immediately after the commencement of the total obscuration, red protuberances, resembling flames, appear to issue from the edge of the moon's

disk. These appearances, which were first noticed by Vassenius, on the occasion of the total solar eclipse which was visible at Göttenberg on 3rd May, 1733, have been re-observed on the occurrence of every total solar eclipse which has taken place since that time, and constitute one of the most curious and interesting effects attending this class of phenomena.

2929. *Solar eclipse of* 1851.—A total eclipse of the sun took place on the 28th July, 1851, which became a subject of systematic observation by the most eminent astronomers of the present day. A considerable number of English observers, aided by several foreigners, distributed themselves in parties at different points along the path of the shadow, so that the chances of the impediments that might arise from unfavourable conditions of the atmosphere might be diminished. The reports and drawings of these various observers have been collected by the Royal Astronomical Society, and published in their transactions.

The Astronomer Royal, with two assistants, Messrs. Dunkin and Humphreys, authorised by the Board of Admiralty, selected certain parts of Sweden and Denmark as the most eligible station. Professor Airy observed at Göttenberg, Mr. Dunkin at Christiana, and Mr. Humphreys, assisted by Mr. Miland, at Christianstad.

2930. *Observations of the Astronomer Royal.*—The weather on the whole proved favourable at Göttenberg. We take from the report of the Astronomer Royal the following highly interesting particulars of the progress of the phenomenon.

"The approach of the totality was accompanied with that indescribably mysterious and gloomy appearance of the whole surrounding prospect which I have seen on a former occasion. A patch of clear blue sky in the zenith became purple-black while I was gazing at it. I took off the higher power, with which I had scrutinized the sun, and put on the lowest power (magnifying about 34 times). With this I saw the mountains of the moon perfectly well. I watched carefully the approach of the moon's limb to the sun's limb, which my graduated dark glass enabled me to see in great perfection; I saw both limbs perfectly well defined to the last, and saw the line becoming narrower and the cusps becoming sharper without any distortion or prolongation of the limbs. I saw the moon's serrated limb advance up to the sun's, and the light of the sun glimmering through the hollows between the mountain peaks, and saw these glimmering spots extinguished one after another in extremely rapid succession, but without any of the appearances which Mr. Baily has described. I saw the sun covered, and

immediately slipping off the dark glass, *instantly* saw the appearances represented at *a b c d*, *fig.* 1. Pl. XIII.

"Before alluding more minutely to these, I must advert to the darkness. I have no means of ascertaining whether the darkness really was greater in the eclipse of 1842; I am inclined to think that in the wonderful, and I may say appalling, obscurity, I saw the grey granite hills within sight of Hvalās more distinctly than the darker country surrounding the Superga. But whether because in 1851 the sky was much less clouded than in 1842 (so that the transition was from a more luminous state of sky to a darkness nearly equal in both cases), or from whatever cause, the suddenness of the darkness in 1851 appeared to me much more striking than in 1842. My friends who were on the upper rock, to which the path was very good, had great difficulty in descending. A candle had been lighted in a lantern about a quarter of an hour before the totality; Mr. Haselgren was unable to read the minutes of the chronometer-face without having the lantern held close to the chronometer.

"The corona was far broader than that which I saw in 1842: roughly speaking, its breadth was little less than the moon's diameter; but its outline was very irregular. I did not remark any beams projecting from it which deserved notice as much more conspicuous than the others; but the whole was beamy, radiated in structure, and terminated (though very indefinitely) in a way which reminded me of the ornament frequently placed round a mariner's compass. Its colour was white, or resembling that of *Venus*. I saw no flickering or unsteadiness of light. It was not separated from the moon by any dark ring, nor had it any annular structure; it looked like a radiating luminous cloud behind the moon.

"The form of the prominences was most remarkable. That which I have marked (*a*) reminded me of a bomerang. Its colour for at least two-thirds of its breadth, from the convexity towards the concavity, was full lake-red, the remainder was nearly white. The most brilliant part of it was the swell farthest from the moon's limb; this was distinctly seen by my friends and myself with the naked eye. I did not measure its height; but judging generally by its proportion to the moon's diameter, it must have been 3′. This estimation perhaps belongs to a later period of the eclipse. The prominence (*b*) was a pale white semi-circle based on the moon's limb. That marked (*c*) was a red detached cloud, or balloon, of nearly circular form, separated from the moon's limb by a space (differing in no way from the rest of the corona) of nearly its own breadth. That marked (*d*) was a small triangular or conical red mountain, perhaps a little white in the interior. These were the appearances seen instantly after the formation of the totality.

"I employed myself in an attempt to delineate roughly the appearances on the western limb, and I took a hasty view of the country; and I then examined the moon a second time. I believe (but I did not carefully remark) that the prominences *a b c* had increased in height; but (*d*) had now disappeared, and a new one (*e*) had risen up. It was impossible to see this change without feeling the conviction that the prominences belonged to the sun and not to the moon.

"I again looked round, when I saw a scene of unexpected beauty. The southern part of the sky, as I have said, was covered with uniform white

cloud; but in the northern part were detached clouds upon a ground of clear sky. This clear sky was now strongly illuminated, to the height of 30° or 35°, and through almost 90° of azimuth, with rosy red light shining through the intervals between the clouds. I went to the telescope, with the hope that I might be able to make the polarization-observation, (which, as my apparatus was ready to my grasp, might have been done in three or four seconds,) when I saw that the *sierra*, or rugged line of projections, shown at (*f*), had arisen. This *sierra* was more brilliant than the other prominences, and its colour was nearly scarlet. The other prominences had perhaps increased in height, but no additional new ones had arisen. The appearance of this *sierra*, nearly in the place where I expected the appearance of the sun, warned me that I ought not now to attempt any other physical observation. In a short time the white sun burst forth, and the corona and every prominence vanished.

"I withdrew from the telescope and looked round. The country seemed, though rapidly, yet half unwillingly, to be recovering its usual cheerfulness. My eye, however, was caught by a duskiness in the south-east, and I immediately perceived that it was the eclipse-shadow in the air travelling away in the direction of the shadow's path. For at least six seconds this shadow remained in sight, far more conspicuous to the eye than I had anticipated."

2931. *Observations of Messrs. Dunkin and Humphreys.*— Owing to the unfavourable state of the atmosphere, the observations of the other members of the Admiralty party were not so satisfactory as those of its chief. Nevertheless, both observers saw the red prominences, though imperfectly, as compared with the results of the observations of the Astronomer Royal. Baily's beads were seen by Mr. Dunkin, as well before as after the total obscuration. Their appearance was of intense brilliancy, compared by the observer to a diamond necklace. Their effect on the observer was "quite overpowering," being unprepared for a sight so magnificent.

At Christianstad, the planets VENUS, MERCURY, and JUPITER, and the stars ARCTURUS and VEGA, were visible during the totality of the eclipse.

2932. *Observations of W. Gray, at Tune, near Sarpsborg.* —This gentleman also saw Baily's beads, both before and after the total obscuration. He saw four of the red projections, three of which are represented in *fig.* 2. pl. xiii., the fourth resembling *c* and *d* in form, and diametrically opposite to *a* in position on the moon's limb. The apparent height of *a* was estimated at $1\frac{1}{4}'$, and its breadth $62''$, but the altitude of this afterwards increased to $1\frac{3}{4}'$. There was a dark shade in the curved portion, which gave it a resemblance to a gas flame. The remainder,

however, was rose-red, not uniform, and very pale, like the innermost parts of the petals of a rose. The red prominence opposite to *a* had an apparent altitude of 1′, and a deeper red colour. The prominence *c* and *d* were estimated at about 50″ in size.

During the totality, the light seemed like that of an evening in August at an hour and a half after sunset.

2933. *Observations of Messrs. Stephenson and Andrews at Fredrichsvaarn.* — Baily's beads were seen both before and after the total obscuration. The crescent, before disappearing, was seen as a fine thread of light, which broke up into fragments, and when it re-appeared, it gave the idea of globules of mercury rushing amongst each other along the edge of the moon. In a second or two after the disappearance of the crescent, a rose-coloured flame shot out from the limb of the moon, which in form resembled a sickle, see *fig.* 3. It increased rapidly, and then two other rose-coloured prominences, above and below it, started out, differing in shape, but evidently of the same character. Besides these, there were, as well between them as elsewhere, around the moon's edge other lurid points and other indistinct lines. The height of the principal prominence was estimated at about the twentieth of the moon's diameter, that is, about 1½′. The chief prominences looked like burning volcanoes, and the lurid points and lines reminded the observers of dull streams of cooling lava.

2934. *Observations of Mr. Lassell at Trollhättan Falls.* — Having heard the red prominences seen in former total eclipses described as faint appearances, the astonishment of the observer may be imagined when he saw around the dark disk of the moon, after the commencement of total obscuration, prominences of the most brilliant lake colour, — a splendid pink, quite defined and hard, *fig.* 4. They appeared not to be absolutely quiescent. The observer judged from their appearance that they belonged to the sun, and not to the moon.

2935. *Observations of Mr. Hind at Ravelsborg, near Engelholm.* — Baily's beads were seen, both before and after the total obscuration, in such a manner as to leave no doubt of their cause being that already explained. In five seconds after the commencement of the total obscuration, the corona or glory around the moon's disk was seen. Its colour seemed to be that of tarnished silver, brightest next the moon's limb, and gradually

fading to a distance equal to one-third of her diameter, where it became confounded with the general tint of the heavens. Appearances of radiation are mentioned, similar to those described by Professor Airy.

"On first viewing the sun" says Mr. Hind "without the dark glass after the commencement of totality, three rose-coloured prominences immediately caught my eye, and others were seen a few seconds later (*fig.* 5.). The largest and most remarkable of them was situate about 5° north of the parallel of declination, on the western limit of the moon; it was straight through two-thirds of its length, but curved like a sabre near the extremity, the concave edge being towards the horizon. The edges were of a full rose-pink, the central parts plainer, though still pink.

"Twenty seconds, or thereabouts, after the disappearance of the sun, I estimated its length at 45″ of arc, and on attentively watching it towards the end of totality, I saw it materially lengthened (probably to 2′), the moon having apparently left more and more of it visible as she travelled across the sun. It was always curved, and I did not remark any change of form, nor the slightest motion during the time the sun was hidden. I saw this extraordinary prominence *four seconds after the end of totality*, but at this time it appeared detached from the sun's limb, the strong white light of the corona intervening between the limb and the base of the prominence.

"About 10° south of the above object I saw, during the totality, a detached triangular spot of the same rose colour, suspended, as it were, in the light of the corona, which gradually receded from the moon's dark limb, as she moved onwards, and was, therefore, clearly connected with the sun. Its form and position, with respect to the large prominence, continued exactly the same so long as I observed it. On the south limb of the moon appeared a long range of rose-coloured flames, which seemed to be affected with a tremulous motion, though not to any great extent.

"The bright rose-red of the tops of these projections gradually faded towards their bases, and along the moon's limb appeared a bright narrow line of a deep violet tint: not far from the western extremity of this long range of red flames was an isolated prominence, about 40″ in altitude, and another of similar size and form, at an angle of 145° from the north towards the east: the moon was decidedly reddish-purple at the beginning of totality, but the reddish tinge disappeared before its termination, and the disk assumed a dull purple colour. A bright glow, like that of twilight, indicated the position where the sun was about to emerge, and three or four seconds later the beads again formed, this time instantaneously, but less numerous, and even more irregular, than before. In five seconds more the sun reappeared as a very fine crescent on the sudden extinction of the beads."

2936. — *Observations of Mr. Dawes near Engelholm.* — Mr. Dawes observed the beads, and found all the circumstances attending their appearance such as to leave no doubt as to the truth of the cause generally assigned to them. He observed the corona

a few seconds after the commencement of the totality, and estimated its extreme breadth at half the moon's diameter, the brightness being greatest near the moon's limb, and gradually decreasing outwards. The phenomena of the red protuberances, witnessed by Mr. Dawes, are so clearly and satisfactorily described by him, that we think it best here to give the account of them in his own words, —

"Throughout the whole of the quadrant, from north to east, there was no visible protuberance, the corona being uniform and uninterrupted. Between the east and south points, and at an angle of about 175° from the north point, appeared a large red prominence of a very regular conical form, *fig.* 6. When first seen, it might be about 1½′ in altitude from the edge of the moon, but its length diminished as the moon advanced.

"The position of this protuberance may be inaccurate to a few degrees, being more hastily noticed than the others. It was of a deep rose colour, and rather paler near the middle than at the edges.

"Proceeding southward, at about 145° from the north point commenced a low ridge of red prominences, resembling in outline the tops of a very irregular range of hills. The highest of these probably did not exceed 40″. This ridge extended through 50° or 55°, and reached, therefore, to about 197° from the north point, its base being throughout formed by the sharply-defined edge of the moon. The irregularities at the top of the ridge seemed to be permanent, but they certainly appeared to undulate from the west towards the east; probably an atmospheric phenomenon, as the wind was in the west.

"At about 220° commenced another low ridge of the same character, and extending to about 250°, less elevated than the other, and also less irregular in outline, except that at about 225° a very remarkable protuberance rose from it to an altitude of 1¼′, or more. The tint of the low ridge was a rather pale pink; the colour of the more elevated prominence was decidedly deeper, and its brightness much more vivid. In form it resembled a *dog's tusk*, the convex side being northwards, and the concave to the south. The apex was somewhat acute. This protuberance, and the low ridge connected with it, were observed and estimated in height towards the end of the totality.

"A small double-pointed prominence was noticed at about 255°, and another low one with a broad base, at about 263°. These were also of the rose-coloured tint, but rather paler than the large one at 225°.

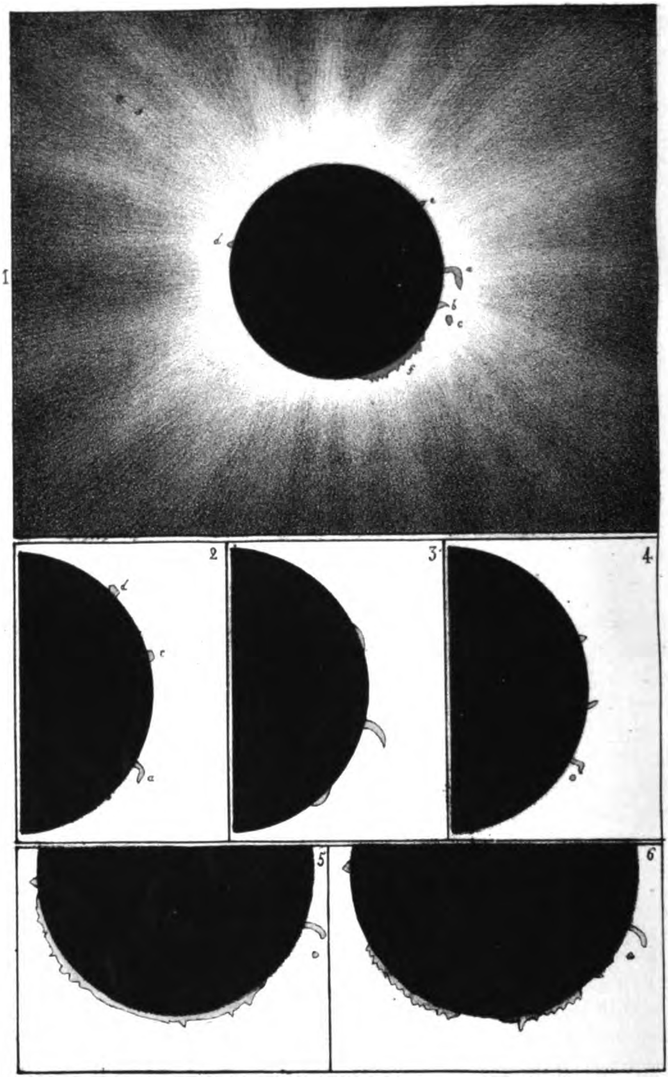
"Almost directly preceding, or at 270°, appeared a bluntly triangular pink body, *suspended*, as it were, in the corona. This was separated from the moon's edge when first seen, and the separation increased as the moon advanced. It had the appearance of a large conical protuberance, whose base was hidden by some intervening soft and ill-defined substance, like the upper part of a conical mountain, the lower portion of which was obscured by clouds or thick mist. I think the apex of this object must have been at least 1′ in altitude from the moon's limb when first seen, and more than

1½′ towards the end of total obscuration. Its colour was pink, and I thought it paler in the middle.

"To the north of this, at about 280° or 285°, appeared the most wonderful phenomenon of the whole. A red protuberance, of vivid brightness and very deep tint, arose to a height of, perhaps, 1½′ when first seen, and increased in length to 2′, or more, as the moon's progress revealed it more completely. In shape it somewhat resembled a *Turkish cimeter*, the northern edge being convex, and the southern concave. Towards the apex it bent suddenly to the south, or upwards, as seen in the telescope. Its northern edge was well defined, and of a deeper colour than the rest, especially towards its base. I should call it a *rich carmine*. The southern edge was less distinctly defined, and decidedly paler. It gave me the impression of a somewhat conical protuberance, partly hidden on its southern side by some intervening substance of a soft or flocculent character. The apex of this protuberance was paler than the base, and of a purplish tinge, and it certainly had a flickering motion. Its base was, from first to last, sharply bounded by the edge of the moon. To my great astonishment, this marvellous object *continued visible for about five seconds*, as nearly as I could judge, *after the sun began to reappear*, which took place many degrees to the south of the situation it occupied on the moon's circumference. It then rapidly faded away, *but it did not vanish instantaneously*. From its extraordinary size, curious form, deep colour, and vivid brightness, this protuberance absorbed much of my attention; and I am, therefore, unable to state precisely what changes occurred in the other phenomena towards the end of the total obscuration.

"The arc, from about 283° to the north point, was entirely free from prominences, and also from any roseate tint.

2937. *Effects of total obscuration on surrounding objects and scenery.*— Although the different parties of observers scattered over the path of the moon's shadow were not equally fortunate in having a clear unclouded sky, they were all enabled to observe and record the effects of the total obscuration upon the surrounding objects and country. Dr. Robertson of Edinburgh, Dr. Robinson of Armagh, and some others, witnessed the eclipse from an island off the coast of Norway, in lat. 61° 21′, at a point in the path of the axis of the shadow. The precursory phenomena corresponded with those described by other observers. The atmosphere was, however, obscured by clouds, which appeared to rush down in streams from the place of the sun. The sea-fowl flocked to their customary places of rest and shelter in the rocks. The darkness at the moment of total obscuration was sudden, but not absolute; for the clouds had left an open strip of the sky, which assumed a dark lurid orange, which changed to greenish colour in another direction, and shed upon persons and objects a faint and unearthly light. Lamps



Telescopic views of the rose coloured emanations.
1. *Astronomer Royal*. 2. *Mr Grey* 3. *Mr Slephenson*. 4. *Mr Lassell*
5. *Mr Hind*. 6 *Mr Dawes*.

and candles, seen at fifty or sixty yards distance, were as visible as in a dark night, and the redness of their light presented a strange contrast with the general green hue of every thing around them. "The appearance of the country," says Dr. Robertson, "seen through the lurid opening under the clouds, was most appalling. The distant peaks of the Tostedals and Dorieffeld mountains were seen still illuminated by the sun, while we were in utter darkness. Never before have we observed all the lights of heaven and earth so entirely confined to one narrow stripe along the horizon,—never that peculiar greenish hue, and never that appearance of outer darkness in the place of observation, and of excessive distance in the verge of the horizon, caused in this case by the hills there being more highly illuminated as they receded by a less and less eclipsed sun."

Mr. Hind says, that during the obscuration "the entire landscape was overspread with an unnatural gloom; persons around him assumed an unearthly cadaverous aspect; the distant sea appeared of a lurid red; the southern heavens had a sombre purple hue, the place of the sun being indicated only by the CORONA; the northern heavens had an intense violet hue, and appeared very near. On the east and west of the northern meridian, bands of light of a yellowish crimson colour were seen, which gradually faded away into the unnatural purple of the sky at greater altitudes, producing an effect that can never be effaced from the memory, though no description could give a just idea of its awful grandeur."

At several places in Prussia, where the heavens were unclouded during the total obscuration, a great number of the more conspicuous stars, as well as the planets Jupiter, Venus, and Mercury, were visible. Several flowering plants were observed to close their blossoms, birds which had been previously flying about disappeared, and domestic fowls went to roost.

2938. *Evidence of a solar atmosphere.*—Many of the phenomena attending total solar eclipses afford strong corroboratory evidence of the existence of a solar atmosphere, extending to a vast height above the luminous coating of the sun, the probability of which has been already shown (2550.).

The corona, or bright ray or glory, surrounding the dark disk of the moon where it covers the sun, is observed to be concentric

with the moon only at the moment when the latter is concentric with the sun. In other positions of the moon's disk, it appears to be concentric with the sun. This would be the effect produced by a solar non-luminous atmosphere faintly reflecting the sun's light.

The corona supplies no exact data by which the height of the solar atmosphere thus faintly reflecting light can be ascertained; but Sir J. Herschel thinks, that from the manner in which the diminution of light is manifested on the sun's disk, being by no means sudden on approaching the borders, but extending to some distance within the disk, the height must be not only great in an absolute sense, but must even be a very considerable fraction of the sun's semi-diameter, and this inference is strongly confirmed by the luminous corona surrounding the eclipsed disk.

2939. *Probable causes of the red emanations in total solar eclipses.*—It appears to be agreed generally among astronomers that the red emanations above described are solar, and not lunar. If they be admitted then to be solar, it is scarcely possible to imagine them to be solid matter, notwithstanding the apparent constancy of their form in the brief interval during which at any one time they are visible, for the entire duration of their visibility has never yet been so much as four minutes. To admit the possibility of their being solar mountains projecting above the luminous atmosphere surrounding the sun, and rising to the height in the exterior and non-luminous atmosphere forming the corona necessary to explain their appearance, we must suppose their height to amount to nearly a twentieth part of the sun's diameter, that is, to 44,000 miles.

The fact that they are gaseous and not solid matter appears, therefore, to be conclusively established by their enormous magnitude, the great height above the surface of the sun at which they are placed, their faint degree of illumination, and the circumstances of their being sometimes detached at their base from the visible limb of the sun. These circumstances render it probable that these remarkable appearances are produced by cloudy masses of extreme tenuity, supported, and probably produced in an extensive spherical shell of non-luminous gaseous matter, surrounding and rising above the luminous surface of the sun to a great altitude.

II. Lunar Eclipses.

2940. *Cause of lunar eclipses.*—When the moon is in opposition, its apparent distance from the plane of the ecliptic or its latitude, varying from 0° to upwards of 5°, is at times less than the apparent semi-diameter, *o m*, *fig.* 794., of the section of the earth's conical shadow, in which case, falling more or less within the shadow, it will be deprived of the sun's light, and will therefore be eclipsed.

The circumstances and conditions attending such a phenomenon depend evidently on the dimensions of the earth's shadow, the magnitude of its section at the moon's distance, and the position of the moon in relation to it.

2941. *Dimensions of the earth's shadow.*—Let s, h, s', and h' express, as before, the apparent semi-diameters and horizontal parallaxes of the sun and moon, and let S express the semi-angle $e f a'$ of the conical shadow, and S' the apparent semi-diameter $o\ a'\ n$ of the section of the shadow at the moon's distance. By the common principles of geometry we shall then have

$$\mathrm{S} = s - h \qquad \mathrm{S}' = h' - \mathrm{S} = h + h - s.$$

Let L express the length $a' f$ of the shadow, in semi-diameters, of the earth. We shall then have (2297.)

$$\mathrm{L} = \frac{206265}{\mathrm{S}} = \frac{206265}{s - h},$$

s and h being expressed in seconds.

The mean value of $s - h$ being 952″, we shall have

$$\mathrm{L} = 206.$$

2942. *Conditions which determine lunar eclipses.*—As the earth moves in its orbit round the sun, this conical shadow is therefore constantly projected in a direction contrary to that of the sun. The axis $\mathrm{E}\,f$, *fig.* 794., of the cone is always in the plane of the ecliptic, and its vertex f describes an orbit which lies in the plane of the ecliptic, outside that of the earth, at a distance somewhat above 206 semi-diameters of the earth. Any body, therefore, which may happen to be in the plane of the ecliptic, or sufficiently near to it, and within this distance of the path of the earth, will be deprived of the sun's light while it is within the limits of the cone. The moon being the only body in the universe which passes within such a distance of the earth, is therefore the only one which can be thus obscured.

At the distance of the moon, which is less than one-third of the length of the shadow, the section of the dark cone is a circular disk, the apparent semi-diameter of which, according to what has just been proved, subtends an angle, the mean and extreme values of which will be found by giving to the quantities which enter the preceding formula for s', their mean and extreme values.

The greatest value of s' will be found by subtracting the least value of the sun's semi-diameter from the greatest value of the sum of the horizontal parallaxes, and it is therefore

$$s' = 61' \, 27'' - 15' \, 45' = 45' \, 42''.$$

Its least value is found by subtracting the greatest value of the sun's semi-diameter from the least values of the horizontal parallaxes, and it is therefore

$$s' = 54' \, 7'' - 16' \, 18'' = 37' \, 49''.$$

The mean value of s' is, in like manner,

$$s' = 57' \, 47'' - 16' \, 2'' = 41' \, 45''.$$

The section of the shadow may therefore be regarded as a dark disk, whose apparent semi-diameter varies between 37′ 49″ and 45′ 42″, and the true place of whose centre is a point on the ecliptic 180° behind the centre of the sun. A lunar eclipse is produced by the superposition, partial or total, of this disk on that of the moon, and the circumstances and conditions which determine such an eclipse are investigated upon the principles already explained.

By the solar tables, the apparent position of the centre of the sun, from hour to hour, may be ascertained, and the position of the centre of the section of the shadow may thence be inferred. From the lunar tables, the position of the moon's centre being in like manner determined, the distance between the centres of the section of the shadow and the moon's disk can be ascertained. When this distance is equal to the sum of the apparent semi-diameters of the moon's disk and the section of the shadow, the eclipse will begin; the moment when the distance is least will be the middle of the eclipse, and the line of greatest obscuration; and when the distance between the centres increasing becomes again equal to the sum of the apparent semi-diameters, the eclipse will terminate. The computation of all these conditions, and the

time of their occurrence, presents no other difficulty than those of ordinary arithmetical calculation.

The magnitude of the eclipses is measured, like that of the sun, by the difference between the sum of the semi-diameters and the distance between the centres.

The occurrence of a total eclipse, and the moment of its commencement, if it take place, are determined by the distance between the centre of the shadow and that of the moon becoming equal to the difference between the semi-diameter of the shadow and that of the moon. Thus, a total eclipse will take place if the moon's latitude L in opposition be less than

$$L = S' - s' = (h + h') - (s + s');$$

that is, less than the difference between the sum of the horizontal parallaxes and the sum of the semi-diameters.

Since the sum of the horizontal parallaxes, even when least, is much greater than the sum of the apparent semi-diameters, even when greatest, a total eclipse of the moon is always possible, provided the centre of the moon approaches near enough to the centre of the shadow, and for the same reason an annular lunar eclipse is impossible.

2943. *Lunar ecliptic limits.*—That a lunar eclipse may take place, it is necessary that the moon, when in opposition, should approach the ecliptic within a distance less than the sum of the apparent semi-diameters of the moon and the section of the shadow. Let its latitude in opposition be L, the limiting value of this will be

$$L' = h + h' + s' - s.$$

If the latitude of the moon be less than this (which is the sum of the semi-diameters of the moon and shadow) an eclipse must take place.

But, as in the case of solar eclipses, the quantities composing this being variable, the limit itself is variable. If such values be assigned to the component quantities as to render L′ the greatest possible, we shall obtain the latitude within which an eclipse is possible. If such values be assigned, as will render L′ the least possible, we shall obtain the latitude within which an eclipse is inevitable.

To obtain the major limit, we must take the greatest values of h, h', and s', and the least value of s. This will give

$$L' = 61' \, 27'' + 16' \, 46'' - 15' \, 45'' = 1^\circ \, 2' \, 28'';$$

and to obtain the minor limit, we must assign the least values to h, h', and s', and the greatest to s, which will give

$$\text{L}' = 54' \, 7'' + 14' \, 44'' - 16' \, 18'' = 52' \, 33''.$$

The corresponding distances from the node, determined in the same manner as in the case of the solar ecliptic limits, will be, for the major limit,

$$\text{N S} = 11^\circ \, 34' \, 38'',$$

and for the minor limit,

$$\text{N}' \text{S} = 9^\circ \, 24' \, 22''.$$

If the moon in opposition be within 11° 34′ 38″ of its node, therefore, a lunar eclipse *may* take place, and *will* do so, if the apparent diameters and parallaxes have the necessary values; but if it take place within 9° 24′ 22″ of the node, an eclipse *must* take place, because the same quantities must be within the requisite limits.

Since the sun moves through these limits on each side of the node, in from $18\frac{1}{2}$ to $22\frac{1}{2}$ days, it may happen that within the time no opposition may take place at either node, and consequently that no lunar eclipse may take place within the year.

2944. *Limits for a total eclipse.*—It has been explained that a total eclipse can only take place when the moon's latitude in opposition is less than

$$\text{L} = (h + h') - (s + s'').$$

To determine the limit within which a total eclipse is *possible*, we must assign to $h + h'$ its greatest, and $s + s'$ its least, value. These give

$$\text{L}' = (61' \, 27'') - (30' \, 29'') = 30' \, 58''.$$

The distance from the node corresponding to this is therefore

$$\text{N S} = (30' \, 58'') \times \frac{57 \cdot 30}{5 \cdot 15} = 5^\circ \, 44' \, 21'.$$

To determine the limit within which a total eclipse is inevitable, we must assign to $h + h'$ its least, and to $s + s'$ its greatest, value. These give

$$\text{L}'' = (54' \, 7'') - (33' \, 4'') = 21' \, 3'',$$

and the corresponding distance from the node is

$$\text{N}' \text{S}' = (21' \, 3'') \times \frac{57 \cdot 30}{5 \cdot 15} = 3^\circ \, 65' \, 5''.$$

When the distance from the node at opposition, therefore, is greater than $5^{\circ}\ 44'\ 21''$, a total eclipse cannot, and when less than $3^{\circ}\ 54'\ 5''$, it must, take place. Between these limits it may or may not occur, according to the magnitude of the parallaxes and apparent diameters.

2945. *Greatest duration of total eclipse.*—The duration of a total eclipse depends on the distance over which the centre of the moon's disk moves relatively to the shadow while passing from the first to the last internal contact. This may vary from 0 to twice the greatest possible distance of the moon's centre from the centre of the shadow at the moment of internal contact, that is, to

$$2\,\mathrm{L}' = 2\,(h + h') - 2\,(s + s'),$$

and this at its greatest value is, as has been already shown,

$$2\,\mathrm{L}' = 2 \times (30'\,58'') = 61'\,56'';$$

and since the moon's centre moves synodically through half a minute of space in each minute of time, the interval necessary to move over $61'\ 56''$ will be two hours and four minutes, which is therefore the greatest possible duration of a total lunar eclipse.

2946. *Relative number of solar and lunar eclipses.*—It will be evident, from what has been explained, that the frequency of solar is much greater than that of lunar eclipses, since two at least of the former *must*, and five *may*, take place within the year, while not one of the latter may occur. Nevertheless, the number of lunar which are exhibited at *any given place* on the earth is greater than that of solar eclipses, because, although the latter occur with so much greater frequency, they are seen only within particular limits on the earth's surface.

2947. *Effects of the earth's penumbra.*—Long before the moon enters within the sides of the cone of the shadow it enters the penumbra, and is partially deprived of the sun's light, so as to render the illumination of its surface sensibly more faint. When once it passes within the line *a′ p′ fig.* 794 forming the external limit of the penumbra, it ceases to receive light from that part of the sun which is near the limb *b*. As it advances closer to *a′ f*, the edge of the true shadow, more and more of the solar rays are intercepted by the earth; and when it approaches the edge, it is only illuminated by a thin crescent of the sun, visible from the moon over the edge of the earth at *a′*. It might be

thus inferred, that the obscuration of the moon is so extremely gradual, that it would be impossible to perceive the limitation of the shadow and penumbra. Nevertheless, such is the splendour of the solar light, that the thinnest crescent of the sun, to which the part of the moon's surface near the edge of the earth's shadow is exposed, produces a degree of illumination which contrasts so strongly with the shadow as to render the boundary of the latter so distinct, that the phenomenon presents one of the most striking evidences of the rotundity of the earth, the form of the shadow being accurately that which one globe would project upon another.

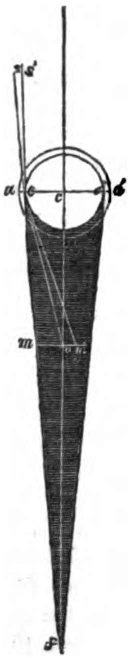


Fig. 804.

2948. *Effects of refraction of the earth's atmosphere in total eclipse.* — If the earth were not surrounded with an atmosphere capable of refracting the sun's light, the disk of the moon would be absolutely invisible after entering within the edge of the shadow. For the same reason, however, that we continue to see the sun's disk, and receive its rays after it has really descended below the horizon, an observer placed upon the moon, and therefore the surface of the moon itself, must continue to receive the sun's rays after the interposition of the edge of the earth's disk as seen from the moon. This refracted light falling upon the moon after it has entered within the limits of the shadow, produces upon it a peculiar illumination, corresponding in faintness and colour to the rays thus transmitted through the earth's atmosphere.

To render this more clear, let *e e'*, *fig.* 804., represent a diameter of the earth at right angles to the axis *f* of the shadow; and let *a a'* represent the limits of the atmosphere. Let *s e f* be the ray proceeding from the edge of the sun, and forming therefore the boundary of the shadow, considered without reference to the atmosphere. But the solar rays in passing through the convex shell of air, between *a* and *e*, are affected as they would be by a convex lens composed of a transparent refracting medium (1028i), and are therefore rendered convergent, so that the ray *s e*, instead of passing directly to *m*, will be bent inwards towards *m'*, while the ray which really passes from *e* to *f* is one

which comes in the direction *s' e*, and therefore from a point within the sun's disk. The moon's disk, therefore, or any point of it which is within the angle *m e m'*, will receive this refracted light, and will be illuminated by it in accordance with its colour and intensity.

The deflection which a solar ray suffers in passing through the atmosphere towards *e* on the side of the sun, is equal to the horizontal refraction, and as, according to the principles of optics (1034.), it suffers an equal refraction in passing out on the other side, the total deflection, which is measured by the angle *m e m'*, is twice the horizontal refraction. But the mean value of the horizontal refraction being 33′, the mean value of the angle *m e m'* will be 66′. But since the greatest value of *m e o* is 45′ 42″ (2942.), it follows that the refracted ray *e m'* will fall upon the section of the shadow at a point beyond its centre; and since the same will take place at all points around the shadow, it follows that the entire section will be more or less illuminated by the light thus refracted: the intensity of such illumination increases from the centre towards the borders.

2949. *The lunar disk visible during total obscuration.*— When the moon's limb first enters the shadow at *m*, the contrast and glare of the part of the disk still enlightened by the direct rays of the sun render the eye insensible to the more feeble illumination produced upon the eclipsed part of the disk by the refracted rays. As, however, the eclipse proceeds, and the magnitude of the part of the disk directly enlightened decreases, the eye, partly relieved from the excessive glare, begins to perceive very faintly the eclipsed limb, which is nevertheless visible from the beginning in a telescope, in which it appears with a dark grey hue. When the entire disk has passed into the shadow, it becomes distinctly visible, showing a gradation of tints from a bluish or greenish on the outside to a gradually increasing red, which, further in, changes to a colour resembling that of incandescent iron when at a dull red heat. As the lunar disk approaches the centre of the shadow, this red line is spread all over it. Its illumination in this position is sometimes so strong as to throw a sensible shadow, and to render distinctly visible in the telescope the lineaments of light and shadow upon its surface.

These effects are altogether similar to the succession of tints developed in our atmosphere at sunset, and arise, in fact, from

the same cause, operating, however, with a two-fold intensity. The solar rays traversing twice the thickness of air, the blue and green lights are more effectually absorbed, and a still more intense red is imparted to the tints transmitted. Without pursuing these consequences further here, the student will find no difficulty in tracing them in the effects of sunset and of sunrise, and of evening and morning twilight.

III. Eclipses, Transits, and Occultations of the Jovian System.

2950. The motions of Jupiter and his satellites, as seen from the earth, exhibit, from time to time, all the effects of interposition.

Let J J′, *fig.* 805., represent the planet, J *f* J′ its conical shadow, *s s′* the sun, E and E′ the positions of the earth when the planet is in quadrature, in which position the shadow J *f* J′ is presented with least obliquity to the visual line, and therefore least foreshortened, and most distinctly seen. Let *b b′d′d* represent the orbit of one of the satellites, the plane of which coincides nearly with that of the planet's orbit, and, for the purposes of the present illustration, the latter may be considered as coinciding with the ecliptic without producing sensible error.

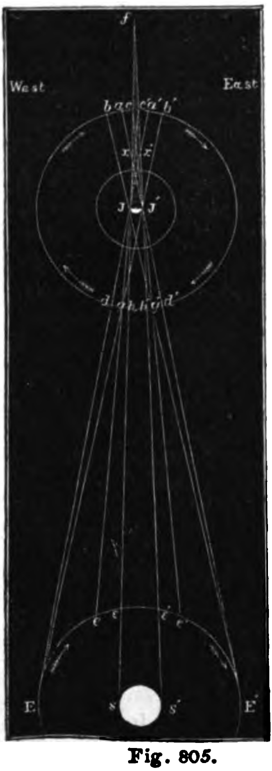


Fig. 805.

From E suppose the visual lines E J and E J′ to be drawn, meeting the path of the satellite at *d* and *g*, and at *a′* and *b′*, and, in like manner, let the corresponding visual lines from E′ meet it at *d′* and *g′*, and at *a′* and *b′*. Let *c* and *c′* be the points where the path of the satellite crosses the limits of the shadow, and *h* and *h′* the points where it crosses the extreme solar rays which pass along those limits.

If *l* express the length J *f* of the shadow, *d* the distance of the planet

from the sun in semi-diameters of the planet, and r and r' the semi-diameters of the sun and the planet respectively, we shall have (2917.)

$$l = d \times \frac{r'}{r - r'}.$$

But

$$d = 11227 \qquad r = 441000 \qquad r' = 4400,$$

and therefore

$$l = 11227 \times \frac{44}{441-44} = 1247;$$

that is to say, the length of the shadow is 1247 semi-diameters of the planet. Now, since the distance of the most remote satellite is not so much as 27 semi-diameters of the planet (2760.), and since the orbits of the satellites are almost exactly in the plane of the orbit of the planet, it is evident that they will necessarily pass through the shadow, and almost through its axis, every revolution, and the lengths of their paths in the shadow will be very little less than the diameter of the planet.

The fourth satellite, in extremely rare cases, presents an exception to this, passing through opposition without entering the shadow. In general, however, it may be considered that all the satellites in opposition pass through the shadow.

2951. *Effects of interposition.*—The planet and satellites exhibit, from time to time, four different effects of interposition.

2952. 1st. *Eclipses of the satellites.* — These take place when the satellites pass behind the planet. Their entrance into the shadow, called the *immersion*, is marked by their sudden extinction. Their passage out of the shadow, called their *emersion*, is manifested by their being suddenly relighted.

2953. 2nd. *Eclipses of the planet by the satellites.*—When the satellites, at the periods of their conjunctions, pass between the lines s J and s' J$'$, their shadows are projected on the surface of the planet in the same manner as the shadow of the moon is projected on the earth in a solar eclipse, and in this case the shadow may be seen moving across the disk of the planet, in a direction parallel to its belts, as a small round and intensely black spot.

2954. 3rd. *Occultations of the satellites by the planet.*—When a satellite, passing behind the planet, is between the tangents E J a' and E J$'$ b', drawn from the earth, it is concealed from the observer on the earth by the interposition of the body of the

planet. It suddenly disappears on one side of the planet's disk, and as suddenly reappears on the other side, having passed over that part of its orbit which is included between the tangents. This phenomenon is called an occultation of the satellite.

2955. 4th. *Transits of the satellites over the planet.*—When a satellite, being between the earth and planet, passes between the tangents E J and E J′, drawn from the earth to the planet, its disk is projected on that of the planet, and it may be seen passing across, as a small brown spot, brighter or darker than the ground on which it is projected, according as it is projected on a dark or bright belt. The entrance of the satellite upon the disk, and its departure from it, are denominated its *ingress* and *egress*.

2956. *All these phenomena manifested at quadrature.*—When the planet is in quadrature, and the shadow therefore presented to the visual ray with least effect of foreshortening, all these several phenomena may be witnessed in the revolution of each satellite.

The earth being at E or E′, the visual line E J or E′ J′ crosses the boundary x' or x of the shadow at a distance x' J′ or x J from the planet, which bears the same ratio to its diameter as the distance of Jupiter from the sun bears to the distance of the earth from the sun, as is evident from the figure. But Jupiter's distance from the sun being five times that of the earth, it follows that the distance x J is five diameters, or ten semi-diameters, of the planet. But since the distance of the first satellite is only six, and that of the second somewhat less than ten, semi-diameters of the planet, it follows that the paths of these two will lie within the distance x J or x' J′.

The planet being in quadrature 90° behind the sun, the earth will be at E, and the entire section c c' of the shadow, at the distances of the third and fourth satellites (which are 15 and 27 semi-diameters of the planet respectively), will be visible to the west of the planet, so that when these satellites, moving from b, as indicated by the arrow, pass through the shadow, their immersion and emersion will be both manifested on the west of the planet, by their sudden disappearance and reappearance on entering and emerging from the shadow at c and c'. But the section of the shadow, at the distances of the first and second satellites, being nearer to the planet than x x', will be visible only at its western edge, the planet intercepting the

visual ray directed to the eastern edge. The immersion, therefore, of these will be manifested by their sudden disappearance on the west of the planet, at the moment of their immersion, but the view of their immersion will be intercepted by the body of the planets, and they will only reappear after having passed behind the planet.

The third and fourth satellites, after emerging from the shadow at c', and appearing to be relighted, will again be extinguished when they come to the visual ray E J a', which touches the planet. The moment of passing this ray is that of the commencement of their occultation by the planet. They will continue invisible until they arrive at the other tangential visual ray E J′ b', when they will suddenly reappear to the east of the planet, the occultation ceasing.

In the cases of the first and second satellites, the commencement of the occultation preceding the termination of the eclipse, it is not perceived, the satellite at the moment of the interposition of the edge of the planet not having yet emerged from the shadow. In these cases, therefore, the disappearance of the satellite at the commencement of the eclipse, and its reappearance at the termination of the occultation, alone are perceived, the emersion from the shadow being concealed by the occultation, which has already commenced, and the disappearance at the commencement of the occultation being prevented by the eclipse not yet terminated.

When the satellite, proceeding in its orbit, arrives at h', its shadow falls upon the planet, and is seen from the earth at E, to move across its disk as a small black spot, while the planet moves from h' to h.

When the planet arrives at g, it passes the visual ray E J′, and while it moves from g to d, its disk is projected on that of the planet, and a transit takes place, as already described.

Thus, at quadrature, the third and fourth satellites present successively all the phenomena of interposition: 1st, an eclipse of the satellite to the west of the planet shows both immersion and emersion; 2nd, an occultation of the satellite by the planet, the disappearance and reappearance being both manifested; 3rd, the eclipse of the planet by the satellite; and 4th, the transit of the satellite over the planet.

2957. *Effects modified at other elongations.* — There is a certain limit, such as e, at which the emersion of the third and

fourth satellites is intercepted, like that of the first, by the body of the planet. This is determined by the place of the earth from which the visual ray e J c' is directed to the eastern edge of the section of the shadow at the planet's distance. Within this limit the phenomena for the third and fourth satellites are altogether similar to those already explained in the case of the first and second satellites seen from E.

When the earth is between ϵ and ϵ' no eclipses can be witnessed. Those of the satellites are rendered invisible by the interposition of the planet, and those of the planet by the interposition of the satellites.

When the earth is at e' and E$'$, the phenomena are similar to those manifested at e and E, but they are exhibited in a different order and direction. The occultation of the satellite precedes its eclipse, and the latter takes place to the east of the planet. In like manner, the transit of the satellite precedes the eclipse of the planet.

The student, aided by the diagram, and what has been explained, will find no difficulty in tracing these and other consequences.

2958. *Phenomena predicted in Nautical Almanack.*—The times of the occurrence of all these several phenomena are calculated and predicted with the greatest precision, and may be found registered in the Nautical Almanack, with the diagrams for each month, to aid the observer. The mean time of their occurrence at Greenwich is there given, so that if the time at which any of them are observed to occur in any other place be observed, the difference of such local time and that registered in the Almanack will give the longitude of the place east or west of the meridian of Greenwich.

2959. *Motion of light discovered, and its velocity measured by means of these eclipses.*—Soon after the invention of the telescope, Roemer, an eminent Danish astronomer, engaged in a series of observations, the object of which was the discovery of the exact time of the revolution of one of these bodies around Jupiter. The mode in which he proposed to investigate this was, by observing the successive eclipses of the satellite, and noticing the time between them.

Now if it were possible to observe accurately the moment at which the satellite would, after each revolution, either enter the shadow, or emerge from it, the interval of time between

these events would enable us to calculate exactly the velocity and motion of the satellite. It was, then, in this manner that Roemer proposed to ascertain the motion of the satellite. But, in order to obtain this estimate with the greatest possible precision, he proposed to continue his observations for several months.

Let us, then, suppose that we have observed the time which has elapsed between two successive eclipses, and that this time is, for example, forty-three hours. We ought to expect that the eclipse would recur after the lapse of every successive period of forty-three hours.

Imagine, then, a table to be computed in which we shall calculate and register before hand the moment at which every successive eclipse of the satellite for twelve months to come shall occur, and let us conceive that the earth is at A, at the commencement of our observations: we shall then, as Roemer did, observe the moments at which the eclipses occur, and compare them with the moments registered in the table.

Let the earth, at the commencement of these observations, be supposed at E, *fig.* 752., where it is nearest to Jupiter. When the earth has moved to E″, it will be found that the occurrence of the eclipse is *a little later* than the time registered in the table.

As the earth moves from E″ towards E‴, the actual occurrence of the eclipse is more and more retarded beyond the time of its computed occurrence, until at E‴, in conjunction, it is found to occur about sixteen minutes later than the calculated time.

By observations such as these, Roemer was struck with the fact that his predictions of the eclipses proved in every case to be wrong. It would at first occur to him that this discrepancy might arise from some errors of his observations; but, if such were the case, it might be expected that the result would betray that kind of irregularity which is always the character of such errors. Thus it would be expected that the predicted time would sometimes be later, and sometimes earlier, than the observed time, and that it would be later and earlier to an irregular extent. On the contrary, it was observed, that while the earth moved from E to E‴, the observed time was continually later than the predicted time, and, moreover, that the interval by which it was later continually and regularly increased. This

was an effect, then, too regular and consistent to be supposed to arise from the casual errors of observation; it must have its origin in some physical cause of a regular kind.

The attention of Roemer being thus attracted to the question, he determined to pursue the investigation by continuing to observe the eclipses. Time accordingly rolled on, and the earth, transporting the astronomer with it, moved from E‴ to E′.

It was now found, that though the time observed was later than the computed time, it was not so much so as at E‴; and as the earth again approached opposition, the difference became less and less, until, on arriving at E, the position of opposition, the observed eclipse agreed in time exactly with the computation.

From this course of observation it became apparent that the lateness of the eclipse depended altogether on the increased distance of the earth from Jupiter. The greater that distance, the later was the occurrence of the eclipse as apparent to the observers, and on calculating the change of distance, it was found that the delay of the eclipse was exactly proportional to the increase of the earth's distance from the place where the eclipse occurred. Thus, when the earth was at E‴, the eclipse was observed sixteen minutes, or about 1000 seconds, later than when the earth was at E. The diameter of the orbit of the earth, E E‴, measuring about two hundred millions of miles, it appeared that that distance produced a delay of a thousand seconds, which was at the rate of two hundred thousand miles per second. It appeared, then, that for every two hundred thousand miles that the earth's distance from Jupiter was increased, the observation of the eclipse was delayed one second.

Such were the facts which presented themselves to Roemer. How were they to be explained? It would be absurd to suppose that the actual occurrence of the eclipse was delayed by the increased distance of the earth from Jupiter. These phenomena depend only on the motion of the satellite and the position of Jupiter's shadow, and have nothing to do with, and can have no dependence on, the position or motion of the earth, yet unquestionably the time they *appear* to occur to an observer upon the earth, has a dependance on the distance of the earth from Jupiter.

To solve this difficulty, the happy idea occurred to Roemer that the moment at which we see the extinction of the satellite

by its entrance into the shadow is not, in any case, the very moment at which that event takes place, but sometime afterward, viz., such an interval as is sufficient for the light which left the satellite just before its extinction to reach the eye. Viewing the matter thus, it will be apparent that the more distant the earth is from the satellite, the longer will be the interval between the extinction of the satellite and the arrival of the last portion of light which left it at the earth; but the moment of the extinction of the satellite is that of the commencement of the eclipse, and the moment of the arrival of the light at the earth is the moment the commencement of the eclipse is observed.

Thus Roemer, with the greatest felicity and success, explained the discrepancy between the calculated and the observed times of the eclipses; but he saw that these circumstances placed a great discovery at his hand. In short, it was apparent that light is propagated through space with a certain definite speed, and that the circumstances we have just explained supply the means of measuring that velocity.

We have shown that the eclipse of the satellite is delayed one second more for every two hundred thousand miles that the earth's distance from Jupiter is increased, the reason of which obviously is, that light takes one second to move over that space; hence it is apparent that the velocity of light is at the rate, in round numbers, of two hundred thousand miles per second.

By more exact observation and calculation the velocity is found to be 192,000 miles per second, the time taken in crossing the earth's orbit being 16m. 26·6s.

2960. *Eclipses of Saturn's satellites not observable.*—Owing to the obliquity of the orbits of the Saturnian satellites to that of the primary, eclipses only take place at or near the equinoxes of the planet, the satellites revolving nearly in the common plane of the equator and the ring. When they do take place, these eclipses are so difficult of observation as to be practically useless for the determination of longitudes, and have, consequently, received but little attention.

IV. Transits of the Inferior Planets.

2961. *Conditions which determine a transit.*—When an inferior planet, being in inferior conjunction, has a less latitude

or distance from the ecliptic than the sun's semi-diameter, it will be less distant from the sun's centre than such semi-diameter, and will therefore be within the sun's disk. In this case, the planet being between the earth and sun, its dark hemisphere being turned towards the earth, it will appear projected upon the sun's disk as an intensely black round spot. The apparent motion of the planet being then retrograde, it will appear to move across the disk of the sun from east to west in a line sensibly parallel to the ecliptic.

Such a phenomenon is called a TRANSIT, and as it can only take place with planets which pass between the earth and sun, it is limited to Venus and Mercury.

Notwithstanding the very small obliquity of the orbits of these planets, it is evident that transits can only take place when the planet is within an extremely small distance of its node. Let N be the node, *fig.* 806, *s* the centre of the sun's disk on the ecliptic, at the distance N*s* from the node at which the edge *p* of the disk just touches the orbit N*p* of the planet. It is evident that a transit can only take place when the sun's centre is at a less distance than N*s* from the node.

Fig. 806.

Let O express the obliquity *p*N*s* of the planet's orbit. The mean value of the semi-diameter *sp* of the sun being 16′ or 0°·266, we shall have (2294.)

$$\text{N}s = 0°{\cdot}266 \times \frac{57{\cdot}3}{\text{O}} = \frac{15°{\cdot}24}{\text{O}}.$$

The obliquity of Mercury's orbit being 7″, we shall have

$$\text{N}s = \frac{15°{\cdot}24}{7} = 2°{\cdot}18 = 2°\ 10',$$

and that of Venus' orbit being 3°·39, we shall have

$$\text{N}s = \frac{51°{\cdot}24}{3{\cdot}39} = 4°{\cdot}5 = 4°\ 30'.$$

Thus the distances from the node within which transits take place, the planet being in conjunction, are 2° 11′ for Mercury, and 4° 30′ for Venus.

2962. *Intervals of the occurrence of transits.* — The transits of Mercury and Venus are phenomena of rare occurrence,

especially those of Venus, and they are separated by very unequal intervals. The following are the dates of the successive transits of Mercury during the latter half of the present century:—

1845 - - -	May 8.
1848 - - -	Nov. 9.
1861 - - -	Nov. 11.
1868 - - -	Nov. 4.
1878 - - -	May 6.

Those of Venus occur only at intervals of 8, 122, 8, 105, 8, 122, &c. years. Two only will take place in the present century—in 1874 and 1882.

2963. *The sun's distance determined by the transit of Venus.*—The transits of Venus have acquired immense interest and importance, from the circumstance of their supplying data by which the sun's distance from the earth can be determined with far greater precision than by any other known method. The transits of Mercury would supply like data, but owing to the greater distance of that planet from the earth when in inferior conjunction, the conditions affecting the data are not nearly so favourable as those supplied by Venus.

The details of this celebrated problem are much too complicated, and involve calculations too long and intricate, to admit of being fully explained here, but there is no difficulty in rendering the principle and spirit of the solution intelligible.

It will be observed that although the exact determination of the absolute distances of the planets from the sun be attended with some difficulty, there is none in the determination of their relative distances. The observation of their synodic motions, which may be made with great precision, supplies the data necessary for the solution of this problem (2593.), and it is thus ascertained that the distances of the earth and Venus from the sun are in the ratio of 1000 to 723.

It follows, therefore, that when Venus is directly interposed between the earth and sun, as she always must be when a transit takes place, the ratio of her distances from the earth and sun is that of 277 to 723.

Let V, *fig.* 807., represent the place of the planet at conjunction, and let *e e* and s s′ be two lines taken at right angles to the plane of the ecliptic, and, therefore, to the direction of

the planet's motion. The planet V, viewed from any points, such as p and p', upon the line $e\,e'$, will be seen as projected on corresponding points P and P' of the line S S'. Now it is evident that the distance between the points P and P' will bear to the distance between the points p and p' the same ratio as V P bears to V p, that is, 723 to 277, or 2·61 to 1.

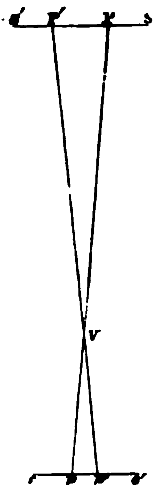


Fig. 807.

If, therefore, the distance between the points p and p' upon the earth be known, or can be ascertained (which it always may be), the distance between the corresponding points P and P' on the sun will be

$$\text{PP}' = pp' \times 2{\cdot}61.$$

If the points P and P' were visibly marked upon the sun so that the apparent distance between them could be exactly measured with the micrometer, the linear value of 1″ at the sun would be ascertained; for if a express, in seconds, the apparent distance between P and P' thus ascertained, we should have for the linear value s of 1″

$$s = \frac{\text{PP}'}{a} = \frac{pp' \times 2{\cdot}61}{a}.$$

Supplied with this datum, the distance d of the sun from the earth would be (2294.)

$$d = \frac{pp' \times 2{\cdot}61}{a} \times 206265.$$

If the places of observation p and p' be not placed upon a line at right angles to the ecliptic, its projection on such a line can be ascertained by computation, and may be substituted for it.

It is evident, therefore, that the problem is reduced to the determination of the apparent angle subtended at the earth by the two points of the sun's disk P and P', upon which the planet is projected when viewed from the two places p and p' upon the earth.

But since the black spot formed by the projection of the planet is in continual motion on the disk of the sun, it would be

impracticable to determine its position at any given moment with the degree of precision necessary to compare observations of this delicate kind made at distant places on the earth. Besides which, to render the observations made at precisely the same moment of absolute time comparable, it would be necessary that the difference of the longitudes of the stations be known with a degree of precision not attainable under the circumstances.

A happy expedient, however, has been imagined, by which this difficulty has been effectually surmounted. It will be remembered that the motion of the planet V is at right angles to the plane of the diagram, and therefore to the line S S′ supposed to be drawn upon the disk of the sun. The apparent paths of the projections P and P′ of the planet on the sun's disk will therefore be also at right angles to this line S S′, and parallel to each other, being, in fact, both parallel to the ecliptic.

Let S S′, *fig.* 808., represent the disk of the sun which is at right angles to the line joining the earth and planet with the sun's centre, and therefore to the plane of *fig.* 807. Let P and P′, *fig.* 808., represent two points, upon which the planet is simultaneously projected, as viewed from *p* and *p′*, *fig.* 807. Let S S′, *fig.* 808., be that diameter of the sun's disk which is in the plane of the ecliptic. The apparent paths of the projections of the planet on the sun's disk will be parallel to S S′, and will therefore be *m n* and *m′ n′*. Now, if the planet, as it moves, were to leave a permanent mark upon the disk of the sun, indicating the line its projection followed, seen from each place, it would be an easy matter to measure the apparent distance P P′ between these lines, and thus solve the problem. No such permanent mark, nor any other visible indication, however, exists.

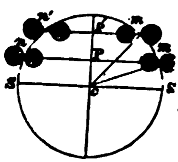


Fig. 808.

The synodic motion of the planet, that is to say, its motion relatively to the sun, however, being computed and known with the utmost precision, this motion has, with the greatest felicity and success, been used as a means of estimating the distances between the chords *m n* and *m′ n′* of the sun's disk, along which the planet appears to move. The time taken at each place by the planet to move over the chords *m n* and *m′ n′* being exactly observed, and the rate of the apparent motion of the planet being

exactly known, the number of seconds in each of the chords can be ascertained. From these data the length of P P′ may be computed. We shall have

$$c\,P^2 = c\,m^2 - m\,P^2$$
$$c\,P'^2 = c\,m'^2 - m'\,P'^2.$$

The distances cP and c P′ being thus determined, their difference will be P P′, which will therefore be known.

To determine with the necessary precision the duration of the transit at each place of observation, it is necessary to ascertain the exact moment at which the centre of the planet's disk crosses the limb of the sun at the beginning and end of the transit, but as the centre of the planet's disk is not marked by any visible or distinguishable point, this cannot be directly observed. It is ascertained by noting, as precisely as possible, the times of external and internal contact of the planet's disk with the limb of the sun, both at the beginning and end of the transit. The middle of the interval between external and internal contact in each case, is the moment of the passage of the centre of the planet's disk over the limb.

It is evident that the solution of this problem includes the determination of the horizontal parallax of the sun, for the linear value of 1″ at the sun as seen from the earth is the same as the linear value 1″ at the earth as seen from the sun. By the method just explained, this is found to be at the mean distance, 466 miles, and since the horizontal parallax π of the sun is the angle which the semi-diameter of the earth subtends at the sun, it is

$$\pi = \frac{3963}{466} = 8''\cdot5,$$

and the distance itself of the sun is

$$d = 206265 \times 466 = 96{,}119{,}490 \text{ miles.}$$

In the practical application of this method various circumstances are taken into account, such as the effects of the diurnal rotation of the earth, &c.

V. Occultations.

2964. *Occultation defined.*—When any celestial object, the sun excepted, is concealed by the interposition of another, it is

said to be OCCULTED, and the phenomenon is called an OCCULTATION.

Strictly speaking, a solar eclipse is an occultation of the sun by the moon, but usage has given to it, by exception, the name of an eclipse.

2965. *Occultations by the moon.* — The phenomena of this class, which possess greatest astronomical interest, are those of stars and planets by the moon. That body, measuring about half a degree in diameter, moves in her monthly course so as to occult every object on the firmament which is included in a zone extending to a quarter of a degree at each side of the apparent path of her centre. All the stars whose places lie in this zone are successively occulted, and disappearances and reappearances of the more conspicuous ones, as well as those of the planets which may be found within the limits of the same zone, present some of the most striking effects which are witnessed by observers.

The astronomical amateur will find in the Nautical Almanack a table in which all the principal occultations, both of stars and planets, are predicted.

The disappearance takes place always at the limb of the moon, which is presented in the direction of its motion.

From the epoch of full moon to that of new moon the moon moves with the enlightened edge foremost, and from new moon to full moon with the dark edge foremost. During the former interval, therefore, the objects occulted disappear at the enlightened edge, and reappear at the dark edge, and during the latter period they disappear at the dark, and reappear at the enlightened edge.

The disappearances and reappearances when the moon is a crescent are especially remarkable. If the disappearance take place at the convex edge, notice of its approach is given by the visible proximity of the star, which, at the moment of contact, is suddenly extinguished. Its reappearance is more startling, for it seems to be suddenly lighted up at a point of the firmament nearly half a degree from the concave edge of the crescent. If the disappearance take place at the dark edge it is much more striking, the star appearing to "go out" of itself at a point of the sky where nothing interferes with it.

The moon's horizontal parallax amounting to nearly twice its diameter, the part of the firmament on which it is projected and

which is its apparent place, differs at different parts of the earth. In different latitudes the moon, therefore, in the course of the month, appears to traverse different zones of the firmament, and consequently to occult different stars. Stars which are occulted in certain latitudes are not occulted at all at others, and of those which are occulted, the durations of the occultation and the moments and places of disappearance and reappearance are different.

To render this more intelligible, let N S, *fig.* 809., represent the earth, N being its north, and S its south pole. Let *m m'* represent the moon, and *m** and *m'** the direction of a star which is occulted by it. It must be observed, that the distance of the star being practically infinite compared with the diameter of the moon, the lines and *m** and *m'** are parallel. Let these lines be supposed to be continued to meet the earth at *l* and *l'*. Let similar lines, parallel to these, be imagined to be drawn through all points of a section of the moon made by a plane at right angles to the direction of the star passing through the moon's centre. Such lines would form a cylindrical surface, the base of which would be the section of the moon, and it would be intersected by the surface of the earth, a portion of which would be included within it, one half of which is represented by the darkly shaded part of the earth between *l* and *l'*. It is clear that the star will be occulted by the moon to all observers situated within this space.

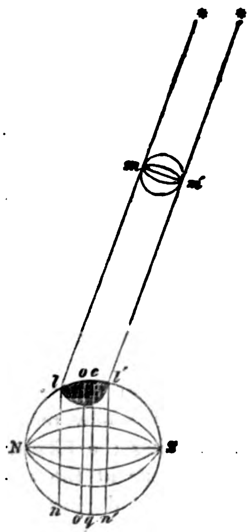


Fig. 809.

While this cylindrical space is carried by the moon's orbital motion from west to east, the surface of the earth included between the parallels of latitude *l n* and *l' n'*, is also carried from west to east, but much more rapidly, by the diurnal rotation, so that the places between these parallels are continually overtaking the cylindrical space which limits the occultation.

It is evident that beyond *l n* and *l' n'*, which are called the "limiting parallels," no occultation can take place. At *l* and *l'* the star is seen just to touch the moon's limb without being

occulted, but within those limits it will be occulted. The middle parallel $o\ o'$, between the limiting parallels, is that at which a central occultation is seen, and where therefore the duration is greatest.

The occultation *may* be seen from any place upon the earth which lies within the shaded zone, and *will* be seen provided the phenomenon occur during the night, and that the star at the time be above the horizon at such an altitude as to render the event observable.

In the Nautical Almanack these "limiting parallels" for every conspicuous occulation are tabulated, as well as the data necessary to enable an observer at any proposed latitude to ascertain previously whether any particular occultation will be observable.

2966. *Determination of longitudes by lunar occultations.* — In common with all phenomena which can be exactly predicted, and whose manifestation is instantaneous, occultations of stars by the moon are eminently useful for the exact determination of longitudes. The frequency of their occurrence greatly increases their utility in this respect, and although, for nautical purposes, the observer cannot always choose his time of observation, and therefore cannot be left dependent on them, they come in aid of the lunar method as verifications; and for geographical purposes on land, are among the best means which science has supplied. The times of the disappearance and reappearance, as observed, being compared with the Greenwich times tabulated in the Nautical Almanack, the difference of the longitudes is inferred after applying the necessary corrections.

2967. *Occultations indicate the presence or absence of an atmosphere around the occulting body.*—When a star is occulted by the disk of the moon or planet, its brightness, previously to its disappearance, would be more or less dimmed by the atmosphere surrounding such object, if it existed. Such a gradual decrease of brightness previously to disappearance, as well as a like increase of brightness after reappearance, is observable in occultations by the disks of planets, but never by the disk of the moon.

It is hence inferred that the planets have, and the moon has not, an atmosphere.

It might be objected that the lunar atmosphere may not have sufficient density to produce any sensible diminution of bright-

ness. Another test has, however, been found in the effect which the refraction of an atmosphere would have in decreasing the duration of an occultation (2483.). No such decrease being observed, it is inferred that no atmosphere exists around the moon.

2968. *Singular visibility of a star after the commencement of occultation.*—Some observers, of sufficient weight and authority to command general confidence, have occasionally witnessed a phenomenon in occultations which has been hitherto unexplained. According to them, it sometimes happens that after the occulted star has passed behind the limb of the moon it continues to be seen, and even for a considerable time, notwithstanding the actual interposition of the body of the moon. If this be not an optical illusion, and if the visual rays actually come straight to the observer, they must pass through a deep fissure in the moon. Such a supposition is compatible with the rare, and apparently fortuitous, occurrence of the phenomenon.

2969. *Suggested application of lunar occultations to resolve double stars.*—Sir J. Herschel thinks that these occultations would supply means of ascertaining the double character of some stars, the individuals suspected to compose which are too close together to be divided by any telescope. He thinks, nevertheless, that they might disappear in perceptible succession behind the edge of the moon's disk. It does not seem to be easy to conceive how such an effect can be expected in a case where the most powerful telescopes have failed to resolve the stars.

2970. *Occultations by Saturn's rings.*—In the case of stars occulted by Saturn's rings, a reappearance and second disappearance may be seen in the open space between the ring and the planet. It has been affirmed also, that a momentary reappearance of a star, in the space which intervenes between the rings, has been witnessed. This observation does not, however, seem to have been repeated, notwithstanding the recent improvements in the telescope, and the increased number of observers. The passage of the planet, in a favourable phase of the ring, through the neighbourhood of the milky way, which is so thickly strewed with stars, would afford an opportunity of testing this, and might also supply decisive evidence, positive or negative, upon the question of the existence of more than two concentric rings. If other black streaks

seen upon the surface of the ring be, like the principal one, real openings between a multiple system of rings, the stars sprinkled in such countless numbers over the regions of the galaxy, and the adjacent parts of the firmament, would be seen to flash between ring and ring, as the planet passes before them. Such observations, however, would require in the telescope the very highest attainable degree of optical perfection.

CHAP. XVII.

TABULAR SYNOPSIS OF THE SOLAR SYSTEM.

2971. *Planetary data.*—Having explained, individually, the circumstances attending the physical condition and motion of the bodies of the system, it remains to bring them into juxtaposition, to view them collectively, and to supply, in tabulated forms, those numerical data which at any given time determine their positions and motions.

Such data may be resolved into three classes:—

I. Those which determine the orbit.

II. Those which determine the place of the body in the orbit.

III. Those which determine the conditions which are independent of the orbit.

I. *Data which determine the form, magnitude, and position of the orbits of the planets.*

2972. *Form of the orbit determined by the eccentricity.*—It is well understood (2606.) that the form of an ellipse depends on the eccentricity, all ellipses with the same eccentricity, however they may differ in magnitude, having the same form.

Let a = the mean distance, c = the distance of the centre of the orbit from the centre of the sun, and e = the eccentricity. We shall then have

$$e = \frac{c}{a}, \qquad c = a \times e.$$

The values of e, in the cases of all the principal planets,

Mercury excepted, are less than $\frac{1}{10}$. In the case of Mercury, it is about $\frac{1}{5}$, and in the larger planets $\frac{1}{20}$.

In the case of the planetoids, the eccentricities are subject to great and exceptional variation, amounting in one case to $\frac{1}{3}$, and in others being less than $\frac{1}{20}$.

2973. *Magnitude determined by semi-axis major.*—As the eccentricity determines the form, the semi-axis determines the magnitude, of the orbit. This quantity forms in other respects a very important planetary element, since upon it is dependent also, by the Harmonic law, the periodic time, and, consequently, the mean angular and mean linear velocity in the orbit.

2974. *Position of the plane of the orbit.*—The plane of the orbit must always pass through the centre of the sun, which is therefore the common point at which the planes of all the planetary orbits intersect. But to define the position of the plane of any orbit something more is necessary. If the plane of the earth's orbit be provisionally assumed as a fixed plane, (which however it is not, as will appear hereafter), the positions of the planes of the orbits of the planets, severally, with relation to it, will be determined, 1st, by the angle at which they intersect it, and, 2ndly, by the direction of the line of intersection.

2975. *Inclinations of the orbits.*—The angles which the planes of planets' orbits form with the plane of the ecliptic are generally less than 3°. Of the principal planets, Mercury forms an exception to this, having an inclination of 7°. The planetoids are also exceptional, the orbit of one having an inclination of 34½°; the inclinations of the others varying between 16° and 1°.

2976. *Line of nodes.*—The inclination is not enough to determine the position of the plane of the orbit, for it is evident that an infinite variety of different planes may be inclined at the same angle to the ecliptic. If, however, the direction of the line of intersection of the plane of the orbit with the plane of the ecliptic (which line must always pass through the centre of the sun) be also defined, the position of the plane of the orbit will be determined. This line of intersection is called the *line of nodes*, being the direction in which the nodes of the planet's orbit are seen from the sun. If an observer be imagined to be stationed at the centre of the sun, he will be in this line, and the nodes will be viewed by him in opposite directions along this line, the ascending node (2625.) being

viewed in one direction, and the descending node in the other.

2977. *Longitude of ascending node.*—It has been customary to define the direction of the line of nodes by the angle which the direction of the ascending node, seen from the sun, makes with the direction of the "first point of Aries," or, what is the same, by its heliocentric longitude.

The position of the plane of the orbit is, therefore, determined by its *inclination* and the *longitude of the ascending node*.

2978. *Longitude of perihelion.*—These data, however, are still insufficient to determine the position of the orbit. They would be sufficient if the orbit were circular, since a circle is symmetrical with relation to its centre. But the orbit being an ellipse, the major axis may have an infinite variety of different directions, all of which shall pass through the sun's centre, and all of which shall be in the same plane. After defining, therefore, the position of the plane of the orbit, it is necessary to determine the position of the orbit upon that plane, and this is determined by the direction of its major axis, just as the plane itself was determined by the direction of the line of nodes, and as the latter was determined by the heliocentric longitude of the ascending node; the position of the orbit upon its plane is determined by the heliocentric longitude of perihelion (2608).

2979. *Five elements which determine the orbit.*—The orbit of a planet is, therefore, determined, in form, magnitude, and position, by the five following data, which are called its ELEMENTS:—

1. The semi-axis, or mean distance - -	a
2. The eccentricity - - - -	e
3. The inclination - - - -	i
4. The longitude of the ascending node - -	ν
5. The longitude of perihelion - - -	π

The eccentricity is sometimes expressed by the angle ϕ, of which e is the sine, which is called the "angle of eccentricity."

2980. *Elements subject to slow variation—Epoch.*—If the elements of the orbit were invariable, they would be always known when once ascertained. But it will appear hereafter, that although for short intervals of time they may, without sensible error, be regarded as constant, some of them are

TABLE I.

Data which determine the magnitude, form

Planet.	Sign.	Semi-axis.	Eccentricity.			
		a	e	ϕ		
				°	′	″
MERCURY - -	☿	0·3870985	0·2056063	11	51	50
VENUS - - -	♀	0·7233317	0·0068618	0	23	30
EARTH - - -	⊖	1·0000000	0·01679226	0	57	50
MARS - - -	♂	1·523691	0·0932168	5	20	10
PLANETOIDS:						
Flora - - -		2·201727	0·1567974	9	1	15·0
Melpomene - -		2·295713	0·2169123	12	28	5
Victoria (Clio)} -		2·335008	0·2181980	12	35	18·2
Thetis - -		2·343192	0·0464569	6	24	39·83
Vesta - - -	⚶	2·361702	0·0888410	5	5	55·6
Massalia - -		2·375851	0·1338916	7	41	40
Iris - - -		2·385310	0·2323515	13	26	16·2
Metis - - -		2·386897	0·1228221	7	1	26·4
Hebe - - -		2·425368	0·2020077	11	38	44·4
Fortuna - -		2·445902	0·1555438	9	3	55·67
Parthenope - -		2·448097	0·058 302	5	43	27·0
Astræa - -		2·577400	0·1887517	10	52	47·8
Irene - - -		2·581951	0·1697575	9	46	20
Egeria - -		2·582492	0·0862748	4	57	0
Lutetia - -		2·604770	0·3398105	19	52	0
Eunomia - -		2·650918	0·1893392	10	47	29·24
Juno - - -	⚵	2·669095	0·2560780	14	50	34·3
Ceres - - -	⚳	2·766921	0·0763660	4	27	18·3
Pallas - -	⚴	2·772896	0·2394280	13	49	15·7
Psyche - -		2·932951	0·1309378	7	31	20
Hygeia - -		3·151388	0·1009159	5	47	30·9
Calliope - -		?	?	?	—	—
Thalia - -		?	?	?	—	—
Phocea * - -		?	?	?	—	—
JUPITER - -	♃	5·202767	0·0481621	2	45	40
SATURN - -	♄	9·538850	0·0561505	3	13	8
URANUS - -	♅	19·1824	0·0466	2	40	15
NEPTUNE - -	♆	30·04	0·0087195	0	29	58·5

* This planetoid was discovered on the 6th of April, 1853 (since p. 327. of this volume and about the same day another was discovered by M. Gasparis at Naples.

TABLE I.

and position of the Planetary Orbits.

Inclination.			Longitude of ascending Node.			Longitude of Perihelion.			Epoch.
i			☊			ϖ			M. T. Paris.
°	′	″	°	′	″	°	′	″	
7	0	5	45	57	38	74	20	42	1 January, 1800.
3	23	29	74	51	41	128	43	6	,,
0	0	0	0	0	0	99	30	29	,,
1	51	6	47	59	38	332	22	51	,,
5	53	3	110	20	53	32	49	45	24·0 March. 1852.
10	10	38	149	54	23	15	35	53	10·0 July, 1852.
8	23	7	235	29	31	301	55	18	0·0 January, 1851.
5	42	32	128	5	50	271	25	26	1·0 May, 1852.
7	8	25	105	23	14	250	44	3	3·0 November, 1852.
0	50	16	203	20	3	144	1	49	0·0 October, 1852.
5	28	16	259	44	5	41	20	22	8·0 June, 1852.
5	35	55	68	28	58	71	33	11	4·0 June, 1852.
14	46	32	138	31	55	15	15	26	13·0 July, 1852.
1	33	18	211	0	9	31	16	13	23·5 September, 1852.
4	36	54	124	59	54	317	3	51	13·0 July, 1852.
5	19	23	141	27	48	135	42	32	29·5 April, 1851.
9	5	33	86	51	33	178	26	58	13·0 July, 1852.
16	33	7	43	17	40	118	17	17	15·0 March, 1852.
3	19	49·9	78	38	43·4	309	53	19·4	0·0 Dec. 1852, M. T. Ber.
11	43	50	293	53	19	27	13	24	13·0 October, 1852.
13	3	17	170	56	28	54	18	55	24·0 September, 1852.
10	37	12	80	49	50	148	2	54	2·0 July, 1852.
34	37	20	172	45	14	121	24	11	2·0 July, 1852.
3	3	37	150	36	43	11	28	9	31·0 March, 1852.
3	47	11	287	38	27	218	2	29	28·5 September, 1851.
?	—	—	?	—	—	?	—	—	
?	—	—	?	—	—	?	—	—	
?	—	—	?	—	—	?	—	—	
1	18	52	98	25	45	11	7	38	1 January, 1800.
2	29	36	111	56	7	89	8	20	,,
0	46	28	72	59	21	167	30	24	,,
1	46	59	130	6	52	47	14	37	,,

was printed), by M. Chacornac (the discoverer of Massalia), at the Marseilles Observatory,

subject to slow variations, which, after long intervals, such, for example, as centuries, completely change the orbits. These variations have been calculated with surprising precision, and are, moreover, found to be periodical, although their periods are in general of such magnitude, as to surpass not only the limits of human life, but those of all human record.

Since, therefore, the planetary orbits are thus subject to a slow but constant change, it is necessary in assigning their elements, to assign also the date at which the orbits had these elements. When the rates at which the elements severally vary are known, their values at any assigned date being given, their values at any other date, anterior or posterior, can be determined.

The date at which the elements of the orbits have had the values assigned to them is technically called the EPOCH.

2981. *Table of the elements of the orbits.*—In the preceding table are given the elements of the planetary orbits, severally, at the epochs assigned in the last column. Those of the more recently discovered planets must be regarded as provisional, and subject to such corrections as future observations may suggest.

The elements are taken from the tables published by the French Board of Longitude, with the exception of those of the recently discovered planetoid LUTETIA, the elements of which are given, provisionally, from those calculated by M. George Rumker, Jun., of Hamburg. (Comptes Rendus de l'Acad. des Sc. t. xxxv. p. 810.)

To illustrate the relative mean distances of the planets from the sun, and from each other, we have delineated, nearly in their proper proportions, the mean distances of the principal planets, and the planetoids or asteroids, in *fig.* 810.

II. *Data to determine the place of the planet.*

2982. *By the epoch and the mean daily motion.* — The orbit being defined in magnitude, form, and position, it is necessary to supply the data by which the position of the planet in it at any assigned time may be found. It will be sufficient for this to assign the position which the planet had at the EPOCH, and the periodic time, from which the mean daily motion of the planet can be inferred. By means of this motion, the mean

Fig. 810.

place of the planet for any given time anterior or posterior to the epoch can be determined.

2983. *The equation of the centre.*—To find the true place of the planet a further correction, however, is necessary. The angular velocity of the planet referred to the sun is not uniform, being greatest at perihelion and least at aphelion. The difference between the position which the planet would have, as seen from the sun, if its angular motion were uniform and that which it actually has, is called the "equation of the centre," and tables are computed by which this correction for each planet may be made, so that, the mean place of the planet in its orbit being determined, the true place may be found.

2984. *Table of the data necessary to determine the place of the planet.*—In the following table are given the data which are necessary to determine the mean place of each of the planets for any given time. In the first column is given the mean longitude of the planet at the EPOCH assigned in Table I., and in the second column is given the mean daily increment of heliocentric longitude.

The SIDEREAL PERIOD, or the time which the planet takes to make a complete revolution round the sun, is given in days and years in the third and fourth columns.

If the equinoctial points were fixed, the sidereal period would be equal to the interval between two successive returns of the planet to the same equinoctial point. But the equinoctial points are subject, as will appear hereafter, to a very slow retrograde motion, in virtue of which the first point of Aries, from which right ascensions and longitudes are measured,

moves annually from the east to west upon the ecliptic through a space a little less than a degree. A planet, therefore, departing from the vernal equinoctial point, and moving constantly from west to east, will return to that point before it completes its revolution, inasmuch as that point moving in the contrary direction meets it before its return to the point of departure.

It follows from this, that the interval between two successive returns to the vernal equinoctial point is a little less than the sidereal period. This interval is called the *equinoctial period*, and is given in the fifth column of Table II.

The synodic period is given in the last column.

TABLE II.

Data which determine the places of the Planets in their respective Orbits at a given time.

Name.	Longitude at Epoch. ε			Daily Motion. h	Period. Sidereal.		Period. Equinoctial.	Period. Synodic.
	°	′	″	″	Days — P.	Years — P′.	Days.	Days.
MERCURY -	110	13	18	14732·419	87·96926	0·2408	87·9684	115·87
VENUS -	146	44	56	5767·668	224·70080	0·6152	224·6955	583·92
EARTH -	100	53	30	3548·193	365·25637	1·0000	365·242255	
MARS -	233	5	34	1886·518	686·97964	1·8807	686·9297	779·88
PLANETOIDS:								
Flora -	174	46	5	1086·0790	1193·281	3·2668	1193·2	[illegible]·3
Melpomene -	302	14	20	1020·0708	1270·498	3·4785	1270·4	[illegible]·0
Victoria (Clio)	7	42	5	994·4325	1303·255	3·5680	1303·0	507·5
Thetis -	199	3	37	989·2240	1310·116	3·5870	1309·87	506·0
Vesta -	35	59	53	977·6178	1325·669	3·6290	1325·3	504·2
Massalia -	13	17	36	968·8970	1337·601	3·6620	1337·3	502·0
Iris -	85	45	6	963·1396	1345·600	3·6837	1345·5	501·3
Metis -	255	13	26	962·1801	1346·9400	3·6890	1346·7	501·2
Hebe -	47	26	23	939·3772	1379·635	3·7770	1379·4	49[illegible]·8
Fortuna -	355	4	21	927·5728	1397·192	3·8250	1397·0	[illegible]4·0
Parthenope -	86	3	24	926·3257	1399·074	3·8300	1399·0	49[illegible]·8
Astræa -	197	37	33	857·4996	1511·369	4·1380	1511·1	481·6
Irene -	323	47	51	855·2337	1515·373	4·1490	1515·0	481·5
Egeria -	162	29	20	854·9642	1515·850	4·1503	1515·6	481·5
Lutetia -	61	29	233	844·0179	1535·5	4·2040	1535·3	479·0
Eunomia -	47	43	44	822·0764	1576·493	4·3161	1576·3	475·5
Juno -	22	25	8	813·6926	1592·736	4·3550	1592·5	473·8
Ceres -	145	10	55	770·9242	1681·093	4·5995	1680·8	466·3
Pallas -	123	49	27	768·6413	1686·089	4·6160	1679·8	466·1
Psyche -	149	21	19	706·3977	1834·658	5·0230	1833·0	439·0
Hygeia -	356	45	31	634·2404	2043·386	5·5942	2042·9	444·7
Calliope -	?	—	—	?	?	?	?	?
Thalia -	?	—	—	?	?	?	?	?
Phocea -								
JUPITER -	81	54	49	298·989	4332·58480	11·86	4330·6105	398·8
SATURN -	123	6	29	120·435	10759·2198	29·46	10746·7324	378·0
URANUS -	173	30	37	42·233	30686·8205	84·01	30589·3673	369·7
NEPTUNE -	335	8	58	21·554	60127·	164·62	59743·	367·5

2985. *Table of extreme and mean distances from sun and earth.*—In Table I., the mean distances a of the several planets from the sun are expressed in numbers, of which the earth's mean distance is the unit. It is necessary, however, to compute the actual mean distances in some known units, such as miles. To obtain these it is only necessary to multiply the actual mean distances of the earth in miles by the numbers in the column a of Table I.

It is also necessary to assign the actual limits of the varying distances of the planets as well from the earth as from the sun. These are easily determined by the data in Table I.

2986. *Perihelion and aphelion distances.*—Let the extreme and mean distances of the earth from the sun, expressed in millions of miles, be

$$d = \text{mean distance}$$
$$d' = \text{least distance}$$
$$d'' = \text{greatest distance}:$$

we shall then have, according to what has been already explained and proved,

$$d = 95 \quad d' = 95 \times (1 - e) \quad d'' = 95 \times (1 + e),$$

the value of e in the case of the earth being 0·01679226.

Let the mean and extreme distances of a planet from the sun be in like manner expressed by $\text{D}, \text{D}', \text{D}''$ in million of miles, and we shall have

$$\text{D} = 95\,a \quad \text{D}' = 95\,a \times (1 - e) \quad \text{D}'' = 95\,a \times (1 + e).$$

The distance S of a planet from the earth at superior conjunction being equal to the sum of the distances of the earth and planet from the sun, we shall have

$$\text{S} = \text{D} + d.$$

This will vary, because the distances from the sun vary. It will be greatest when the earth and planet are both in aphelion, and least when they are both in perihelion. If S'', therefore, express the greatest, and S' the least possible, distance of the planet when in conjunction, the mean being expressed by S, we shall have

$$\text{S}'' = \text{D}'' + d'' \qquad \text{S}' = \text{D}' + d'.$$

The distance of an inferior planet from the earth, when in inferior conjunction, is found by subtracting the planet's

distance from the sun from the earth's distance. If O express the mean distance of the planet in inferior conjunction from the earth, we shall have

$$O = d - D.$$

The distance will vary according to the relative positions of the axes of the elliptic orbits, and will evidently be greatest when the earth is in aphelion and the planet in perihelion, and least when the earth is in perihelion and the planet in aphelion. If O'' and O' then express, as before, the greatest and least possible distances of the planet in inferior conjunction, we shall have

$$O'' = d'' - D' \qquad O' = d' - D''.$$

The distance of a superior planet in opposition is found by subtracting the earth's distance from the planet's distance; and it may in like manner be shown that the mean and extreme distances of the planet in opposition from the earth will be

$$O = D - d \qquad O'' = D'' - d' \qquad O' = D' - d''.$$

In the following table the mean and extreme distances of the planets successively from the sun and earth are given as computed by these several formulæ. The method of computation is indicated at the head of each column. (See Table top of next page.)

III. *Conditions affecting the physical and mechanical state of the planet independently of its orbit.*

2987. In the preceding chapters we have explained and illustrated the methods by which the real magnitudes, masses, densities, diurnal rotation, oblateness, and superficial gravity of the planets are severally determined. These data and some others are brought together and arranged in juxtaposition, being expressed numerically, with relation to the most generally useful units in the Table IV.

The methods of computing many of the quantities and magnitudes given in the several columns of Table IV. have been already explained. Some of them, however, require further elucidation.

2988. *Method of computing the extreme and mean apparent diameters.* — The real diameters δ being ascertained by the methods explained in (2299.), the extreme variation of the

TABLE III.

Distances from the Sun and from the Earth in millions of Miles.

Name.	Distance from Sun.			Distance from Earth.					
				At superior Conjunction.			At opposite or inferior Conjunctions.		
	Greatest.	Least.	Mean.	Greatest.	Least.	Mean.	Greatest.	Least.	Mean.
	D'' $= 95a(1+e)$	D' $= 95a(1-e)$	D $= 95a$	δ'' $= D''+d''$	δ' $= D'+d'$	δ $= D+d$	O'' $= d''-D'$ or $= D''-d'$	O' $= d'-D''$ or $= d''-D'$	O $= d-D$ or $= D-d$
MERCURY -	44·32	29·32	36·77	140·91	122·73	131·77	67·27	49·09	58·23
VENUS -	69·18	68·24	68·71	165·77	161·65	163·77	28·35	24·23	26·29
EARTH -	96·59	93·41	95·00						
MARS -	158·24	131·26	144·75	254·83	224 67	239·77	64·83	34·67	49·75
PLANETOIDS:									
Flora -	212·45	205·89	209·17	309·04	299·30	304·77	119·04	109·30	114·17
Melpomene -	265·17	170·99	218·08	361·76	264·40	313·77	177·76	74·40	123·08
Victoria (Clio)	270·21	173·45	221·82	366·80	266·86	316 77	176·80	76·86	126·82
Thetis -	232·94	212·28	222·61	329·53	305·69	317·77	139·53	115·69	127·61
Vesta -	244·28	204·44	224·36	340·87	297·85	319·77	150·87	107·85	129·36
Massalia -	255·93	195·49	225·71	352·52	288·90	320·77	162·52	98·90	130·71
Iris -	279·25	173·97	226·61	375·84	267·38	321·77	125·84	77·38	131·61
Metis -	254·61	198·91	226·76	351·20	292·32	321·77	161·20	102·32	131·76
Hebe -	276·87	183·93	230·42	373·46	277·34	325·77	183 46	87·34	135·42
Fortuna -	268·50	196·24	232·37	365·09	289·65	327·77	175·09	99·65	137·37
Parthenope -	255·36	209·78	232·57	351·95	303·19	327 77	161·95	113·19	137·57
Astræa -	291·17	198·75	244·96	387·76	292·16	339·77	197·76	102·16	149·96
Irene -	286·90	203·66	245·28	383·49	297·07	340·77	193·49	107·07	150·28
Egeria -	266·51	224·17	245·34	363 10	317·58	340·77	173·10	107·58	150·34
Lutetia -	331·54	163·38	247·46	328·15	256·79	342·77	238·13	66·79	152·46
Eunomia -	299·51	204·17	251·84	396·10	297·58	346·77	206·10	107·58	156·84
Juno -	318·48	184·66	253·57	415·07	282·07	248·77	225·07	92·07	158·57
Ceres -	282·94	242·78	262·86	379·53	336·19	357·77	189·53	146·19	167·86
Pallas -	326·49	200·37	263·43	423·08	293·78	358·77	233·08	103·78	168·43
Psyche -	315·10	242·16	278·63	411·69	335·57	373·77	221·69	145·57	183·63
Hygeia -	329·60	269·18	299·39	426·19	362·59	394·77	236·19	172·60	204·39
Calliope -	?	?	?						
Thalia -	?	?	?						
Phocea.									
JUPITER -	518·09	470·45	494·27	614·68	563·86	589·77	424·68	373·86	399·27
SATURN -	959·18	853·20	906·19	1055·77	946·61	1001·77	865·77	756·61	811·19
URANUS -	1907·25	1737·41	1822·33	2003·84	1830·82	1917·77	1813·84	1640·82	1727·33
NEPTUNE -	2878·89	2829·11	2854·00	2975·48	2947·41	2949·77	2785·48	2732·52	2759·00

apparent diameter may be found from a comparison of the real diameter with the extreme and mean distances D'', D', D given in Table III. We have thus (2294.)

$$a'' = \frac{\delta}{D''} \times 206265, \quad a' = \frac{\delta}{D'} \times 206265, \quad a = \frac{\delta}{D} \times 206265.$$

2989. *Surfaces and volumes.*—The surface of the earth consists of 197 millions of square miles, and its volume of 259,800 millions of cubic miles. Let these numbers be expressed respectively by E' and E''. Since, then, the surfaces of

TABLE IV.

Data affecting the Planet, which are independent of its Orbit.

	Real Diameter.		Apparent Diameter.			Surface.		Volume.	
	Earth's = 1.	Miles.	Greatest ″	Least ″	Mean ″	Earth's = 1.	Millions Square Miles.	Earth's = 1	Billions Cubic Miles.
	Δ	δ	$a'' = \frac{\delta}{J''} \times 206265$	$a' = \frac{\delta}{D'} \times 206265$	$a = \frac{\delta}{D} \times 206265$	$\Delta' = \Delta^2$	$\delta' = \Delta' \times E'$	$\Delta'' = \Delta^3$	$\delta'' = \Delta'' \times E''$
MERCURY	0·373	2,950	12·4	4·3	8·4	0·139	2·734	0·0519	0·0138
VENUS	0·986	7,800	66·4	6·3	36·4	0·972	18·840	0.958	0·249
EARTH	1·000	7,912	—	—	—	1·000	19·660	1·000	0·260
MARS	0·518	4,100	24·4	3·3	14·0	0·268	5·271	0·139	0·036
JUPITER	11·180	88,610	45·9	30·0	39·0	125·000	2458·330	1397·400	363·330
SATURN	9·501	75,000	20·5	14·6	17·6	90·250	1775·000	857·400	223·000
URANUS	4·360	34,500	4·3	3·5	3·9	19·000	373·660	82·909	21·550
NEPTUNE	4·740	37,500	2·8	2·6	2·7	22·460	441·700	107·200	27·800
SUN	111·450	882,000	1947·6	1883·3	1915·5	12410·000	244066·000	1384333·000	359925·000
MOON	0·272	2,153	1962·5	1776·5	1873·7	0·074	1·456	0·020	0·0052

	Mass.			Density.		Apparent Diameter of Sun.		Solar Light and Heat.	Rotation.			Inclination of Axis to Orbit.			Oblateness.
	Earth's = 1.	Sun's = 1.	Trillions Tons.	Earth's = 1.	Water's = 1.	At Earth = 1.	″	At Earth = 1.	T			°	′	″	
	$\Delta''' = \frac{3'''}{354936}$	δ'''	$\delta''' = \Delta''' \times 6069$	$s = \frac{\Delta'''}{\Delta''}$	$s' = s \times 5{\cdot}67$	$\frac{1}{a}$	$\frac{1915}{a}$	$\gamma = \frac{1}{a^2}$	h	m	s				
MERCURY	0·175	$\frac{1}{2025810}$	1063	3·45	19·56	2·580	4947	6·67	24	5	0 ?	?	—	—	?
VENUS	0·885	$\frac{1}{401211}$	5369	0·92	5·22	1·380	2647	1·91	23	21	21	?	—	—	?
EARTH	1·000	$\frac{1}{354936}$	6069	1·00	5·67	1·000	1915	1·00	23	56	4	23	28	0	$\frac{1}{299}$
MARS	0·132	$\frac{1}{2680337}$	804	0·95	5·39	0·656	1256	0·43	24	37	22	28	27	0	$\frac{1}{16}$?
JUPITER	338·475	$\frac{1}{1048{\cdot}70}$	2054200	0·24	1·36	0·192	368	0·037	9	55	26	3	5	30	$\frac{1}{13}$
SATURN	101·066	$\frac{1}{3512}$	613300	0·12	0·68	0·105	201	0·011	10	29	17	26	48	40	$\frac{1}{11}$
URANUS	14·255	$\frac{1}{24900}$	86505	0·17	0·97	0·052	100	0·003	9	30	0 ?	?	—	—	?
NEPTUNE	18·900	$\frac{1}{18780}$	114700	0·17	0·97	0·033	64	0·001	?	—	—	?	—	—	?
SUN	354936·000	1.	2154100000	0·26	1·47	—	—	—	607	4	0	—	—	—	0
MOON	0·0125	$\frac{1}{28394880}$	38·8	0·62	3·52	1·000	1915	1·00	655	44	0	—	—	—	0

	Superficial Gravity.		Orbital Velocity.			Velocity of Rotation at Equator.		Gravitation towards Sun.	
	Earth's = 1. $g' = \frac{\Delta'''}{\Delta^2}$	Fall Feet in 1 Second. $f = g' \times 16{\cdot}08$	Earth's = 1. $V = \frac{1}{\sqrt{a}}$	Miles per Hour. $V' = V \times 68890$	Feet per Second. $V'' = V' \times \frac{528}{360}$	Miles per Hour. $v = \frac{\delta \times 3{\cdot}1415}{T}$	Feet per Second. $v' = v \times \frac{528}{360}$	Terrestrial Gravity = 1. $G = \frac{g}{1626 \times a^2}$	Fall Thousandths of Inch in 1 Second. $F = 192{\cdot}960 \times G$
MERCURY	0·50	8·04	1·60	110725	162400	370	543	$\frac{1}{244}$	792·
VENUS	0·90	14·63	1·1	81000	118800	1050	1540	$\frac{1}{851}$	227
EARTH	1·00	16·08	1·00	68890	101066	1040	1525	$\frac{1}{1626}$	119
MARS	0·54	8·04	0·81	55812	81860	523	768	$\frac{1}{6775}$	51
JUPITER	2·62	39·40	0·44	30203	44297	28128	41266	$\frac{1}{44020}$	4·4
SATURN	1 12	18·00	0·32	22306	32715	22440	32910	$\frac{1}{147930}$	1·3
URANUS	0·75	12·05	0·23	15730	23070	11410	16735	$\frac{1}{598160}$	0·323
NEPTUNE	0·84	13·56	0·18	12570	18435	?	?	$\frac{1}{1467333}$	0·132
SUN	28·58	459·47	—	—	—	4664	6694	—	—
MOON	0·169	2·72	0·033	2265	3338	10·3	15·12	$\frac{1}{1626}$	119

spheres are as the squares, and their volumes as the cubes, of their diameters, if Δ' express the surface and Δ'' the volume of a planet related to those of the earth as an unit, and δ' the surface in millions of square miles, and δ'' the volume in billions of cubic miles, we shall have

$$\Delta' = \Delta^2, \qquad \Delta'' = \Delta^3$$
$$\delta' = \Delta' \times E' \qquad \delta'' = \Delta'' \times E''.$$

2990. *The masses.*—The masses of the planets in relation to the sun being ascertained by the several methods explained in (2633), *et seq.*, and the ratio of that of the sun to the earth being ascertained to be 354936 to 1, let Δ''' express the mass related to that of the earth, and Σ''' to that of the sun as the unit. We shall then have

$$\Delta''' = \frac{\Sigma'''}{354936}.$$

By which Δ''' may be inferred from Σ'''.

The actual weight of the earth in trillions of tons being 6069 (2394.), let the weight of any other mass in trillions of tons be δ''', and we shall have

$$\delta''' = \Delta''' \times 6069.$$

2991. *The densities.*—The mean densities being the quotients obtained by dividing the volumes by the masses, and the mean density of the earth related to that of water as the unit being 5·67 (2393.), let the mean density of any of the other bodies related to that of the earth as the unit be x, and related to water x', and we shall have

$$x = \frac{\Delta'''}{\Delta''} \qquad x' = x \times 5{\cdot}67.$$

2992. *Certain data not exactly ascertained.*—It will be useful to observe that in the determination of several of these the results of the observations and computations of astronomers are to a certain extent at variance, and a corresponding uncertainty attends such data, as well as all conditions which depend on them or are derived by calculation from them. This is more especially the case with the masses of those planets which are unaccompanied by satellites, and consequently with the densities which are ascertained by dividing the masses by the volumes.

2993. *Example of the masses and densities of some planets.*—As an example of the character and extent of these discrepancies we give the following estimates of the masses of some of the principal planets expressed as fractions of the mass of the sun. The column E contains the values assigned by Professor Encké, from a comparison of all the authorities, except that of Neptune, which is given on the authority of Professor Pierce. The column F contains the values adopted by the French Board of Longitude, and the columns L and M the values given in the treatises lately published in Germany by Professors Littrow and Mädler.

	E.	F.	L.	M.
MERCURY . . .	$\frac{1}{4865751}$	$\frac{1}{2025810}$	$\frac{1}{2025810}$	$\frac{1}{4670335}$
VENUS	$\frac{1}{401839}$	$\frac{1}{401847}$	$\frac{1}{405871}$	$\frac{1}{401718}$
EARTH	$\frac{1}{359551}$	$\frac{1}{354936}$	$\frac{1}{355000}$	$\frac{1}{355499}$
MARS	$\frac{1}{2680337}$	$\frac{1}{2680337}$	$\frac{1}{2546320}$	$\frac{1}{2680500}$
URANUS . . .	$\frac{1}{24905}$	$\frac{1}{21000}$	$\frac{1}{21000}$	$\frac{1}{24516}$
NEPTUNE . . .	$\frac{1}{18780}$	$\frac{1}{14446}$	$\frac{1}{19000}$	$\frac{1}{14153}$

It will be observed that in Table IV., as well as in the preceding tables, the quantities are in all cases reduced to, and expressed in, those actual standard measures and weights with which all persons are familiar. The utility of this was very forcibly expressed and very happily illustrated by the Astronomer Royal, in the popular lectures delivered by him at Ipswich.

2994. *Intensity of solar light and heat.*—Since the intensity of solar radiation decreases as the square of the distance from the sun decreases, if γ expresses its intensity at the mean distance of any planet relative to its intensity at the earth as the unit, we shall have

$$\gamma = \frac{1}{a^2}.$$

2995. *Superficial gravity.*—The superficial gravity of a spherical body being in proportion to its mass, divided by the square of its semi-diameter, and the height through which a body falls upon the surface of the earth in one second being 16·08 feet, let g' express the superficial gravity of a spherical body related to that of the earth as the unit, and let f' express the height through which a body submitted to it would fall in one second, and we shall have

$$g' = \frac{\Delta'''}{\Delta^2}, \qquad f' = g' \times 16{\cdot}08.$$

2996. *Orbital velocities.*—It is easy to show that it follows as a necessary consequence of the harmonic law, that the mean orbital velocities of the planets are in the inverse ratio one to another of the square roots of the distances; for since these velocities are proportional to the circumferences, or, what is the same, the semi-diameters of the orbits, divided by the periods, they are proportional to $\frac{a}{\text{P}}$; but since, by the harmonic law, P^2 is proportional to a^3, the velocities will be proportional to $\frac{a}{\sqrt{a^3}}$, or, what is the same, to $\frac{1}{\sqrt{a}}$, that is, inversely proportional to the square roots of the mean distances.

This being understood, and the mean orbital velocity of the earth expressed in miles per hour being 68,890, let V be the

mean velocity of a planet related to that of the earth as the unit, and v′ its mean velocity in miles per hour, and we shall have

$$\text{v} = \frac{1}{\sqrt{a}}, \qquad \text{v}' = \text{v} \times 68890;$$

and since the ratio of miles per hour to feet per second is that of 5280 to 3600, if v″ be the velocity in feet per second, we shall have

$$\text{v}'' = \frac{528}{360} \times \text{v}'.$$

2997. *Superficial velocity of rotation.* — The superficial velocity of a planet at its equator in virtue of its diurnal rotation is found by comparing the circumference of its equator with the time of its rotation. By the elementary principles of geometry, the circumference of a circle whose diameter is δ, is $\delta \times 3{\cdot}1415$, and if T express the time of rotation in hours, we shall have for v, the velocity of rotation in miles per hour

$$v = \frac{\delta \times 3{\cdot}1415}{\text{T}};$$

which may be reduced to feet per second, as before, by

$$v' = v \times \frac{528}{360}.$$

2998. *Solar gravitation.*—The general law of gravitation supplies easy and simple means by which the force of the sun's attraction at the mean distance of each of the planets may be brought into immediate comparison with the known force of gravity at the surface of the earth.

Let this latter force be expressed by g. It will decrease in the same ratio as the square of the distance of the body affected by it increases. The distance of the sun being 24,000 semidiameters of the earth, the intensity of the attraction which the earth's mass would exert at that distance would be

$$\frac{g}{24000 \times 24000} = \frac{g}{576{,}000000}.$$

But the mass of the sun being 354,936 times that of the earth, it will at the same distance exert an attraction 354,936 times

greater. The intensity of the attraction, therefore, which the sun exerts at the earth's mean distance will be

$$g \times \frac{354936}{24000^2} = \frac{g}{1626};$$

and the intensity of its attraction at the mean distance of the other planets being still inversely as the squares of the distances, will be found by dividing this by a^2. So that if G express this attraction, and F the height, in thousandths of an inch, through which a body placed at each distance would fall in one second, we shall have

$$G = \frac{g}{1626\,a^2}, \qquad F = 16080 \times 12 \times G = 192960\,G.$$

By the numbers given in the column G, it is there to be understood that a mass of matter which, placed upon the surface of the earth, would weigh the number of pounds expressed by the denominators of the fractions severally, would, if submitted only to the sun's attraction at the respective mean distances of the planets, gravitate to the sun with the force of one pound. Thus, a mass which on the earth's surface would weigh 1626 lbs. would weigh only one pound if exposed to the sun's attraction in the absence of the earth. In like manner, a mass which upon the earth's surface would weigh 1467333 lbs., or 655 tons, would, if exposed to the sun's attraction at the mean distance of Neptune, weigh only one pound, so extremely is the intensity of solar attraction enfeebled by the enormous increase of distance.

The numbers given in the column F have a more absolute sense, and express in thousandths of an inch the actual spaces through which a body would be drawn in one second of time by the sun's attraction at the mean distances of the planets severally.

IV. Tabulated Elements of the Satellites.

2999. The elements of the orbits, and other physical data relating to the satellites of Jupiter, Saturn, and Uranus, so far as they have been discovered, are given in Tables V., VI., and VII. After the explanations which have been given above respecting the corresponding data of the primary planets, no

difficulty will be found in comprehending those tables, and the manner of computing them.

TABLE V.

Elements of the Jovian System.

	I.	II.	III.	IV.
	M. T. G.	M. T. G.	M. T. G.	M. T. G.
Epoch - - - - - - -	1 Jan. 1801.	1 Jan. 1801	1 Jan. 1801.	1 Jan. 1801.
Time of revolution (days) - - -	1·7691	3·5512	7·1546	16·6888
Greatest elongation - - -	135″	215″	346″	585″
Distance from Jupiter in semi-diameter of planet - - - - - -	6·0485	9·6235	15·3502	25·9983
" " in miles - -	269800	4·26500	680000	1158000
Eccentricity - - - - -	0	0	Small and variable	Small and variable
Apparent diameter seen from earth -	1″·191	1″·070	1″·747	1″·495
" " " from Jupiter -	33′ 40″	19′ 30″	18′ 16″	8′ 39″
Real diameter in semi-diameter, Jupiter	0·0326	0·0233	0·038	0·0326
" in miles - - - -	2309	2069	3378	2891
Mass—Earth's = 1 - - - -	1/1736	1/1294	1/34	1/70
" Jupiter's = 1 - - - -	1/57800	1/43100	1/11300	1/23420
Density, Earth's = 1 - - - -	1/49·6	1/35·2	1/11·3	1/25·5
" Water's = 1 - - - -	1/8·75	1/5·85	1/2·52	1/4·49
Orbital velocity, miles per hour - -	34772	30716	24513	17743
" " feet per second - -	56865	45133	36952	26022
Gravitation towards Jupiter, ter. gr. = 1	1/14	1/36·5	1/90	1/258
" " fall inches in 1 second -	13·82	5·46	2·146	0·748
Inclination of orbit to a fixed plane proper to each - - - - - -	0° 0′ 0″	0° 27′ 50″	0° 12′ 20″	0° 14′ 58″
Inclination of the fixed plane to Jupiter's equator - - - - - -	0° 0′ 6″	0° 1′ 5″	0° 5′ 2″	0° 24′ 4″
Time of retrograde revolution of nodes, years - - - - - -	—	29·9142	141·7390	531·0000

TABLE VI.

Elements of the Saturnian System.

	I. Mimas.	II. Enceladus.	III. Tethys.	IV. Dione.	V. Rhea.	VI. Titan.	VII. Hyperion.	VIII. Japetus.
	M. T. G.	M. T. G.	M. T. G.	M. T. G.	M. T. G.	M. T. G.	M. T. G.	M. T. G.
Epoch - - -	1790·0	1836·0	1836·0	1836·0	1836·0	1830·0	—	1790·0
Time of revolution (days) - - -	0·943	1·370	1·888	2·739	4·517	15·945	22·5 ?	79·330
Greatest elongation (in opposition) -	34″·1	43″·1	53″·4	68″·4	95″·5	221″·4	280″ ?	645·6
Distance from Saturn in semi-diameter planet - - -	3·3607	4·3125	5·3396	6·8398	9·5528	22·1450	28· ?	64·359
Distance from Saturn in miles - -	126020	161720	200235	256500	358225	830410	1050000	2414060
Eccentricity - -	?	?	0·01 ?	0·02 ?	0·02 ?	0·029314	?	?
Longitude of peri-Saturnium - -	?	?	54° ?	42° ?	95° ?	256° 38′ 11″	?	?
Orbital velocity, miles per hour - -	34986	30975	27776	24516	20763	13635	12215	7968
Orbital velocity, feet per second - -	51313	45325	40723	35957	30152	19998	17920	11960
Gravitation towards Saturn, terrestrial gravity = 1 - -	1/10·09	1/16·6	1/25·45	1/41·77	1/81·48	1/438	1/700	1/3702
Gravitation towards Saturn, inches in 1 second - - -	19·13		7·58	4·62	2·37	0·441	0·276	0·052
Mean long. at epoch	256° 58′ 48″	67° 41′ 36″	313° 43′ 48″	327° 40′ 18″	353° 44′ 0″	137° 21′ 24″	?	269° 37′ 48″

TABLE VII.

Elements of the Uranian System.

	I.	II.	III.	IV.	V.	VI.
Time of revolution (days)	5·893	8·707	10·961	13·456	38·075	107·694
Greatest elongation	13″·12	33″·19	38″·71	44″·36	88″·74	177″·80
Distance from Uranus in semi-diameter planet	13·12	17·02	19·85?	22·75	45·51?	91·01?
Distance from Uranus in miles	226320	293690	342416	392450	785060	1570000
Orbital velocity, miles per hour	10056	8828	8178	7636	5398	3816
„ „ feet per second	14746	12945	11995	11200	7917	5520
Gravitation towards Uranus, terrestrial gravity = 1	$\frac{1}{230}$	$\frac{1}{383}$	$\frac{1}{530}$	$\frac{1}{697}$	$\frac{1}{2762}$	$\frac{1}{8772}$
Gravitation, fall in thousandths of inch in 1 second	841	500	364	277	70	20

CHAP. XVIII.

COMETS.

I. COMETARY ORBITS.

3000. *Prescience of the astronomer.* For the civil and political historian the past alone has existence — the present he rarely apprehends; the future never. To the historian of science it is permitted, however, to penetrate the depths of past and future with equal clearness and certainty: facts to come are to him as present, and not unfrequently more assured than facts which are passed. Although this clear perception of causes and consequences characterises the whole domain of physical science, and clothes the natural philosopher with powers denied to the political and moral inquirer, yet foreknowledge is eminently the privilege of the astronomer. Nature has raised the curtain of futurity, and displayed before him the succession of her decrees, so far as they affect the physical universe, for countless ages to come; and the revelations of which she has made him the instrument, are supported and verified by a never-ceasing train of predictions fulfilled. He "shows us the things which will be hereafter," not obscurely shadowed out in figures and in parables, as must necessarily be the case with other revelations, but attended with the most minute precision of time, place, and circumstance. He converts the hours as they roll into an ever-

present miracle, in attestation of those laws which his Creator through him has unfolded; the sun cannot rise—the moon cannot wane—a star cannot twinkle in the firmament, without bearing witness to the truth of his prophetic records. It has pleased the "Lord and Governor" of the world, in his inscrutable wisdom, to baffle our inquiries into the nature and proximate cause of that wonderful faculty of intellect—that image of his own essence which he has conferred upon us; nay, the springs and wheelwork of animal and vegetable vitality are concealed from our view by an impenetrable veil, and the pride of philosophy is humbled by the spectacle of the physiologist bending in fruitless ardour over the dissection of the human brain, and peering in equally unproductive inquiry over the gambols of an animalcule. But how nobly is the darkness which envelopes metaphysical inquiries compensated by the flood of light which is shed upon the physical creation! *There* all is harmony, and order, and majesty, and beauty. From the chaos of social and political phenomena exhibited in human records—phenomena unconnected to our imperfect vision by any discoverable law, a war of passions and prejudices, governed by no apparent purpose, tending to no apparent end, and setting all intelligible order at defiance—how soothing and yet how elevating it is to turn to the splendid spectacle which offers itself to the habitual contemplation of the astronomer! How favourable to the development of all the best and highest feelings of the soul are such objects! the only passion they inspire being the love of truth, and the chiefest pleasure of their votaries arising from excursions through the imposing scenery of the universe—scenery on a scale of grandeur and magnificence, compared with which whatever we are accustomed to call sublimity on our planet dwindles into ridiculous insignificancy. Most justly has it been said, that nature has implanted in our bosoms a craving after the discovery of truth, and assuredly that glorious instinct is never more irresistibly awakened than when our notice is directed to what is going on in the heavens. "Quoniam eadem Natura cupiditatem ingenuit hominibus veri inveniendi, quod facillime apparet, cum vacui curis, etiam quid in cœlo fiat, scire avemus; his initiis inducti omnia vera diligimus; id est, fidelia, simplicia, constantia; tum vana, falsa, fallentia odimus."*

* Cic. de Fin. Bon. et Mal. ii. 14.

3001. *Strikingly illustrated by cometary discovery.*—Such reflections are awakened by every branch of the science which now engages us, but by none so strongly as by the history of cometary discovery. No where can be found so marvellous a series of phenomena foretold. The interval between the prediction and its fulfilment has sometimes exceeded the limits of human life, and one generation has bequeathed its predictions to another, which has been filled with astonishment and admiration at witnessing their literal accomplishment.

3002. *Motion of comets explained by gravitation.*—In the vast framework of the theory of gravitation constructed by Newton, places were provided for the arrangement and exposition not only of all the astronomical phenomena which the observation of all preceding generations had supplied, but also for a far greater mass which the more fertile and active research of the generations which succeeded him have furnished. By this theory, as we have seen, all the known planetary motions were explained, and planets previously unseen were felt by their effects, their places ascertained, and the telescope of the observer guided to them.

But transcendently the greatest triumph of this celebrated theory was the exposition it supplied of the physical laws which govern the motions of comets as distinguished from those which prevail among the planets.

3003. *Conditions imposed on the orbits of bodies which are subject to the attraction of gravitation.*—It is proved in the propositions demonstrated in the first book of Newton's Principia, which propositions form in substance the ground-work of the entire theory of gravitation, that a body which is under the influence of a central force, the intensity of which decreases as the square of the distance increases, must move in one or other of the curves known to geometers as the "CONIC SECTIONS," being those which are formed by the intersection of the surface of a cone by a plane, and that the centre of attraction must be in the FOCUS of the curve; and in order to prove that such curves are compatible with no other law of attraction, and may therefore be taken as conclusive evidence of the existence of this law, it is further demonstrated that whenever a body is observed to move round a centre of attraction in any one of these curves, that centre being its focus, the law of the attraction will be that of gravitation; that is to say, its intensity will vary in the inverse

proportion of the square of the distance of the moving body from the centre of force.

Subject to these limitations, however, a body may move round the sun in any orbit, at any distance, in any plane, and in any direction whatever. It may describe an ellipse of any eccentricity, from a perfect circle to the most elongated oval. This ellipse may be in any plane, from that of the ecliptic to one at right angles to it, and the body may move in such ellipses either in the same direction as the earth or in the contrary direction. Or the body thus subject to solar attraction may move in a parabola with its point of perihelion at any distance whatever from the sun, either grazing its very surface or sweeping beyond the orbit of Neptune, or, in fine, it may sweep round the sun in an hyperbola, entering and leaving the system in two divergent directions.

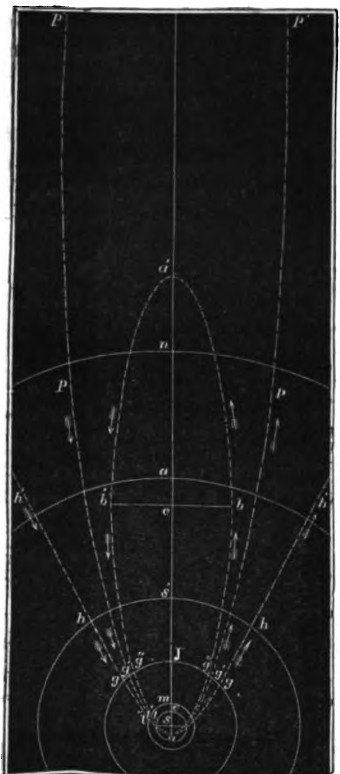


Fig. 811.

To render these explanations, which are of the greatest interest and importance in relation to the subject of comets, more clearly understood, we have represented, in *fig.* 811., the forms of a very eccentric ellipse, *a b a′ b′*, a parabola *a p p′*, and an hyperbola *a h h′*, having *s* as their common focus, and it will be convenient to explain in the first instance the relative magnitude of some important lines and distances connected with these orbits.

3004. ***Elliptic orbit.*** — Ellipses or ovals vary without limit in their eccentricity. A circle is regarded as an ellipse whose eccentricity is nothing. The orbits of the planets generally are ellipses, but having eccentricities so small that, if

described on a large scale in their proper proportions on paper, they would be distinguishable from circles only by measuring accurately the dimensions taken in different directions, and thus ascertaining that they are longer in a certain direction than in another at right angles to it. A very eccentric and oblong ellipse is delineated in *fig.* 811., of which $a a'$ is the major axis. The focus being s, the perihelion distance d is $s\,a$, and the aphelion distance d' is $s\,a'$, the mean distance a being $s\,c$, or half the major axis. The eccentricity e, being expressed by the numerical ratio of the distance of the focus s from the centre c to the semi-axis, we shall have

$$\frac{s\,c}{a} = e,\ s\,c = a \times e.$$

It is evident from what has been just stated then that we shall have

$$d = a - a \times e = a \times (1 - e),$$

and consequently

$$a = \frac{d}{1 - e};$$

and also

$$d' = a + a \times e = a \times (1 + e) = d \times \frac{1 + e}{1 - e}.$$

Hence it is evident that if the perihelion distance and eccentricity be given, the semi-axis a and the aphelion distance d' can be computed.

By the properties of the ellipse, the distance of any point from the focus s can be computed if the perihelion distance d, the eccentricity e, and the angular distance of the point from perihelion be given. Let α express this angular distance, which is, in fact, the angle formed by two lines, one d, drawn from s to a, and the other z from s to the actual place of the body in the ellipse. It is proved in geometry that the value of z may be determined in all cases by the formula,

$$z = d \times \frac{1 + e}{1 + e \times \cos.\ \alpha};$$

that is to say, if the perihelion distance be first multiplied by the eccentricity increased by 1, and the product then divided by the number found by adding to 1 the product of the eccentricity and the cosine of the angular distance of the body from perihelion, the quotient will be the distance of the body from the focus s.

From this general formula, therefore, the expression for the distance of the body from the focus in every position can be found. Thus at 90° from perihelion we have

$$\cos a = \cos. 90° = 0,$$

and therefore the distance $s\,l$, which we shall call l, is

$$l = d \times (1 + e)$$

If we suppose $a = 180°$, we shall reproduce the expression for the aphelion distance d', for cos. 180° = − 1 ; and therefore

$$d' = d \times \frac{1 + e}{1 - e}.$$

It will be useful also to observe that the cos. a is positive when a is less, and negative when a is greater, than 90°.

The number which is expressed by e is necessarily less than 1, since it is a fraction whose numerator $s\,c$ is less than its denominator $c\,a$. The more eccentric the ellipse is, the more nearly equal will $s\,c$ be to $c\,a$, and consequently the more nearly equal to 1 will be the eccentricity e.

The distance l, being greater than the perihelion distance d, in the ratio of $1 + e$ to 1, it will therefore be always less than twice this distance, inasmuch as $1 + e$ is always less than 2; but the more eccentric the ellipse is the more nearly will l approach to twice the perihelion distance d. Thus, for example, if $e = 0{\cdot}999$, we should have $l = d \times 1{\cdot}999$, which falls short of twice the perihelion distance by not more than the 1000th part of that distance.

The curvature of the ellipse continually increases from the mean distance to perihelion, and constantly decreases from perihelion to the mean distance, being equal at equal angular distances from perihelion as seen from the sun.

It is evident that if a body move in a very eccentric ellipse, such as that represented in *fig.* 811., whose plane coincides exactly or nearly with the common plane of the planetary orbits, it may intersect the orbits of several or all of the planets, as it is represented to do in the figure, although its mean distance from the sun may be less than the mean distance of several of those which it thus intersects. The aphelion distance of such a body may, therefore, greatly exceed that of any planet; while its mean distance may be less than that of the more distant planets.

3005. *Parabolic orbits.* — The form of a parabolic orbit having the same perihelion distance as the elliptic orbit is represented at *a p p*, in *fig.* 811. This orbit consists of two indefinite branches, similar in form, which unite at perihelion *a*. Departing from this point on opposite sides of the axis *a a'*, their curvature regularly and rapidly decreases, being equal at equal distances from perihelion. The two branches have a constant tendency to assume the direction and form of two straight lines parallel to the axis *a a'*. To actual parallelism, and still less to convergence, these branches, however, never attain, and consequently they can never reunite. They extend, in fine, like parallel straight lines, to an unlimited distance without ever reuniting, but assuming directions when the distance from the focus bears a high ratio to the perihelion distance, which are practically undistinguishable from parallelism.

It is demonstrated in geometry, that if d express the perihelion distance and z the distance of the body at the angular distance a from perihelion, we shall have

$$z = \frac{2\,d}{1 + \cos.\,a};$$

that is to say, the distance z is found by dividing twice the perihelion distance by 1 added to the cosine of the angular distance from perihelion.

It must be mentioned, however, that when a is greater than 90°, the cosine is negative, and its value must then be subtracted from 1 to obtain the divisor.

To find the distance l of the body at 90° from perihelion, let $a = 90°$, and consequently $\cos.\,a = 0$. Therefore

$$l = 2\,d;$$

that is to say, the distance at 90° from perihelion is twice the perihelion distance.

One parabolic orbit differs from another in its perihelion distance. The less this distance is, the less will be the separation at a given distance from *s* between the parallel directions to which the indefinite branches *p p'* tend. This distance may have any magnitude. The body in its perihelion may graze the surface of the sun, or may pass at a distance from it greater than that of the most remote of the planets, so that, although it be subject to solar attraction, it would in that case never enter within the limits of the solar system at all.

A body moving in such an orbit, therefore, would not make, like one which moves in an ellipse, a succession of revolutions round the sun; nor can the term periodic time be applied at all to its motion. It enters the system in some definite direction, such as $p'p$, as indicated by the arrow from an indefinite distance. Arriving within the sensible influence of solar gravitation, the effects of this attraction are manifested in the curvation of its path, which gradually increases as its distance from the sun decreases, until it arrives at perihelion, where the attractive force, and consequently the curvature, attain their maxima. The extreme velocity which the body attains at this point produces, in virtue of the inertia of the moving mass, a centrifugal force, which counteracts the gravitation, and the body, after passing perihelion, begins to retreat; the solar gravitation and the curvature of its path decreasing together, until it issues from the system in a direction $p\,p'$, as indicated by the arrows, which is nearly a straight line, and parallel to that in which it entered. In such an orbit a body therefore visits the system but once. It enters in a certain direction from an indefinite distance, and, passing through its perihelion, issues in a parallel direction, passing to an unlimited distance, never to return.

3006. *Hyperbolic orbits.*—This class of orbits, like the parabolas, consist of two indefinite branches, which unite at perihelion, which at equal distances from perihelion have equal curvatures, and which, as the distance from perihelion increases, approach indefinitely in direction and form to straight lines, but, unlike the parabolic orbits, the straight lines to whose direction the two branches approximate are divergent and not parallel.

Such an orbit having the same perihelion distance as the ellipse and parabola, is represented by $a\,h\,h'$, *fig.* 811.

If z express, as before, the distance of the body from s, when its angular distance from perihelion is a, and if e express a certain number, which, instead of being less, as in the case of the ellipse, is greater than 1, we shall have

$$z = d \times \frac{1 + e}{1 + e \times \cos. a},$$

a formula identical with that which expresses the value of z in the case of the ellipse, the difference being merely in the relation which the number e bears to 1.

It is easy to perceive that when the perihelion distance is the same, the value of z for any proposed angular distance from perihelion is greater in the hyperbola than in the ellipse, and that its value in the parabola is intermediate, being less than in the hyperbola and greater than in the ellipse.

The parabola is, therefore, included between the ellipse and hyperbola, and the less the number e falls short of 1 for the ellipse, and exceeds it for the hyperbola, the nearer will the three orbits be in relation to each other at those parts which are not very far removed from perihelion.

The distance l of the body which moves in the hyperbola at 90° from perihelion, is

$$l = d\,(1 + e),$$

the same as for the ellipse. If l express this distance for the ellipse, l' for the parabola, and l'' for the hyperbola, and e' express the value of e for the hyperbola, we shall have

$$\begin{aligned} l &= d \times (1 + e) \\ l' &= 2\,d \\ l'' &= d \times (1 + e'), \end{aligned}$$

and consequently the distances between the parabola, and each of the other orbits at 90° from perihelion, measured in the direction of $s\,l\,l'$, will be

$$\begin{aligned} l' - l &= d \times (1 - e) \\ l'' - l' &= d \times (e' - 1). \end{aligned}$$

It is evident, therefore, that, whatever be the fraction by which e falls short of 1, and by which e' exceeds 1, the same will be the fraction of the perihelion distance by which the parabola at this point is separated from the other orbits.

It will be recollected that, when a exceeds 90°, the cos. a becomes negative, and, consequently, we would then have

$$z = d \times \frac{1 + e}{1 - e \times \cos. a.}$$

Now, as the cos. a, which is 0, when $a = 90°$ increases, the product $e \times \cos. a$, which, after passing 90°, is very minute, will also increase, and, consequently, the divisor $1 - e \times \cos. a$ will decrease. This will evidently cause a proportionate increase of z. As the product $e \times \cos. a$ approaches to 1, the divisor decreasing, the value of z rapidly increases; and when $e \times \cos. a$ becomes actually $= 1$, z becomes infinite.

The geometrical interpretation of this analytical contingency is, that each branch of the orbit, in receding from the centre of attraction *s*, approaches indefinitely to coincidence with certain straight lines, which make, with the direction of perihelion, an angle, the cosine of which has such a value, that $e \times \cos. a = 1$, that is, an angle whose cosine is $\frac{1}{e}$.

If two straight lines, therefore, be drawn from *s*, making an angle with the axis *a a'*, whose cosine is $\frac{1}{e}$, these lines will be the directions into which the two branches of the hyperbolic orbit have a constant tendency to run. They will, therefore, be the limit of the divergence of the branches *h h'*.

It is evident, therefore, that the more the number *e* exceeds 1, the greater will be the angle of divergence of the two branches *h h'*, and the less it exceeds 1, the nearer will the hyperbolic branches *h h'* approach to the parabolic branches *p p'*.

In the *fig.* 811., the orbits circular, elliptic, parabolic, and hyperbolic are necessarily represented as being all in the same plane. It must, however, be understood, that, so far as any conditions are imposed upon them by the law of gravitation, they may severally be in any planes whatever, inclined each to the other, at any angles whatever, from 0° to 90°, with their mutual intersections or lines of nodes in any directions whatever, and that the bodies may move in these several orbits, in any directions, how opposed soever to each other.

3007. *Planets observe in their motions order not exacted by the law of gravitation.*—When the theory of gravitation was first propounded by its illustrious author, no other bodies, save the planets and satellites then discovered, were known to move under the influence of such a central attraction. These bodies, however, supplied no example of the play of that celebrated theory in its full latitude. They obeyed, it is true, its laws, but they did much more. They displayed a degree of harmony and order far exceeding what the law of gravitation exacted. Permitted by that law to move in any of the three classes of conic sections, their paths were exclusively elliptical; permitted to move in ellipses infinitely various in their eccentricities, they moved exclusively in such as differed almost insensibly from circles; permitted to move at distances subordinated to no regular law, they moved in a series of orbits at

distances increasing in a regular progression; permitted to move at all conceivable angles with the plane of the ecliptic, their paths are inclined to it at angles limited in general to a few degrees; permitted, in fine, to move in either direction, they all agreed in moving in the direction in which the earth moves in its annual course.

Accordance so wondrous, and order so admirable, could not be fortuitous, and, not being enjoined by the conditions of the law of gravitation, must either be ascribed to the immediate dictates of the Omnipotent Architect of the universe above all general laws, or to some general laws superinduced upon gravitation, which had escaped the sagacity of the discoverer of that principle. If the former supposition were adopted, some bodies, different in their physical characters from the planets, primary and secondary, and playing different parts and fulfilling different functions in the economy of the universe, might still be found, which would illustrate the play of gravitation in its full latitude, sweeping round the sun in all forms of orbit, eccentric, parabolic, and hyperbolic, in all planes, at all distances, and indifferently in both directions. If the latter supposition were accepted, then no other orbit, save ellipses of small eccentricity, with planes coinciding nearly with that of the ecliptic, would be physically possible.

3008. *Comets observe no such order in their motions.*—The theory of gravitation had not long been promulgated, nor as yet been generally accepted, when the means of its further verification were sought in the motion of comets. Hitherto these bodies had been regarded as exceptional and abnormal, and as being exempt altogether from the operation of the law and order which prevailed in a manner so striking among the members of the solar system. So little attention had been given to comets that it had not been certainly ascertained whether they were to be classed as meteoric or cosmical phenomena; whether their theatre was the regions of the atmosphere, or the vast spaces in which the great bodies of the universe move. Their apparent positions in the heavens on various occasions of the appearances of the most conspicuous of them had nevertheless been from time to time for some centuries observed and recorded with such a degree of precision as the existing state of astronomical science permitted; and even when their places were not astronomically ascertained, the date of their appearance was generally preserved

in the historic records, and in many cases the constellations through which they passed were indicated, so that the means of obtaining at least a rude approximation to their position in the firmament were thus supplied.

3009. *They move in conic sections, with the sun for the focus.* — Such observations, vague, scattered, and inexact as they were, supplied, however, data by which, in several cases, it was possible to compute the real motion of these bodies through space, their positions in relation to the sun, the earth, and the planets, and the paths they followed in moving through the system, with sufficiently approximate accuracy to conclude with certainty that they were one or other of the conic sections, the place of the sun being the focus.

This was sufficient to bring these bodies under the general operation of the attraction of gravitation.

It still remained, however, to determine more exactly the specific character of these orbits. Are they ellipses more or less eccentric? or parabolas? or hyperbolas?—Any of the three classes of orbits would, as has been shown, be equally compatible with the law of gravitation.

3010. *Difficulty of ascertaining in what species of conic section a comet moves.*—It might be supposed that the same course of observation as that by which the orbit of a planet is traced would be applicable equally to comets. Many circumstances, however, attend this latter class of bodies, which render such observations impossible, and compel the astronomer to resort to other means to determine their orbits.

A spectator stationed upon the earth keeps within his view each of the other planets of the system throughout nearly the whole of its course. Indeed, there is no part of the orbit of any planet in which, *at some time or other*, it may not be seen from the earth. Every point of the path of each planet can therefore be observed; and, although without waiting for such observation, its course might be determined, yet it is material here to attend to the fact, that the whole orbit may be submitted to direct observation. The different planets, also, present peculiar features by which each may be distinguished. Thus, as has been explained, they are observed to be spherical bodies of various magnitudes. Their surfaces are marked by peculiar modes of light and shade, which, although variable and shifting, still, in each case, possess some prevailing and permanent characters by

which the identity of the object may be established, even were there no other means of determining it.

Unlike planets, comets do not present to us those individual characters above mentioned, by which their identity may be determined. None of them have been satisfactorily ascertained to be spherical bodies, nor indeed to have any definite shape. It is certain that many of them possess no solid matter, but are masses consisting of some nearly transparent substances; others are so surrounded with this apparently vaporous matter, that it is impossible, by any means of observation which we possess, to discover whether this vapour enshrouds within it any solid mass. The same vapour which thus envelopes the body (if such there be within it) also conceals from us its features and individual character. Even the limits of the vapour itself, if vapour it be, are subject to great change in each individual comet. Within a few days they are sometimes observed to increase or diminish some hundred-fold. A comet appearing at distant intervals presents, therefore, no very obvious means of recognition. A like extent of surrounding vapour would evidently be a fallible test of identity; and not less inconclusive would it be to infer diversity from a different extent of nebulosity.

If a comet, like a planet, revolved round the sun in an orbit nearly circular, it might be seen in every part of its path, and its identity might thus be established independently of any peculiar characters in its appearance. But such is not the course which comets are observed to take.

In general a comet is visible only throughout an arc of its orbit, which extends to a certain limited distance on each side of its perihelion. It first becomes apparent at some point of its path, such as g, g' or g'', *fig.* 811.; it approaches the sun and disappears after it passes a corresponding point g, g' or g'' in departing from the sun. The arc of its orbit in which alone it is visible would therefore be $g\,a\,g$, $g'\,a\,g'$, or $g''\,a\,g''$.

If this arc, extending on either side of perihelion, could always be observed with the same precision as are the planetary orbits, it would be possible, by the properties of the conic sections, to determine not only the general character of the orbit, whether it be an ellipse, or parabola, or an hyperbola, but even to ascertain the individual curve of the one kind or the other in which the comet moves, so that the course it followed before it became visible, as well as that which it pursues after it ceases

to be visible, would be as certainly and precisely known as if it could be traced by direct observation throughout its entire orbit.

3011. *Hyperbolic and parabolic comets not periodic.*—If it be ascertained that the arc in which the comet moves while it is visible is part of an hyperbola, such as *g a g*, it will be inferred that the comet coming from some indefinitely distant region of the universe, has entered the system in a certain direction, *h′h*, which can be inferred from the visible arc *g a g*, and that it must depart to another indefinitely distant region of the universe following the direction *h h′*, which is also ascertained from the visible arc *g a g*.

If, on the other hand, it be ascertained that the visible arc, such as *g′ a g′*, be part of a parabola, then, in like manner by the properties of that curve, it will follow that it entered the system coming from an indefinitely distant region of the universe in a certain direction, *p′ p*, which can be inferred from the visible arc *g′ a g′*, and that after it ceases to be visible, it will issue from the system in another determinate direction, *p p′*, parallel to that by which it entered.

The comet, in neither of these cases, would have a periodic character. It would be analogous to one of those occasional meteors which are seen to shoot across the firmament never again to reappear. The body, arriving from some distant region, and coming, as would appear, fortuitously within the solar attraction, is drawn from its course into the hyperbolic or parabolic path, which it is seen to pursue, and escapes from the solar attraction, issuing from the system never to return. The phenomenon would in each case be occasional, and, in a certain sense, accidental, and the body could not be said properly to belong to the system. So far as relates to the comet itself, the phenomenon would consist in a change of the direction of its course through the universe, operated by the temporary action of solar gravitation upon it.

3012. *Elliptic comets periodic like the planets.*—But the case is very different, the tie between the comet and the system much more intimate, and the interest and physical importance of the body transcendently greater when the arc, such as *g″ a g″*, proves to be part of an ellipse. In that case, the invisible part of the orbit being inferred from the visible, the major axis *a a′* would be known. The comet would possess the periodic cha-

racter, making successive revolutions like the planets, and returning to perihelion *a* after the lapse of its proper periodic time, which could be inferred by the harmonic law from the magnitude of its major axis.

Such a body would then not be, like those which follow hyperbolic or parabolic paths, an occasional visitor to the system, connected with it by no permanent relation, and subject to solar gravitation only accidentally and temporarily. It would, on the contrary, be as permanent, if not as strictly regular, a member of the system as any of the planets, though invested, as will presently appear, with an extremely different physical character.

It will therefore be easily conceived with what profound interest comets were regarded before the theory of gravitation had been yet firmly established or generally accepted, and while it was, so to speak, upon its trial. These bodies were, in fact, looked for as the witnesses whose testimony must decide its fate.

3013. *Difficulties attending the analysis of cometary motions.* —Difficulties, however, which seemed almost insurmountable, opposed themselves to a satisfactory and conclusive analysis of their motions. Many causes rendered the observations upon their apparent places few in number and deficient in precision. The arcs $g\ a\ g$, $g'\ a\ g'$, and $g''\ a\ g''$ of the three classes of orbit in any of which they might move without any violation of the law of gravitation were very nearly coincident in the neighbourhood of the place of perihelion *a*. It was, for example, in almost all the cases which presented themselves, possible to conceive three different curves, an eccentric ellipse, such as $a\,b\,a'\,b'$, a parabola, such as $p'\ p\ a$, and an hyperbola, such as $h'\ h\ a$, so related that the arcs $g\ a\ g$, $g'\ a\ g'$, and $g''\ a\ g''$, would not deviate one from another to an extent exceeding the errors inevitable in cometary observations. Thus any one of the three curves within the limits of the visible path of the comet might with equal fidelity represent its course. In such cases, therefore, it was impossible to infer, from the observations alone, whether the comet belonged to the class of hyperbolic or parabolic bodies, which have no periodic character, or to the elliptic, which has.

3014. *Periodicity alone proves the elliptic character.*—The character of periodicity itself, which belongs exclusively to elliptic orbits, supplied the means of surmounting this difficulty. If any observed comet have an elliptic motion, it must return to

perihelion after completing its revolution, and it must have been visible on former returns to that position. Not only ought it to be expected, therefore, that such a comet would re-appear in future after absences of equal duration (depending on its periodic time), but that its previous returns to perihelion would be found by searching among the recorded appearances of such objects for any, the dates of whose appearance might correspond with the supposed period, and whose apparent motions, if observed, might indicate a real motion in an orbit, identical or nearly so with that of the comet in question.

If the motion of such a body were not affected by any other force except the solar attraction, it would re-appear after each successive revolution at exactly the same point; would follow, while visible, exactly the same arc $g'' a g''$; would move in the same plane, inclined at the same angle to the ecliptic, the nodes retaining the same places; and would arrive at its perihelion at exactly the same point a, and after exactly equal intervals.

Now, although the disturbing actions of the planets near which it might pass, in departing from and returning to the sun, must be expected to be much more considerable than when one planet acts upon another, as well because of the extreme comparative lightness of the comet, as of the great eccentricity of its orbit, which sometimes actually or nearly intersects the paths of several planets, and especially those of the larger ones, yet still such planetary attractions are *only* disturbances, and cannot be supposed to efface that character which the orbit receives from the predominant force of the immense mass of the sun. While therefore we may be prepared for the possibility, and even the probability, that the same periodic comet on the occasion of its successive re-appearances, may follow a path $g'' a g''$ in passing to and from its perihelion, differing to some extent from that which it had followed on previous appearances, yet in the main such differences cannot, except in rare and exceptional cases, be very considerable, and for the same reason the intervals between its successive periods, though they may differ, cannot be subject to any very great variation.

3015. *Periodicity, combined with the identity of the paths while visible, establishes identity.*—If then, on examining the various comets whose appearances have been recorded, and whose places while visible have been observed, and on computing from the apparent places the arc of the orbit through which they

moved, it be found that two or more of them, while invisible, moved in the same path, the presumption will be that these were the same body re-appearing after having completed its motion in an elliptic orbit; nor should this presumption of identity be hastily rejected because of the existence of any discrepancies between the observed paths, or any inequality of the intervals between its successive re-appearances, so long as such discrepancies can fairly be ascribed to the possible disturbances produced by planets which the comet might have encountered in its path.

3016. *Many comets recorded—few observed.*—Many comets, however, have been *recorded*, but not *observed*. Historians have mentioned, and even described, their appearances, and in some cases have indicated the chief constellations through which such bodies passed, although no observations of their apparent places have been transmitted by which any close approximation to their actual paths could be made. Nevertheless, even in these cases, some clue to their identification is supplied. The intervals between their appearances alone is a highly probable test of identity. Thus if comets were regularly recorded to have appeared at intervals of fifty years (no circumstance affording evidence of the diversity of these objects), they might be assumed, with a high degree of probability, to be the successive returns of an elliptic comet having that interval as its period.

3017. *Classification of the cometary orbits.*—The appearances of about 400 comets had been recorded in the annals of various countries before the end of the seventeenth century, the epoch signalised by the discoveries and researches of Newton. In most cases, however, the only circumstance recorded was the appearance of the object, accompanied in many instances with details bearing evident marks of exaggeration respecting its magnitude, form, and splendour. In some few cases, the constellations through which the object passed successively, with the necessary dates, are mentioned, and in some, fewer still, observations of a rough kind have been handed down. From such scanty data, eagerly sought for in the works preserved in different countries, sufficient materials have been collected for the computation, with more or less approximation, of the elements of the orbits of about sixty of the 400 comets above mentioned.

Since the time of Newton, Halley, and their contemporaries,

observers have been more active, and have had the command of instruments of considerable and constantly increasing power; so that every comet which has been visible from the northern hemisphere of the earth since that time, has been observed with continually increasing precision, and data have been in all cases obtained, by which the elements of the orbits have been calculated. Since the year 1700, accordingly, about 140 have been observed, the elements of the orbits of which have been ascertained with great precision.

It appears, therefore, that of the entire number of comets which have appeared in the firmament, the orbits of about 200 have been ascertained. Of this number forty have been ascertained, some conclusively, others with more or less probability, to revolve in elliptic orbits.

Seven have passed through the system in hyperbolas, and consequently will not visit it again, unless they be thrown into other orbits by some disturbing force.

One hundred and sixty have passed through the system either in parabolic orbits, or in ellipses of such extreme eccentricity as to be undistinguishable from parabolas by any data supplied by the observations.

II. Elliptic Comets revolving within the orbit of Saturn.

3018. *Encké's comet.*—In 1818, a comet was observed at Marseilles, on the 26th of November, by M. Pons. In the following January, its path being calculated, M. Arago immediately recognised it as identical with one which had appeared in 1805. Subsequently, M. Encké of Berlin succeeded in calculating its entire orbit—inferring the invisible from the visible part—and found that its period was about twelve hundred days. This calculation was verified by the fact of its return in 1822, since which time the comet has gone by the name of *Encké's comet*, and returned regularly.

It may be asked, How it could have happened that a comet which made its revolution in a period so short as three years and a quarter, should not have been observed until so recent an epoch as 1818? This is explained by the fact that the comet is so small, and its light so feeble even when in the most favourable position, that it can only be seen with the aid of the

telescope, and not even with this except under certain conditions which are not fulfilled on the occasion of every perihelion passage. Nevertheless, the comet was observed on three former occasions, and the general elements of its path recorded, although its elliptic, and consequently periodic character, was not recognised.

On comparing, however, the elements then observed with those of the comet now ascertained, no doubt can be entertained of their identity.

3019. *Table of the elements of the orbit.*—In the following table are given the elements of the orbit of this comet, as computed from the observations made upon it at each of its three appearances in 1786, 1795, and 1805, before its periodic character was discovered, and at its eleven subsequent appearances up to 1852.

TABLE I.

Elements of the Orbit of Encké's Comet to 1852.

	Mean Distance, Earth's =1.	Eccentricity.	Perihelion Distance.	Aphelion Distance.	Longitude of Perihelion.			Longitude of Ascending Node.			Inclination.			Time of Perihelion Passage.		
	a	e	$d' = a \times (1-e)$	$d'' = a \times (1+e)$	ϖ			ν			i					
					°	′	″	°	′	″	°	′	″		h.	m.
1786	2·2080	0·8484	0·3348	4·0812	156	38		334	8		13	36		Jan. 30.	21	7
1795	2·2130	0·8489	0·3344	4·0916	156	41	20	334	39	22	13	42	30	Dec. 21.	10	44
1805	2·2131	0·8462	0·3404	4·0860	156	47	24	334	20	10	13	33	30	Nov. 21.	12	9
1819	2·2141	0·8486	0·3353	4·0929	156	59	12	334	33	19	13	36	54	Jan. 27.	6	18
1822	2·2244	0·8445	0·3460	4·1028	157	11	44	334	25	9	13	20	17	May 23.	23	16
1825	2·2233	0·8449	0·3449	4·1017	157	14	31	334	27	30	13	21	24	Sept. 16.	6	43
1829	2·2239	0·8446	0·3455	4·1023	157	17	53	334	29	32	13	20	34	Jan. 9.	18	3
1832	2·2219	0·8454	0·3435	4·1003	157	21	1	334	32	9	13	22	9	May 23.	23	34
1835	2·2227	0·8450	0·3444	4·1010	157	23	29	334	34	59	13	21	15	Aug. 26.	8	49
1838	2·2222	0·8452	0·3440	4·1004	157	27	4	334	36	41	13	21	28	Dec. 19.	0	27
1842	2·2229	0·8448	0·3450	4·0998	157	29	27	334	39	10	13	20	26	April 12.	0	35
1845	2·2215	0·8474	0·3381	4·1049	157	44	21	334	19	34	13	7	34	Aug. 9.	15	11
1848	2·2147	0·8478	0·3371	4·0923	157	47	8	334	22	12				Nov. 26.	3	+ 0
1852	2·2152	0·8477	0 3374	4·0930	157	51	2	334	23	21	13	7	55	Mar. 14.	20	
															M.T.B.	

The motion of this comet is direct; and its period in 1852 was 3·29616 years, which is subject to a slight variation.

It is evident that between 1786 and 1795 there were two, between 1795 and 1805 two, and, in fine, between 1805 and 1819 three, unobserved returns to perihelion.

It appears, therefore, that, excepting the oval form of the orbit, the motion of this body differs in nothing from that of a planet whose mean distance from the sun is that of the nearest of the planetoids. Its eccentricity is such, however, that when in perihelion it is within the orbit of Mercury, and when in

aphelion it is outside the most distant of the planetoids, and at a distance from the sun equal to four-fifths of that of Jupiter.

3020. *Indications of the effects of a resisting medium.* — A fact altogether anomalous in the motions of the bodies of the solar system, and indicating a consequence of the highest physical importance, has been disclosed in the observation of the motion of this comet. It has been found that its periodic time, and consequently its mean distance, undergoes a slow, gradual, and apparently regular decrease. The decrease is small, but not at all uncertain. It amounted to about a day in ten revolutions, a quantity which could not by any means be placed to the account either of errors of observation or of calculation; and, besides, this increase is incessant, whereas errors would affect the result sometimes one way and sometimes the other. The period of the comet between 1786 and 1795 was $1208\frac{1}{3}$ days; between 1795 and 1805 it was $1207\frac{9}{10}$ days; between 1805 and 1819 it was $1207\frac{4}{10}$ days; in 1845 it was $1205\frac{1}{4}$ days; and, in fine, in 1852 it was 1204 days.

The magnitude of the orbit thus constantly decreasing (for the cube of its greater axis must decrease in the same proportion as the square of the period), the actual path followed by the comet must be a sort of elliptic spiral, the successive coils of which are very close together, every successive revolution bringing the comet nearer and nearer to the sun.

Such a motion could not arise from the disturbing action of the planets. These forces have been taken strictly into account in the computation of the ephemerides of the comet, and there is still found this residual phenomenon, which cannot be placed to their account, but which is exactly the effect which would arise from any physical agency by which the tangential motion of the comet would be feebly but constantly resisted. Such an agency, by diminishing the tangential velocity, would give increased efficacy to the solar attraction, and, consequently, increased curvature to the comet's path; so that, after each revolution, it would revolve at a less distance from the centre of attraction.

3021. *The luminiferous ether would produce such an effect.* — It is evident that a resisting medium, such as the luminiferous ether (1225.) is assumed to be in the hypothesis which forms the basis of the undulatory theory of light, would produce just such a phenomenon, and, accordingly the motion of this

comet is regarded as a strong evidence tending to convert that hypothetical fluid into a real physical agent.

It remains to be seen whether a like phenomenon will be developed in the motion of other periodic comets. The discovery of these bodies, and the observation of their motions, are as yet too recent to enable astronomers, notwithstanding their greatly multiplied number, to pronounce decisively upon it.

3022. *Comets would ultimately fall into the sun.*—If the existence of this resisting medium should be established by its observed effects on comets in general, it will follow that, after the lapse of a certain time (many ages, it is true, but still a definite interval), the comets will be successively absorbed by the sun, unless, as is not improbable, they should be previously vaporized by their near approach to the solar fires, and should thus be incorporated with his atmosphere.*

3023. *Why like effects are not manifested in the motion of the planets.*—It may be asked, If the existence of a resisting medium be admitted, whether the same ultimate fate must not await the planets? To this inquiry it may be answered that, within the limits of past astronomical record, the ethereal medium, if it exist, has had no sensible effect on the motion of any planet. That it might have a perceptible effect upon comets, and yet not upon planets, will not be surprising, if the extreme

* In the efforts by which the human mind labours after truth, it is curious to observe how often that desired object is stumbled upon by accident, or arrived at by reasoning which is false. One of Newton's conjectures respecting comets was, that they are "the aliment by which suns are sustained;" and he therefore concluded that these bodies were in a state of progressive decline upon the suns, round which they respectively swept; and that into these suns they from time to time fell. This opinion appears to have been cherished by Newton to the latest hours of his life: he not only consigned it to his immortal writings; but, at the age of eighty-three, a conversation took place between him and his nephew on this subject, which has come down to us. "I cannot say," said Newton, "when the comet of 1680 will fall into the sun: possibly after five or six revolutions; but whenever that time shall arrive, the heat of the sun will be raised by it to such a point, that our globe will be burnt, and all the animals upon it will perish. The new stars observed by Hipparchus, Tycho, and Kepler, must have proceeded from such a cause, for it is impossible otherwise to explain their sudden splendour." His nephew then asked him, "Why, when he stated in his writings that comets would fall into the sun, did he not also state those vast fires they must produce, as he supposed they had done in the stars?"—"Because," replied the old man, "the conflagrations of the sun concern us a little more directly. I have said, however," added he, smiling, "enough to enable the world to collect my opinion."

lightness of the comets compared with their bulk be considered. The effect in the two cases may be compared to that of the atmosphere upon a piece of swan's down and upon a leaden bullet moving through it. It is certain that whatever may be the nature of this resisting medium, it will not, for many hundred years to come, produce the slightest perceptible effect upon the motions of the planets.

3024. *Corrected estimate of the mass of Mercury.*—The masses of comets in general are, as will be explained, incomparably smaller than those of the smallest of the planets; so much so, indeed, as to bear no appreciable ratio to them. A consequence of this is, that while the effects of their attraction upon the planets are altogether insensible, the disturbing effects of the masses of the planets upon them are very considerable. These disturbances, being proportional to the disturbing masses, may then be used as measures of the latter, just as the movement of the pith-ball in the balance of torsion supplies a measure of the physical forces to which that instrument is applied.

Encké's comet near its perihelion passes near the orbit of Mercury; and when that planet at the epoch of its perihelion happens to be near the same point, a considerable and measurable disturbance is manifested in the comet's motion, which being observed supplies a measure of the planet's mass.

This combination of the motions of the planet and comet took place under very favourable circumstances, on the occasion of the perihelion passage of the comet in 1838, the result of which, according to the calculations of Professor Encké, was the discovery of an error of large amount in the previous estimates of the mass of the planet. After making every allowance for other planetary attractions, and for the effects of the resisting medium, the existence of which it appears necessary to admit, it was inferred that the mass assigned to Mercury by Laplace was too great in the proportion of 12 to 7.

This question is still under examination, and every succeeding perihelion passage of the comet will increase the data by which its more exact solution may be accomplished.

3025. *Biela's comet.*—On February 28th, 1826, M. Biela, an Austrian officer, observed in Bohemia a comet, which was seen at Marseilles at about the same time by M. Gambart. The path which it pursued, was observed to be similar to that of

comets which had appeared in 1772 and 1806. Finally, it was found that this body moved round the sun in an oval orbit, and that the time of its revolution was about 6 years and 8 months. It has since returned at its predicted times, and has been adopted as a member of our system, under the name of Biela's comet.

Biela's comet moves in an orbit whose plane is inclined at a small angle to those of the planets. It is but slightly oval, the length being to the breadth in the proportion of about four to three. When nearest to the sun, its distance is a little less than that of the earth; and when most remote from the sun, its distance somewhat exceeds that of Jupiter. Thus it ranges through the solar system, between the orbits of Jupiter and the earth.

This comet had been observed in 1772 and in 1806; but the elliptic form of its orbit, and consequently its periodicity, was not discovered. Its return to perihelion was predicted and observed in 1832, in 1846, and in 1852; but that which took place in 1838 escaped observation, owing to its unfavourable position and extreme faintness.

The elements of the orbit, deduced from the observations made on each of its appearances to 1846 inclusive, are given in Table II. We have not yet obtained calculations of its elements from observations made in 1852. It was first seen in that year by Professor Secchi at Rome, on September 16th, and continued to be seen for three weeks. It was preceded in right ascension about two minutes of time by a still fainter comet, whose real distance from it must have been about a million and a quarter miles.

TABLE II.

Elements of the Orbit of Biela's Comet to 1846.

	Mean Distance, Earth's = 1.	Eccentricity.	Perihelion Distance.	Aphelion Distance.	Longitude of Perihelion.			Longitude of Ascending Node.			Inclination.			Time of Perihelion Passage.		
	a	e	$d' = a \times (1 - e)$	$d'' = a \times (1 + e)$	ϖ			ν			ι					
					°	′	″	°	′	″	°	′	″		h.	m.
1772	2·8222	0·6769	0·9118	4·73	97	21	0	263	24	0	17	39	0	Feb. 8.	1	0
1806	3·5671	0·7458	0·9068	6·23	109	32	23	251	15	15	13	33	45	Jan. 1.	23	32
1826	3·5613	0·7466	0·9025	6·23	109	45	50	251	28	12	13	33	51	March 18.	10	3
1832	3·5389	0·7516	0·8792	6·20	110	0	25	248	15	18	13	12	47	Nov. 26.	1	41
1846	3·5159	0·7563	0·8567	6·18	109	5	31	245	47	51	12	39	45	Feb. 11.	0	43

3026. *Possibility of the collision of Biela's comet with the*

earth.—One of the points at which the orbit of Biela's comet intersects the plane of the ecliptic, is at a distance from the earth's orbit less than the sum of the semi-diameters of the earth and the comet. It follows, therefore (2905.), that if the comet should arrive at this point at the same moment at which the earth passes through the point of its orbit which is nearest to it, a portion of the globe of the earth must penetrate the comet.

It was estimated on the occasion of the perihelion passage of this comet in 1832, that the semi-diameter of the comet (that body being nearly globular, and having no perceptible tail) was 21,000 miles, while the distance of the point at which its centre passed through the plane of the ecliptic, on the 29th October in that year, from the path of the earth was only 18,600 miles. If the centre of the earth happened to have been at the point of its orbit nearest to the centre of the comet on that day, the distance between the centres of the two bodies would have been only 18,600 miles, while the semi-diameter of the comet was 21,000 miles; and the semi-diameter of the earth being in round numbers 4000 miles, it would follow that in such a contingency the earth would have plunged into the comet to a depth of

$$21{,}000 + 4000 - 18{,}600 = 6400 \text{ miles},$$

a depth exceeding three-fourths of the earth's diameter.

The possibility of such a catastrophe having been rumoured, great popular alarm was excited before the expected return of the comet in 1832. It was, however, shown that on the 29th October the earth would be about five millions of miles from the point of danger, and that on the arrival of the earth at that point the comet would have moved to a still greater distance.

3027. *Resolution of Biela's comet into two.*—One of the most extraordinary phenomena of which the history of astronomy affords any example, attended the appearance of this comet in 1846. It was on that occasion seen to resolve itself into two distinct comets, which, from the latter end of December, 1845 to the epoch of its disappearance in April, 1846, moved in distinct and independent orbits. The paths of these two bodies were in such optical juxtaposition that both were always seen together in the field of view of the telescope, and the greatest visual angle between their centres did not amount at

any time to 10′, the variation of that angle arising principally from the change of direction of the visual line, relatively to the line joining their centres, and to the change of the comet's distance from the earth.

M. Plantamour, director of the Observatory of Geneva, calculated the orbits of these two comets, considered as independent bodies; and found that the real distance between their centres was, subject to but little variation while visible, about thirty-nine semi-diameters of the earth, or two-thirds of the moon's distance. The comets moved on thus side by side, without manifesting any reciprocal disturbing action; a circumstance no way surprising, considering the infinitely minute masses of such bodies.

3028. *Changes of appearance attending the separation.*— The original comet was apparently a globular mass of nebulous matter, semi-transparent at its very centre, no appearance of a tail being discoverable. After the separation, both comets had short tails, parallel in their direction, and at right angles to the line joining their centres; both had nuclei. From the day of their separation the original comet decreased, and the companion increased in brightness until (on the 10th February) they were sensibly equal. After this the companion still increased in brightness, and from the 14th to the 16th was not only greatly superior in brightness to the original, but had a sharp and starlike nucleus compared to a diamond spark. The change of brightness was now reversed, the original comet recovering its superiority, and acquiring on the 18th the same appearance as the companion had from the 14th to the 16th. After this the companion gradually faded away, and disappeared previously to the final disappearance of the original comet on 22nd April.

It was observed also that a thin luminous line or arc was thrown across the space which separated the centres of the two nuclei, especially when one or the other had attained its greatest brightness, the arc appearing to emanate from that which for the moment was the brighter.

After the disappearance of the companion, the original comet threw out three faint tails, forming angles of 120° with each other, one of which was directed to the place which had been occupied by the companion.

It is suspected that the faint comet which was observed by Prof. Secchi to precede Biela's comet in 1852, may have been the

companion thus separated from it, and if so, the separation must be permanent, the distance between the parts being greater than that which separates the earth from the sun.

3029. *Faye's comet.*—On the 22nd November, 1843, M. Faye, of the Paris Observatory, discovered a comet, the path of which soon appeared to be incompatible with the parabolic character. Dr. Goldschmidt showed that it moved in an ellipse of very limited dimensions, with a period of 7½ years. It was immediately observed as being extraordinary, that, notwithstanding the frequent returns to perihelion which such a period would infer, its previous appearances had not been recorded. M. Faye replied by showing that the aphelion of the orbit passed very near to the path of Jupiter, and that it was possible that the violent action of the great mass of that planet, in such close proximity with the comparatively light mass of the comet, might have thrown the latter body into its present orbit, its former path being either a parabola or an ellipse, with such elements as to prevent the comet from coming within visible distance. M. Faye supported these observations by reference to a more ancient comet, which we shall presently notice, to which a like incident is supposed with much probability, if not certainty, to have occurred.

3030. *Re-appearance in* 1850–1 *calculated by M. Le Verrier.*—The observations which had been made in 1843, at several observatories, but more especially those made by M. Struve at Pultowa, who continued to observe the comet long after it ceased to be observed elsewhere, supplied to M. Le Verrier the data necessary for the calculation of its motion in the interval between its perihelion in 1843 and its expected re-appearance in 1850–1, subject to the disturbing action of the planets, and predicted its succeeding perihelion for the 3rd of April, 1851.

Aided by the formulæ of M. Le Verrier, Lieutenant Stratford calculated a provisional ephemeris in 1850, by which observers might be enabled more easily to detect the comet, which was the more necessary as the object is extremely faint and small, and not capable of being seen except by means of the most perfect telescopes. By means of this ephemeris, Professor Challis, of Cambridge, found the comet on the night of the 28th November very nearly in the place assigned to it in the tables. Two observations only were then made upon it, which, however, were sufficient to enable M. Le Verrier to give still

greater precision to his formulæ, by assigning a definite numerical value to a small quantity which before was left indeterminate. Lieutenant Stratford, with the formula thus corrected, calculated a more extensive and exact ephemeris, extending to the last day of March, and published it in January, 1851, in the Nautical Almanack.

The comet, though extremely faint and small, and consequently difficult of observation, continued to be observed by Professor Challis with the great Northumberland telescope at Cambridge, and by M. Struve at Pultowa, and it was found to move in exact accordance with the predictions.

3031. *De Vico's comet.*—On the 22nd August, 1844, M. de Vico, of the Roman Observatory, discovered a comet whose orbit was soon afterwards proved by M. Faye to be an ellipse of moderate eccentricity, with a period of about 5½ years. It arrived at its perihelion on the 2nd of September, and continued to be observed until the 7th of December. The return of this comet to perihelion was predicted for March, 1851; but, owing most probably to its apparent proximity to the sun, it was not seen. It will be more favourably situate on its next return in 1855. Dr. Brünnow has computed the effects of the planetary perturbations upon it; so that an ephemeris of its positions in the heavens for that year, will be placed in the hands of observers. It will pass through its perihelion on the 6th August, 1855.

M. Le Verrier has made some computations, which render it somewhat probable that a comet which passed its perihelion in August, 1678, is identical with that discovered by De Vico.

3032. *Brorsen's comet.*—On the 26th of February, 1846, M. Brorsen, of Kiel, discovered a faint comet, which was soon found to move in an elliptic orbit, with a period of about 5½ years. Its position in the heavens not being favourable, the observations upon it were few, and the resulting elements, consequently, not ascertained with all the precision that might be desired. Its re-appearance on its approach to the succeeding perihelion, was expected from September to November, 1851. It escaped observation however, owing to its unfavourable position in relation to the sun. Its next perihelion passage will take place in 1857.

3033. *D'Arrest's comet.*—On the 27th of June, 1851, Dr. d'Arrest, of the Leipsic Observatory, discovered a faint comet,

which M. Villarceaux proved to move in an elliptic orbit, with a period of about 6¼ years. The next perihelion passage of this comet will take place in the end of 1857, or the beginning of 1858.

3034. *Elliptic comet of* 1743.—A revision of the recorded observations of former comets by the more active and intelligent zeal of modern mathematicians and computers, has led to the discovery of the great probability of several among them having revolved in elliptic orbits, with periods not differing considerably from those of the comets above mentioned. The fact that these comets have not been re-observed on their successive returns through perihelion, may be explained, either by the difficulty of observing them, owing to their unfavourable positions, and the circumstance of observers not expecting their re-appearance, their periodic character not being then suspected; or because they may have been thrown by the disturbing action of the larger planets into orbits such as to keep them continually out of the range of view of terrestrial observers.

Among those may be mentioned a comet which appeared in 1743, and was observed by Zanetti at Bologna; the observations indicate an elliptic orbit, with a period of about 5½ years.

3035. *Elliptic comet of* 1766.—This comet, which was observed by Messier, at Paris, and by La Nux, at the Isle of Bourbon, revolved, according to the calculations of Burckhardt, in an ellipse with a period of 5 years.

3036. *Lexell's comet.*—The history of astronomy has recorded one singular example of a comet which appeared in the system, made two revolutions round the sun in an elliptic orbit, and then disappeared, never having been seen either before or since.

This comet was discovered by Messier, in June 1770, in the constellation of Sagittarius between the head and the northern extremity of the bow, and was observed during that month. It disappeared in July, being lost in the sun's rays. After passing through its perihelion, it re-appeared about the 4th of August, and continued to be observed until the first days of October, when it finally disappeared.

All the attempts of the astronomers of that day failed to deduce the path of this comet from the observations, until six years later, in 1776, Lexell showed that the observations were explained, not, as had been assumed previously, by a parabolic

path, but by an ellipse, and one, moreover, without any example at that epoch, which indicated the short period of 5½ years.

It was immediately objected to such a solution, that its admission would involve the consequence that the comet, with a period so short, and a magnitude and splendour such as it exhibited in 1770, must have been frequently seen on former returns to perihelion; whereas no record of any such appearance was found.

To this Lexell replied, by showing that the elements of its orbit, derived from the observations made in 1770, were such that at its previous aphelion, in 1767, the comet must have passed within a distance of the planet Jupiter fifty-eight times less than its distance from the sun; and that consequently it must then have sustained an attraction from the great mass of that planet, more than three times more energetic than that of the sun; that consequently it was thrown out of the orbit in which it previously moved, into the elliptic orbit in which it actually moved in 1770; that its orbit previously to 1767 was, according to all probability, a parabola; and, in fine, that consequently moving in an elliptic orbit from 1767 to 1770, and having the periodicity consequent on such motion, it nevertheless moved only for the first time in its new orbit, and had never come within the sphere of the sun's attraction before this epoch.

Lexell further stated, that since the comet passed through its aphelion, which nearly intersected Jupiter's orbit, at intervals of somewhat above 5½ years, and it encountered the planet near that point in 1767, the period of the planet being somewhat above 11 years, the planet after a single revolution and the comet after two revolutions must necessarily again encounter each other in 1779; and, that since the orbit was such that the comet must in 1779 pass at a distance from Jupiter 500 times less than its distance from the sun, it must suffer from that planet an action 250 times greater than the sun's attraction, and that therefore it would in all probability be again thrown into a parabolic or hyperbolic path; and if so, that it would depart for ever from our system to visit other spheres of attraction. Lexell, therefore, anticipated the final disappearance of the comet, which actually took place.

In the interval between 1770 and 1779, the comet returned once to perihelion; but its position was such that it was above

the horizon only during the day, and could not in the actual state of science be observed.

3037. *Analysis of Laplace applied to Lexell's comet.*—At this epoch analytical science had not yet supplied a definite solution of the problem of cometary disturbances. At a later period the question was resumed by Laplace who in his celebrated work, the *Mécanique céleste* gave the general solution of the following problem.

"The actual orbit of a comet being given, what was its orbit before, and what will be its orbit after being submitted to any given disturbing action of a planet near which it passes?"

3038. *Its orbit before* 1767 *and after* 1770 *calculated by his formulæ.*—Applying this to the particular case of Lexell's comet, and assuming as data the observations recorded in 1770, Laplace showed that, before sustaining the disturbing action of Jupiter in 1767, the comet must have moved in an ellipse of which the semi-axis major was 13·293 and consequently that its period, instead of being 5½ years, must have been 48½ years; and that the eccenticity of the orbit was such that its perihelion distance would be but little less than the mean distance of Jupiter, and that consequently it could never have been visible. It followed also that, after suffering the disturbing action of Jupiter in 1779, the comet passed into an elliptic orbit whose semi-axis major was 7·3, that its period was consequently 20 years, and that its eccentricity was such that its perihelion distance was more than twice the distance of Mars, and that in such an orbit it could not become visible.

3039. *Revision of these researches by M. Le Verrier.*—This investigation has recently been revised by M. Le Verrier*, who has shown that the observations of 1770 were not sufficiently definite and accurate to justify conclusions so absolute. He has shown, that the orbit of 1770 is subject to an uncertainty comprised between certain definite limits; that tracing the consequences of this to the positions of the comet in 1767 and 1779, these positions are subject to still wider limits of uncertainty. Thus he shows that, compatibly with the observations of 1770, the comet might in 1779 pass either considerably outside, or considerably inside Jupiter's orbit, or might, as it was supposed to have done, have passed actually within the orbits of his satellites. He deduces in fine the following general conclusions:—

* See Mém. Acad. des Sciences, 1847, 1848.

1. That if the comet had passed within the orbits of the satellites, it must have fallen down upon the planet and coalesced with it; an incident which he thinks highly improbable, though not absolutely impossible.

2. The action of Jupiter may have thrown the comet into a parabolic or hyperbolic orbit, in which case it must have departed from our system altogether, never to return, except by the consequence of some disturbance produced in another sphere of attraction.

3. It may have been thrown into an elliptic orbit, having a great axis and long period, and so placed and formed that the comet could never become visible; a supposition within which comes the solution of Laplace.

4. It may have had merely its elliptic elements more or less modified by the action of the planet, without losing its character of short periodicity; a result which M. le Verrier thinks the most probable, and which would render it possible that this comet may still be identified with some one of the many comets of short period, which the activity and sagacity of observers are every year discovering.

To facilitate such researches M. Le Verrier has given a table, including all the possible systems of elliptic elements of short period which the comet could have assumed, subject to the disturbing action of Jupiter in 1779, and taking the observations of 1770 within their possible limits of error.

He further demonstrates, that the orbit in which the comet moved antecedently to the disturbing action of Jupiter upon it in 1767, not only could not have been a parabola or hyperbola, but must have been an ellipse, whose major axis was considerably less than that which Laplace deduced from the insufficient observations of Messier. He shows that, before that epoch, the perihelion distance of the comet could not, under any possible supposition, have exceeded three times the earth's mean distance, and most probably was included between 1½ and 2 times that distance; and that the semi-axis major of the orbit could not have exceeded 4½ times the earth's mean distance, a magnitude 3 times less than that assigned to it by the calculations of Laplace.

3040. *Process by which the identification of periodic comets may be decided.*—It must not, however, be supposed that it is suf-

ficient to compare the actual elements of each periodic comet thus discovered, with the elements given in the table of M. Le Verrier, and to infer the absence of identity from their discordance. Such an inference would only be rendered valid by showing that in past ages, the comet in question had suffered no serious disturbing action by which the elements of its orbit could be considerably changed. To decide the question a much more laborious and difficult process must be encountered; a process from which the untiring spirit of M. Le Verrier has not shrunk. It is necessary, in fine, to the satisfactory and conclusive solution of such a problem, that the periodic comet in question should be traced back through all its previous revolutions up to 1779, that all the disturbances which it suffered from the planets which it encountered in that interval be calculated and ascertained, and that by such means the orbit which it must have had previous to such disturbances, in 1779, be determined. Such orbit would then be compared with the table of possible orbits of Lexell's comet, as given by M. le Verrier; and if it were found to be identical with any of them, the identity of the comet in question with that of Lexell, would be inferred with the highest degree of probability; but if, on the other hand, such descrepancies were found to prevail as must exceed all supposable errors of observation or calculation, the diversity of the comets would follow.

3041. *Application of this process by M. Le Verrier to the comets of Faye, de Vico and Brorsen and that of Lexell.—Their diversity proved.*—M. Le Verrier has applied these principles to the comets of Faye, De Vico, and Brorsen; tracing back their histories during their unseen motions for three-quarters of a century, and ascertaining the effects of the disturbing actions which they must severally have sustained from revolution to revolution, until he brought them to the epoch of 1779. On comparing the orbits thus determined with those of the table of possible orbits of Lexell's comet he has shown that none of them can be identical with it, however strongly some of the elements of their present orbits may raise such a presumption.

3042. *Probable identity of De Vico's comet with the comet of 1678.*—The comet of De Vico having presented striking analogies with a comet which was observed by Tycho Brahe and Rothmann in 1585, and one observed by La Hire in 1678, M. Le

Verrier has applied like principles to the investigation of these questions.

MM. Laugier and Mauvais observed that the elements of De Vico's comet presented such a resemblance to that of Tycho Brahe, as almost to decide the question of their identity. M. Le Verrier tracing back the comet of De Vico to 1585, has shown that its orbit at that epoch was so different from that of the comet of Tycho, as to be incompatible with any plausible inference of their identity.*

He has shown, however, by like reasoning, that there is a high degree of probability that the comet of De Vico is identical with that observed by La Hire in 1678.

3043. *Blainplan's comet of* 1819.—M. Blainplain discovered a comet at Marseilles on 28th November 1819, which was observed at Milan until 25th January 1820. The observations reduced and calculated by Prof. Encké gave an elliptic orbit with a period a little short of 5 years. Clausen conjectures that this comet may be identical with that of 1743. It has not been seen since 1820.

3044. *Pons's comet of* 1819.—A comet was discovered by M. Pons on June 12th 1819, which was observed until July 19th Prof. Encké assigned to it an elliptic orbit, with a period of 5¼ years.

3045. *Pigott's comet of* 1783.—A comet, discovered by Mr. Pigott at New York in 1783, was shown by Buckhardt to have an elliptic orbit, with a period of 5½ years.

3046. *Peters's comet of* 1846.—On the 26th June 1846, a comet was discovered at Naples by M. Peters, which was subsequently observed at Rome by De Vico, and continued to be seen until 21st July. An elliptic orbit is assigned to this comet, with a period of from 13 to 16 years, some uncertainty attending the observations. The re-appearance of this comet may be expected in 1859, 1860.

3047. *Tabular synopsis of the orbits of the comets which revolve within Saturn's orbit.*—In Table III. we have given the elements of the thirteen comets above mentioned.

* Mém. Acad. des Sciences, 1847.

TABLE III.

Synopsis of the Motion of the Elliptic Comets which revolve within the Orbit of Saturn.

Designation.	Mean Distance, Earth's = 1.	Eccentricity.	Perihelion Distance, Earth's = 1.	Aphelion Distance, Earth's = 1.	Daily Motion. At Mean Distance.	Daily Motion. At Perihelion.	Daily Motion. At Aphelion.
	a	e	$d' = a \times (1 - e)$	$d'' = a \times (1 + e)$	a	a'	a''
					″	″	″
1. Encké - -	2·2148	0·8477	0·3370	4·0927	1076·5	46497	315·3
2. Biela - -	3·5245	0·7570	0·8564	6·1926	536·3	9082	173·7
3. Faye - -	3·8118	0·5559	1·6926	5·9310	476·8	2418·5	196·9
4. De Vico - -	3·1028	0·6173	1·1863	5·0194	648·8	5264·1	247·8
5. Brorsen - -	3·1465	0·7945	0·6501	5·6429	635·7	14893	197·7
6. D'Arrest - -	3·4618	0·6609	1·1740	5·7497	550·9	4789·7	199·7
7. Clausen (1743) -	3·0913	0·7213	0·8615	5·3211	652·8	8406	220·4
8. Burckhardt (1766) -	2·9337	0·8640	0·3990	5·4670	706·1	38175	203·3
9. Lexell (1770) -	3·1560	0·7861	0·6745	5·6375	632·8	13855	198·3
10. Blainpain (1819) -	2·8490	0·6867	0·8926	4·8060	737·8	7516·4	259·3
11. Pons (1819) - -	3·1602	0·7552	0·7736	5·5468	631·6	10300	200·3
12. Pigott (1783) -	4·6496	0·6784	1·4953	7·8039	353·9	3422·7	79·3
13. Peters (1846) -	6·3206	0·7567	1·5377	11·1020	221·9	3748·3	71·9

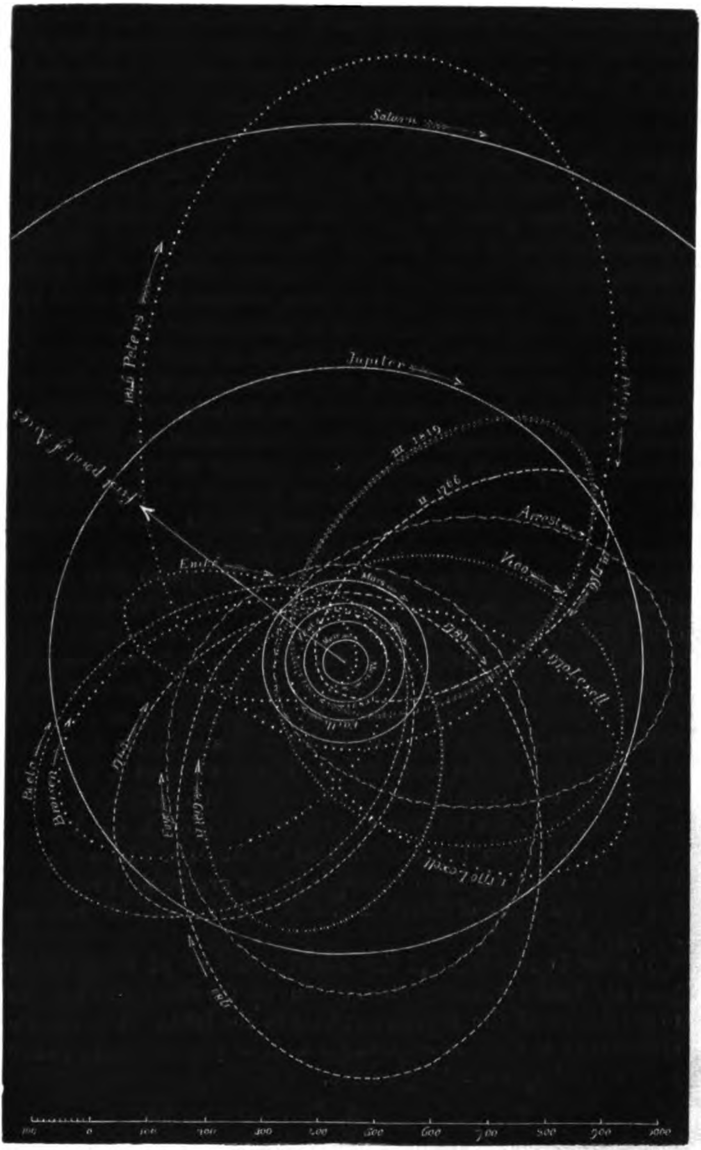
Designation.	Period, Years.	Longitude of Perihelion.			Longitude of Ascending Node.			Inclination.			Time of Perihelion Passage.			Direction of Motion.
	P	ϖ			ν			i			M. T. G.			
		°	′	″	°	′	″	°	′	″		h.	m.	
1. Encké - -	3·296	157	51	2	334	23	21	13	7	24	Mar. 14, 1852.	18	58	D
2. Biela - -	6·617	109	2	20	245	54	39	12	34	53	Feb. 10, 1846.	23	42	D
3. Faye - - -	7·441	49	34	19	209	29	19	11	22	31	Oct. 17, 1843.	3	33	D
4. De Vico - -	5·469	342	31	15	63	49	31	2	54	45	Sept. 2, 1844.	11	28	D
5. Brorsen - -	5·581	116	28	24	102	37	41	30	57	51	Feb. 25, 1846.	7	57	D
6. D'Arrest - -	6·641	322	59	46	148	27	20	13	56	12	July 8, 1851.	16	48	D
7. Clausen (1743) -	5·435	93	19	35	86	54	29	1	53	43	Jan. 8, 1743.	4	39	D
8. Burckhardt (1766) -	5·025	251	13	0	74	11	0	8	1	45	Apr. 26, 1766.	23	44	D
9. Lexell (1770) -	5·607	356	17	12	131	59	17	1	34	28	Aug. 13, 1770.	12	58	D
10. Blainpain (1819) -	4·809	67	18	48	77	13	57	9	1	16	Nov. 20, 1819.	6	3	D
11. Pons (1819) -	5·618	274	40	51	113	10	46	10	42	48	July 18, 1819.	21	46	D
12. Pigott (1783) -	10·025	49	31	55	55	12	0	47	43	0	Nov. 19, 1783.	13	30	D
13. Peters (1846) -	15·990	239	49	51	260	12	25	13	2	14	June 1, 1846.	2	40	D

3048. *Diagram of the orbits.*—In *fig.* 812., the orbits of these thirteen comets brought to a common plane, are represented roughly but in their proper proportions and relative positions, so as to exhibit to the eye their several ellipticities, and the relative directions of their axes.* All these bodies, without one exception, revolve in the common direction of the planets.

3049. *Planetary character of their orbits.*—It is not alone, however, in the direction of their motions that the orbits of these bodies have an analogy to those of the planets. Their

* In the diagram, to prevent confusion, the orbits of the different comets are indicated by dotted or broken lines of different kinds.

Fig. 812.



inclinations, with one exception, are within the limits of those of the planets. Their eccentricities, though incomparably greater than those of the planets, are, as will presently appear, incomparably less than those of all other comets yet discovered. Their mean distances and periods (with the exception of the last two in the table) are within the limits of those of the planetoids.

3050. *Planetary character of their orbits.* — The comparison of the numbers given in Table III. with those which will be given hereafter, in the tables of the elements of other elliptic comets, and the comparison of the diagrams of their orbits with those of others, will show in a striking manner, to how great an extent the orbits of this group of comets possess the planetary character. Besides moving round the sun in the common direction, their inclinations, with a single exception, are within the limits of those of the planets. It is true that their eccentricities have an order of magnitude much greater; but on the other hand, it will be seen presently that they are incomparably less than the eccentricities of all other periodic comets yet discovered. Their mean distances and periods place them in direct analogy with the planetoids.

Moderate, as are the eccentricities as compared with those of other comets, they are sufficiently great to impart a decided oval form to the orbits, and to produce considerable differences between the perihelion and aphelion distances, as will be apparent by inspecting the numbers in the columns d' and d''. It appears by these, that while the perihelion of Encké's comet lies within the orbit of Mercury, its aphelion lies outside the orbit of the most remote of the planetoids, and not far within that of Jupiter. The perihelion of Biela's comet, in like manner, lies between the orbits of the earth and Venus, while its aphelion lies outside that of Jupiter. In the case of Faye's comet, the least eccentric of the group, the perihelion lies near the orbit of Mars, and the aphelion outside that of Jupiter.

It must be remembered that the elliptic form of these orbits has only been verified by observations on the successive returns to perihelion of the first five comets in the table. The elliptic elements of the others may, so far as is at present known, have been effaced by disturbing causes.

The angular motions at the mean and extreme distances from the sun, given in the columns α, α' and α'', have been computed, on the principles already explained, by the formulæ

$$\alpha=\frac{1,296,000}{365{\cdot}25\times \mathrm{P}},\quad \alpha'=\alpha\times\frac{a^2}{d'^2},\quad \alpha''=\alpha\times\frac{a^2}{d''^2}.$$

The same numbers which express these angular motions, also express in all cases the intensities of solar light and heat in the several positions of the comet; and also the apparent motion of the sun, as seen from the comet; and a comparison of these with the corresponding numbers related to any of the planets, will illustrate in a striking manner how different are the physical conditions by which these two classes of bodies are affected; and this will be more and more striking, when the other groups of comets have been noticed.

Taking the comet of Encké as an example, it appears that while its mean daily motion is 1076″ or 18′, its motion in aphelion is only 5′, and in perihelion nearly 13°. Its motion in perihelion, the light and heat it receives from the sun, and the apparent motion of the sun as seen from it, are therefore severally more than 150 times greater in perihelion than in aphelion.

III. Elliptic Comets, whose mean distances are nearly equal to that of Uranus.

3051. *Comets of long periods first recognised as periodic.*—It might be expected, that comets moving in elliptic orbits of small dimensions, and consequently having short periods, would have been the first in which the character of periodicity would be discovered. The comparative frequency of their returns to those positions near perihelion, where alone bodies of this class are visible from the earth, and the consequent possibility of verifying the fact of periodicity, by ascertaining the equality of the intervals between their successive returns to the same heliocentric position, to say nothing of the more distinctly elliptic form of the arcs of their orbits in which they can be immediately observed, would afford strong ground for such an expectation;

nevertheless in this case, as has happened in so many others in the progress of physical knowledge, the actual results of observation and research have been directly contrary to such an anticipation; the most remarkable case of a comet of large orbit, long period, and rare returns, being the first, and those of small orbits, short periods, and frequent returns, the last whose periodicity has been discovered.

3052. *Newton's conjectures as to the existence of comets of long periods.* — It is evident that the idea of the possible existence of comets with periods shorter than those of the more remote planets, and orbits circumscribed within the limits of the solar system, never occurred to the mind either of Newton or any of his contemporaries or immediate successors.

In the third book of his PRINCIPIA, he calls comets a species of planets, revolving in elliptic orbits of a very oval form. But he continues, "I leave to be determined by others the transverse diameters and periods, by comparing comets which return *after long intervals of time* to the same orbits."

It is interesting to observe the avidity with which minds of a certain order snatch at such generalisations, even when but slenderly founded upon facts. These conjectures of Newton were soon after adopted by Voltaire: "Il y a quelque apparence," says he, in an essay on comets, "qu'on connaîtra un jour un certain nombre de ces autres planètes qui sous le nom de comètes tournent comme nous autour du soleil, mais il ne faut pas espérer qu'on les connaisseent toutes."

And again, elsewhere, on the same subject: —

> "Comètes, que l'on craint à l'égal du tonnerre,
> Cessez d'épouvanter les peuples de la terre;
> Dans une ellipse immense achevez votre cours,
> Remontez, descendez près de l'astre des jours."

3053. *Halley's researches.* — Extraordinary as these conjectures must have appeared at the time, they were soon strictly realised. Halley undertook the labour of examining the circumstances attending all the comets previously recorded, with a view to discover whether any, and which of them, appeared to follow the same path. He found that a comet which had been observed by himself, by Newton, and their contemporaries in 1682, followed a path while visible, which coincided so nearly

with those of comets which had been observed in 1607, and in 1531, as to render it extremely probable, that these objects were the same identical comet, revolving in an elliptic orbit of such dimensions, as to cause its return to perihelion at intervals of 75—76 years.

The comet of 1682 had been well observed by La Hire, Picard, Hevelius, and Flamstead, whose observations supplied all the data necessary to calculate its path while visible. That of 1607 had been observed by Kepler and Longomontanus; and that of 1531, by Pierre Apian at Ingolstadt, the observations in both cases being also sufficient for the determination of the path of the body, with all the accuracy necessary for its identification.

3054. *Halley predicts its re-appearance in* 1758–9. — Of the identity of the paths while visible on each of these appearances Halley entertained no doubt, and announced to the world the discovery of the elliptic motion of comets, as the result of combined observation and calculation, and entitled to as much confidence as any other consequence of an established physical law; and predicted the re-appearance of this body, on its succeeding return to perihelion in 1758-9. He observed, however, that as in the interval between 1607 and 1682 the comet passed near Jupiter, its velocity must have been augmented, and consequently its period shortened by the action of that planet. This period, therefore, having been only seventy-five years, he inferred that the following period would probably be seventy-six years or upwards; and consequently that the comet ought not to be expected to appear until the end of 1758, or the beginning of 1759. It is impossible to imagine any quality of mind more enviable than that which, in the existing state of mathematical physics, could have led to such a prediction. The imperfect state of science rendered it impossible for Halley to offer to the world a demonstration of the event which he foretold. "He therefore," says M. de Pontecoulant, "could only announce these felicitous conceptions of a sagacious mind as mere intuitive perceptions, which must be received as uncertain by the world, however he might have felt them himself, until they could be verified by the process of a rigorous analysis."

Subsequent researches gave increased force to Halley's prediction; for it appeared from the ancient records of observers,

that comets had been seen in 1456* and 1378, whose elements were identical with those of the comet of 1682.

3054. *Great advance of mathematical and physical sciences between* 1682 *and* 1759.—In the interval of three quarters of a century which elapsed between the announcement of Halley's prediction and the date of its expected fulfilment, great advances were made in mathematical science; new and improved methods of investigation and calculation were invented; and, in fine, the theory of gravitation was pursued with extraordinary activity and success through its consequences in the mutual disturbances produced upon the motions of the planets and satellites, by the attraction of their masses one upon another. As the epoch of the expected return of the comet to its perihelion approached therefore, the scientific world resolved to divest, as far as possible, the prediction, of that vagueness which necessarily attended it owing to the imperfect state of science at the time it was made, and to calculate the exact effects of those planets whose masses

* The appearance of this comet in 1456, was described by contemporary authorities to have been an object of "unheard-of magnitude;" it was accompanied by a tail of extraordinary length, which extended over sixty degrees (a third of the heavens), and continued to be seen during the whole of the month of June. The influence which was attributed to this appearance, renders it probable that in the record there exists more or less of exaggeration. It was considered as the celestial indication of the rapid success of Mohammed II., who had taken Constantinople, and struck terror into the whole Christian world. Pope Calixtus II. levelled the thunders of the Church against the enemies of his faith, terrestrial and celestial, and in the same bull exorcised the Turks and the comet; and in order that the memory of this manifestation of his power should be for ever preserved, he ordained that the bells of all the churches should be rung at midday—a custom which is preserved in those countries to our times. It must be admitted that, notwithstanding the terrors of the Church, the comet pursued its course with as much ease and security as those with which Mohammed converted the church of St. Sophia into his principal mosque.

The extraordinary length and brilliancy which was ascribed to the tail upon this occasion, have led astronomers to investigate the circumstances under which its brightness and magnitude would be the greatest possible; and, upon tracing back the motion of the comet to the year 1456, it has been found that it was then actually under the circumstances of position with respect to the earth and sun most favourable to magnitude and splendour. So far, therefore, the results of astronomical calculation corroborate the records of history.

were sufficiently great, in accelerating or retarding its motion while passing near them.

3056. *Exact path of the comet on its return and time of its perihelion calculated and predicted by Clairaut and Lalande.* —This inquiry, which presented great mathematical difficulties and involved enormous arithmetical labour, was undertaken by Clairaut and Lalande: the former, a mathematician and natural philosopher, who had already applied with great success the principles of gravitation to the motions of the moon, undertook the purely analytical part of the investigation, which consisted in establishing certain general algebraical formulæ, by which the disturbing actions exerted by the planets on the comet were expressed; and Lalande, an eminent practical astronomer, undertook the labour of the arithmetical computations, in which he was assisted by a lady, Madame Lepaute, whose name has thus become celebrated in the annals of science.

These elaborate calculations being completed, Clairaut presented the result of their joint labours, in a memoir to the Academy of Sciences of Paris*, in which he predicted the next arrival of the comet at perihelion, on the 18th April 1759; a date, however, which, before the re-appearance of the

* When it is considered that the period of Halley's comet is about seventy-five years, and that every portion of its course, for two successive periods, was necessary to be calculated separately in this way, some notion may be formed of the labour encountered by Lalande and Madame Lepaute. "During six months," says Lalande, "we calculated from morning till night, sometimes even at meals; the consequence of which was, that I contracted an illness which changed my constitution for the remainder of my life. The assistance rendered by Madame Lepaute was such, that without her we never could have dared to undertake this enormous labour, in which it was nécessary to calculate the distance of each of the two planets, Jupiter and Saturn, from the comet, and their attraction upon that body, separately, for every successive degree, and for 150 years."

The name of Madame Lepaute does not appear in Clairaut's memoir; a suppression which Lalande attributes to the influence exercised by another lady to whom Clairaut was attached. Lalande, however, quotes letters of Clairaut, in which he speaks in terms of high admiration of "la savante calculatrice." The labours of this lady in the work of calculation (for she also assisted Lalande in constructing his *Ephemerides*) at length so weakened her sight that she was compelled to desist. She died in 1788, while attending on her husband, who had become insane. See the articles on comets, by Prof. de Morgan, in the *Companion to the British Almanac* for the year 1833.

comet, he found reason to change to the 11th April; and assigned the path which the comet would follow while visible as determined by the following data:—

Inclination.	Long. of node.	Long. of perih.	Perihel. dist.	Direction.
17° 37′	53° 50′	303° 10′	0·58	retrograde.

3057. *Remarkable anticipation of the discovery of Uranus.*— In announcing his prediction Clairaut stated, that the time assigned for the approaching perihelion might vary from the actual time to the extent of a month; for that independently of any error either in the methods or process of calculation, the event might deviate more or less from its predicted occurrence, by reason of the attraction of *an undiscovered planet of our system revolving beyond the orbit of Saturn.* In twenty-two years after this time, this conjecture was realised by the discovery of the planet Uranus, by the late Sir William Herschel, revolving round the sun one thousand millions of miles beyond the orbit of Saturn!

3058. *Prediction of Halley and Clairaut fulfilled by reappearance of the comet in* 1758–9.—The comet, in fine, appeared in December 1758, and followed the path predicted by Clairaut, which differed but little from that which it had pursued on former appearances as will be seen by a comparison of the elements as given above with those since ascertained. It passed through perihelion on the 13th March, within 22 days of the time, and within the limit of the possible errors assigned by Clairaut.

3059. *Disturbing action of a planet on a comet explained.*— The general effects of a planet in accelerating or retarding the motion of a comet are easily explained, although the exact details of the disturbances are too complicated to admit of any exposition here.

Let P, *fig.* 813, represent the place of the disturbing planet, and C that of the comet. The attraction of the planet on the comet will then be a force directed from C towards P, and by the principle of the composition of forces is equivalent to two components, one C *m* in the direction of the comet's path, and the other C *n* perpendicular to that path. If the motion of the comet

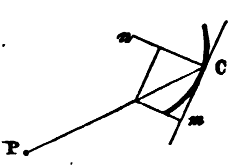


Fig. 813.

be directed from C towards *m*, it will be accelerated; and if it be directed from C towards *m*′, it will be retarded by that component of the planet's attraction which is directed from C to *m*. The other component C *n* being at right angles to the comet's motion, will have no direct effect either in accelerating or retarding it.

It appears, therefore, in general that, if the direction of the comet's motion C *m* make an acute angle with the line C P drawn to the planet, the planet's attraction will accelerate it; and if its direction C *m*′ make an obtuse angle with the line C P, it will retard it.

This being understood, the disturbing action of a planet such as Jupiter or Saturn on a comet such as Halley's may be easily comprehended. In *fig.* 814. the orbit of the comet is represented at A *c* P *c*″ in its proper proportions, A P being the major axis, P the place of perihelion, A that of aphelion, and S that of the focus in which the sun is placed. The small circle described round S represents in its proper proportions the orbit of the earth, whose distance is about twice that of the comet when the latter is at perihelion. The circle *p p*′ *p*″ represents in its proper proportions the orbit of Jupiter which, for illustration, we shall consider as the disturbing planet.

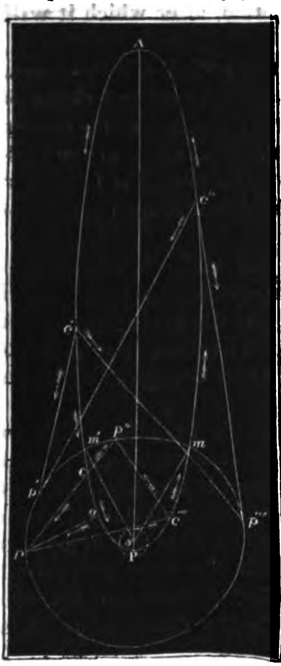


Fig. 814.

It will be apparent on the mere inspection of the diagram, that lines drawn from the planet whatever be its place, to any point whatever of the comet's path between its aphelion A and the point *m*′ where it arrives at the orbit of the planet in approaching the sun, will make acute angles with the direction of the comet's motion; and that, consequently, the comet will be accelerated by the action of the planet. In like manner, it is apparent that lines drawn from the planet whatever be its

place, to any point whatever of the comet's path between *m* and aphelion A, will make obtuse angles with the direction of the comet's motion; and, consequently, the comet will be retarded by the action of the planet, in departing from the sun, from *m* to A.

In that part of the comet's path which lies within the planet's orbit, the action of the planet alternately accelerates and retards it, according to their relative position. If the planet be at *p*, suppose *p o* drawn so as to be at right angles to the path of the comet. Between *m'* and *o* the action of the planet at *p* will accelerate the comet, and after the comet passes *s* it will retard it. In like manner if the planet be at *p''*, it will first retard the motion of the comet proceeding from *m'* towards A, and will continue to do so until the line of direction becomes perpendicular to that of the comet's motion, after which it will accelerate it.

It appears, therefore, that during the period of the comet, the disturbing action of the planet is subject to several changes of direction, owing partly to the change of position of the comet and partly to that of the planet; and the total effect of the disturbing action of the planet on the comet's period is found by taking the difference between the total amount of all the accelerating and all the retarding actions.

In the case of the planet Jupiter and Halley's comet, the former makes nearly seven complete revolutions in a single period of the comet; and consequently its disturbing action is not only subject to several changes of direction, but also to continual variation of intensity, owing to its change of distance from the comet.

Small as the arc *m'* P *m* of the comet's path is which is included within the orbit of Jupiter, the fraction of the period in which this arc is traversed by the comet is much smaller, as will be apparent by considering the application of the principle of equable areas (2599.) to this case. The time taken by the comet to move over the arc *m'* P *m* is in the same proportion to its entire period, as the area included between the arc *m'* P *m* and the lines *m'* S and *m* S is to the entire area of the ellipse A P.

To simplify the explanation the orbit of the comet has here been supposed to be in the plane of that of the disturbing planet. If it be not, the disturbing action will have another

component at right angles to the plane of the comet's orbit, the effect of which will be a tendency to vary the inclination.

3060. *Effect of the perturbing action of Jupiter and Saturn on Halley's comet between* 1682 *and* 1758.—The result of the investigation by Clairaut showed, that the total effect of the disturbing action of Jupiter and Saturn on Halley's comet between its perihelions in 1682 and in 1759, was to increase its period by 618 days as compared with the time of its preceding revolution, of which increase, 100 days were due to the action of Saturn, and 518 to that of Jupiter.

Clairaut did not take into account the disturbing action of the earth, which was not altogether inconsiderable, and could not allow for those of the undiscovered planets Uranus and Neptune. The effects of the action of the other planets, Mars, Venus, Mercury, and the planetoids, are in these cases insignificant.

3061. *Calculations of its return in* 1835–6.—In the interval of three quarters of a century which preceded the next re-appearance of this comet, science continued to progress, and instruments of observation and principles and methods of investigation were still further improved; and, above all, the number of observers was greatly augmented. Before the epoch of its return in 1835, its motions, and the effects produced upon them by the disturbing action of the several planets, were computed by MM. Damoiseau, Pontecoulant, Rosenberger, and Lehmann, who severally predicted its arrival at perihelion:—

Damoiseau - - -	4th Nov. 1835.
Pontecoulant - - -	7th "
Rosenberger - - -	11th "
Lehmann - - - -	26th "

3062. *Predictions fulfilled.*—These predictions were all published before July 1835. The comet was seen at Rome on the 5th August, in a position within *one degree* of the place assigned to it for that day, in the ephemeris of M. Rosenberger. On the 20th August, it became visible to all observers, and pursued the course with very little deviation, which had been assigned to it in the ephemerides, arriving at its perihelion on the 16th Nov., being very nearly a mean between the four epochs assigned in the predictions.

After this, passing south of the equator, it was not visible in northern latitudes, but continued to be seen in the southern hemisphere until the 5th of May 1836, when it finally disappeared, not again to return until the year 1911.

3063. *Tabular synopsis of the motion of Halley's comet.*— In Table IV. is given a synopsis of the elements of the orbit of this comet, deduced from the observations made on each of its seven successive returns to perihelion, between 1378 and 1835 inclusive.

TABLE IV.

Elements of Halley's Comet to 1835.

	Mean Distance, Earth's = 1.	Eccentricity.	Perihelion Distance.	Aphelion Distance.	Longitude of Perihelion.	Longitude of Ascending Node.	Inclination.	Time of Perihelion Passage.
	a	e	$d' = a \times (1 - e)$	$d'' = a \times (1 + e)$	ϖ	ν	i	M.T.G.
					° ′ ″	° ′ ″	° ′ ″	d. h. m.
1378	. . .	. . .	0·5835	. . .	299 31	47 17	17 56	Nov. 8. [illegible] [illegible]
1456	. . .	. . .	0·5853	. . .	301 0	48 30	17 56	June 8. 22 10
1531	17·7845	0·9674	0·5799	34·99	301 12	45 30	17 0	Aug. 25. 19 10
1607	17·9420	0·9671	0·5880	35·28	301 38 10	48 20 28	17 2 17	Oct. 26. 17 20
1682	18·1701	0·9679	0·5829	35·76	301 55 37	51 11 18	17 44 45	Sept. 14. 19 14
1759	18·0278	0·9677	0·5845	35·48	301 10 28	53 50 27	17 36 52	March 12. 13 21
1835	17·9875	0·9674	0·5866	35·39	304 31 32	55 9 59	17 45 4	Nov. 15. 22 41

It appears that the mean distance of this comet is about eighteen times that of the earth, and that it is consequently a little less than the mean distance of Uranus. When in perihelion, its distance from the sun is about half the earth's distance, while its distance in aphelion is above thirty-five times the earth's distance, and therefore seventy times its perihelion distance.

3064. *Pons's comet of* 1812.—On the 20th of July 1812, a comet was discovered by M. Pons, whose orbit was calculated by Professor Encké, and was found to be an ellipse of such dimensions as to give a period of 75½ years, equal to that of Halley's comet.

3065. *Olbers's comet of* 1815.—On the 6th of March 1815, Dr. Olbers discovered at Bremen, a comet whose orbit, calculated by Professor Bessel, proved to be an ellipse, with a period of 74 years. The next perihelion passage of this comet is predicted for the 9th of February 1887.

3066. *De Vico's comet of* 1846.—On the 28th of February 1846, M. de Vico discovered a comet at Rome, whose orbit,

calculated by MM. van Deinse and Pierce, appears to be an ellipse, with a period of 72–73 years.

3067. *Brorsen's comet of* 1847. — A comet was discovered by M. Brorsen at Altona, on the 20th of July 1847; the orbit of which, calculated by M. d'Arrest, appears to be an ellipse, with a period of 75 years.

3068. *Westphal's comet of* 1852. — On the 27th of June 1852, a comet was discovered by M. Westphal at Gottingen, and was soon after wards observed by M. Peters at Constantinople. The calculation of its orbit proves it to be an ellipse, with a period of about 70 years.

3069. *Tabular synopsis of the motions of these six comets.* — In Table V. are presented the data necessary to determine the motions of these six comets: —

TABLE V.

Synopsis of the Motion of the Elliptic Comets, whose mean Distances are nearly equal to that of Uranus.

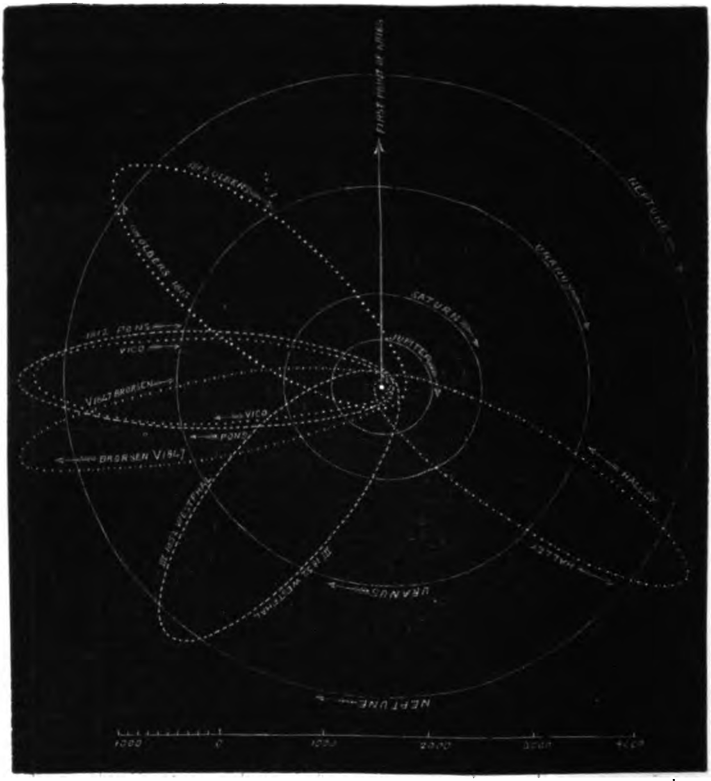
Designation.	Mean Distance, Earth's = 1.	Eccen-tricity.	Perihelion Distance.	Aphelion Distance.	Daily Motion At Mean Distance.	Daily Motion At Perihe-lion.	Daily Motion At Aphe-lion.
	a	e	$d' = a \times (1 - e)$	$d'' = a \times (1 + e)$	a	a'	a''
					″	″	″
1. Halley - -	17·9875	0·9674	0·5866	35·3660	46·3	43510	12
2. Pons (1812) - -	17·0955	0·9545	0·7771	33·4140	50·7	24506	13·3
3. Olbers (1815) -	17·6338	0·9312	1·2129	34·0560	47·9	10125	12·8
4. De Vico, (1846) -	14·5386	0·9544	0·6631	34·3510	48·4	23283	8·7
5. Brorsen (1847) -	17·7795	0·9726	0·4879	35·0710	47·3	62850	12·2
6. Westphal (1852) -	16·6200	0·9248	1·2510	31·9700	52·4	22955	14·2

Designation.	Period, Years.	Longitude of Perihelion.			Longitude of Ascend-ing Node.			Inclination.			Time of Perihelion Passage.			Direction of Motion.
	P	ϖ			ν			i						
		°	′	″	°	′	″	°	′	″		h.	m.	
1. Halley - -	76·680	304	31	32	55	9	59	17	45	5	Nov. 15, 1835.	22	41	R
2. Pons (1812) - -	70·068	92	18	44	253	1	2	73	57	3	Sept. 15, 1812.	7	41	D
3. Olbers (1815) -	74·050	149	1	56	83	28	34	44	29	55	April 25, 1815.	25	58	D
4. De Vico (1846) -	73·250	90	34	46	77	35	36	84	57	13	Mar. 5, 1846.	14	1	D
5. Brorsen (1847) -	74·970	79	12	46	309	48	49	19	8	25	Sept. 9, 1847.	13	11	D
6. Westphal (1852) -	67·770	43	12	16	346	13	25	40	58	32	Oct. 12, 1852.	15	6	D

3070. *Diagram of their orbits.* — In *fig.* 815., is presented a plan of the orbits, brought upon a common plane, and drawn according to the scale indicated. This figure shows, in a manner sufficiently exact for the purposes of illustration, the relative

magnitudes and the forms of the six orbits, as well as the directions of their several axes with relation to that of the first point of Aries.

Fig. 815.



3071. *Planetary characters are nearly effaced in these orbits.* — By comparing the elements given in this table, and the forms and magnitudes of the orbits shown in the diagram, with those of the first group of elliptic comets given in Table III. and drawn in *fig.* 812., it will be perceived that the planetary characteristics noticed in the latter group, are nearly effaced. Five of the six comets composing the second group, revolve in the common direction of the planets, and this is the only planetary

character observable among them. The inclinations, no longer limited to those of the planetary orbits, range from 18° to 74°. The eccentricities are all so extreme, that the arc of the orbit near perihelion approximates closely to the parabolic form, and, in fine, the most remarkable body of the group, the comet of Halley, revolves in a direction contrary to the common motion of the planets.

But it is more than all in the elongated oval form of their orbits, that this group of comets differs, not only from the planets, but from the first group. While their perihelia are at distances from the sun, between those of Mars and Mercury, their aphelia are from two to five hundred millions of miles outside the orbit of Neptune. Thus, the comet of Halley, for example, in perihelion, is at a distance from the sun less than that of Venus; but at its aphelion, its distance exceeds that of Neptune by a space greater than Jupiter's distance from the sun. The mean angular motion of this comet is nearly the same as that of Uranus; but its angular motion in perihelion is three times that of Mercury; while its angular motion in aphelion is little more than half that of Neptune.

The corresponding variations of solar light and heat, and of the apparent magnitude and motion of the sun as seen from the comet, may be easily inferred.

IV. Elliptic Comets, whose mean distances exceed the limits of the solar system.

3072. *Tabular synopsis of twenty-one elliptic comets, of great eccentricity and long period.* — Although the periodicity of this class of comets has not yet in any instance been certainly established by observations made upon their successive returns to perihelion, the observations made upon them during a single perihelion passage indicate an arc of their orbit, which exhibits the elliptic form so unequivocally, as to supply computors and mathematicians with the data necessary to obtain, with more or less approximation, the value of the eccentricity, which, combined with the perihelion distance, gives the form and magnitude of the comet's orbit.

By calculations conducted in this manner, and applied to the observations made on various comets which have appeared since the latter part of the seventeenth century, the elliptic orbits of

twenty-one of these bodies have been computed, and are given in the order of the dates of their perihelion passages in the following table.

TABLE VI.

Synopsis of the Motions of the Elliptic Comets whose mean Distances exceed the Limits of the Solar System.

	Mean Distance.		Eccentricity.	Minor Axis, Major Axis = 1.	Perihelion Distance Earth's = 1.	Aphelion Distance.		Daily Motion.		
	Earth's = 1.	Neptune's = 1.				Earth's = 1.	Neptune's = 1.	At Mean Distance.	At Perihelion.	At Aphelion.
	a		e		$d' = a \times (1 - e)$	$d'' = a \times (1 + e)$		a	a'	a''
								"	°	"
1. 1680 -	427·6437	14·2547	0·99998		0·00622	855·2	28·5	0·4	928333	0·001
2. 1683 -	33·0310	1·1010	0·98330		0·56340	65·5	2·18	18·7	18·5	4·75
3. 1763 -	217·4074	7·2469	0·99540		0·49830	433·8	14·5	0·11	58·5	0·28
4. 1769 -	163·4575	5·4485	0·99920		0·12280	326·8	10·9	1·7	116·8	0·42
5. 1780 -	1787·9100	59·5973	0·99994		0·09630	3575·5	119·1	0·047	4479·8	0·012
6. 1793 -	56·2000	1·9000	0·97340		1·4951	110·9	3·7	8·43	3·4	2·16
7. 1807 -	143·8362	4·7952	0·99550		0·6604	287·0	9·56	2·06	27·9	0·52
8. 1811. I.	211·0125	7·0340	0·99510		1·0354	421·0	14·0	1·16	13·8	0·37
9. 1811. II.	91·5088	3·0505	0·98270		1·5821	181·4	6·0	4·06	3·8	1·05
10. 1822 -	309·5000	10·3200	0·99630		1·1451	617·7	20·6	0·65	13·2	1·16
11. 1825 -	267·9440	8·9314	0·99540		1·2408	534·6	17·8	0·81	13·21	0·26
12. 1827 -	189·6187	6·3206	0·99880		0·1378	379·0	12·6	1·36	714·71	0·34
13. 1830 -	1536·0000	51·2000	0·99940		0·9215	3070·0	102·3	0·059	46·5	0·015
14. 1840. I.	577·1099	19·2370	0·99790		1·2214	1153·0	38·4	0·26	20·0	0·08
15. 1840. II.	49·1110	1·6370	0·96990		1·4808	96·7	3·2	10·3	3·1	2·66
16. 1843 -	56·0000	18·6000	0·99990		0·0056	111·9	3·7	9·44	202125·0	2·36
17. 1844 -	2138·0000	71·2600	0·99960		0·8554	4274·0	142·5	0·36	61·6	0·009
18. 1845 -	39·7600	1·3200	0·98990		0·4016	79·1	2·6	1·41	3·8	0·36
19. 1846. I.	194·8000	64·9000	0·99240		1·4807	388·1	12·9	1·3	6·3	0·33
20. 1846. II.	54·4192	1·8139	0·98840		0·6334	108·7	3·6	8·85	18·1	2·22
21. 1849 -	164·2000	5·4733	0·99780		0·8947	328·0	10·9	0·42	3·9	0·1

	Period, Years.	Longitude of Perihelion.			Longitude of Ascending Node.			Inclination.			Time of Perihelion Passage.			Direction.
	P	ϖ			γ			i						
		°	'	"	°	'	"	°	'	"		h.	m.	
1. 1680 -	8813	262	49	5	272	9	29	60	40	16	Dec. 17. 1680.	23	55	D
2. 1683 -	190	86	31	15	173	17	48	83	47	46	July 12. 1683.	17	36	R
3. 1763 -	3205	84	57	27	356	17	38	72	34	10	Nov. 1. 1763.	21	4	D
4. 1769 -	2089	144	11	29	175	3	59	40	45	50	Oct. 7. 1769.	15	3	D
5. 1780 -	75838	246	35	59	123	51	18	54	25	12	Sept. 30. 1780.	22	25	R
6. 1793 -	421	71	54	3	2	0	12	51	31	10	Nov. 28. 1793.	5	6	D
7. 1807 -	1725	270	54	42	266	47	11	63	10	28	Sept. 18. 1807.	17	53	D
8. 1811. I.	3065	75	0	34	140	24	44	73	2	21	Sept. 12. 1811.	6	20	R
9. 1811. II.	875	47	27	27	93	1	52	31	17	11	Nov. 10. 1811.	23	56	D
10. 1822 -	5444	271	40	17	92	54	[illegible]	52	39	30	Oct. 23. 1822.	18	28	R
11. 1825 -	4386	318	46	41	215	43	14	33	32	39	Dec. 10. 1825.	16	31	R
12. 1827 -	2611	250	57	12	149	39	11	54	4	42	Sept. 11. 1827.	16	47	R
13. 1830 -	60200	212	18	22	206	21	35	21	16	5	April 9. 1830.	7	15	D
14. 1840. I.	13·864	80	18	10	236	49	6	59	13	20	Mar. 12. 1840.	23	56	R
15. 1840. II.	344	22	31	40	248	56	22	57	57	23	Nov. 13. 1840.	15	37	D
16. 1843 -	376	278	39	51	1	12	24	35	41	9	Feb. 27. 1843.	9	47	R
17. 1844 -	100000	180	24	3	31	39	5	48	36	1	Oct. 17. 1844.	8	15	R
18. 1845 -	251	262	2	56	337	48	56	48	41	59	June 5. 1845.	16	10	R
19. 1846. I.	2719	89	6	22	111	8	26	47	26	6	Jan. 22. 1846.	2	15	D
20. 1846. II.	401	162	0	54	261	51	14	29	18	47	June 5. 1846.	12	35	R
21. 1849 -	8375	267	3	16	30	31	48	66	59	2	June 8. 1849.	4	10	D

Of this group the least eccentric is No. 15., which passed its perihelion in 1840. This comet was discovered at Berlin by M. Bremiker, and its orbit was calculated by Götze, and proved

to be an ellipse, having the elements given in the table, subject to no greater uncertainty than $\frac{1}{61}$th of the value assigned to the mean distance. The eccentricity, and consequently the form of the orbit, is similar to that of Halley, but the major axis is $2\frac{2}{3}$ times, and the period nearly five times greater. Its perihelion distance is equal to that of Mars, and its aphelion distance more than three times that of Neptune.

No. 6, which passed its perihelion in 1793, has an orbit, according to the calculations of D'Arrest, nearly similar both in form and magnitude, as will be seen by comparing the numbers given in the table. More uncertainty, however, attends the estimation of these elements.

The comets which approached nearest to the sun were the great comets of 1680 and 1843, Nos. 1 and 16 in the table, both memorable for their extraordinary magnitude and splendour.

The elements of that of 1680, given in the table, are those which have resulted from the calculations of Professor Encké, based on all the observations of the comet which have been recorded. The elements of the great comet of 1843 have resulted from the computations of Mr. Hubbard. Both are subject to considerable uncertainty, and must be accepted only as the best approximations that can be obtained.

What is not subject, however, to the same uncertainty, is the extraordinary proximity of these bodies to the sun at their respective perihelia. The perihelion distance of the comet of 1680 was about 576,000 miles, and that of 1843, 538,000 miles. Now the semidiameter of the sun being 441,000 miles, it follows that the distance of the centres of those comets respectively from the surface of the sun at perihelion must have been only 235,000 and 97,000; so that if the semidiameter of the nebulous envelope of either of them exceeded this distance, they must have actually grazed the sun.

The velocity of the orbital motion of these bodies in aphelion appears by the table to be such, that the comet of 1680 would have revolved round the sun in a minute, and that of 1843 in little less than two minutes, if they retained the same angular motion undiminished.

The distance to which the comet of 1680 recedes in its aphelion is $28\frac{1}{2}$ times greater than that of Neptune. The apparent

diameter of the sun seen from that distance would be 2″, and the intensity of its light and heat would be 730,000 times less than at the earth; while their intensity at the perihelion distance would be 26,000 times greater, so that the light and heat received by the comet in its aphelion would be $26{,}000 \times 730{,}000 = 18{,}980$ million times less than in perihelion.

The greatest aphelion distances in the table are those of Nos. 5, 13, and 17, the comets of 1780, 1830, and 1844, amounting to from 100 to 140 times the distance of Neptune; the eccentricities differing from unity by less than $\frac{1}{1000}$. These orbits, though strictly the results of calculation, must be regarded as subject to considerable uncertainty.

3073. *Plan of the form and relative magnitude of the orbits.* — To convey an idea of the form of the orbits of the comets of this group, and of the proportion which their magnitude bears to the dimensions of the solar system, we have drawn in *fig.* 816, an ellipse, which may be considered as representing the form of the orbits of the comets Nos. 15, 6, 9, 12, and 1, of the Table VI.

If the ellipse represent the orbit of the comet No. 15, the circle *a* will represent on the same scale the orbit of Neptune.

If the ellipse represent the orbit of the comet No. 6, the circle *b* will represent the orbit of Neptune.

If the ellipse represent the orbit of No. 9, the circle *c* will represent the orbit of Neptune.

If the ellipse represent the orbit of No. 12, the circle *d* will represent the orbit of Neptune.

If the ellipse represent the orbit of No. 1, the circle *e* will represent the orbit of Neptune.

V. Hyperbolic Comets.

3074. *Tabular synopsis of hyperbolic comets.* — In the annexed Table (VII.) are given the elements of seven comets, which appear by the results of calculations made upon the observations to have passed through the system in hyperbolic orbits.

TABLE VII.

Synopsis of the Motions of the Hyperbolic Comets.

Time of Perihelion Passage.			Perihelion Distance, Earth's = 1.	Eccentricity.	Longitude of Perihelion.			Longitude of Ascending Node.			Inclination.			Direction.
			d	e	ϖ			γ			i			
	h.	m.			°	′	″	°	′	″	°	′	″	
1. June 13. 1729.	6	19	4·0435	1·0050	320	31	22	310	38	0	77	5	18	D
2. April 19. 1771.	5	16	0·9035	1·0094	104	3	16	57	51	55	11	15	19	D
3. Aug. 15. 1774.	20	5	1·4329	1·0283	317	27	40	180	44	34	83	20	26	D
4. Dec. 5. 1818.	0	56	0·8550	1·0116	101	47	30	90	0	51	63	0	24	R
5. Sept. 29. 1824.	1	24	1·0500	1·0017	4	31	7	279	15	39	54	36	59	D
6. Jan. 4. 1840.	10	23	0·6184	1·0002	192	11	50	119	57	46	53	5	32	D
7. May 6. 1843.	1	30	1·6163	1·0002	281	29	43	157	14	54	52	44	46	D

VI. Parabolic Comets.

3075. *Tabular synopsis of the parabolic comets.*—In the annexed Table (VIII.) are given the elements of the orbits of 161

Fig. 816.

comets, whose paths in passing through the system have been either parabolas or ellipses of eccentricities so extreme as to be undistinguishable from parabolas in that part of their orbits at which they were capable of being observed.

TABLE VIII.

Elements of the Orbits of Comets ascertained or presumed to be parabolic.

Time of Perihelion Passage.	h.	m.	Perihelion Distance, Earth's = 1.	Longitude of Perihelion. °	′	″	Longitude of Ascending Node. °	′	″	Inclination. °	′	″	Direction.
1. B. C. 370. Winter -			Very small	150 to 210			270 to 330			Above 30	0	0	R
2. 136. April 29 -	0	0	1·01	230	0	0	220	0	0	20	0	0	R
3. 68. July - -	0	0	0·80	300 to 330			150 to 180			70	0	0	D
4. 11. October 8	19	12	0·58	280	0	0	28	0	0	10+	—0	0	R
5. A. D. 66. January 14	4	48	0·445	325	0	0	32	40	0	40	30	0	R
6. 141. March 29 -	2	24	0·720	251	55	0	12	50	0	17	0	0	R
7. 240. November 9 -	23	51	0·372	271	0	0	159	0	0	44	0	0	D
8. 539. October 20 -	14	51	0·341	313	30	0	58 or 238			10	0	0	D
9. 565. July 14 - -	11	51	0·832	80	0	0	159	30	0	59	0	0	R
10. 568. August 28 -	6	29	0·889	316	47	0	294	36	0	4	0	0	D
11. 574. April 7 -	6	43	0·963	113	39	0	128	17	0	46	31	0	D
12. 770. June 6 - -	15	22	0·603	2	8	0	88	54	0	59	31	0	R
13. 837. February 28 -	23	51	0·580	289	3	0	206	33	0	10 to 12			R
14. 961. December 30 -	3	50	0·552	268	3	0	350	35	0	79	33	0	R
15. 989. September 11 -	23	51	0·568	264	0	0	84	0	0	17	0	0	R
16. 1066. April 1 -	0	0	0·740	264	55	0	25	50	0	17	0	0	
17. 1092. February 15 -	0	0	0·928	156	20	0	125	40	0	28	55	0	D
18. 1097. September 21 -	21	27	0·738	332	30	0	207	30	0	73	30	0	D
19. 1231. January 30 -	7	13	0·948	134	48	0	13	30	0	6	5	0	D
20. 1264. July 15 -	23	51	0·430	272	30	0	175	30	0	30	25	0	D
21. 1299. March 31 -	7	29	0·318	3	20	0	107	8	0	68	57	0	R
22. 1301. October 23 -	23	51	0·640	312	0	0	138	0	0	13	0	0	R
23. 1337. June 22 -	19	12	0·937	330	22	0	99	6	0	42	51	0	R
24. 1351. November 25 -	23	51	1·000	69	0	0	—			—			D
25. 1362. March 11 -	4	51	0·456	219	0	0	249	0	0	21	0	0	R
26. 1366. October 13 -	0	0	0·958	66	0	0	212	0	0	6	0	0	
27. 1385. October 16 -	6	14	0·774	101	47	0	268	31	0	52	15	0	
28. 1433. November 5 -	4	34	0·329	262	1	0	110	9	0	77	14	0	R
29. 1457. September 3 -	16	48	2·103	92	48	0	256	6	0	20	20	0	D
30. 1466. October 7 -	9	50	0·853	356	3	0	61	15	0	44	19	0	R
31. 1472. February 28 -	5	13	0·539	48	3	0	207	32	0	1	55	0	R
32. 1490. December 24 -	11	17	0·738	58	40	0	288	45	0	51	37	0	D
33. 1506. September 3 -	15	53	0·386	250	37	0	132	50	0	45	1	0	R
34. 1532. October 19 -	22	13	0·5091	111	7	0	80	27	0	32	36	0	D
35. 1533. June 14 -	21	11	0·3269	217	40	0	299	19	0	28	14	0	D?
35.* 1533. June 16 -	19	31	0·2028	104	12	0	125	44	0	35	49	0	R?
36. 1556. April 22 -	0	34	0·5049	274	14	54	175	25	48	30	12	12	D
37. 1558. August 10 -	12	25	0·5773	329	49	0	332	36	0	73	29	0	R
38. 1577. October 25 -	22	45	0·1775	129	42	0	25	20	24	75	9	42	R
39. 1580 November 28 -	13	45	0·5955	109	12	0	19	8	0	64	52	0	D
40. 1582. May 6 - -	16	0	0·2257	245	23	0	231	7	0	61	28	0	R
41. 1585. October 8 -	2	45	1·0954	9	15	29	37	43	52	6	5	4	D
42. 1590. February 8 -	0	39	0·5677	217	57	12	165	36	56	29	29	44	
43. 1593. July 18 -	13	39	0·0891	176	19	0	164	15	0	87	58	0	
44. 1496. July 25 -	5	9	0·5672	270	54	35	330	20	49	51	58	10	
45. 1618. August 17 -	3	3	0·5130	318	20	0	293	25	0	21	28	0	D
46. 1618. November 8 -	8	25	0·3896	3	5	21	75	44	10	37	11	31	D
47. 1652. November 12 -	15	41	0·8475	28	18	40	88	10	0	79	28	0	D
48. 1661. January 26 -	21	9	0·4427	115	16	8	81	54	0	33	0	55	D
49. 1664. December 4 -	11	53	1·0258	130	41	25	81	14	0	21	18	30	R
50. 1665. April 24 -	5	16	0·1065	71	54	30	228	2	0	76	5	0	R
51. 1668. February 24 -	18	46	0·2511	40	9	0	193	26	0	27	7	0	D?
51. 1668. February 28 -	19	12	0·0048	277	2	0	357	17	0	35	50	0	R?
52. 1672. March 1 -	8	38	0·6974	46	59	30	297	30	30	83	22	10	D
53. 1677. May 6 -	0	38	0·2806	137	37	5	236	49	10	79	3	15	R·
54. 1678. August 26 -	14	4	1·2380	327	46	0	161	40	0	3	4	20	D
55. 1684. June 8 -	10	17	0·9602	238	52	0	268	15	0	65	48	40	D
56. 1686. September 16 -	14	34	0·3250	77	0	30	350	34	40	31	21	40	D
57. 1689. December 1 -	14	56	0·0169	263	44	45	323	45	20	69	17	0	R
58. 1695. November 9 -	16	51	0·8435	60	0	0	216	0	0	22	0	0	D

Time of Perihelion Passage.				Perihelion Distance, Earth's = 1.	Longitude of Perihelion.			Longitude of Ascending Node.			Inclination.			Direction.
		h.	m.		°	′	″	°	′	″	°	′	″	
59.	1698. October 18	16	58	0·6913	270	51	15	267	44	15	11	46	0	R
60.	1699. January 13	8	23	0·7440	212	31	6	321	45	35	69	20	0	R
61.	1701. October 17	9	51	0·5926	133	41	0	298	41	0	41	39	0	R
62.	1702. March 13	14	33	0·6468	138	46	34	188	59	10	4	24	41	D
63.	1706. January 30	4	23	0·4258	72	29	10	13	11	40	55	14	10	D
64.	1718. January 14	21	41	1·0254	121	39	55	127	55	29	31	8	6	R
65.	1723. September 27	15	4	0·9988	42	52	35	14	14	17	50	0	18	R
66.	1729. June 12	17	51	4·0431	320	27	36	310	38	0	77	5	18	D
67.	1737. January 30	8	21	0·2228	325	55	0	226	22	0	18	20	45	D
68.	1737. June 8	7	39	0·8670	262	36	39	123	53	43	39	14	5	D
69.	1739. June 17	10	0	0·6736	102	38	40	207	25	14	55	42	44	R
70.	1742. February 8	4	59	0·7657	217	35	13	185	38	29	66	59	14	R
71.	1743. January 10	20	20	0·8382	92	57	51	67	31	57	2	16	16	D
72.	1743. September 20	21	17	0·5216	246	33	52	5	16	35	45	48	21	R
73.	1744. March 1	8	17	0·2221	197	12	55	45	45	20	47	8	36	D
74.	1746. February 15	0	0	0·95	140	0	0	335	0	0	6	0	0	D
75.	1747. March 3	7	11	2·1985	277	2	0	147	18	50	79	6	20	R
76.	1748. April 28	18	41	0·8404	215	23	29	232	51	50	85	28	23	R
77.	1748. June 18	21	18	0·6254	278	47	10	33	8	29	67	3	28	D
78.	1757. October 21	7	55	0·3375	122	58	0	214	12	50	12	50	20	D
79.	1758. June 11	3	18	0·2154	267	38	0	230	50	0	68	19	0	D
80.	1759. November 27	2	19	0·7985	53	24	20	139	39	21	78	59	22	D
81.	1759. December 16	21	4	0·9660	138	24	35	79	50	45	4	51	32	R
82.	1762. May 28	8	2	1·0091	104	2	0	348	33	5	85	38	13	D
83.	1764. February 12	13	42	0·5552	15	14	52	120	4	33	52	53	31	R
84	1766. February 17	8	41	0·5053	143	15	25	244	10	50	40	50	20	R
85.	1770. November 22	5	39	0·5282	208	22	44	108	42	10	31	25	55	R
86.	1773. September 5	14	34	1·1269	75	10	58	121	5	30	61	14	17	D
87.	1779. January 4	2	4	0·7132	87	14	27	25	4	10	32	30	57	D
88.	1780. November 28	20	21	0·5153	246	52	0	141	1	0	72	3	30	R
89.	1781. July 7	4	32	0·7758	239	11	25	83	0	38	81	43	26	D
90.	1781. November 29	12	32	0·9610	16	3	28	77	22	57	27	13	8	R
91.	1784. January 21	4	47	0·7079	80	44	24	56	49	21	51	9	12	R
92.	1785. January 27	7	49	1·1434	109	51	56	264	12	15	70	14	12	D
93.	1785. April 8	8	59	0·4273	297	29	33	64	33	36	87	31	54	R
94.	1786 July 7	21	51	0·4101	159	25	36	194	22	40	50	54	28	D
95.	1787. May 10	19	49	0·3489	7	44	9	106	51	35	48	15	51	R
96.	1788. November 10	7	25	1·0630	99	8	7	156	56	43	12	27	40	R
97.	1788. November 20	7	16	0·7573	22	49	54	352	24	26	64	30	24	D
98.	1790. January 15	5	6	0·7581	60	14	32	176	11	46	31	54	15	R
99.	1790. January 28	7	36	1·0633	111	44	37	267	8	37	56	58	13	D
100.	1790. May 21	5	47	0·7980	273	43	27	33	11	2	65	52	27	R
101.	1792. January 13	13	35	1·2930	36	29	42	190	46	15	39	46	55	R
102.	1792. December 27	6	5	0·9663	135	59	24	283	15	17	49	1	45	R
103.	1793. November 4	20	12	0·4034	228	42	0	108	29	0	60	21	0	R
104.	1796. April 2	19	48	1·5782	192	44	13	17	2	16	64	54	33	R
105.	1797. July 9	2	31	0·5266	49	27	8	329	15	37	50	40	34	R
106.	1798. April 4	11	32	0·4848	104	59	0	122	9	0	43	52	16	D
107.	1798. December 31	13	17	0·7795	34	27	27	219	30	34	42	26	4	R
108.	1799. September 7	5	39	0·8399	3	39	46	99	32	47	50	56	27	R
109.	1799. December 25	21	31	0·6258	190	20	12	326	49	11	77	1	38	R
110.	1801. August 8	13	23	0·2617	183	49	0	44	28	0	21	20	0	R
111.	1802. September 9	21	23	1·0941	332	9	4	310	15	39	57	0	47	D
112.	1804. February 13	15	31	1·0723	148	53	32	176	49	47	56	44	20	D
113.	1806. December 28	22	21	1·0816	97	2	3	322	19	15	35	2	50	R
114.	1808. May 12	22	52	0·3899	69	12	57	322	58	56	45	43	7	R
115.	1808. July 12	5	16	0·6073	252	31	39	24	27	33	39	17	24	R
116.	1810. October 5	19	45	0·9691	63	9	10	308	53	4	62	46	17	D
117.	1813. March 4	12	38	0·6991	69	56	8	60	48	24	21	13	35	R
118.	1813. May 19	10	3	1·2161	197	43	46	42	40	40	81	2	28	R
119.	1816. March 1	8	18	0·0485	267	35	33	323	14	56	43	5	26	D
120.	1818. February 25	23	1	1·1978	182	45	22	70	26	11	89	43	48	D
121.	1818. December 4	22	26	0·8551	101	55	2	89	59	53	63	5	29	R
122.	1819. June 27	17	11	0·3410	287	5	54	273	42	52	80	44	44	D
123.	1821. March 21	12	53	0·0918	239	29	25	48	40	56	73	33	7	R
124.	1822. May 5	14	33	0·5044	192	43	51	177	26	56	53	37	21	R
125.	1822. July 16	12	45	0·8367	218	32	56	97	40	3	38	12	39	R
126.	1823. December 9	10	39	0·2265	274	34	30	303	3	0	76	11	57	R
127.	1824. July 11	12	9	0·5912	260	16	32	234	19	1	54	34	19	R
128.	1825. May 30	13	9	0·8891	273	55	1	20	6	5	56	41	6	R
129.	1825. August 18	17	4	0·8834	10	14	25	192	56	10	89	41	47	D
130.	1826. April 21	23	4	2·0111	116	54	40	197	38	9	40	2	33	D
131.	1826. April 29	0	56	0·1881	35	48	13	40	29	13	5	17	2	R
132.	1826. October 8	22	51	0·8528	57	48	24	44	6	28	25	57	18	D
133.	1826. November 18	9	54	0·0269	315	31	34	235	7	44	89	22	10	R
134.	1827. February 4	22	7	0·5065	33	30	16	184	27	49	77	35	35	R
135.	1827. June 7	20	9	0·8081	297	31	42	318	10	28	43	38	45	R
136.	1830. December 27	15	51	0·1258	310	59	19	337	53	7	44	45	30	R
137.	1832. September 25	12	31	1·1836	227	55	36	72	26	42	43	18	3	R
138.	1833. September 10	4	28	0·4584	222	51	17	323	0	51	7	21	2	D
139.	1834. April 2	15	5	0·5150	276	33	49	226	48	52	5	56	52	D
140.	1835. March 27	13	50	2·0143	207	42	55	58	19	46	9	7	39	R
141.	1840. April 2	12	54	0·7421	324	20	24	186	4	24	75	51	24	D
142.	1842. December 15	22	58	0·5044	327	17	32	207	49	39	73	34	4	R
143.	1844. December 13	16	23	0·2513	296	0	32	118	23	24	45	36	34	D
144.	1845. January 8	3	44	0·9052	91	19	39	336	44	30	46	50	30	D
145.	1845. April 21	0	45	1·2547	192	13	19	347	6	45	56	23	36	D

Time of Perihelion Passage.			Perihelion Distance, Earth's = 1.	Longitude of Perihelion.			Longitude of Ascending Node.			Inclination.			Direction.
	h.	m.		°	′	″	°	′	″	°	′	″	
146. 1846. May 27 -	21	57	1·3763	82	32	57	161	18	49	57	35	50	R
147. 1846. October 29 -	17	56	0·8306	98	35	60	4	41	4	49	41	17	D
148. 1847. March 30 -	6	28	0·0180	276	11	50	21	49	31	48	39	49	D
149. 1847. June 4 -	16	31	2·1151	141	27	10	173	57	40	79	33	45	R
150. 1847. August 9 -	8	17	1·4848	21	17	30	76	43	23	32	38	47	R
151. 1847. August 9 -	10	37	1·7672	216	41	34	338	17	31	83	27	1	R
152. 1847. November 14 -	4	14	0·3300	274	26	11	190	55	56	72	10	51	R
153. 1848. September 8 -	1	6	0·3199	310	34	39	211	32	29	84	24	60	R
154. 1849. January 19 -	8	21	0·9597	63	14	15	215	12	51	85	2	54	D
155. 1849. May 26 -	11	54	1·1593	235	43	21	202	32	56	67	9	39	D
156. 1850. July 23 -	12	28	1·0815	273	23	58	92	52	56	68	12	8	D
157. 1850. October 19 -	8	10	0·5655	89	13	54	215	59	24	40	5	37	D
158. 1851. August 26 -	7	20	0·9814	311	12	52	223	9	11	37	43	57	D
159. 1851. September 30 -	19	12	0·1413	338	45	18	44	25	37	73	59	44	D
160. 1852. April 19 -	13	52	0·9050	280	0	33	317	8	22	48	52	51	R

VII. Distribution of Cometary Orbits in Space.*

3076. *Distribution of the cometary orbits in space.*—In reviewing the vast mass of data collected by the labours of observers, ancient and modern, and which, so far as we have been enabled to see grounds for classification, are marshalled in the series of tables which are given above, it is natural to look for some evidence of a prevalent law in the motions of these bodies. The absence of all analogy to the planetary orbits, except in the case of the first group of elliptic comets consigned to Table III., has been already indicated; but, although no analogy to the planetary motions may exist, it does not follow that the cometary motions may not be governed by some laws of their own, the nature and character of which can only be discovered by carefully conducted induction.

3077. *Relative numbers of direct and retrograde comets.*—It has been shown that of the nineteen comets included in the first and second groups, which possess in the most marked degree the planetary character, one only is retrograde. Here is, then, the indication of a law, so far as regards the direction of the motion of the comets of these groups.

To ascertain whether traces of the same law are discoverable in the other classes of comets, let the other tables be examined and compared.

Of the twenty-one comets included in Table VI. there are,

Direct - - -	10
Retrograde - - -	11
	21

* This section contains the substance of a paper by the author, which has been printed in the Proceedings of the Royal Astronomical Society.

There is, therefore, among these no indication of the prevalence of any law in relation to the direction of the motion.

Of the seven hyperbolic comets in Table VII. there are,

Direct - - -	6
Retrograde - - -	1
	7

Here the law of direct motion reappears; but the number is too small to supply ground for any safe induction.

Of 160 parabolic comets included in Table VIII. there are,

Direct - - -	70
Retrograde - - -	86
	156
Direction unascertained -	4
	160

Here the tendency leans to retrograde motion, but still not in a degree sufficiently decided to supply safe ground for a conclusion. If, instead of taking as the basis of the induction the 160 parabolic comets, we take the entire number of 203 comets of which the direction is ascertained, we shall find,

Direct - - -	104
Retrograde - - -	99
	203

It must, therefore, be concluded that, notwithstanding the considerable number of comets whose motions have been observed, no general trace of any law governing the direction of motion is discoverable.

3078. *Inclination of the orbits.*—Taking all the orbits of which the inclinations have been ascertained, it will be found that, of every hundred, the inclinations are distributed as follows:—

° °	
0 to 10 - -	10·30
10 „ 20 - -	8·80
20 „ 30 - -	6·85
30 „ 40 - -	11·25
40 „ 50 - -	15·75
50 „ 60 - -	14·70
60 „ 70 - -	11·30
70 „ 80 - -	12·75
80 „ 90 - -	8·30
	100·00

There are here evident indications of a tendency of the planes of the cometary orbits to collect round a plane whose inclination to the plane of the ecliptic is 45°, or if a cone be imagined to be formed having a semi-angle of 45°, and its axis at right angles to the plane of the ecliptic, the planes of the cometary orbits betray a tendency to take the position of tangent planes to the surface of such a cone.

3079. *Directions of the nodes and perihelia.*—Taking, in like manner, the longitudes of the nodes and the perihelia, we find that those of every hundred comets are distributed in longitude as follows:—

	Number of Nodes.	Number of Perihelia.
0° to 30°	8·35	6·80
30 60	8·85	7·30
60 90	11·85	12·25
90 120	8·35	11·70
120 150	1·35	8·25
150 180	7·90	2·90
180 210	9·90	5·85
210 240	8·90	7·80
402 270	7·40	10·70
270 300	4·90	12·75
300 330	8·35	10·30
330 360	6·90	3·40
	100·00	100·00

An uniform distribution would give 8·33 nodes to each arc of 30°. The number in the third sign between 60° and 90° is nearly 12, and in the seventh sign between 180° and 210° nearly 10, both considerably exceeding the mean share.

The distribution of the perihelia is still more unequal. There is an evident tendency to crowd into the arcs between 60° and 120°, and between 240° and 300°. The number of perihelia due to an arc of 60° is 16·66. Now the actual number found between 60° and 120° is 23·95, about 50 per cent. above the mean. Between 240° and 330° there are 33·75 perihelia, where, as the number due to an arc of 90° is 25, the actual number being 35 per cent. above the mean.

3080. *Distribution of the points of perihelion.*—Considering how much the visibility of a comet from the earth depends on its perihelion distance, and that beyond a certain limit of such distance a comet cannot be expected to be seen at all, it cannot be expected that the law, if any such there be, which governs the distribution of the points of perihelion round the sun can be

discovered with any degree of certainty. Nevertheless, it will not be without interest to show the distribution of the points of perihelion of the known comets in relation to their distances from the sun.

If the centre of the sun be imagined to be surrounded by spheres having semi-diameters increasing successively by a constant increment of 20 millions of miles, the number out of every hundred known comets whose perihelia lie between sphere and sphere, will be as follows:—

					Number of Perihelia.
within	20				8·65
between	20	and	40		11·70
	40	„	60		20·30
	60	„	80		17·20
	80	„	100		20·80
	100	„	120		8·65
	120	„	140		4·55
	140	„	160		4·05
	160	„	180		2·00
	180	„	220		2·55
	220	„	420		0·55
					100·00

It is evident that the small proportion of the perihelia which lie outside the sphere, whose radius is 120 millions of miles, must be ascribed to the fact that comets moving in such orbits will mostly escape observation: but it may, perhaps, be assumed that, *ceteris paribus*, the comets whose perihelia lie within a sphere through the earth's orbit have nearly equal chances of being observed. If this be assumed, then it will follow that the numbers of such comets which have been observed are nearly proportional to their total numbers, and therefore that the numbers within this limit in the preceding table do actually represent approximately the distribution of the points of perihelia round the sun.

If we compare then the number of perihelia situate between the equidistant spheres indicated in the preceding table with the cubical spaces through which they are respectively distributed, we shall obtain an approximate estimate of the *density* of their distribution in relation to the distances from the sun. I have computed the following table with this view. In the second column I have given the number of comets per cent.

whose perihelia are included between the equidistant spheres; in the third column the numbers express the cubical spaces between sphere and sphere, the volume of the sphere whose radius is 20 millions of miles, being the cubical unit; and in the fourth column the numbers are the quotients of those in the second divided by those in the third, and therefore express the successive densities of the perihelia between sphere and sphere.

	Numbers of Perihelia.	Cubical Space.	Density of Perihelia.
0 to 20 - -	8·65	1	8·65
20 40 - -	11·70	7	1·67
40 60 - -	20·30	19	1·06
60 80 - -	17·20	37	0·47
80 100 - -	20·40	61	0·34
100 120 - -	8·65	91	0·095

It is evident then that the density of the perihelia increases rapidly in approaching the sun. If the numbers in the last column of the table be compared with the inverse powers of the distance, it will be found that this increase of density is more rapid than the inverse distance, but less so than the inverse distance squared.

VIII. Physical constitution of comets.

3081. *Apparent form — Head and Tail.* — Comets in general, and more especially those which are visible without a telescope, present the appearance of a roundish mass of illuminated vapour or nebulous matter, to which is often, though not always, attached a train more or less extensive, composed of matter having a like appearance. The former is called the HEAD, and the latter the TAIL of the comet.

3082. *Nucleus.*—The illumination of the head is not generally uniform. Sometimes a bright central spot is seen in the nebulous matter which forms it. This is called the NUCLEUS.

The nucleus sometimes appears as a bright stellar point, and sometimes presents the appearance of a planetary disk seen through a nebulous haze. In general, however, on examining the object with high optical power, these appearances are changed, and the object seems to be a mere mass of illuminated vapour from its borders to its centre.

3083. *Coma.*—When a nucleus is apparent, or supposed to be so, the nebulous haze which surrounds it and forms the exterior part of the head is called the *coma*.

3084. *Origin of the name.*—These designations are taken from the Greek word κομή (komé) hair, the nebulous matter composing the coma and tail being supposed to resemble hair, and the object being therefore called κομήτης (kometes), a hairy star.

3085. *Magnitude of the head.*—As the brightness of the coma gradually fades away towards the edges, it is impossible to determine with any great degree of precision its real dimensions. These, however, are obviously subject to enormous variation, not only in different comets compared one with another, but even in the same comet during the interval of a single perihelion passage. The greatest of those which have been submitted to micrometrical measurement was the great comet of 1811, Table VI. No. 8, the diameter of the head of which was found to be not less than 1⅛ millions of miles, which would give a volume greater than that of the sun in the ratio of about 2 to 1. The diameter of the head of Halley's comet when departing from the sun, in 1836, at one time measured 357,000 miles, giving a volume more than sixty times that of Jupiter. These are, however, the greatest dimensions which have been observed in this class of objects, the diameter rarely exceeding 200,000 miles, and being generally less than 100,000.

3086. *Magnitude of the nucleus.*—Attempts have been made where nuclei were perceivable, to estimate their magnitude, and diameters have been assigned to them, varying from 100 to 5000 miles. For the reasons, however, already explained, these results must be regarded as very doubtful.

Those who deny the existence of solid matter within the coma, maintain that even the most brilliant and conspicuous of those bodies, and those which have presented the strongest resemblance to planets, are more or less transparent. It might be supposed that a fact so simple as this, in this age of astronomical activity, could not remain doubtful; but it must be considered, that the combination of circumstances which alone would test such a question, is of rare occurrence. It would be necessary that the centre of the head of the comet, although very small, should pass critically over a star, in order to ascertain whether such star is visible through it. With comets having

extensive comæ without nuclei, this has sometimes occurred; but we have not had such satisfactory examples in the more rare instances of those which have distinct nuclei.

In the absence of a more decisive test of the occultation of a star by the nucleus, it has been maintained that the existence of a solid nucleus may be fairly inferred from the great splendour which has attended the appearance of some comets. A mere mass of vapor could not, it is contended, reflect such brilliant light. The following are the examples adduced by Arago: —

In the year 43 before Christ, a comet appeared which was said to be visible to the naked eye by daylight. It was the comet which the Romans considered to be the soul of Cæsar transferred to the heavens after his assassination.

In the year 1402 two remarkable comets were recorded. The first was so brilliant that the light of the sun at noon, at the end of March, did not prevent its nucleus, or even its tail, from being seen. The second appeared in the month of June, and was visible also for a considerable time before sunset.

In the year 1532, the people of Milan were alarmed by the appearance of a star which was visible in the broad daylight. At that time Venus was not in a position to be visible, and consequently it is inferred that this star must have been a comet.

The comet of 1577 was discovered on the 13th of November by Tycho Brahe, from his observatory on the isle of Huene, in the Sound, before sunset.

On the 1st of February, 1744, Chizeaux observed a comet more brilliant than the brightest star in the heavens, which soon became equal in splendour to Jupiter, and in the beginning of March it was visible in the presence of the sun. By selec.ing a proper position for observation, on the 1st of March it was seen at one o'clock in the afternoon without a telescope.

Such is the amount of evidence which observation has supplied respecting the existence of a solid nucleus. The most that can be said of it is, that it presents a plausible argument, giving some probability, but no positive certainty, that comets have visited our system which have solid nuclei, but, meanwhile, this can only be maintained with respect to few: most of those which have been seen, and all to which very accurate observations have been directed, have afforded evidence of being mere masses of semi-transparent matter.

3087. *The tail.* — Although by far the great majority of comets are not attended by tails, yet that appendage, in the popular mind, is more inseparable from the idea of a comet than any other attribute of these bodies. This proceeds from its singular and striking appearance, and from the fact that most

comets visible to the naked eye have had tails. In the year 1531, on the occasion of one of the visits of Halley's comet to the solar system, Pierre Apian observed that the comet generally presented its tail in the direction opposite to that of the sun. This principle was hastily generalized, and is even at present too generally adopted. It is true that in most cases the tail extends itself from that part of the comet which is most remote from the sun; but its direction rarely corresponds with the direction which the shadow of the comet would take. Sometimes it has happened that the tail forms with a line drawn to the sun a considerable angle, and cases have occurred when it was actually at right angles to it.

Another character which has been observed to attach to the tails of comets, which, however, is not invariable, is, that they incline constantly toward the region last quitted by the comet, as if in its progress through space it were subject to the action of some resisting medium, so that the nebulous matter with which it is invested, suffering more resistance than the solid nucleus, remains behind it and forms the tail.

The tail sometimes appears to have a curved form. That of the comet of 1744 formed almost a quadrant. It is supposed that the convexity of the curve, if it exists, is turned in the direction from which the comet moves. It is proper to state, however, that these circumstances regarding the tail have not been clearly and satisfactorily ascertained.

The tails of comets are not of uniform breadth or diameter; they appear to diverge from the comet, enlarging in breadth and diminishing in brightness as their distance from the comet increases. The middle of the tail usually presents a dark stripe, which divides it longitudinally into two distinct parts. It was long supposed that this dark stripe was the shadow of the body of the comet, and this explanation might be accepted if the tail was always turned from the sun; but we find the dark stripe equally exists when the tail, being turned sideward, is exposed to the effect of the sun's light.

This appearance is usually explained by the supposition that the tail is a hollow, conical shell of vapor, the external surface of which possesses a certain thickness. When we view it, we look through a considerable thickness of vapor at the edges, and through a comparatively small quantity at the middle. Thus upon the supposition of a hollow cone, the greatest brightness

would appear at the sides, and the existence of a dark space in the middle would be perfectly accounted for.

The tails of comets are not always single; some have appeared at different times with several separate tails. The comet of 1744, which appeared on the 7th or 8th of March, had six tails, each about 4° in breadth, and from 30° to 44° in length. Their sides were well defined and tolerably bright, and the spaces between them were as dark as the other parts of the heavens.

The tails of comets have frequently appeared, not only of immense real length, but extending over considerable spaces of the heavens. It will be easily understood that the apparent length depends conjointly upon the real length of the tail, and the position in which it is presented to the eye. If the line of vision be at right angles to it, its length will appear as great as it can do at its existing distance; if it be oblique to the eye, it will be foreshortened, more or less, according to the angle of obliquity. The real length of the tail is easily calculated when the apparent length is observed and the angle of obliquity known.

In respect of magnitude, the tails are unquestionably the most stupendous objects which the discoveries of the astronomer have ever presented to human contemplation.

The following are the results of the observation and measurement of a few of the more remarkable.

Table.	No.	Date of Appearance.	Greatest observed Length of Tail.
			miles.
VIII	148	1847	5,000,000
—	73	1744	19,000,000
VI	4	1769	40,000,000
VIII	46	1618	50,000,000
VI	1	1680	100,000,000
—	8	1811	100,000,000
—	9	1811	130,000,000
—	16	1843	200,000,000

The magnitude of these prodigious appendages is even less amazing than the brief period in which they sometimes emanate from the head. The tail of the comet of 1843, long enough to stretch from the sun to the planetoids, was formed in less than twenty days.

3088. *Mass, volume, and density of comets.* — The masses of comets, like those of the planets, would be ascertained if the reciprocal effects of their gravitation, and those of any known

bodies in the system could be observed. But although the disturbing action of the planets on these bodies is conspicuous, and its effects have been calculated and observed, not the slightest effect of the same kind has ever been ascertained to be produced by them, even upon the smallest bodies in the system, and those to which comets have approached most nearly.

In fine, notwithstanding the enormous number of comets, observed and unobserved, which constantly traverse the solar system in all conceivable directions; notwithstanding the permanent revolution of the periodic comets, whose presence and orbits have been ascertained; notwithstanding the frequent visits of comets, which so thoroughly penetrate the system as almost to touch the surface of the sun at their perihelion, the motions of the various bodies of the system, great and small, planets major and minor, planetoids and satellites, go on precisely as if no such bodies as the comets approached their neighbourhood. Not the smallest effects of the attraction of such visitors are discoverable.

Now since, on the other hand, the disturbing effects of the planets upon the comets are strikingly manifest, and since the comets move in elliptic, parabolic or hyperbolic orbits, of which the sun is the common focus, it is demonstrated that these bodies are composed of ponderable matter, which is subject to all the consequences of the law of gravitation. It cannot, therefore, be doubted that the comets do produce a disturbing action on the planets, although its effects are inappreciable even by the most exact observation. Since, then, the disturbances mutually produced are in the proportion of the disturbing masses, it follows that the masses of the comets must be smaller beyond all calculation than the masses even of the smallest bodies among the planets primary or secondary.

The volumes of comets in general exceed those of the planets in a proportion nearly as great as that by which the masses of the planets exceed those of the comets. The consequence obviously resulting from this, is that the density of comets is incalculably small.

Their densities in general are probably thousands of times less than that of the atmosphere in the stratum next the surface of the Earth.

3089. *Light of comets.* — That planets are not self-luminous, but receive their light from the sun, is proved by their phases,

and by the shadows of their satellites, which are projected upon them, when the latter are interposed between them and the sun. These tests are inapplicable to comets. They exhibit no phases, and are attended by no bodies to intercept the sun's light. But, unless it could be shewn that a comet is a solid mass, impenetrable to the solar rays, the non-existence of phases is not a proof that the body does not receive its light from the sun.

A mere mass of cloud or vapor, though not self-luminous, but rendered visible by borrowed light, would still exhibit no effect of this kind: its imperfect opacity would allow the solar light to affect its constituent parts throughout its entire depth—so that, like a thin fleecy cloud, it would appear not superficially illuminated, but receiving and reflecting light through all its dimensions. With respect to comets, therefore, the doubt which has existed is, whether the light which proceeds from them, and by which they become visible, is a light of their own, or is the light of the sun shining upon them, and reflected to our eyes like light from a cloud. Among several tests which have been proposed to decide this question, one suggested by Arago merits attention.

It has been already shown (1131 *et seq.*), that the apparent brightness of a visible object is the same at all distances, supposing its real brightness to remain unchanged. Now if comets shone with their proper light, and not by light received from the sun, their apparent brightness would not decrease as they would recede from the sun, and they would cease to be visible, not because of the faintness of their light, but because of the smallness of their apparent magnitude. Now the contrary is found to be the case. As the comet retires from the sun its apparent brightness rapidly decreases, and it ceases to be visible from the mere faintness of its light, while it still subtends a considerable visual angle.

3090. *Enlargement of magnitude on departing from the sun.*—It will doubtless excite surprise, that the dimensions of a comet should be enlarged as it recedes from the source of heat. It has been often observed in astronomical inquiries, that the effects, which at first view seem most improbable, are nevertheless those which frequently prove to be true; and so it is in this case. It was long believed that comets enlarged as they approached the sun; and this supposed effect was naturally and probably ascribed to the heat of the sun expanding their dimen-

sions. But more recent and exact observations have shown the very reverse to be the fact. Comets increase their apparent volume as they recede from the sun; and this is a law to which there appears to be no well-ascertained exception. This singular and unexpected phenomenon has been attempted to be accounted for in several ways. Valz ascribed it to the pressure of the solar atmosphere acting upon the comet; that atmosphere being more dense near the sun, compresses the comet and diminishes its dimensions; and, at a greater distance, being relieved from this coercion, the body swells to its natural bulk. A very ingenious train of reasoning was produced in support of this theory. The density of the solar atmosphere and the elasticity of the comet, being assumed to be such as they might naturally be supposed, the variations of the comet's bulk are deduced by strict reasoning, and show a surprising coincidence with the observed change in the dimensions. But this hypothesis is tainted by a fatal error. It proceeds upon the supposition that the comet, on the one hand, is formed of an elastic gas or vapor; and, on the other, that it is impervious to the solar atmosphere through which it moves. To establish the theory, it would be necessary to suppose that the elastic fluid composing the comet should be surrounded by a *nappe* or envelope as elastic as the fluid composing the comet, and yet wholly impenetrable by the solar atmosphere.

After several ingenious hypotheses* having been proposed and successively rejected for explaining this phenomenon, it seems now agreed to ascribe it to the action of the varying temperature to which the vapour which composes the nebulous envelope is exposed. As the comet approaches the sun, this vapour is converted by intense heat into a pure, transparent, and therefore invisible elastic fluid. As it recedes from the sun, the temperature decreasing it is partially and gradually condensed, and assumes the form of a semitransparent visible cloud, as steam does escaping from the valve of a steam boiler. It becomes more and more voluminous as the distance from the source of heat, and therefore the extent of condensation is augmented.

3091. *Professor Struve's drawings of Enché's comet.* — Professor Struve made a series of observations on the comet of

* For several of these, see Sir J. Herschel's memoir, Proceedings of Astronomical Society, vol. vi. p. 104.

Encké, at the period of its reappearance in 1828, and by the aid of the great Dorpat telescope, made the drawings given in Pl. XIV. *figs.* 1. and 2.

Fig. 1. represents the comet as it appeared on the 7th November, the diameters *a b* and *c d* measuring each 18′. The brightest part of the comet extended from α to κ, and was consequently eccentric to it, the distance of the centre of brightness from the centre of magnitude being κ K. Between the 7th and the 30th November, the magnitude of the comet decreased from that represented in *fig.* 1. to that represented in *fig.* 2.; but the apparent brightness was so much increased, that at the latter date it was visible to the naked eye as a star of the 6th magnitude. The apparent diameter was then reduced to 9′.

On November 7th a star of the 11th magnitude was seen through the comet, so near the centre κ of brightness that it was for a moment mistaken for a nucleus. The brightness of the star was not in the least perceptible degree dimmed by the mass of cometary matter through which its light passed.

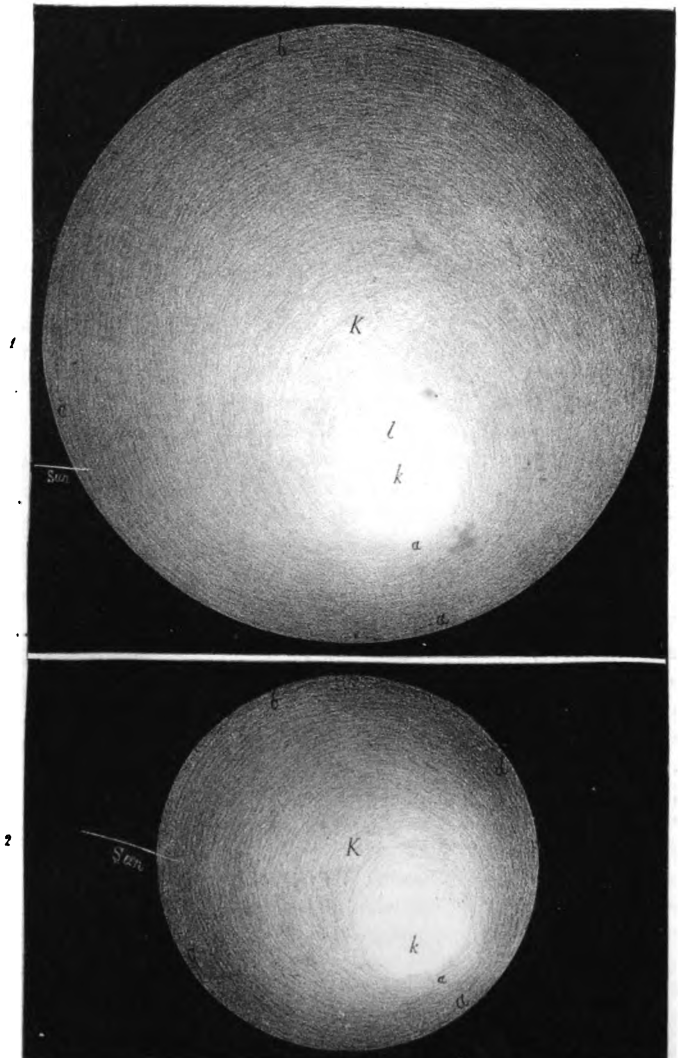
It was evident that the increase of the brightness of the comet on the 30th November, must be ascribed to the contraction, and consequent condensation, of the nebulous matter composing it in receding from the sun, for its distance from the earth on the 7th November, when it subtended an angle of 18′, was 0·515 (the earth's mean distance from the sun being =1); while its distance on the 30th, when it subtended an angle of 9′, was only 0·477. Its cubical dimensions must, therefore, have been diminished, and the density of the matter composing it augmented in more than an eight-fold proportion.

3092. *Remarkable physical phenomena manifested by Halley's comet.* — The expectation so generally entertained, that, on the occasion of its return to perihelion in 1835, this comet would afford observers occasion for obtaining new data, for the foundation of some satisfactory views respecting the physical constitution of the class of which it is so striking an example, was not disappointed. It no sooner reappeared than phenomena began to be manifested, preceding and accompanying the gradual formation of the tail, the observation of which has been most justly regarded as forming a memorable epoch in astronomical history.

Happily, these strange and important appearances were observed with the greatest zeal, and delineated with the most ela-

ENCKE'S COMET 1828

Approaching the sun Telescopic drawings by Struve. XIV



1 Nov 7 2 Nov 30.

borate and scrupulous fidelity by several eminent astronomers in both hemispheres. MM. Bessel at Konigsburg, Schwabe at Dessau, and Struve at Pultowa, and Sir J. Herschel and Mr. Maclear at the Cape of Good Hope, have severally published their observations, accompanied by numerous drawings, exhibiting the successive transformations presented under the physical influence of varying temperature, in its approach to and departure from the sun.

The comet first became visible as a small round nebula, without a tail, and having a bright point more intensely luminous than the rest eccentrically placed within it. On the 2d October, the tail began to be formed, and, increasing rapidly, acquired a length of about 5° on the 5th; on the 20th it attained its greatest length, which was 20°. It began after that day to decrease, and its diminution was so rapid, that on the 29th it was reduced to 3°, and on the 5th November, to 2½°. The comet was observed on the day of its perihelion by M. Struve, at the Observatory of Pultowa, when no tail whatever was apparent.

The circumstances which accompanied the increase of the tail from 2d October, until its disappearance, were extremely remarkable, and were observed with scrupulous precision, simultaneously by Bessel at Konigsburg, by Struve at Pultowa, and by Schwabe at Dessau, all of whom made drawings from time to time, delineating the successive changes which it underwent.

On the 2nd, the commencement of the formation of the tail took place by the appearance of a violent ejection of nebulous matter from that part of the comet which was presented towards the sun. This ejection was, however, neither uniform nor continuous. Like the fiery matter issuing from the crater of a volcano, it was thrown out at intervals. After the ejection, which was conspicuous, according to Bessel, on the 2nd, it ceased, and no efflux was observed for several days. About the 8th, however, it recommenced more violently than before, and assumed a new form. At this time Schwabe noticed an appearance which he denominates a "second tail," presented in a direction opposed to that of the original tail, and, therefore, towards the sun. This appearance seems, however, to be regarded by Bessel merely as the renewed ejection of nebulous matter which was afterwards turned back from the sun, as smoke would be by a current of air blowing from the sun in the direction of the original tail.

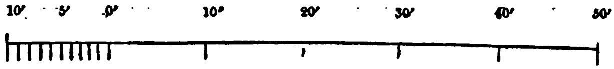
From the 8th to the 22nd, the form, position, and brightness of the nebulous emanations underwent various and irregular changes, the last alternately increasing and decreasing.

At one time two, at another three, nebulous emanations were observed to issue in divergent directions. These directions were continually varying, as well as their comparative brightness. Sometimes they would assume a swallow-tailed form, resembling the flame issuing from a fan gas-burner. The principal jet or tail was also observed to oscillate on the one side and the other of a line drawn from the sun through the centre of the head of the comet, exactly as a compass needle oscillates between the one and the other side of the magnetic meridian. This oscillation was so rapid, that the direction of the jets was visibly changed from hour to hour. The brightness of the matter composing them, being most intense at the point at which it seemed to be ejected from the nucleus, faded away as it expanded into the coma, curving backwards, in the direction of the principal tail, like steam or smoke before the wind.

3093. *Struve's drawings of the comet approaching the sun in* 1835. — These curious phenomena will, however, be more clearly conceived by the aid of the admirable drawings of M. Struve, which we have reproduced with all practicable fidelity, in Plates XV. XVI. and XVII. These drawings were executed by M. Kruger, an eminent artist, from the immediate observation of the appearances of the comet with the great Fraunhoffer telescope, at the Pultowa Observatory. The sketches of the artist were corrected by the astronomer, and only adopted definitively after repeated comparisons with the object. The original drawings are preserved in the library of the observatory.

3094. *Its appearance 29th September.*—Plate XV. *fig.* 1. represents the appearance of the comet on the 29th September. The tail was difficult to be recognised, appearing to be composed of very feeble nebulous matter. The nucleus passed almost centrically over a star of the 10th magnitude, without in the slightest degree affecting its apparent brightness. The star was distinctly seen through the densest part of the comet. Another transit of a star took place with a like result.

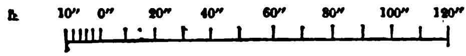
Annexed is the scale according to which this drawing has been made.



3095. *Appearance on October* 3.—This is represented in *fig.* 2. on the same scale.

The comet changed not only its magnitude and form, but also its position, since September 29. On that day the direction of the tail was that of the parallel of declination through the head. On October 3. it was inclined from that parallel towards the north at a small angle, and, instead of being straight, was curved. The diameter of the head was increased in the ratio of 2 to 3, and the length of the tail in the ratio of nearly 1 to 3.

3096. *Appearance on October* 8.—Plate XVI. *fig.* 1. This drawing is made on the subjoined scale of seconds.



On the 5th, 6th, and 7th the comet underwent several changes: the nucleus became more conspicuous. On the 6th, a fan-formed flame issued from it, which disappeared on the 7th, and reappeared on the 8th with increased splendour, as represented in the figure. The nucleus appeared like a burning coal, of oblong form, and yellowish colour. The extent of the flame-like emanation was about 30″. The feeble nebula surrounding the nuclei extended much beyond the limits of the drawing, but, being overpowered by the moonlight, could not be measured.

3097. *Appearance on October* 9.—Plate XVI. *fig.* 2., same scale, represents the nucleus and flame-like emanation, which entirely changed their form and magnitude since the preceding night. The tail (not included in the drawing) measured very nearly 2°. The flame consisted of two parts, one resembling that seen on the 8th, and the other issuing like the jet from a blow-pipe in a direction at right angles to it. The figure represents the nucleus and flame as they appeared at 21^h sid. time, with a magnifying power of 254.

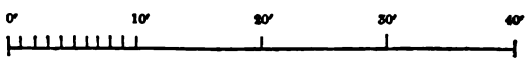
3098. *Appearance on October* 10.—Plate XVI. *fig.* 3. on the same scale. The tail, which still measured nearly 2°, was now much brighter, being visible to the naked eye, notwithstanding strong moonlight. The coma was evidently broader than the tail. The flaming nucleus is represented in the drawing as it

appeared under a magnifying power of 86, with a field of 18′ diameter, the entire of which was filled with this coma. The diameter of the latter must, therefore, have been more than 18′. The drawing was taken at 21*h*. s. t.

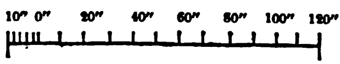
3099. *Appearance on October* 12.—Plate XVI. *fig*. 4. on the same scale. The comet appeared at 0 *h* — 25 *m*. s. t. for a short interval in uncommon splendour, the nucleus and flame, however, alone being visible, as represented in the drawing. The greatest extent of the flame measured 64″·7. Its appearance was most beautiful, resembling a jet streaming out from the nucleus, like flame from a blow-pipe, or the flame from the discharge of a mortar, attended with the white smoke driven before the wind.

3100. *Appearance on October* 14.—Plate XVI. *fig*. 5. on the same scale. The principal flame was now greatly enlarged, extending to the apparent length of 134″. Its deflection and curved form were most remarkable.

3101. *Appearance on October* 29.—A cloudy sky prevented all observation for 12 days. On the 27th, the comet appeared to the naked eye as bright as a star of the third magnitude, the tail being distinctly visible. The coma surrounding the nucleus appeared as a uniform nebula. The tail was curved and of great length; but, owing to the low altitude at which the observation was taken, it could not be measured. On the 29th, however, the comet was presented under much more favourable conditions, and the drawings, Plate XV. *fig*. 3. and Plate XVII. *fig*. 1. were made. The former represents the entire comet, including the whole visible extent of the tail, and is drawn to



the annexed scale of minutes. The latter represents the head of the comet only, and is drawn to the annexed scale of seconds.



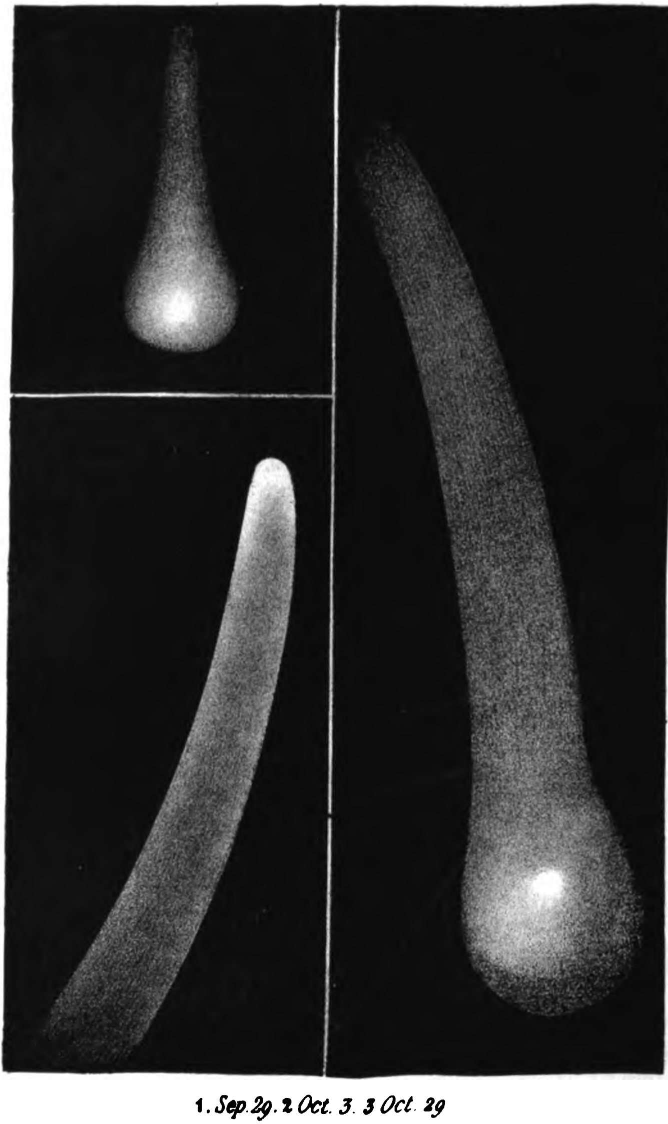
At 20^h 30^m s. t., the head presented the appearance represented in Plate XVII. *fig*. 1. The chief coma was almost exactly circular, and had a diameter of 165″. With a power of 198, the nucleus appeared as in the figure, the diameter being about 1″25 to 1·50. The flame issuing from the nucleus, curved

HALLEY'S COMET 1835

Approaching the sun from drawings by Struve

XV

1



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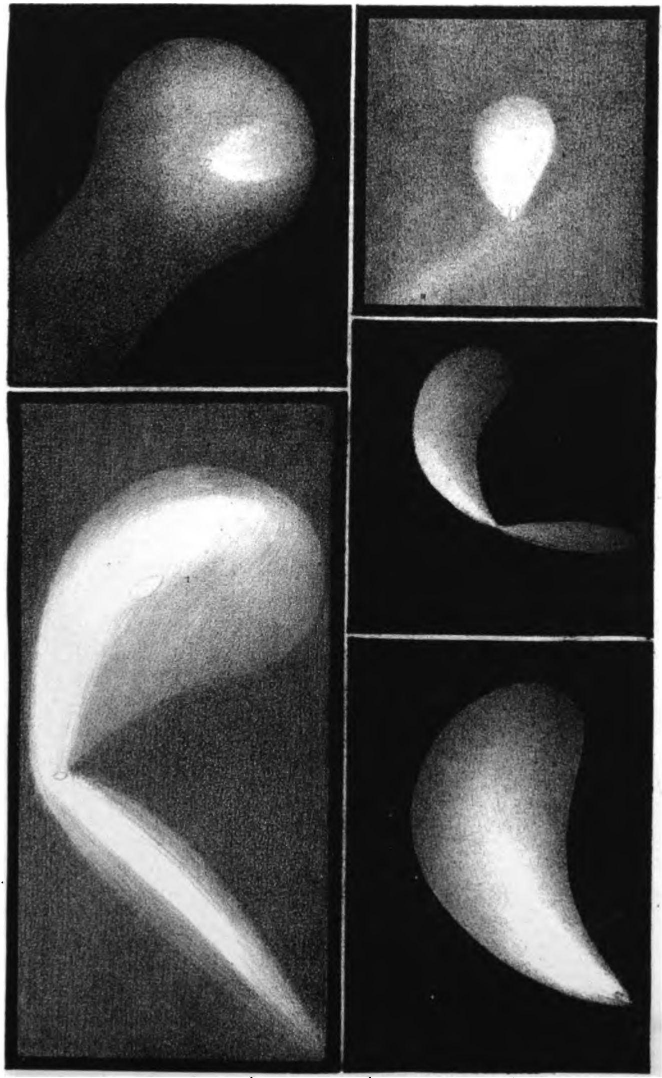
2

1. Sep. 29. 2 Oct. 3. 3 Oct. 29

HALLEY'S COMET 1835

Approaching the sun - from drawings by Struve.

XVI



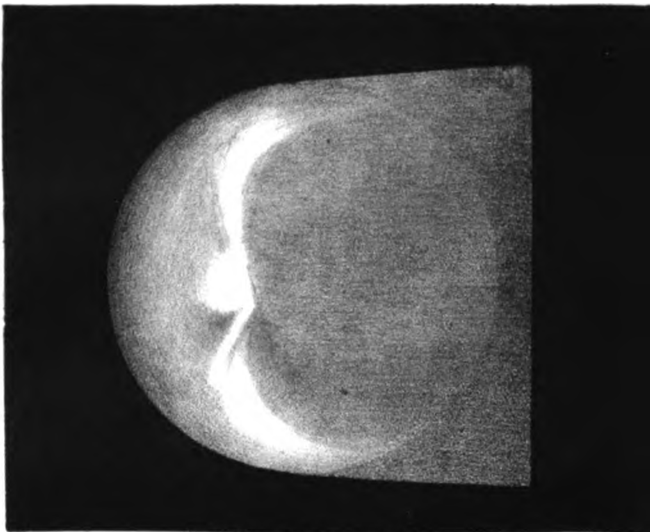
1 Oct 8 2 Oct. 9. 3 Oct 10 4 Oct. 12. 5 Oct. 14

XVII

HALLEY'S COMET 1835

Approaching the sun · from drawings by Struve

1

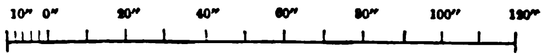


2

1 Oct 29. 2 Nov 5.

back like smoke before the wind, was very conspicuous. The appearance of the formation of the tail as it issues from the nucleus was remarkably developed.

3102. *Appearance on November 5.*—Plate XVII. *fig.* 2. This drawing represents the nucleus and flame issuing from it on the annexed scale of seconds.



The proper nucleus was found to measure about 2″·3. Two flames were seen issuing from it in nearly opposite directions, and both curved towards the same side. The brighter flame, directed towards the north, was marked by strongly defined edges. The other, directed towards the south, was more feeble and ill-defined.

3103. *Sir J. Herschel's deductions from these phenomena.*—Sir J. Herschel, who also observed this comet himself at the Cape of Good Hope, makes from all these observations the following inferences.

1. That the matter of the comet vaporised by the sun's heat escapes in jets, throwing the comet into irregular motion by its reaction, and thus changing its own direction of ejection.

2. That this ejection takes place principally from the part presented to the sun.

3. That thus ejected it encounters a resistance from some unknown force by which it is repulsed in the opposite direction, and so forms the tail.

4. That this acts unequally on the cometary matter, which is not all vaporised, and of that which is a considerable portion, is retained so as to form the head and coma.

5. That this force cannot be solar gravitation, being contrary to that in its direction, and very much greater in its intensity, as is manifest by the enormous velocity with which the matter of the tail is driven from the sun.

6. That the matter thus repelled to a distance so great from a body whose mass is so small must to a great extent escape from the feeble influence of the gravitation of the mass composing the head and coma, and, unless there be some more active agency in operation, a large portion of such vaporised matter must be lost in space, never to reunite with the comet. This would lead to the consequence, that at every passage

through its perihelion the comet would lose more and more of its vaporisable constituents, on which the production of the coma and tail depends, so that, at each successive return, the dimensions of these appendages would be less and less, as they have in fact been found to be.

3104. *Appearance of the comet after perihelion.*—On receding from the sun after its perihelion, the comet was observed under very favourable circumstances at the Cape by Sir J. Herschel and Mr. Maclear. It first reappeared there on the 24th of January, under an aspect altogether different from that under which it was seen before its perihelion. It had evidently, as Sir J. Herschel thinks, undergone some great physical change, which had operated an entire transformation upon it.

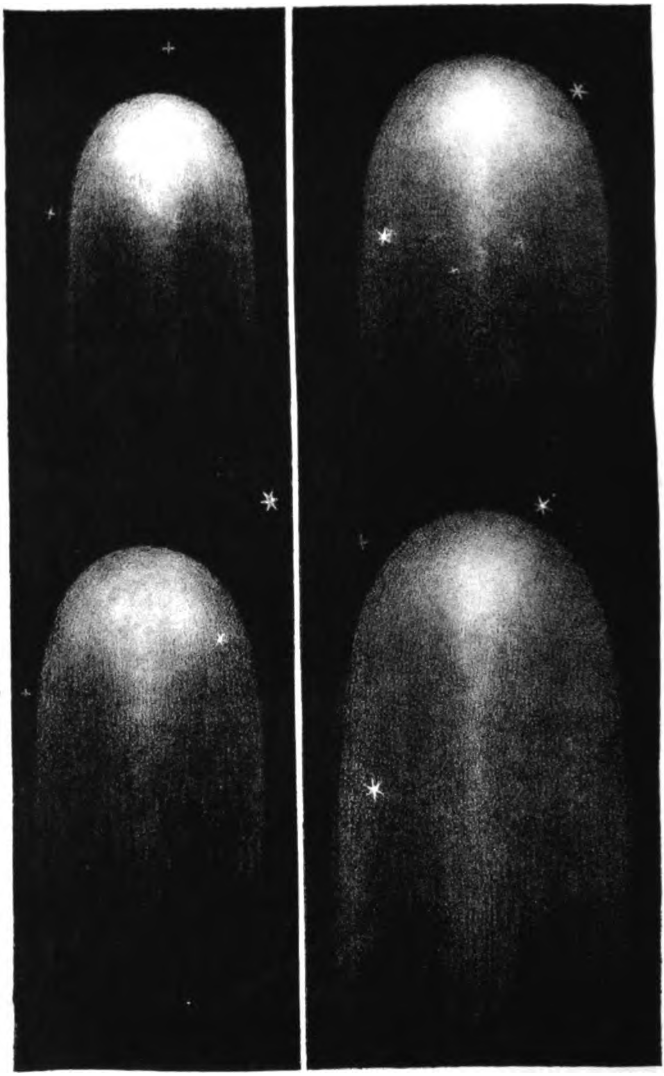
"Nothing could be more surprising than the total change which had taken place in it since October. . . . A new and unexpected phenomenon had developed itself, quite unique in the history of comets. Within the well-defined head, somewhat eccentrically placed, was a vivid nucleus resembling a miniature comet, with a head and tail of its own, perfectly distinct from and considerably exceeding in intensity the nebulous disk or envelope which I have above called the 'head.' A minute bright point, like a small star, was distinctly perceived within it, but which was never quite so well defined as to give the positive assurance of the existence of a solid sphere, much less could any phase be discerned." *

3105. *Observations and drawings of Messrs. Maclear and Smith.*—The phenomena and changes which the comet presented from its reappearance on the 24th of January, until its final disappearance, have been described with great clearness by Mr. Maclear, and illustrated by a beautiful series of drawings by that astronomer and his assistant, Mr. Smith, in a memoir which appeared in the tenth volume of the Transactions of the Royal Astronomical Society, from which we reproduce the series of illustrations given on Plates XVIII. and XIX.

3106. *Appearance on January* 24.—The comet appeared, as in *fig.* I., visible to the naked eye as a star of the second magnitude. The head was nearly circular, and presented a pretty well-defined planetary disc, encompassed by a coma or halo of delicate gossamer-like brightness. The diameter of the head,

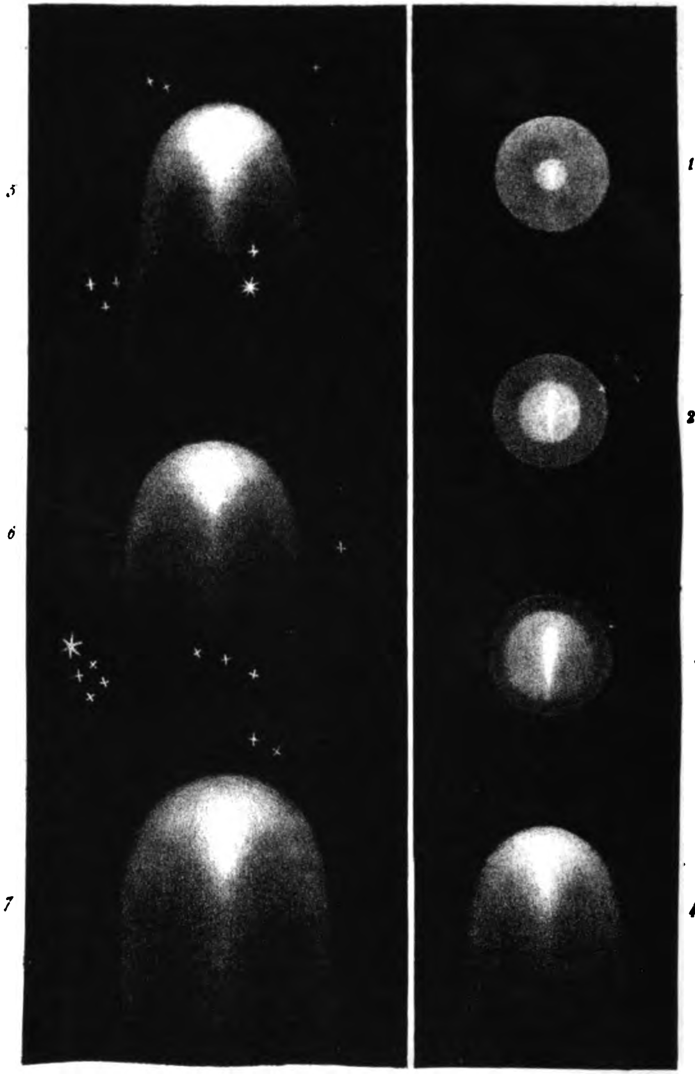
* Cape Observations, p. 397.

Halley's Comet departing from the sun in 1836
Telescopic Views by Mess. Maclear & Smith. XIX



1. Feb 7. 2. Feb 10. 3. Feb. 16. 4. Feb 25

Halley's Comet departing from the sun in 1836
Telescopic Views by Mess. Maclear & Smith. XVIII.



1. Jan. 24 . 2 Jan 25 3. Jan 26 4 Jan 27
5 Jan 28 6 Jan 30 . 7 Feb 1

without the halo or coma, measured 131″, and with the latter 492″.

3107. *Appearance on January* 25.—*Fig.* 2. Circular form broken, and magnitude increased. Three stars seen through the coma, and one through the head.

3108. *Appearance on January* 26.—*Fig.* 3. Magnitude again increased, but coma diminished.

3109. *Appearance on January* 27.—*Fig.* 4. Comet began to assume the parabolic form, and increase of magnitude continued.

3110. *Appearance on January* 28.—*Fig.* 5. The coma or halo quite invisible, but the nucleus appeared like a faint small star. The magnitude of the comet continued to increase. The observer fancied he saw the faint outline of a tail.

3111. *Appearance on January* 30.—*Fig.* 6. The form of the comet now became decidedly parabolic. The breadth across the head was 702″, being greater than on the 24th in the ratio of 49 to 70, or 7 to 10, which corresponds to an increase of volume in the ratio of 1 to 3, supposing the form to remain unchanged; but it was estimated that the extension in length gave a superficial increase in the ratio of 35 to 1, which would correspond to a much greater augmentation of volume.

3112. *Appearance on February* 1.—*Fig.* 7. Further increase of magnitude, the form remaining the same.

3113. *Appearance on February* 7.—Plate XIX. *fig.* 8. The comet was on this night rendered faint by the effect of moonlight.

3114. *Appearance on February* 10.—*Fig.* 9. Further increase of volume. A star visible through the body of the comet.

3115. *Appearance on February* 16. *and* 23.—*Figs.* 10, 11. The magnitude went on increasing, while the illumination became more and more faint, and this continued until the comet's final disappearance; the outline, after a short time, became so faint as to be lost in the surrounding darkness, leaving a bland nebulous blotch with a bright centre enveloping the nucleus.

3116. *Number of comets.*—According to Mr. Hind, the number of comets which have appeared since the birth of Christ in each successive century is as follows: I., 22.; II., 23.; III., 44.; IV., 27.; V., 16.; VI., 25.; VII., 22.; VIII., 16.; IX., 42.; X., 26.; XI., 36.; XII., 26.; XIII., 26.; XIV., 29.; XV., 27.;

XVI., 31.; XVII., 25.; XVIII., 64.; XIX. (first half), 80: Total, 607.

3117. *Duration of the appearance of comets.*— Since comets are visible only near their perihelia, when their velocity is greatest, the duration of their visibility at any single perihelion passage is generally short. The longest appearance on record is that of the great comet of 1811 (No. 8., Table VI.), which continued to be visible for 510 days. The comet of 1825 (No. II., Table VI.) was visible for twelve months, and others which appeared since have been seen for eight months. In general, however, these bodies do not continue to be seen for more than two or three months.

3118. *Near approach of comets to the earth.*—Considering the vast numbers of comets which have passed through the system, such an incident as the collision of one of them with a planet might seem no very improbable contingency. Lexell's comet was supposed to have passed among the satellites of Jupiter; and, if that was the case, it is certain that the motions of these bodies were not in the least affected by it. The nearest approach to the earth ever made by a comet was that of the comet of 1684 (No. 55., Table VIII.), which came within 216 semi-diameters of the earth, a distance not so much as four times that of the moon. We are not aware of any nearer approach than this being certainly ascertained.

CHAP. XIX.

THEORY OF VARIABLE ORBITS.

3119. *Conditions under which elliptic orbits are described.* —If any number of bodies P, P′, P″, &c., moving with any velocities in any directions whatever, be exposed to the influence of the attraction of a central body S, in a fixed position, such attraction varying in its intensity inversely as the square of the distance of the attracted body from S; and if at the same time the several bodies P, P′, P″, &c., exert no attraction either upon each other or upon the central body S; these bodies P, P′, P″, &c., will each of them revolve in an ellipse of which the centre of

the attracting body S is the focus. Each of these elliptic orbits will be invariable in form, magnitude, and position, and so long as such a system would be exposed to the influence of no other force than the central attraction of S, each of the bodies P, P′, P″, &c., would continue to revolve round S in the same invariable orbit.

The general proposition here enunciated was demonstrated by Newton in the first book of his celebrated work entitled the PRINCIPIA.

3120. *Forces other than the central attraction would destroy the elliptic form.*—If, however, the bodies composing such a system were exposed to the influence of any other attraction, whether proceeding from each other or from any cause exterior to and independent of the system, the motions of P, P′, P″, &c., would no longer take place in such elliptic orbits. Their paths would, in that case, depend on the directions, intensities, and law of variation of those other attractions, whether internal or external, to the operation of which they are exposed. If such forces have intensities which bear any considerable proportion to the intensity of the central attraction exercised by S, the elliptic form impressed on the orbits by the latter would be altogether effaced, and the bodies P, P′, P″, &c., would be thrown into new and wholly different paths, the problem to determine which would be one of the greatest physical complexity and mathematical difficulty.

3121. *But when these forces are feeble compared with the central attraction, the elliptic form is only slightly affected.*—But if the forces, whether internal or external, to the operation of which the system is exposed, have intensities incomparably more feeble than the central attraction exercised by S upon P, P′, P″, &c., then, as may be readily conceived, the influence of S in imparting the elliptic form to the paths of P, P′, P″, &c., around it, will still in the main prevail. These paths will not, in strictness, be ellipses, but, owing to the comparatively small effect of the disturbing forces, they will deviate from the elliptic form, which in the absence of such forces they would rigorously assume, in a degree so slight as to be only perceptible when the most exact methods of observation and measurement are brought to bear upon them, and in many cases not even then until the effects of such feeble forces have been allowed to accumulate during a long succession of revolutions.

3122. *This is the case in the system of the Universe.*— Now it happens that the Great Architect of the Universe has so constructed it, that in all cases whatever, without a single known exception, the forces which are independent of the central attraction s, whether they be those which arise between the bodies composing the systems, or whether they proceed from causes exterior to and independent of them, are under the conditions last mentioned. In no case do their intensities exceed very small fractions, such as an hundredth, and more generally not a thousandth, of the central attraction. Hence it is that the ellipticity of the orbits is so preserved, their magnitudes so maintained, and their positions so little variable, that in all cases the most exact means of observation, and in general long periods of time, are necessary to discover and measure the changes produced upon them.

3123. *Hence proceed great facilities of investigation and calculation.*— Hence arise consequences of vast importance in the development of the laws of nature and the progress of physical knowledge. The problem presented by a system subject only to such feeble interferences with the influence of the central attraction, is incomparably more simple and easy of solution than would be that of a system in which interfering attractions of much greater relative intensity might prevail. Methods of investigation and calculation, as well as modes of observation, are applicable in the one case, which would be altogether inadmissible in the other; and results are obtained and laws developed which would, under the more complicated conditions of the other problem, be utterly unattainable.

3124. *Perturbations and disturbing forces.*—The central attraction, therefore, being in all cases regarded as the chief presiding physical power by which the system is held together, and by which its motions in the main are regulated, the orbits of the revolving bodies P, P′, P″, &c., are first calculated as if they depended solely upon the central attraction of S. This gives a first, but very close, approximation to them. The forces by which they are affected, independent of the central attraction of S, are then severally taken into account, and the deviations, minute as they always are, from the elliptic paths first determined, are exactly calculated. These deviations are called DISTURBANCES OR PERTURBATIONS; and the forces which produce them are called DISTURBING OR PERTURBING FORCES.

3125. *Method of variable elements.*—*The instantaneous ellipse.*—Let a body P be supposed to revolve in a certain orbit, subject to the central attraction F of a certain mass S, and at the same time to a disturbing force D much more feeble than F in its intensity. There are two ways in which the problem to determine the exact path of P may be approached. 1st. The body P may be regarded as under the influence of two forces F and D, of given intensities and directions; and its actual path may be investigated by the principles of mathematical and physical analysis. This path would, in every case presented in nature, be a very complicated curve of no regular form, although in its general shape and outline it would differ very little from an ellipse having its focus at S. 2ndly. Instead of attempting to determine the exact geometrical character of this complicated curve, the body P may be regarded as revolving round S in an ellipse, the form, position, and magnitude of which are subject to a slow and continuous variation. To comprehend this method of considering the motion of P, let the disturbing force D be imagined to be suspended at any proposed point of P's path. From the moment of such suspension, P would move in an exact and invariable ellipse, having S as its focus. The form, position, and magnitude of this ellipse, or, what is the same, its *elements*, that is, its major axis, excentricity, longitude of perihelion, inclination, and longitude of node, would be exactly deducible from P's distance from S at the moment of suspension of the disturbing force, its velocity, and the direction of its motion. The problem of its determination in such case would have nothing indeterminate. One, and but one, ellipse could, under such conditions, be described.

If the disturbing force D be imagined to be suspended at another moment and at a different point in the path of P, another and a different ellipse would be in the same manner described by P after such suspension. If the interval between the two moments of such supposed suspension be not considerable, as, for example, when they occur at different parts of a single revolution of P round S, the two ellipses will not in general have any appreciable difference in any of their elements, from which it follows, that when a single or even several revolutions of P round S are only considered, the path of P may in general be regarded as an ellipse of fixed position and invariable form and magnitude, such as P would describe

independently of the influence of D. But if the interval between the two moments of supposed suspension be very great, as, for example, when it extends to a long series of revolutions of P round S, then the disturbing effects of D, having accumulated from revolution to revolution, will become very sensible and measurable, and the two ellipses may differ one from the other in any or all of their elements, and to an extent more or less considerable according to the intensity and direction of the disturbing force D.

The ellipse in which P would thus move, if at any point of its path the action of the disturbing force were thus suspended, is called the "INSTANTANEOUS ELLIPSE."

According to this second method of viewing the effects of disturbing forces, therefore, the body P, which is subject to their action, is regarded as moving in an elliptic orbit of which S is the focus; but this orbit is supposed from moment to moment to change its position, form, and magnitude in a certain minute degree. The perturbation being thus, as it were, transferred from the body P to the orbit which it describes, and the body being supposed to move as a bead would slide on a fine wire, while the wire itself, being flexible, would bend into various forms, the bead still moving along it, the method has been denominated as that of "VARIABLE ELEMENTS."

This is the method of considering the effects of disturbing forces which was adopted by Newton, under the title of moveable orbits, and is still generally adopted as the most simple, clear, and convenient means of investigating and explaining the phenomena of perturbations.

3126. *Feebleness of the disturbing forces in the cases presented in the solar system explained.*—In the cases which are presented in the actual system of the world, the extremely feeble intensities of the disturbing forces, compared with those of the central attractions, arise in some cases from the vastness of the masses of the central compared with those of the disturbing bodies; in others, from the smallness of the distance of the central compared with that of the disturbing from the body which is attracted and disturbed; and, in some cases, from the combination of both these causes.

Thus, when a planet attracted by the sun is disturbed by the attraction of another planet, the enormous preponderance of the mass of the sun, which is more than a thousand times greater

than the larger, and hundreds of thousands of times greater than the smaller planets, gives to the effects of its attraction a predominance which could only be compensated by a greater degree of proximity of the disturbing to the disturbed planet than is consistent with the wise conditions under which the solar system is placed. The differences between the mean distances of the planets, taken in succession from Mercury outwards, are so great, and the excentricities of their several orbits so small, that in no possible position, not even when they are in heliocentric conjunction (2986.), with one in aphelion, and the other in perihelion, can they approach each other so as to give to the disturbing force exerted by any one upon any other an intensity amounting to more than a very minute fraction of the central attraction.

This will be rendered quite apparent hereafter, and we shall assume it provisionally for the present in our exposition of the general effects of the disturbing forces which prevail among the bodies of the system; premising, however, that other conditions besides the consideration of relative masses and proximity will be necessary for the exact estimation of the effects of the disturbing forces.

3127. *Order of exposition.*—In the present chapter we shall then explain generally, without reference to any particular disturbing or disturbed body, the effects produced by disturbing forces upon the elements of the orbit of the disturbed body; and in the succeeding chapters we shall show the application of the general principles thus established to the most important cases of perturbation presented in the solar system.

3128. *Resolution of the disturbing force into rectangular components.*—From whatever cause the disturbing force may arise, it can always be resolved into three components, each of which is at right angles to the other two; and its effects may be investigated by ascertaining the separate effects of each of these components, and then combining the results thus obtained.

The resolution of any force into three rectangular components parallel to three lines or axes, arbitrarily chosen, is a process of great utility in mathematical physics, and one which is based upon the general principles of the composition and resolution of force, formerly so fully explained and illustrated (144.) *et seq.*

To render this more clearly intelligible, let P, *fig.* 817., be

the position of the disturbed body at any proposed time, and let the direction of the disturbing force D be PR, its intensity being such that, if no other force acted on P, it would cause that body to move over PR in the unit of time. Let PZ, PX, and PY be the three axes at right angles to each other, taken at pleasure in any directions along which the disturbing force D is to be resolved. Suppose three lines, RR′, R R″, and R R‴, drawn from R parallel to these three axes PZ, PY, and PX; so as to form the rectangular die-shaped solid called a parallelopiped, represented in the figure.

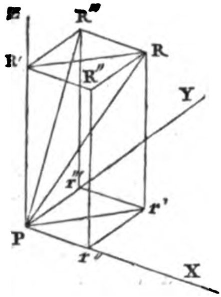


Fig. 817.

Now, by the principles of elementary geometry, it appears that RR′Pr′ is a rectangle of which PR is the diagonal. It follows, therefore (154.), that the disturbing force D represented by PR is equivalent to two forces represented by PR′ and Pr′. But Pr‴r′r″ being also a rectangle, the component Pr′ is equivalent to two components represented by Pr″ and Pr‴, directed along the axes PX and PY respectively.

Thus it appears that the disturbing force D, represented by PR, is resolved into three components, represented by PR′ directed along PZ, Pr″ directed along PX, and Pr‴ directed along PY respectively. If the angles at which the direction PR of the disturbing force is inclined to the three axes PX, PY, and PZ be known, it is easy to obtain arithmetical expressions for these three components of it.

Let the angles RPX, RPY, and RPZ, at which PR is inclined to the axes PX, PY, and PZ respectively, be expressed by α, β, and γ; and let the three components Pr″, Pr‴, and PR′ be expressed by X, Y, and Z respectively. Since the angles Pr″R, Pr‴R, and PR′R are each 90°, it will follow that

$$X = \cos.\,\alpha \times D, \qquad Y = \cos.\,\beta \times D, \qquad Z = \cos.\,\gamma \times D;$$

and since by the figure we have

$$PR^2 = PR'^2 + RR'^2, \qquad RR'^2 = R'R''^2 + RR''^2,$$
$$PR^2 = PR'^2 + R'R''^2 + RR''^2,$$

we shall have

$$D^2 = X^2 + Y^2 + Z^2.$$

Each of the components is therefore found by multiplying the disturbing force by the cosine of the angle which its direction forms with the axis upon which the component is taken, and the sum of the squares of the three components is equal to the square of the disturbing force.

Such is the principle, in its most general expression, by which the resolution of the disturbing force is effected; and if the effect of each of the three components upon the orbit of P be separately ascertained, and the like process be applied to every disturbing force by which P is affected, the actual changes produced in the orbit will be determined.

3129. *Resolution of the disturbing force with relation to the radius vector and the plane of the orbit.*—But for the purposes of elementary exposition, it is found convenient to give to the axes PX, PY, and PZ, in the directions of which the components are assumed, a position having immediate relation to the plane of P's orbit, and to the position of P in its orbit. Since the directions of these axes are altogether arbitrary, being restrained by no other conditions than that of being mutually perpendicular, it is always possible to take one of them, PZ for example, perpendicular to the plane of P's orbit, and in that case the plane of the other two, PX and PY, will coincide with that of P's orbit.

But when this is done, the directions of PX and PY in that plane are still arbitrary; and it has been found convenient to assign to one of them the direction either of the radius vector from P to the central body S, or of the tangent to P's orbit drawn through P's place at the moment the disturbing force is supposed to act, such tangent being in effect the direction of P's motion at that moment.

If one of the axes, PY, for example, be taken in the direction of the radius vector, the other will necessarily be that of a line drawn through P in the plane of the orbit perpendicular to the radius vector.

If one of the axes, PX, for example, be taken in the direction of the tangent, the other will necessarily be that of the normal to the orbit, at the point at which the disturbing force is supposed to act.

3130. *Orthogonal component.*—In referring to these several methods of resolving the disturbing force, it will conduce to brevity and clearness to give the several components distinct

designations indicative of the directions which they have in relation to the plane of the orbit, and to the position of the disturbed body in its orbit. For this purpose we shall adopt the designations which have been already proposed for them by elementary writers.

The component which is perpendicular to the plane of the orbit of the disturbed body will then be distinguished as the ORTHOGONAL COMPONENT of the disturbing force.

3131. *Radial and transversal components.*—If the other two components be taken in the directions of the radius vector, and of a line in the plane of the orbit at right angles to it, we shall call the former the RADIAL and the latter the TRANSVERSAL COMPONENT.

3132. *Tangential and normal components.*—If the other two components be taken in the directions of the tangent and normal of the orbit, we shall call the former the TANGENTIAL and the latter the NORMAL COMPONENT.

3133. *Orthogonal component affects the inclination and the nodes.*—It is evident that of these components the orthogonal alone can have any disturbing effect upon the plane of P's orbit. The other components being all in that plane, can have no tendency to move the disturbed body P into any other plane. The orthogonal component, however, being at right angles to the plane in which P is moving, must have a direct tendency to carry P out of that plane, on the one side or the other, according to the direction in which it acts.

This component therefore, and this alone, affects the inclination of P's orbit, and the longitude of its node. The kind of effect it produces on these elements will be explained hereafter.

3134. *Radial and transversal components affect the central*

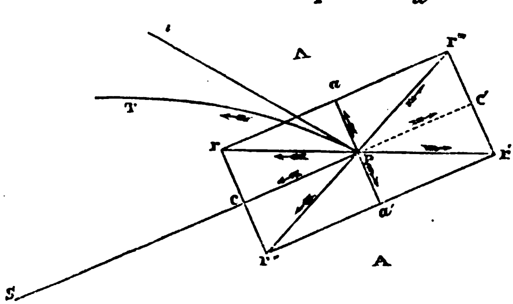


Fig. 818.

attraction and angular motion.—To explain in general the effect produced on P's motion by the radial and transversal components of the disturbing force, let P *c*, *fig.* 818., be the radial, and P *a* the transversal component, the diagonal P *r* being therefore the part of the disturbing force which acts in the plane of P's orbit.

It is evident that the radial component P *c*, as here represented, will have the effect of augmenting the attraction by which P is drawn towards S, and that the transversal component P *a*, being at right angles to the radius vector, will have a tendency to increase the angular velocity of P round S, since this angular velocity is measured by the motion of P, at right angles to P S, the radius P S being supposed to be given.

These components, however, may be otherwise directed in relation to the radius vector. If, for example, the element of the disturbing force which acts in the plane of P's orbit have the direction P *r'*, its radial component will be P *c'*, and its transversal P *a'*. The former, acting directly against S's attraction on P, would have a tendency to diminish that attraction; and the latter, being contrary to the direction of P's motion, would have a tendency to diminish its angular velocity. The two components therefore, in this case, would produce effects on P directly the reverse of those produced in the former case.

If the element of the disturbing force have the direction P *r''*, the radial component P *c* will tend to augment the central attraction, and the transversal P *a'* to diminish the angular velocity.

If, in fine, the disturbing element have the direction P *r'''*, the radial component P *c'* will tend to diminish the central attraction, and the transversal P *a* to augment the angular velocity.

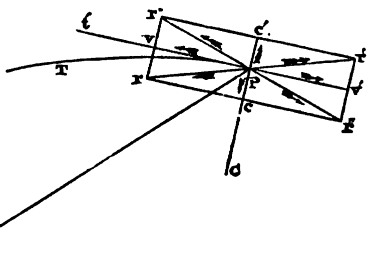


Fig. 819.

3135. *Tangential and normal components affect the linear velocity and curvature.*—The effects of the tangential and normal components are subject to the same varieties. It may be shown, that if a rectangle P *v r c*, *fig.* 819., be drawn, of which the disturbing element P *r* is the diagonal, the element P *r* may be replaced by the two sub-components, P *v* in the direction of the tangent, and P *c* in the direction of the normal.

These are subject to the same variety of conditions, with reference to the direction of the central attraction, and the direction of the motion of P, as have been explained in relation to the radial and transversal sub-components, and which will be easily comprehended by the *fig.* 819.

3136. *Positive and negative components.*—It will conduce at once to brevity and clearness to distinguish each of the components of the disturbing force according to its direction, with reference to the motion of the disturbed body, the direction of the central attraction, and the plane of the disturbed orbit. We shall therefore consider the transversal or tangential component as positive or + when it acts in the direction of P's motion, and therefore tends to accelerate it; and negative or − when it acts in the contrary direction, and therefore tends to retard it. We shall in like manner consider the radial or normal component as positive or + when it acts towards the concave side of P's orbit, and therefore tends to augment the central attraction, and negative or − when it has the contrary direction, and tends to diminish the central attraction. In fine, we shall consider the orthogonal component positive or + when it is directed towards the plane which is adopted as the plane of reference, and negative or − when it is directed from that plane.

3137. *In slightly elliptic orbits, the normal and radial components, and the tangential and transversal, coincide.*—It is evident that when the elliptic orbit is but slightly excentric, and therefore very nearly circular, the radial and normal components are very nearly identical in their direction; and, in like manner, the transversal and tangential components are nearly coincident. As this is the case with all the orbits to be considered in this chapter, we shall, without again recurring to this point, consider these components as practically identical.

We shall then explain, in the first instance, without special reference to any particular disturbing body, the effects which

are produced upon the orbit of P, 1st, by the radial component; 2ndly, by the transversal component; and 3rdly, by the orthogonal component of any disturbing force whatever.

I. EFFECTS OF THE RADIAL COMPONENT OF THE DISTURBING FORCE.

3138. *Equable description of areas not disturbed by it.*—The equable description of areas round the centre being independent of the law of the central attraction, and involving no other condition, except that the revolving body should be affected by no forces except such as have directions passing through the fixed centre (2599.), it will not be affected by the radial component, the direction of which necessarily passes through that point.

3139. *Its effect on the mean distance and period.*—If M express the central mass, a the mean distance, and P the period proper to the instantaneous ellipse, we shall have, according to what has been proved (2634.), $M = \frac{a^3}{P^2}$. Now the radial component either augments or diminishes the central attraction, according as it is positive or negative. This is equivalent to a momentary increase or diminution of the central mass M, which would be attended by a corresponding increase or diminution of $\frac{a^3}{P^2}$; that is, of the ratio of the cube of the mean distance to the square of the periodic time in the instantaneous ellipse.

3140. *If the radial component vary according to any conditions which depend solely on the distance, it will not change the form or magnitude of the instantaneous ellipse.*—It has been established as a principle of high generality by mathematicians, that if the variation of the central force depend only on the distance of the revolving body from the centre of attraction, the orbital velocity of the body, or the space it moves through in the unit of time, will also depend solely on the distance. Now, from this, combined with the general principle of equable areas, it may be inferred that, under such conditions, the apsides of the instantaneous ellipse will be always at the same distances from the centre of attraction. For the apsides must be always at those particular distances, and no other, at which the velocity (which by the supposition depends on the

distance) multiplied by the distance is equal to twice the area which the revolving body describes in the unit of time, that being necessarily the case by the common principles of elementary geometry when the direction of the motion is at right angles to the radius vector. This will therefore always give the same values for the radii vectores which are at right angles to the tangent, or, what is the same, to the distances of the apsides of the instantaneous ellipse from the focus.

But it is clear that, if the distances d', d'' of the apsides from the focus be always the same, the major axis $2a$, and the excentricity $\frac{c}{a}$, will also be always the same; for we shall have

$$2a=d'+d'', \qquad 2c=d''-d', \qquad e=\frac{d''-d'}{d''+d'}.$$

Although the instantaneous ellipse be thus invariable in its form and magnitude, under the conditions here assumed, it does not, however, follow that it is equally invariable in its position. It may, and does, as will presently appear, in certain cases, revolve round the centre of attraction, its major axis continually shifting its direction, so that the real path of the disturbed body will be such as is represented in *fig.* 820.

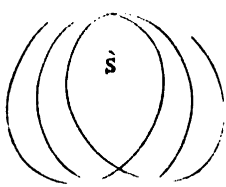


Fig. 820.

3141. *Effect of a gradually increasing or decreasing radial component.* — If the radial component, whether it be positive or negative, were by its continual increase or decrease to cause the effective central attraction continually to increase, the revolving body P would be brought every revolution nearer and nearer to the central body S; and if it caused the effective central attraction continually to decrease, the contrary effect would be produced. In one case, the angular motion of the revolving body would be continually accelerated; and, in the other case, continually retarded.

3142. *Effects on the period and mean motion more sensible than those on the mean distance.*—If the mean distance by means of such radial disturbance as here described should be varied in an extremely small proportion, such, for example, as that of one part in a million in each revolution, such a change would become sensible, even to the most delicate instruments of observation, only after the lapse of centuries. The con-

tinual change effected on the period, and thereby on the mean motion, would, however, tell in a sensible manner on the mean place of the body in a comparatively short time.

3143. *Its effect on the position of the apsides.*—The effect of the radial component on the direction of the major axis of the orbit, will vary with the part of the orbit at which the disturbed body is found at the moment the disturbing force acts upon it.

3144. *It diminishes or increases the angle under the radius vector and tangent, according as it is positive or negative.*—It must be considered that, in general, the radial component, when +, has a tendency to diminish the angle *n*PS, *fig.* 818., formed by the direction P*n* of P's motion and the radius vector SP; and, when −, to increase it. This will be nearly self-evident on inspecting the *fig.* 821. If P*p* represent the arc of the orbit which P is going to describe at the moment that the force R acts upon it; and if R be supposed to be +, and therefore to have a tendency to increase the energy of the force directed to C; it is evident that in the unit of time the force which previously deflected P to *p*, will now deflect it still more to *p′* or *p″*; so that the arc of the new orbit P*p′* or P*p″* will be more inclined to PC, that is, it will make a less angle with it.

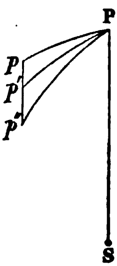


Fig. 821.

If, on the contrary, R be negative, it will tend to diminish the intensity of the central force, and therefore to lessen the obliquity, or increase the angle *n*PC under the tangent and the radius vector.

3145. *It raises or depresses the empty focus of the elliptic orbit above or below the axis, according to the position of the disturbed body in its elliptic orbit.*—Let S, *fig.* 822., be the place of the central body, S′ the empty focus of the instantaneous ellipse, and P, P′, P″, P‴ the disturbed body at different parts of the ellipse. It may be assumed that, in an ellipse of small excentricity, the radial component will produce no sensible effect on the orbital velocity of P, and therefore none (as will more fully appear hereafter) on the magnitude of the major axis of the ellipse.

Now, let us suppose that P, moving in the direction of the arrow, receives the action of a positive radial component. This would deflect the direction of P's motion, and therefore of the

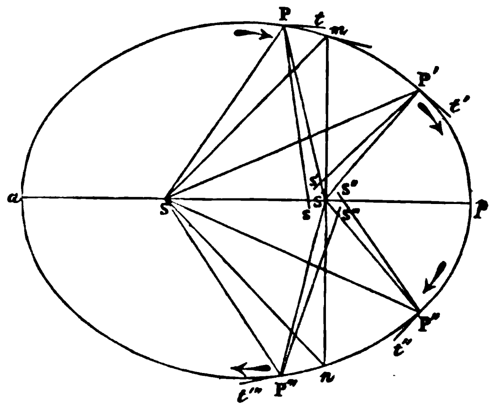


Fig. 822.

tangent Pt, towards the radius vector PS. But since, according to the geometrical properties of the ellipse, the angle tPS′ is half the supplement of SPS′, it follows that the decrease of tPS will cause a decrease of twice the magnitude in the angle SPS′; and, since the direction of SP is fixed, that of PS′ must be deflected towards PS, so that PS′ will change its position to Ps. But, since the major axis of the ellipse is not affected by the disturbing force, and since, by the properties of the ellipse, PS+PS′ is equal to the major axis, it follows that the disturbing force will not change the length of PS′. To comprehend, therefore, the effect of the disturbance, we have only to imagine the line PS′ to turn on P, as a centre towards PS, and to take the position Ps. In effect, the empty focus will be transferred, by the radial component, from S′ to s.

Now, as the disturbed body approaches nearer to mn the line through S′ at right angles to the major axis, the line PS′, drawn from it to the empty focus, approaches more and more to the direction of the perpendicular mS′; and the point to which the empty focus would be transferred by the disturbing force, is less and less removed from the axis; and when the disturbed body is at m, that focus is displaced by the disturbance to another point upon the axis nearer to perihelion p than S′.

After passing m, the disturbed body, at P′, for example, being acted upon as before by a positive radial component, the effect will be to deflect PS′ towards Ps′; but in this case, the angle PS′S

being obtuse, the new position s' of the empty focus will lie *above* the axis.

If we take the disturbed body at P'', the empty focus will be transferred to s'', a point still above the axis. As the disturbed body approaches n, the point to which the empty focus is transferred comes nearer and nearer to the axis, and lies upon it when the disturbed body is at n.

Thus, it appears, that while the disturbed body moves from m, through aphelion a to n, the point to which the empty focus would be transferred by a positive radial component would move from the axis to a certain distance above it, and would again return to the axis when the disturbed body would arrive at n.

After passing n, let us suppose the disturbed body at any point P'''. For the same reasons the line $P'''S'$ will be deflected from $P'''S$, and the empty focus will be transferred to s''', a point below the axis.

Thus it appears, in general, that while the disturbed body moves from m through a to n, the empty focus is transferred to points more or less *above* the axis, and while it moves from n, through p to m, it is transferred to points more or less *below* it.

3146. *Effects of a positive radial component on the apsides.* —It is evident, therefore, that a positive radial component will change the direction of the apsides, or of the major axis of the instantaneous ellipse. The new direction given at each point to the major axis being that of the line drawn from S, through the new position of the empty focus, it will be evident, from what has just been explained, that while the disturbed body moves from m through a to n, the new direction of the axis Ss', Ss'', &c., will be such, that the new perihelion will lie *below* p, and the new aphelion *above* a. These points would, therefore, be removed from their original places, in a direction contrary to the motion of P. The motion imparted to them would then be *regressive.*

While the disturbed body moves from n through p to m, the new direction of the axis Ss, Ss''', &c., must be such that the new perihelion will lie *above* p, and the new aphelion *below* a. These points would, therefore, be removed from their original places, in the direction of the motion of P. The motion imparted to them would then be *progressive.*

Thus it appears, that, with a positive radial component, the axis of the orbit, or the line of apsides, has a progressive motion while the revolving body passes over the arc *npm*, and a regressive motion while it passes over the arc *man*.

3147. *Effects of a negative radial component on the position of the axis.*—If the radial component be negative, it is evident that the effects will be precisely the opposite of those here stated; that is to say, the motion of the axis will be regressive while P moves through the arc *n pm*, and progressive while it moves through the arc *man*.

3148. *Diagram indicating these effects.*—In *fig.* 823. we

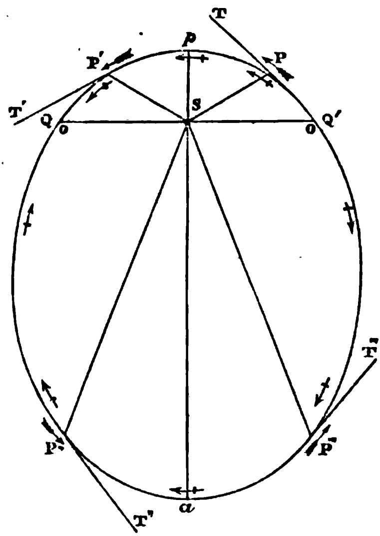


Fig. 823.

have indicated by the feathered arrows the direction of P's motion, and by the arrows with a cross upon their shafts the direction of the motion of the apsides when the radial component is positive. Its motion when negative will be indicated by supposing it to take place in directions contrary to those in which the latter arrows point.

3149. *This motion of the apsides bears a very minute proportion to that of the disturbed body.*—It must not, however, be supposed that these alternate progressive and regressive motions

of the apsides bear any considerable proportion to P's motion: they are, on the contrary, incomparably smaller; so that although, while P moves from Q′ to p, p is moving in the same direction, it is moving with so small a velocity that P arrives at p and passes it almost as soon as if p were not moving at all: and the same observation will apply to the regressive motion.

3150. *Diagram illustrating the motion of the apsides.*—The successive positions assumed by the orbit subject to the disturbing action of a positive radial component, while P passes from p to p''', are represented in *fig.* 824., where p, p', p'', p''' represent the successive positions of the point of perihelion, and a, a', a'', a''' the successive positions of the point of aphelion. If the orbit be imagined to revolve in the other direction from $p''' s a'''$ to $p s a$, the motion will be that which it would receive from the disturbing action of the positive radial component while P moves through the arc Q a Q, *fig.* 824.

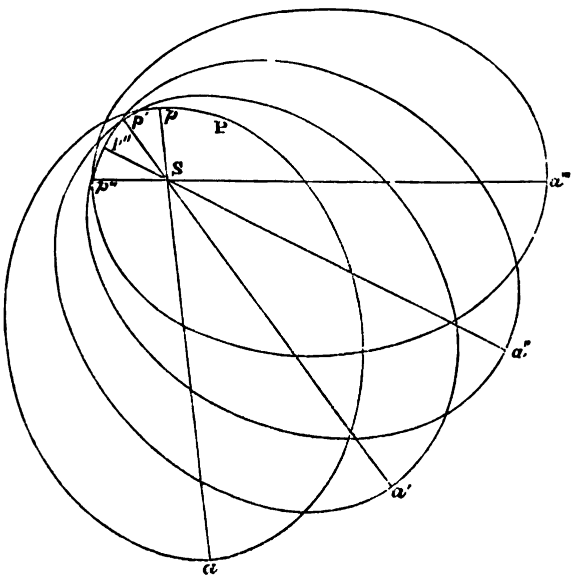


Fig. 824.

3151. *This motion of the apsides increases as the excentricity*

of the orbit diminishes, other things being the same.—It is not difficult to perceive that this effect of the radial component in imparting a motion of revolution progressive or regressive to the line of apsides is greater, *ceteris paribus*, when the orbit of the disturbed body is less excentric.

This will be evident by the mere inspection of *figs.* 825. and 826. In *fig.* 825., the orbit of the disturbed body, of which *p n* is a part, has but small excentricity; and the action of the disturbing force at *p* deflects the body into the arc represented by the dotted line. To find the new position of perihelion, it is only necessary to take C as a centre, *p* C as a radius, and to describe a circle. The middle point *p'* of the dotted arc of the new orbit included within this circle, will be the position of the new perihelion. If we take the case of a much more excentric orbit, represented in *fig.* 826., it will be evident that the arc of the orbit included within the circle for the same degree

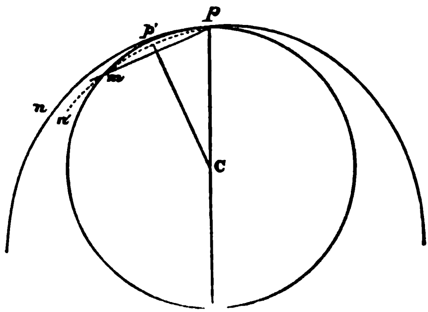


Fig. 825.

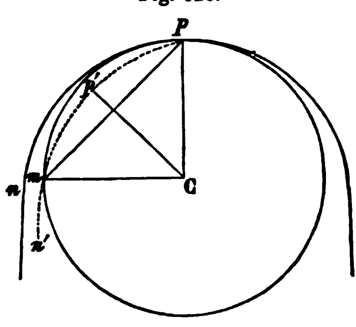


Fig. 826.

of deflexion, will be less than in *fig.* 825.; and consequently its middle point, which is the new perihelion, will be proportionally nearer to *p*.

3152. *Effect of the radial component on the excentricity.*—By what has been explained (3145.), as to the change of position of the empty focus, the effect of the radial component in all cases on the excentricity will be easily understood. The magnitude of the major axis not being affected, the variation of the excentricity will be proportional to that of the distance of the empty focus from S. But, since the distances of P from the two foci are not affected by the disturbing force, the distance between the foci will be increased or diminished, according as the angle SPS′, *fig.* 822., is increased or diminished by the disturbing force. But, from what has been explained it is clear that, while P moves from perihelion *p* to aphelion *a*, the angle SPS′ is diminished; and while it moves from aphelion *a* to perihelion *p*, that angle is increased by the disturbing force.

It follows, therefore, that a positive radial component will diminish the excentricity while P moves from perihelion to aphelion, and will increase it while it moves from aphelion to perihelion; and it is evident that a negative radial component will produce the contrary effect.

The effect produced on the excentricity may also be understood by the following considerations:—While the revolving body P, *fig.* 823., passes from perihelion to aphelion, its distance PS from the central body S continually increases; and while it passes from aphelion to perihelion, on the other hand, its distance P′S from the central body continually diminishes. Now, it is evident that the more excentric the elliptic orbit is, the more rapid will be the increase of the radius vector in the one case, and its decrease in the other. Any influence, therefore, which will have a tendency to diminish that rate of increase or decrease of the radius vector will have the effect of diminishing the excentricity of the orbit; and, on the other hand, any influence which would have the contrary effect of augmenting this rate of increase or decrease, would have the effect of augmenting the excentricity.

Now, it is evident that a positive radial component, having necessarily the effect of increasing the energy of the central attraction, will necessarily diminish the rate at which P departs from C in going from perihelion to aphelion, and will conse-

quently, according to what has been just explained, have the effect of diminishing the excentricity of the orbit; but the same positive radial component, continuing to act while P passes from perihelion to aphelion, will augment the rate at which the radius vector decreases, and will therefore increase the excentricity.

II.—EFFECTS OF THE TRANSVERSAL COMPONENT OF THE DISTURBING FORCE.

3153. *It accelerates or retards the orbital motion.*—In orbits of very small excentricity, such as alone are here considered, the radius vector is never inclined to the tangent drawn in the direction of P's motion, at an angle much less or much greater than 90°. From aphelion to perihelion this angle is always a little less than 90°, being least at the extremity of the minor axis; and from perihelion to aphelion it is always a little greater than 90°, being greatest at the extremity of the minor axis.

Now, the transversal component being always at right angles to the radius rector, it follows that, when it is positive, it forms a very acute, and when negative a very obtuse, angle with the direction of P's motion. In the former case, therefore, very nearly its whole effect is expended in augmenting, and in the latter in diminishing, P's orbital velocity.

In the instantaneous ellipse, the orbital velocity of P constantly increases from aphelion to perihelion, and constantly decreases from perihelion to aphelion. Between the extreme limits of the velocity, there is therefore a certain velocity which corresponds to each particular distance of P from either apse.

3154. *It increases or decreases the major axis, according as it is positive or negative.*—It is demonstrated upon physical principles, combined with the geometrical properties of the ellipse, that if V express the orbital velocity of P, z the radius vector corresponding to any point in the orbit, and a the semiaxis major of the ellipse, we shall have

$$\frac{1}{a}=\frac{2}{z}-V^2.$$

Now, if the transversal component be positive, it will increase V, and will consequently diminish $\frac{1}{a}$, and therefore increase a;

and, on the contrary, if it be negative, it will for like reasons diminish *a*. It follows, therefore, that a positive transversal component will augment, and a negative diminish, the mean distance of P in the instantaneous ellipse.

It follows, by combining this result with the harmonic law, that a positive transversal component will diminish the mean angular motion of P, and that a negative transversal component will increase it,—an effect quite the opposite of what might be at first view expected.

3155. *It produces progression of the apsides from perihelion to aphelion, and regression from aphelion to perihelion, when positive; and the contrary when negative.*—The principle here announced, which is of considerable importance, may be demonstrated and illustrated in several ways.

1. It has just been shown, that a positive transversal component augments the orbital velocity, and that in the instantaneous ellipse the orbital velocity proper to each point continually increases from aphelion to perihelion, and continually decreases from perihelion to aphelion. By augmenting the velocity, therefore, the positive transversal component imparts to P in all cases a velocity proper to a point nearer to perihelion, and therefore in effect brings perihelion nearer to P than it would have been if the disturbing force did not act. It follows, therefore, that when P is moving from aphelion to perihelion, the point of perihelion, being made to approach it, will have a motion contrary to that of P, and therefore regressive; and when P is moving from perihelion to aphelion, the point of perihelion, still approaching P, must follow it, and therefore have a motion in the same direction, or progressive.

It is evident that a negative transversal component must produce the contrary effects, and would therefore impart a progressive motion to the apsides when P moves from aphelion to perihelion, and a regressive motion when it moves from perihelion to aphelion.

It will be apparent, that at the very points of perihelion and aphelion where the effects of a transversal component on the position of the axis, whether positive or negative, change from progression to regression, or *vice versâ*, the actual effect upon the direction of the line of apsides will be nothing.

2. The same principle may be demonstrated as follows:—

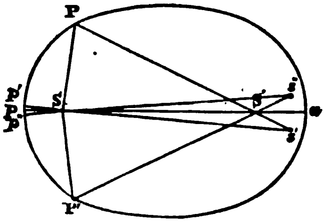


Fig. 827.

By what has been already shown (3154.), a positive transversal component increases the major axis of the instantaneous ellipse. Let p P a P′, *fig.* 827., represent that ellipse, the body being supposed to move in the direction p P′ a P.

Let S be the central body, and S′ the other focus of the ellipse. By the properties of that curve we shall have

$$SP + S'P = 2a.$$

Now, if the velocity of P be augmented by the effect of a positive transversal component, the major axis $2a$, and consequently S′P, the distance of the empty focus of the orbit from P, must also be augmented. The disturbing force will therefore transfer that focus to some such point as s' upon the continuation of the line PS′, and, accordingly the new direction of the major axis will be p'Ss' instead of pSs, and it will therefore revolve round S with a regressive motion.

If the disturbed body be at P′, moving from perihelion to aphelion, the empty focus will in like manner be transferred to s'', and the new direction of the axis being p''Ss'', it will have a progressive motion round S.

It will be evident that a negative transversal component must produce the contrary effects.

3156. *Its effect on the excentricity.*—It is easy to show that a positive transversal component will augment the excentricity when the distance of P from S is less, and will diminish it when greater, than the mean distance, and that a negative transversal component will have the contrary effect.

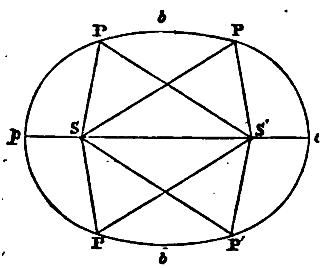


Fig. 828.

It is a property of the ellipse, that for a given position of P with relation to the transverse axis, its distances PS and PS′, *fig.* 828., from the two foci are more less unequal according as the ellipse is more or less excentric.

This property will indeed appear nearly self-evident, if different ellipses having the same major axis be compared.

If then *ap* be the major axis, and *b b* the minor axis, the points *b* being at distances from S equal to half the transverse axis, let a positive transversal component be supposed to act upon P anywhere between *bb* and perihelion. Such a force, according to what has been proved above, will increase the distance of S′ from P; and since this distance is greater than PS, such a change will render the distances PS and PS′ more unequal, and will therefore increase the excentricity.

But if a positive transversal component act anywhere between *bb* and aphelion, as at P′, then by augmenting the distance P′S′, which is less than P′S, it will render the distances P′S and P′S′ less unequal, and will therefore diminish the excentricity.

It is evident that a negative transversal component acting at the same points will produce contrary effects.

It may therefore be inferred, in general, that the excentricity will be increased by a positive transversal component between the mean distance and perihelion, and by a negative one between the mean distance and aphelion; and that it will be diminished by a positive transversal component between mean distance and aphelion, and by a negative one between mean distance and perihelion.

Thus it appears that if a positive transversal component of the disturbing force were to act constantly on a planet during an entire revolution, it would cause the excentricity of the instantaneous ellipse continually to increase from perihelion to the end of the lesser axis, where it would attain its major limit. From that point it would decrease until the planet arrives at aphelion, where it would attain its minor limit. From that it would again increase until the planet would come to the other extremity of the minor axis, where it would again attain its major limit, after which it would again decrease until the planet would arrive at perihelion, where it would attain again its minor limit.

It has been erroneously stated, in some astronomical works, that the points at which the excentricity would attain its major limit by this cause, would be the extremities of a perpendicular to the major axis passing through the empty focus of the orbit.*

* See Herschel's Outlines of Astronomy,—ed. 1849. p. 428.

3157. *Its effects on the major axis at the apsides.*—Let p P Q, *fig.* 829., be the undisturbed orbit, p being perihelion

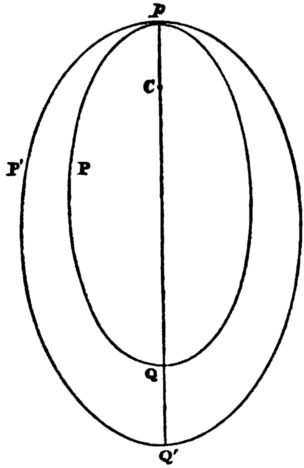


Fig. 829.

and Q aphelion. Now, if we suppose the revolving body at perihelion p to receive the action of the positive transversal component, such action increasing its velocity will throw it into an arc such as p P', which will include p P within it, and the revolving body will consequently move in the orbit p P' Q' instead of the orbit p P Q, in which it would have revolved had it been undisturbed. The orbit, therefore, which would result from the perturbing action of the positive transversal force at P, would be one in which the line of apsides would have the same direction, but in which the point of aphelion Q' would be more remote from C than the point of aphelion Q of the undisturbed orbit. The effect therefore is, that, without changing the direction of the line of apsides, the transverse axis is augmented by the disturbing force.

Let us now consider the effect of a positive transversal component acting upon the revolving body at aphelion. Let Q, *fig.* 830., be the aphelion, and p the perihelion, of the undisturbed orbit of P revolving round the central body C. A positive transversal force acting at Q, accelerating the velocity as before,

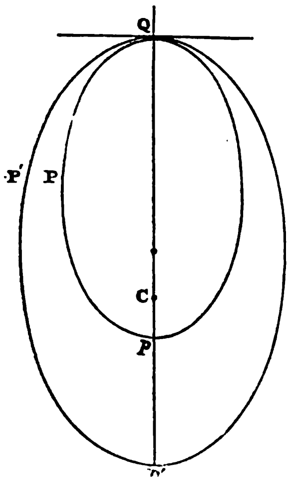


Fig. 830.

will cause the body to move in an orbit Q P′p' having the same point C for its remote focus, and including the undisturbed orbit within it.

It is evident that in this case, while the aphelion distance CQ is the same for both orbits, the perihelion distance Cp' of the disturbed orbit is greater than the perihelion distance Cp of the undisturbed orbit; and, consequently, the transverse axis of the disturbed orbit is greater than that of the undisturbed.

Hence it may be inferred, in general, that a positive transversal force acting upon the revolving body, whether at perihelion or aphelion, will have the effect of augmenting the transverse axis of the orbit, but that when it acts at perihelion it increases, and when it acts at aphelion it diminishes, the excentricity.

It is evident that a negative transversal component will have the contrary effect, diminishing in all cases the transverse axis of the orbit by decreasing the excentricity near perihelion, and increasing it near aphelion.

III.—EFFECTS OF THE ORTHOGONAL COMPONENT OF THE DISTURBING FORCE.

3158. *It changes the plane of the orbit—plane of reference.*—It has been already explained, that the effect of this component is to produce a change in the position of the plane of P's orbit. In order, therefore, to express such change of position, it is necessary that some fixed plane be selected with relation to which the position of the plane of P's orbit may be expressed. Such a plane may be chosen arbitrarily, and we shall denominate it generally the *plane of reference.*

The position of the plane of P's orbit then will depend on

its line of intersection, and its inclination to the plane of reference.

3159. *Nodes of disturbed orbit on plane of reference.*—Let A B A′ and A C A′, *fig.* 831., represent respectively the plane of reference and that of P's orbit, seen in the same manner as we are accustomed to view the equator and ecliptic; A being the ascending node corresponding to the vernal equinoctial point, and A′ the descending node corresponding to the autumnal equinoctial point, and C being that point of the semicircle which corresponds to the solstitial point, and which is, therefore, the point most remote from the plane of reference. We shall call A C the *first quadrant*, counting from the ascending node; C A′ the *second quadrant;* and the corresponding quadrants of the other half of the orbit the *third* and *fourth quadrants*.

3160. *Effect of orthogonal component varies with distance from node.*—Now let us suppose the body subject to perturbation to be at some point such as P in the first quadrant, and to receive there the action of a positive orthogonal component, the direction of P's motion being indicated by the arrow. Since this positive component has a tendency to draw P

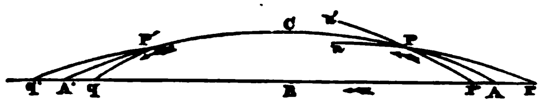


Fig. 831.

towards the plane of reference, it is evident that the motion which P will receive, by its action, must cause it to proceed in a direction between P C and a line drawn from P perpendicular to the plane of reference, that is, in such a direction as P *n* inclined to P C and below it.

If the body subject to perturbation be in the second quadrant, as at P′, it will for a like reason, after the disturbing action, move in a direction P *q*, slightly inclined to P′ A′ and below it. If the orthogonal component be negative, it will be apparent that at P the body would, after the action of the disturbing force, move in the direction P *n*′, slightly inclined to P C and above it; and in like manner, at P′ it would move in the direction P *q*′, slightly inclined to P′ A′ and a little above it.

Now each of these disturbances will change both the nodes and inclination.

3161. *Nodes progress or regress according as the component*

is negative or positive.—If n P be continued backwards, it will intersect the plane of reference at a point r beyond its former node A. It is evident, therefore, that in this case the ascending node would regress upon the plane of reference. If the arc P′ q be in like manner continued to meet the plane of reference at q, this point q will be in like manner behind A′. Thus the new position of the descending node will be behind its former position, and, consequently, in this case also the node regresses.

It appears, therefore, that wherever, in the semi-circle A C A′, a positive orthogonal component acts, its effect will be to impart to the node a regressive motion.

The same reasoning will equally apply to the cases in which P is in the third or fourth quadrant; and therefore generally it may be inferred, that in all parts of P's orbit a positive orthogonal component will produce a regression of the nodes.

By like reasoning it will be perceived, that when P is subject to a negative orthogonal component, and therefore in the first quadrant moves in the arc P n', and in the second in the arc P′ q', the nodes will receive a progressive motion, the new nodes r' and q' being in advance of the original nodes A and A′; and the same reasoning will apply to the third and fourth quadrants. It will follow generally that a negative orthogonal component will everywhere impart a progressive motion to the nodes.

3162. *Effect upon the inclination.*—It will be evident, upon the mere inspection of the diagram, that the obliquity P r B is less, and the obliquity P r' B greater, than the original obliquity P A B, where P is in the first quadrant; and that, on the contrary, the obliquity P′ q B is greater, and P′ q' B is less, than the original obliquity P′ A′ B in the second quadrant. It follows, therefore, that a positive orthogonal component acting in the first quadrant decreases, and in the second increases, the obliquity; and that a negative orthogonal component in the first quadrant increases, and in the second quadrant decreases, the obliquity. It will follow, in like manner, that the effects of these components respectively in the third quadrant are similar to their effects in the first, and that their effects in the fourth quadrant are similar to their effects in the second.

It may then be stated generally, that a positive orthogonal component decreases the obliquity when the disturbed body is

in the first and third quadrants, and increases it when in the second and fourth; and that a negative orthogonal component is attended with the opposite effects.

IV.—GENERAL SUMMARY OF THE EFFECTS OF A DISTURBING FORCE.

3163. *Tabular synopsis of the effects of the several components of the disturbing force.*—

Place of disturbed Body.	Radial.						Transversal.						Orthogonal.			
	+			−			+			−			+		−	
	a	e	ϖ	a	e	ϖ	a	e	ϖ	a	e	ϖ	ϖ	i	ϖ	i
At Perihelion -	.	0	+	.	0	−	+	+	0	−	−	0	.	.	.	.
At Aphelion -	.	0	−	.	0	+	+	−	0	−	+	0	.	.	.	.
From Perihelion to Aphelion -		−	.	.	+	.	+	.	+	−	.	−	.	.	.	.
From Aphelion to Perihelion		+	.	.	−	.	+	.	−	−	.	+	.	.	.	.
From Anomaly 90° to Anom. 270° -		.	−	.	.	+	.	.	.	.	.	.	.	.	.	.
From Anom. 270° to Anom. 90° -		.	+	.	.	−	.	.	.	.	.	.	.	.	.	.
At Anom. 90° or 270° - -		.	0	.	.	0	.	.	.	.	.	.	.	.	.	.
Between Perihelion and mean distance - -		.	.	.	.	.	.	+	.	.	−	.	.	.	.	.
Between Aphelion and mean distance - -		.	.	.	.	.	.	−	.	.	+	.	.	.	.	.
Distance of disturbed body from ascending node: 0°													−	0	+	0
Between 0° and 90° - -													−	−	+	+
90° - -													−	0	+	0
Between 90° and 180° -													−	+	+	−
180° - -													−	0	+	0
Between 180° and 270° -													−	−	+	+
270° - -													−	0	+	0
Between 270° and 0° -													−	+	+	−

It will be convenient, then, to collect and arrange in juxtaposition, for the purpose of reference, the various effects produced by the three components of the disturbing force, as explained in the preceding paragraphs. We have accordingly done this in the above tabular statement. The sense in which the symbols + and — are applied to each of the components has been already explained. As applied to the semi-

axis a, the excentricity e, and the inclination I, $+$ is used here to signify increase, and $-$ diminution, and 0 the positions in which they are not affected by the disturbing force. As applied to the motions of the perihelion (π) and the node (ν), $+$ indicates progressive and $-$ regressive motion. The sign 0 is placed where there is no motion imparted to these points by the disturbing force.

CHAP. XX.

PROBLEM OF THREE BODIES.

3164. *Attraction independent of the mass of the attracted body.* — To obtain clear and distinct ideas of the effects of the disturbing action of masses brought into proximity with bodies moving in orbits round a centre of attraction, it is most necessary, in the first instance, to comprehend clearly the law and conditions which determine such attractions.

The attraction exerted by one body upon another depends solely upon the mass of the *attracting* body, and its distance from the attracted body, and is *altogether independent of the mass of the latter.* It is the more necessary to indicate this, inasmuch as students are apt to confound the effects of attraction with those of forces transmitted by impact, pressure, or other mechanical agency. There is, however, an essential difference between these effects and those of attraction. When a force is transmitted by mechanical agency, the motion which it imparts to the body upon which it acts is so much the less as the mass of that body is greater; but this is not at all the case with attraction. If a body whose mass is M acts upon another P at the distance D, it will impart to P, being free, a certain motion towards M. Now, if another body equal in mass to P be placed in juxtaposition with P, M will impart to it an equal motion in the same direction; and if three, four, or more such bodies be in like manner placed at the same distance D near each other, they will be all equally attracted by M, and will move towards M through equal spaces, the attraction which M excites on any one of them not at all interfering with or diminishing its action upon

the others. If we suppose these several masses to cohere and to form a single mass, this single mass will move towards M with the same velocity. It follows, therefore, that inasmuch as M exercises separate and independent attractions upon every particle composing the mass P, that mass will be moved towards M by the attraction of M at the same rate, *whether it be great or small.*

Thus, for example, it appears (Table IV. page 497.) that the attraction which the sun exerts upon the earth at the distance of ninety-five millions of miles is such as would cause the earth, if it were at rest and free, to move towards the sun through $\frac{109}{1000}$ths of an inch in one second of time. Now, if the mass of the earth were 10 or 100 times greater or less than it is, the sun's attraction exerted at the same distance would produce precisely the same effect upon it..

3165. *Hence the denomination "accelerating force."* — This species of force, the effect of which is exclusively a certain velocity imparted to the body affected, is distinguished in physics from that other sort of force which varies in its effects with the mass of the body moved, by the term *accelerating force;* while the force which varies in its effect with the mass of the body moved, is called *moving force,* or *momentum.*

3166. *Accelerating force of gravitation.*—The accelerating force of gravitation is therefore measured by and proportional to the space through which a body is drawn in the unit of time by the attraction of a given mass at a given distance.

Let the unit of accelerating force be the space through which a body would be drawn in the unit of time by the unit of mass placed at the unit of distance from it. The accelerating force exerted at the unit of distance by any other mass M, will therefore be expressed by the number of units in M; and since the attraction decreases in the same proportion as the square of the distance increases, the accelerating force exerted by M at the distance D being expressed by A, we shall have

$$A = \frac{M}{D^2},$$

—a formula which, properly understood, expresses the law of gravitation in its highest generality.

3167. *Problem of two bodies.*—When a lesser body P revolves in an elliptic orbit round a greater mass S, the place of S being the focus of the ellipse, it has been hitherto assumed that S is

fixed, and not at all affected by P's attraction, the only attraction assumed hypothetically to be in operation being that of S upon P. Now if D be the distance between them, and A the space through which P would be drawn by S in the unit of time, we should have, according to what has been just explained,

$$A = \frac{S}{D^2}.$$

But, by the universal reciprocity of the principle of gravitation, P acts upon S as well as S upon P; and if the accelerating force of P on S be expressed by A′, we shall have

$$A' = \frac{P}{D^2}.$$

It remains, therefore, to consider in what manner this effect of P's attraction will modify the conclusions at which we have arrived upon the supposition that the attraction alone of S was in operation. It is evident that by P's attraction S is drawn towards P, while by S's attraction P is drawn towards S.

If S alone were supposed to act, the distance D would in the unit of time be decreased, supposing the bodies free to move towards each other, by the space A. But now that the action of P is admitted, the distance D between P and S will be diminished by the sum of the spaces through which S and P may be respectively moved in the unit of time, each by the attraction of the other; so that we should have the actual force of mutual attraction between the two bodies —

$$A + A' = \frac{S + P}{D^2}.$$

It appears, therefore, that the mutual attraction of the two bodies is greater than when S alone acted in the proportion of S to S + P; and, consequently, that the relative motion of P round S will be precisely the same as if the mass of S were increased by the addition to it of the mass of P and P were deprived of its reciprocal attraction. Since, then, this increased attraction, like the original attraction of S, is one which increases as the square of the distance decreases, it is subject to the same law as that which determines the elliptic form of the orbits; and it consequently follows, that, subject to this reciprocal attraction of the two bodies, P will still describe an elliptic orbit round S as a focus; but the central attraction being augmented in the ratio of S + P to S, the ratio of the cube of the mean distance

to the square of the periodic time, according to what has been established (2634.), will undergo a corresponding increase; so that if a were the mean distance, and p the period of P moving round S, subject to S's attraction only, and exerting no attraction of its own, and a' were the mean distance and p' the period when P exerts the reciprocal attraction upon S, we shall have

$$S : S + P :: \frac{a^3}{p^2} : \frac{a'^3}{p'^2}$$

Thus it appears that the admission of P's reciprocal attraction on S would augment the distance with the same period, or diminish the period with the same distance; in short, at the same distance the mean motion of the revolving body would be accelerated, or with the same mean motion its distance would be increased.

To this extent, and in this sense only, a perturbation would be produced by the reciprocal attraction of P upon S.

3168. *Problem of three bodies.*—It is, therefore, only when a third body intervenes that a perturbing force is produced which has any tendency to impair the elliptic character of the orbit; and the investigation of the effects of such a combination has acquired great celebrity in the history of science, from the difficulties which it presented, and the importance of the results which followed its solution. This question is generally denominated the PROBLEM OF THREE BODIES.

The three bodies involved in the problem are, 1st, the central body S; 2ndly, the revolving body P, which if undisturbed would describe an elliptic orbit round S as a focus; 3rdly, the disturbing body M.

3169. *Simplified by the comparative feebleness of the forces exerted by the third body.*—In all the cases presented in the great phenomena of the universe, the mass of S is incomparably greater than that of P, and the attraction exerted by M is incomparably more feeble than the central attraction of S on P, either because of the comparative smallness of M's mass, or because of its great distance from the attracted body, or from both these causes combined.

3170. *Attracting force of third body not wholly disturbing.*—But whatever be the force exerted by M, it is most necessary to bear in mind two things respecting it: 1st, that its attraction does not *necessarily* produce a disturbing action at all upon P's orbit; and 2ndly, that when it does, the disturbing action is not

identical with M's attraction upon P, either in intensity or direction. It is never equal to it in intensity, and very rarely identical with it in direction, often having a direction immediately opposed to that of M's attraction.

3171. *Attracting force which would produce no disturbing effect.*—Such a force must be one which would cause S and P to be moved in parallel lines in the same direction through equal spaces in the same time. It is quite evident that such a force, while it would transport S and P with this common motion in a common direction, could not in the least degree derange their relative position or motion. If P, previously to the action of such a force, revolved round S, for example, in a circle with a uniform velocity, it would continue to revolve round it in the same circle with the same velocity when subject to the force here supposed; for the effect would be merely that the two bodies with their circular orbit would be transported in space as bodies would be which might be supposed to have any motion upon the deck of a ship in full sail.

3172. *This would be the case if the third body were enormously distant compared with the second.*—But in order to impart such a force to S and P, the body M must be at such a distance from them that the distance between S and P should subtend at M a visual angle so small as to be insensible; since otherwise the lines of M's attraction, being sensibly convergent, would not be parallel. If the distance, however, of M be enormously great compared with the distance between S and P, then the direction of M's attraction on S and P will be sensibly parallel; and for the same reason, the variation of the intensity of M's attraction will be likewise inconsiderable, since it varies inversely as the square of the distance, and since, by the supposition, the distance of P from S is quite insignificant compared with the distance of M from either of them.

3173. *Example of a force supposed to act on the solar system.*—Thus, if we suppose that the solar system is subject to the attraction of some mass or collection of masses among the fixed stars at a distance so prodigious that, compared with it, the whole dimensions of the system would shrink into a point, such an attraction would have the character here described, and the whole solar system would be moved by it with a common motion in a given direction, all the bodies composing it describing parallel lines with the same velocity, and consequently

preserving, so far as they are affected by this common motion, their relative positions.

3174. *Case in which the distance of the third body is not comparatively great.*—Let us now, however, suppose that the third body, M, is placed at such a distance from P and S that it acts upon them, not only in different directions, but with forces of different intensities, and consequently imparts to them motions which, being neither equal nor parallel, must necessarily derange their relative positions, and which consequently give rise to a disturbing force.

The question then remains to be solved, to determine the intensity and direction of this disturbing force, being given the intensities and directions of the attractions exerted by the mass M upon P and S

Let the distance P S $=r$, M S $=r'$, and M P $=z$. If M S and S P, *fig.* 832., express respectively the masses of the three bodies, we shall have

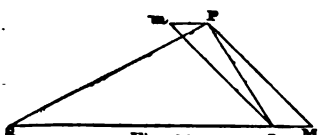


Fig. 832.

M's attraction upon S $=\frac{M}{r'^2}$,

M's attraction upon P $=\frac{M}{z^2}$,

S's attraction upon P $=\frac{S}{r^2}$.

Now let us imagine two forces each equal to M's attraction on S, that is, to $\frac{M}{r'^2}$, to act upon P in opposite directions; one in the direction P m parallel to M S, and the other in the direction immediately opposed to this. These two forces being equal, and immediately opposed, will of course produce no effect whatever upon P, and therefore their introduction is allowable without involving any change in the mechanical conditions of the system. But the force which now acts upon P, parallel to S M, and opposite to Pm, would impel P through a space parallel to S M in the unit of time, exactly equal to that through which M's attraction upon S impels S. These two forces, therefore, would carry P and S in parallel directions through equal spaces in the unit of time, and would therefore produce no disturbing effect (3170.). All the disturbance, therefore, which can be produced upon P by the forces in operation must arise from the combined action of the two forces acting upon P: 1st M's attraction in the direction P M; and 2ndly, a force equal and

contrary to M's attraction on S, acting in the direction P m parallel to M S. If, therefore, we take the line P m in the same proportion to P M as the attraction of M upon S has to the attraction of M upon P, and complete the parallelogram PmxM, the diagonal P x will express, in intensity and direction, the resultant of the forces expressed by the sides P m and P M; in other words, the diagonal P x will express in quantity and direction the resultant of the only two forces which can produce a disturbing effect upon P. Since M's attraction on S, represented by M x, is to M's attraction on P, represented by M P, as the square of M P is to the square of M S, it follows that

$$SM^2 : PM^2 :: PM : Mx;$$

or, if we use the symbols already indicated, we shall have

$$r'^2 : z^2 :: z : Mx;$$

and consequently

$$Mx = \frac{z^3}{r'^2} \ldots\ldots (1.)$$

We shall obtain an expression for S x, the distance of the point x from the central body S, by subtracting M x from S M; consequently,

$$Sx = r' - \frac{z^3}{r'^2} = \frac{r'^3 - z^3}{r'^2}.$$

But since

$$r'^3 - z^3 = (r' - z) \times (r'^2 + r'z + z^2),$$

we shall have

$$Sx = (r' - z) \times \left(1 + \frac{z}{r'} + \frac{z}{r'^2}\right) \ldots\ldots (2.)$$

By either of these formulæ (1) or (2), the direction of the disturbing force can always be determined. It is only necessary to take, upon the line joining the disturbing and central bodies, a space M x or S x, determined by one or the other formulæ (1) or (2).

3175. *To determine the ratio of the disturbing to the central force.*— For this purpose let

A = M's attraction on P.
A′ = M's attraction on S.
A″ = S's attraction on M.
D = the disturbing force Px.
F = S's attraction on P.

We shall then, according to what has been explained above, have—

$$\frac{\text{D}}{\text{A}'}=\frac{\text{P}x}{\text{M}x}=\text{P}x\times\frac{r'^2}{z^3},\qquad \frac{\text{A}'}{\text{A}''}=\frac{\text{M}}{\text{S}},\qquad \frac{\text{A}''}{\text{F}}=\frac{r^2}{r'^2}.$$

By multiplying all these together we shall obtain

$$\frac{\text{D}}{\text{F}}=\frac{\text{M}}{\text{S}}\times\frac{r^2}{z^3}\times\text{P}x. \;\ldots\ldots\; (3.)$$

By this formula the ratio of the disturbing force D to the central attraction F can always be calculated when the masses of the disturbing and central bodies, and their distances from each other and the disturbed body, are known; for in such cases the line Px can be calculated by the common principles of trigonometry; and consequently $\frac{\text{D}}{\text{F}}$, or the ratio of the disturbing to the central attraction, will be found.

3176. *How the direction of the disturbing force varies with the relative distances of the disturbed and the disturbing from the central body.*—By the formula (1), it is evident that Mx will be less or greater than r' according as z is less or greater than r'; and that if z be equal to r', Mx will be equal to MS. It appears, therefore, that when the distance of M from P is less than its distance from S, the direction of the disturbing force will lie between M and S; that when it is greater than P's distance from S, the central body S will lie between the direction of the disturbing force and M; and that when M is equally distant from S and P, the disturbing force will be directed to S.

This appears also from the consideration of the formula (2): for if $z=r'$, Sx=0, which indicates that the disturbing force is in that case directed to S; and according as r' is greater or less than z, $r'-z$, and therefore Sx is positive or negative; showing that in the former case it is to be taken from S towards M, and in the latter case in the opposite direction.

To trace the varying direction of the disturbing force during the synodic revolution of P round S, we must consider successively the cases in which the disturbing body or its projection on the plane of P's orbit lies outside and inside that orbit; the former case representing the disturbing action of a superior upon an inferior, and the latter that of an inferior upon a superior planet. We shall therefore consider, in the first instance, the effects only of that component of the disturbing

force which is in the plane of the disturbed orbit, and afterwards that of the orthogonal component.

3177. First Case. *The component of the disturbing force in the plane of the orbit when the disturbing body is outside the orbit of the disturbed body.*—Let S, *fig.* 833., be the central,

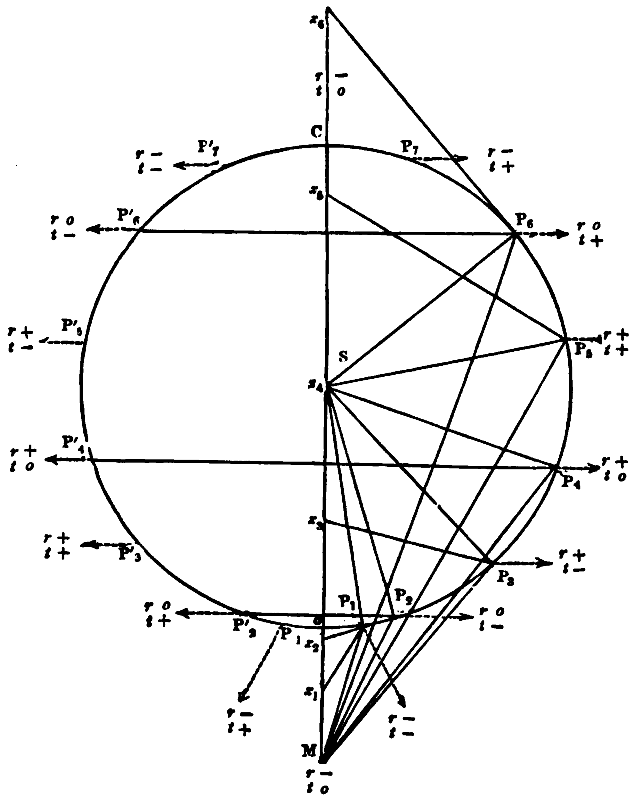


Fig. 833.

and M the disturbing body; and let P_1, P_2, P_3, &c. be the disturbed body in a succession of its synodic positions.

It is evident from the formulæ (1) and (2), that as P moves from o, taking successively the synodic positions P_1, P_2, P_3, &c.,

x, and therefore Mx, constantly increases; and consequently the point where the direction of the disturbing force meets the line MS constantly approaches S. The angle S$P_1 x_1$, obtuse at first, becomes less and less so at P_1 and P_2, until, at a certain point P_2, it is reduced to 90°. There the direction P_2 x_2 of the disturbing force is therefore that of the tangent to P's orbit; and its direction being opposed to P's motion, it is negative.

After passing this point P_2, the angle formed by the disturbing force with the direction PM becomes less than 90°. When the distance PM becomes equal to SM, as at P_4, the direction of the disturbing force, according to what has been shown (3176.), passes through S, and accordingly x_4 coincides with S.

After P passes this position, the direction of the disturbing force passes above S as at P_5 x_5, the angle MPx increasing. At a certain point P_6 this angle again becomes 90°; and the direction P_6 x_6 of the disturbing force being at right angles to SP, becomes tangential. The point x continues to recede from S until P arrives at C.

While P passes from C to o through the other half of its synodic revolution, the direction of the disturbing force undergoes like changes, but in a contrary order, being tangential at P_6' and P_2', and radial at P_4',—points which severally correspond to P_6, P_2, and P_4 in the other half-synodic revolution.

3178. *To determine the sign of each of the components of the disturbing force, in each successive point of the orbit.*— Commencing at o, M's attraction on P will be $\frac{M}{(r'-r)^2}$, and M's attraction on S will be $\frac{M}{r'^2}$. The former being greater than the latter, the disturbing force D, exerted on P, will be

$$D=\frac{M}{(r'-r)^2}-\frac{M}{r'^2}=M\times\left(\frac{1}{(r'-r)^2}-\frac{1}{r'^2}\right);$$

and this force being directed from P to M will be opposed to the central attraction, and will therefore be negative.

While P passes over the arc oP_2 this negative radial component gradually diminishes, and the tangential component, which at o is nothing, gradually increases, being also negative. When P arrives at P_2, the negative radial component vanishes, and the whole disturbing action consists of a negative tangential force.

After passing P_2 the radial component changes its sign and

becomes positive, the tangential component continuing to be negative and to diminish. From P_2 to P_4, as for example at P_3, the negative tangential component gradually diminishes, and vanishes altogether at P_4, while the positive radial component increases, and at P_4 constitutes the whole disturbing force, being then directed to S.

After passing P_4, as for example at P_5, the tangential component changes its sign, and becomes positive, so that both components of the disturbing force are positive, the radial component now decreasing.

When P arrives at P_6, the radial component vanishes, the whole disturbing force becoming tangential.

After passing P_6 the radial component again changes its sign, and becomes negative, as at P_7, and increases continually to C, the positive tangential component at the same time continually decreasing, and becoming $= 0$ at C. At this point C, therefore, the disturbing force is wholly radial, and is negative, being therefore in antagonism not only with the central attraction of S, but with the immediate attraction of M, which is its cause.

It may be naturally asked, how it can happen that the attracting force of M produces a disturbing force in immediate opposition to its own direction? This, however, is easily explained. M's attraction on S being $\frac{M}{r'^2}$, and M's attraction on P at C being $\frac{M}{(r'+r)^2}$, the former being greater than the latter, we shall have the disturbing force

$$D = \frac{M}{(r+r')^2} - \frac{M}{r'^2},$$

which is negative: in fact, S is attracted towards M with a greater accelerating force than P, and, consequently, S and P are caused to separate from each other by a space equal to the difference of the two attractions, which is equivalent to a repulsion acting from S upon P, or, what is the same, a negative disturbing force.

The tangential component vanishing at C changes its sign; the radial component, however, continuing negative. From C to P_6', for example at P_7', therefore, both components are negative, and the radial component gradually diminishes until

P arrives at P'_6, when it vanishes, the whole disturbing action being a negative tangential force.

After passing P_6', as for example at P_5', the radial component changing its sign becomes positive, the tangential component continuing negative and decreasing gradually until it vanishes when P arrives at P'_4, at which point the whole disturbing action is a positive radial force.

After passing P_4', as for example at P_3', the tangential component again changes its sign and becomes positive; both components are therefore positive to P_2', where the radial component becomes nothing, and the whole disturbing action consists of a positive tangential force.

After passing P_2', as for example at P_1', the radial component again changes its sign and becomes negative; the tangential component continuing however positive, until P arriving at *o* completes its revolution.

3179. *Diagram illustrating these changes of directions.*— In the diagram we have indicated in an obvious way these successive changes of the directions of the components of the disturbing force, the radial component being expressed by *r*, and the tangential by *t*, and their quality or value being expressed by the symbols +, −, or o, which follow them. The diagram, though appearing at first view complicated, is really simple and easily understood; and the indications given will be found by the student to be extremely convenient and useful.

3180. SECOND CASE, *in which the disturbing body is within the disturbed orbit.*—Let us now suppose that M is within P's orbit, as in *fig.* 834., and that its distance from the central body S is greater than half the radius of P's orbit. Taking, as before, the disturbed body P in a succession of positions P_1, P_2, P_3, P_4, &c., the direction of the disturbing forces $P_1\,x_1$, $P_2\,x_2$, $P_3\,x_3$, &c. &c., is found in the same manner exactly as before.

Now it will be easy to perceive, by following the lines upon the diagram, how the direction of the disturbing force varies with relation to M and S. At a point such as P_1 it falls at x_1, between M and S; at P_2, where $M\,P_2 = M\,S$, it is directed to S, as has been already proved. At this point, therefore, the disturbing force is wholly radial. When P has advanced to the position P_4, at which the direction of the disturbing force is at right

angles to the radius vector SP_4, it is wholly tangential, the radial force vanishing.

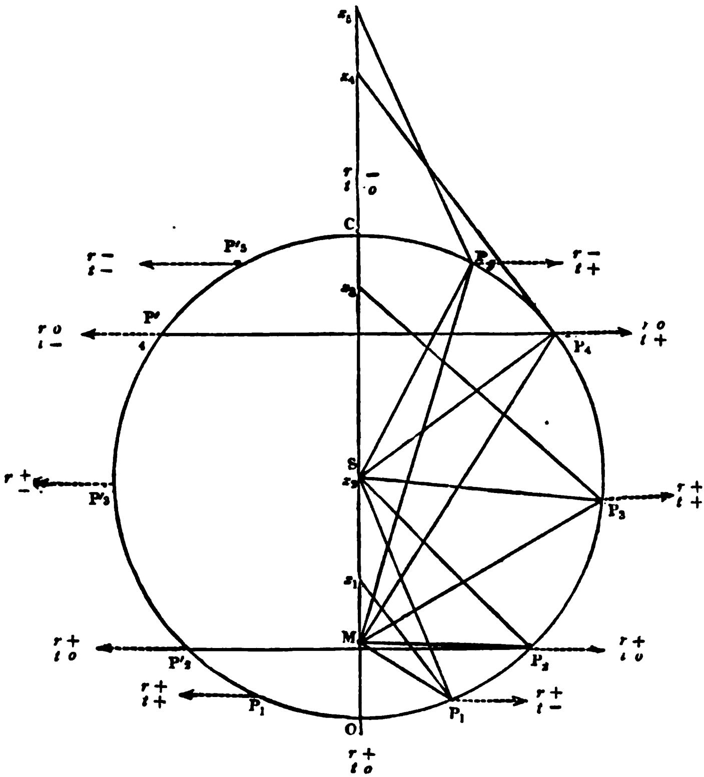


Fig. 834.

At C, where M's attraction is at right angles to the tangent, the tangential force vanishing, the disturbing force is wholly radial. In the semicircle described by P in passing from C to O, there are two points, P'_4 and P'_2, corresponding to the points P_4 and P_2, at the former of which the radial, and at the latter the tangential, component vanishes.

3181. *Changes of sign of the components in this case.*—It will now be easy to trace the successive changes of direction of

each of the components of the disturbing force to which P is subject in the successive positions which it assumes throughout its synodic period.

At O, M's attraction upon P is $\frac{M}{(r-r')^2}$, and M's attraction on S is $\frac{M}{r'^2}$; and since both these attractions are directed towards M, the relative effect upon P and S will be their sum; and therefore P and S will be attracted towards each other by a disturbing force, the value of which will be,

$$D=\frac{M}{(r-r')^2}+\frac{M}{r'^2}=M\times\left(\frac{1}{(r-r')^2}+\frac{1}{r'^2}\right).$$

In this case it is obvious that the disturbing force will be wholly radial and positive, the tangential force being nothing, inasmuch as the direction of the disturbing force is at right angles to the tangent. When the body leaving O moves towards P_2, the radial force, being still positive, gradually decreases, and the tangential force being inclined below the radius, will act against P's motion, and therefore be negative. As P moves forward, the negative tangential force gradually decreases; and when P arrives at P_2, where its distance from M is equal to that of S from M_2, the disturbing force being wholly radial, the tangential force vanishes. After passing this point, as for example at P_3, the tangential force, changing its sign, becomes positive as well as the radial force, and both forces continue to be positive, the radial force gradually decreasing until the body arrives at P_4, where the radial force vanishes, the disturbing force taking the direction of the tangent, and being still positive.

After passing this point, the radial force, changing its sign, becomes negative as at P_5, the tangential force still being positive, and gradually decreasing. This latter vanishes when P arrives at C, where the whole disturbing force is radial and negative. After passing this point, the negative radial force gradually diminishes, and the tangential component, changing its direction as at P_5', becomes, like the radial component, negative. On arriving at P_4', the radial component having gradually decreased, vanishes, the whole disturbing force being tangential and negative.

Passing this point, the radial component changing its sign becomes positive as at P_3', the tangential component being still negative. On arriving at P_2', the tangential component having gradually decreased, vanishes, and the whole disturbing force is radial and positive. After passing this point, as for example at P_1', the tangential component, as well as the radial component, is positive; and, in fine, when the body returns to O, the tangential component vanishes, and the radial component is positive.

All these changes are indicated on the diagram in the same manner as in the former case.

We have here supposed that the distance of M from S is greater than half the distance OS, and we have accordingly seen that there are only two certain points P_2 and P_2' in each semicircle where, the distance of P and S from M being equal, the tangential component vanishes, and the whole disturbing force becomes radial. If M be supposed gradually to approach the middle point of SO, these points P_2 and P_2' will assume positions gradually nearer and nearer to the point O; and when, in fine, M coincides with the middle point of SO, these two points P_2 and P_2' will coalesce with the point O. In this case there will be no point in either semicircle at which the disturbing force will be wholly radial; but there will still be the points P_4 and P_4', at which it will be wholly tangential, and accordingly the orbit in this case will be divided into only four arcs, separated by the points P_4 and P_4' at which the disturbing force is exclusively tangential, and the points O and C at which it is exclusively radial.

The same observations, *mutatis mutandis*, will be applicable when M lies between the middle point of the radius SO and S.

3182. *General summary of the changes of direction of the radial and transversal components of the disturbing force during a synodic period of the disturbed body.*—It will be convenient, as well for the purposes of reference as to impress them on the memory, to collect here, and arrange in juxtaposition, the several changes of direction which the components of the disturbing force in the plane of the disturbed orbit undergo during a synodic period of P. This is done in the following table, the signs retaining the significations already given to them:—

Disturbing Body outside the Orbit of the disturbed Body.			Disturbing Body within the Orbit of the disturbed Body.					
r' greater than r.			r' less than r, but greater than $\frac{1}{2}r$.			r' not greater than $\frac{1}{2}r$.		
Place of P.	R	T	Place of P.	R	T	Place of P.	R	T
At O	−	0	At O	+	0	At O	+	0
From O to P_2	−	−	From O to P_2	+	−	From O to P_4	+	+
At P_2	0	−	At P_2	+	0	At P_4	0	+
From P_2 to P_4	+	−	From P_2 to P_4	+	+	From P_4 to C	−	+
At P_4	+	0	At P_4	0	+	At C	−	0
From P_4 to P_6	+	+	From P_4 to C	−	+	From C to P_4'	−	−
At P_6	0	+	At C	−	0	At P_4'	0	−
From P_6 to C	−	+	From C to P_4'	−	−	From P_4' to O	+	−
At C	−	0	At P_4'	0	−			
From C to P_6'	−	−	From P_4' to P_2'	+	−			
At P_6'	0	−	At P_2'	+	0			
From P_6' to P_4'	+	−	From P_2' to O	+	+			
At P_4'	+	0						
From P_4' to P_2'	+	+						
At P_2'	0	+						
From P_2' to O	−	+						

3183. *Varying effects of the orthogonal component during a synodic revolution.*—If a triangle be imagined to be formed by lines drawn joining the places of the central body S, the disturbed body P, and the disturbing body M, the plane of this triangle will in general be inclined at some angle to the plane of P's orbit. The direction of the disturbing force, according to what has been shown, will be that of a line drawn from P in the plane of this triangle, meeting the line MS either between M and S, at S, or beyond S, according to the relative magnitude of MS and MP. If MP be less than MS, then the direction of the disturbing force will meet MS between M and S; if MP=MS, its direction will be that of the line PS; and, in fine, if MP be greater than MS, its direction will meet the prolongation of MS beyond S.

On considering these various directions of the disturbing force, it will be evident that when MP is less than MS its direction will lie on the same side of the plane of P's orbit with the disturbing body M, and the orthogonal component being therefore directed towards that side, will have a tendency to bring the plane of the disturbed orbit nearer to M.

When MP is greater than MS, on the contrary, the direction of the disturbing force meeting the prolongation of the line MS beyond S, must lie on the side of the plane of P's orbit, different from that at which M is placed; and the orthogonal component being in that case directed to the same side,

would have a tendency to deflect the plane of P's orbit *from* M.

In the former case, the orthogonal component would decrease, and in the latter it would increase the angle at which the line M S is inclined to the plane of P's orbit.

When M P = M S, the disturbing force, being directed from P to S, would act in the plane of P's orbit, and would consequently have no effect in changing that plane.

If M be situate anywhere in the plane of P's orbit, which will be the case whenever it passes through the nodes of its own and P's orbit, the three bodies being in the same plane, the orthogonal component of the disturbing force will be nothing.

Thus it appears that the orthogonal component will vanish at each passage of M through P's nodes; and also at each of the points, if any such there be, at which M and P are equally distant from S.

3184. *Periodic and secular perturbations.*—From what has been explained in the present and the preceding chapter, it appears that the several components of the disturbing force produce contrary effects on the elements of the disturbed orbit when they have contrary signs, and that they are severally subject to a succession of changes of sign during the synodic revolution of the disturbed and the disturbing bodies. The several elements of the disturbed orbit will therefore be in a continual state of oscillation, from these alternate actions of the components of the disturbing force in one direction and the other.

If, in all cases, the sum of the effects produced by the action of each component while positive were equal to the sum of its effects while negative, the elements would oscillate round a fixed and invariable state. They would pass through all their variations when the disturbed and disturbing bodies would have assumed all their possible varieties of position with relation to the central body, and they would then recommence the same succession of changes.

The mean values of all the elements of the disturbed orbit would in this case be constant. Thus the major axis would alternately increase and decrease between fixed limits, but its mean value would be always the same. In like manner, the excentricity and inclination would alternately increase and de-

crease, having a constant mean value; and the like would be true of the other elements.

But if, on the contrary, it were found, on comparing the sum of the positive with the sum of the negative effects of any of the components, that the sum of all the effects while positive were not exactly equal to the sum of the effects while negative, but that a certain minute difference or residual phenomenon always remains uneffaced, it is evident that this residuum, however small it may be, must accumulate until at length it will attain a magnitude which will tell in a very sensible manner, and, if a still longer series of periods be waited for, would totally change the elements of the disturbed orbit.

It must be observed that the interval which leaves this residuum uncompensated, is one during which the disturbed and disturbing bodies pass through all possible varieties of configuration, taking into the account not merely their directions with relation to the central body, but their positions with relation to the apsides of their respective orbits. It is only after assuming all the varieties of relative position and distance, arising as well from the relative changes of direction of M and P as viewed from S, as from their changes of position in their respective orbits with relation to their points of perihelion and aphelion, that the components of the disturbing force complete their round of effects, and recommence another series of actions. It is the residual phenomena which remain uneffaced after this interval, which, accumulating for a long succession of ages, produce the ultimate changes of the elements now referred to.

It appears, therefore, that there are two extremely different classes of INEQUALITIES as they are called, which arise from the operation of the disturbing force.

1st. Those which vary with and depend on the configuration of the disturbed and disturbing bodies with relation to the central body, taking into the account not only their relative directions as seen from the central body, and their varying distance from each other, but also their varying distances from the central body, owing to the elliptic form of their orbits.

These inequalities necessarily pass through all their phases, complete their periods, and recommence the same succession of changes, when the disturbed and disturbing bodies have passed through all their possible varieties of configuration; and, conse-

quently, their several periods must be less than the interval within which this succession of configurations is completed.

These are denominated PERIODIC INEQUALITIES.

2nd. Those which arise from the accumulation of the residual phenomena already mentioned, as being in some cases unextinguished by the opposite effects of the positive and negative components of the disturbing force, acting during that interval of time within which the bodies assume all their possible varieties of configuration. It will appear hereafter that these residual phenomena are generally of very small value,—so small as to be rarely sensible to observation until they have been allowed to accumulate for a long succession of periods.

It is therefore evident that these latter inequalities are incomparably slower in their progressive development than the former, and consequently the intervals of time within which they complete their changes are proportionally more protracted. These have accordingly been denominated SECULAR INEQUALITIES.

It must not, however, be imagined that these are less really periodic than the former. The only difference in that respect between the two classes of phenomena is in the length of their periods. Where those of the former may be expressed by years, those of the latter will be expressed by centuries.

CHAP. XXI.

LUNAR THEORY.

3185. *Lunar theory an important case of the problem of three bodies.*—The most remarkable example, and in many respects the most interesting, of the application of the principles explained in the last Chapter for the solution of the problem of three bodies, is unquestionably that which is presented by the the lunar theory, in which the central body S is the earth, the disturbed body P the moon, and the disturbing body M the sun.

This application of the theory of perturbations is interesting,

not so much because of the physical importance of the moon, as because, by the proximity of that body, perturbations produced upon it become conspicuously observable, which in any other body of the solar system would be inappreciable by reason of its remoteness.

The lunar perturbations, moreover, present another feature of interest, as compared with those of the planets, by reason of the comparative shortness of their cycles,—phenomena, which, in the case of the planets, require a succession of ages to complete their periods, passing, in the case of the moon, through all their phases, and returning upon themselves, in the course of a few years.

3186. *It supplies striking proof of the truth of the theory of gravitation.*— But transcendantly the most interesting and important circumstance attending the lunar theory is, the remarkable evidence it has afforded in support of the theory of gravitation. The perturbations of the lunar motions being, for the most part, of considerable magnitude, and recurring after short intervals, were observed, and the laws of many of them ascertained, at a very early epoch in the progress of astronomical science, and long before the discovery of gravitation had disclosed their physical causes. Several of these phenomena were explained by the illustrious author of that theory, some perfectly, others imperfectly; and those which remained unexplained or imperfectly explained by him have since been fully and satisfactorily accounted for, upon the principles which form the foundation of his physical theory.

3187. *The sun alone sensibly disturbs the moon.*—The only body in the system, which produces a sensible disturbing effect upon the moon, is the sun; for although several of the planets when in opposition or inferior conjunction, come within less distances of the earth, their masses are too inconsiderable to produce any sensible disturbing effect upon the moon's motion. The mass of the sun, on the contrary, is comparatively so prodigious that, although the radius of the moon's orbit bears so small a ratio to the sun's distance, and although lines drawn from the sun to any part of that orbit may be regarded as sensibly parallel, the difference between the forces exerted by the sun upon the moon and earth, so far from being insensible, produces on the contrary those perturbations which it is our purpose in the present Chapter to explain.

3188. *Lines of syzygy and quadrature.* — Let S, *fig.* 835.,

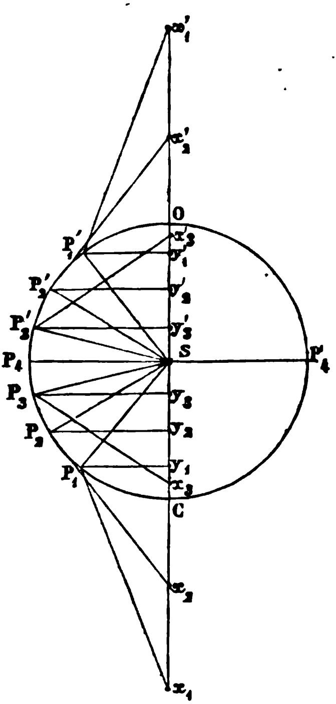


Fig. 835.

represent the place of the earth; P_1 P_2 P_3 &c., representing successive positions of the moon in its orbit, which, for the present, we shall consider to be circular. Let C O be that diameter of the lunar orbit which is directed to the sun, C being the point of conjunction, and O the point of opposition. Let P_4 S P'_4 be drawn through S at right angles to O C; the points P_4 P'_4 will then be the points of quadrature.

The line O C is called the LINE OF SYZYGIES; and the line P_4 P'_4 the LINE OF QUADRATURES.

We shall, for the present, consider only that component of the sun's disturbing force which is in the plane of the moon's orbit; and shall, therefore, speak of the sun as if it were placed in the prolongation of the line S C.

The moon being supposed to move synodically in the direction C P_1 P_2, &c., we shall distinguish C P_4 as the first quadrant, P_4 O as the second, O P'_4 as the third, and P'_4 C as the fourth quadrant of the synodic revolution.

3189. *Direction of the disturbing force of the sun.* — If we suppose the moon at any point, such as P_1 in the first quadrant, and if we draw P_1 y_1 at right angles to S C, the direction of the sun's disturbing force acting at P_1 will be found by taking S x_1, $= 3$ S y_1, and drawing P_1 x_1 which will then be the direction of the disturbing force.

To prove this, we have to observe that P_1 and y'_1 may be considered as equally distant from the sun. Let their common distance be expressed by z, and let the sun's distance from S be expressed by r', and the moon's distance by r.

We shall then by the formula [2] (3174.) have

$$s x_1 = s y_1 \times (1 + \frac{z}{r'} + \frac{z^2}{r'^2})$$

But since the moon's distance bears an insignificant proportion to that of the sun, being only a 400th part of it, we may consider $z = r'$ and consequently $\frac{z}{r'} = 1$, and therefore we shall have

$$s x_1 = 3 s y_1.$$

Whatever, therefore, may be the synodic position of the moon, this supplies an easy and simple method of determining the direction of the sun's disturbing force upon it. Thus, if the moon be at P_2, draw $P_2 y_2$ at right angles to $s c$, and let $s x_2 = 3 s y_2$ and the line $P_2 x_2$ will be the direction of the disturbing force at P_2.

It is evident that, as the moon approaches its western quadrature, the distance of the perpendicular $P_2 y_2$, $P_3 y_3$, &c., from s constantly diminishes; and, therefore, the distance $s x_2$, $s x_3$, &c. which is always three times that distance, also constantly diminishes, so that the point where the direction of the disturbing force meets the line of syzygies, constantly approaches s, as the moon approaches P_4; and when the moon arrives at P_4, the perpendicular on $O C$ passes through s. When the moon arrives at its western quadrature, the disturbing force of the sun is therefore directed along the line $P_4 s$ to the earth. After the moon passes the western quadrature, and moves through the second quadrant, the perpendiculars $P'_3 y'_3$, $P'_2 y'_2$, $P'_1 y'_1$, &c., meet the line of syzygies above s; and as the moon advances the distance of y'_3, y'_2, y'_1, &c., from s, constantly increases. Now, the direction of the disturbing force being still determined by the same principle, it will in all cases meet the line of syzygies at a point above s, at a distance from s always three times the distance, $s y'_3$, $s y'_2$, $s y'_1$, &c. Thus, when the moon takes successively the positions P'_3, P'_2, P'_1, &c., the disturbing force takes successively the directions $P'_3 x'_3$, $P'_2 x'_2$, $P'_1 x'_1$, &c.

Without pursuing these considerations further, it will be evident that, in moving through the first quadrant, the direction of the disturbing force intersects the line of syzygies below s; and in moving through the second quadrant, intersects it above

S; and the same reasoning will show that, in moving through the fourth quadrant, the disturbing force intersects the line of syzygies below S in the same manner as in the first quadrant; and in moving through the third quadrant, it intersects it above S in the same manner as in the second quadrant; and it is further evident, that at corresponding points in the third and fourth quadrants the direction of the disturbing force intersects the line of syzygies at the same points as in the second and first quadrants.

3190. *Points where the disturbing force is wholly radial and wholly tangential.*—From what has been explained it appears, that the disturbing force at quadratures being directed to the earth, is wholly radial and positive.

It is easy to show that at syzygies it is also wholly radial, but negative; for at this place the sun's attraction is obviously at right angles to the tangents of the moon's orbit at C and O, and as the whole attraction is thus perpendicular to the tangent, and the disturbing force is equal to the difference of the attracting force exerted by the sun upon the moon and upon the earth, it is clear that the disturbing force is also perpendicular to the tangent, and therefore radial, since the moon's orbit is here assumed to be sensibly circular.

To determine the points at which the disturbing force of the sun assumes the direction of a tangent to the moon's orbit, let P_2 be that point in the first quadrant. We shall then have the angle $SP_2x_2 = 90°$, and consequently

$$Sy_2 : P_2y_2 :: P_2y_2 : y_2x_2$$

and consequently,

$$\frac{P_2y_2}{Sy_2} = \sqrt{\frac{y_2x_2}{Sy_2}} = \sqrt{2}.$$

But

$$\frac{P_2y_2}{Sy_2} = \tan. P_2SC = \sqrt{2}.$$

And it appears by the trigonometrical tables that the angle whose tangent is $\sqrt{2}$ is 54° 44′ 7″.

Thus it follows that the synodic position of the moon in the first quadrant at which the disturbing force is wholly tangential,

acting in the direction $P_2 x_2$ against the moon's motion, is at the distance of 54° 44′ 7″ from conjunction.*

It will follow in the same manner, that the disturbing force in the second quadrant will be wholly tangential at a point P′, whose distance from opposition O is 54° 44′ 7″; and, in like manner, it will be tangential at points in the third and first quadrants which are at like distances from opposition and conjunction.

Thus it appears that in the synodic orbit of the moon there are four points, viz., opposition, conjunction, and quadratures, at which the sun's disturbing force is wholly radial, being directed from the earth, and therefore negative at opposition, and conjunction, and directed towards it, and therefore positive at quadratures; and four other points at distances of 54° 44′ 7″ on either side of opposition and conjunction, at which it is wholly tangential, being in the direction of the moon's motion and therefore positive in the second and fourth quadrants, and contrary to the moon's motion and therefore negative in the first and third quadrants.

3191. *Intensities of the disturbing forces at syzygies and quadratures.*—Let the intensity of the disturbing force at conjunction be D′, at opposition D″, and at quadratures D.

If we take the radius of the moon's orbit as the unit, the distance of the sun from the moon in quadrature will be 400, in conjunction 399, and in opposition 401, and the sun's attraction upon the moon in these three positions will be $\frac{S}{400^2}$, $\frac{S}{399^2}$, and $\frac{S}{401^2}$; and since the disturbing force at conjunction and opposition are the differences between the whole attractions exerted by the sun upon the moon and upon the earth, we shall have,

$$D' = S \times \left(\frac{1}{399^2} - \frac{1}{400^2}\right) = 0{\cdot}0000000315 \times S$$

$$D'' = S \times \left(\frac{1}{400^2} - \frac{1}{401^2}\right) = 0{\cdot}0000000310 \times S.$$

The disturbing force D exerted at P_4 will be to the sun's entire attraction as $P_4 S$ is to the distance of the earth from

* Sir John Herschel gives for this angle the value 64° 14′ which is certainly erroneous.—See Outlines of Astronomy, page 434, edit. 1849.

the sun, or, what is the same, as r to r', and consequently we shall have,

$$D = \frac{S}{r'^2} \times \frac{r}{r'} = S \times \frac{r}{r'^3} = S \times \frac{1}{400^3},$$

and, consequently,

$$D = 0{\cdot}0000000156 \times S.$$

It appears from these values of D', D'', and D, that the three radial disturbing forces exerted by the sun upon the moon at conjunction, opposition, and quadrature, are in the following proportion,

$$D' : D'' : D :: 63 : 62 : 31{\cdot}2.$$

Thus it appears that the disturbing force at conjunction is greater than at opposition in the proportion of 63 to 62, and that the negative disturbing force at syzygies is about double the positive disturbing force at quadratures.

In other words, it appears that the disturbing force of the sun acts against the moon's attraction upon the earth at conjunction more energetically than at opposition, in the proportion of 63 to 62; and that, in both cases, its action in diminishing the earth's attraction on the moon is twice as great as its action in increasing the earth's attraction upon it in quadratures.

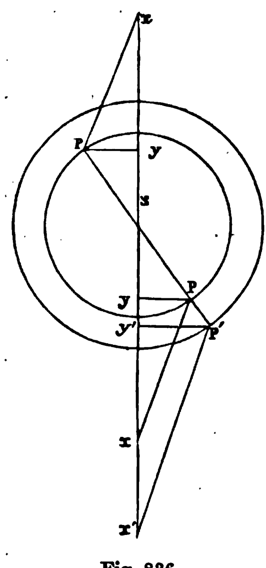


Fig. 836.

3192. *The sun's disturbing force at equal angles with syzygy varies in the direct ratio of the moon's distance from the earth.* — It is easy to show that if the moon's distance from the earth be supposed to vary, while the sun's distance remains the same, and still bears a high ratio to the moon's distance, the sun's disturbing force will vary in the direct ratio of the moon's distance at equal angular distances from syzygy.

Let P and P′ *fig.* 386. represent the moon at two different distances, S P, and S P′, from the earth. To find the

lines representing the disturbing force at P and P′, draw P y and P′ y' at right angles to the line drawn from S to the sun, and let S x = 3 S y, and S x' = 3 S y', and draw the lines P x and P′ x'. These two lines will then represent in quantity and direction the disturbing forces of the sun upon the moon at P and P′.

But it is evident, since S x is three times S y, and S x' is three times S y', that the lines P x and P′ x' are parallel, and are therefore proportional to S P and S P′; that is, the disturbing forces at P and P′ are proportional to the distances of the moon from the earth at these two points.

It will be seen hereafter that this principle is attended with some important consequences in the lunar theory.

3193. *Analysis of the variations of sign of the components of the disturbing force during a synodic period.*—From what has been explained in the preceding paragraphs, it will be easy to trace the successive changes of direction and sign of the radial and tangential components of the disturbing force of the sun, during an entire lunation or synodic period of the moon.

Let C *fig.* 837. represent the position of the moon in conjunction; O, its position in opposition; P_2, its position in western, and P'_2, in eastern quadrature.

Let P_1 be its position in the first quadrant when the radial component vanishes, and the whole disturbing force is tangential, this point being at the distance of 54° 44′ 7″ from C. Let P_3, P'_3, and P'_1 be the corresponding points in the second, third, and fourth quadrants.

From what has been already explained it appears that, the disturbing force at C being at right angles to the tangent, the tangential component is nothing; and since, through the first quadrant C P_2, the direction of the disturbing force forms an obtuse angle with the direction of the moon's motion, its tangential component will be in a direction contrary to the moon's motion, and it will therefore be negative. This component vanishes at P_2, where the whole disturbing force becomes radial, and therefore at right angles to the tangent, and after passing P_2, the direction of the disturbing force forming an acute angle with the direction of the moon's motion, the tangential component is in the direction of the moon's motion, and therefore positive; and continues to be positive throughout the second

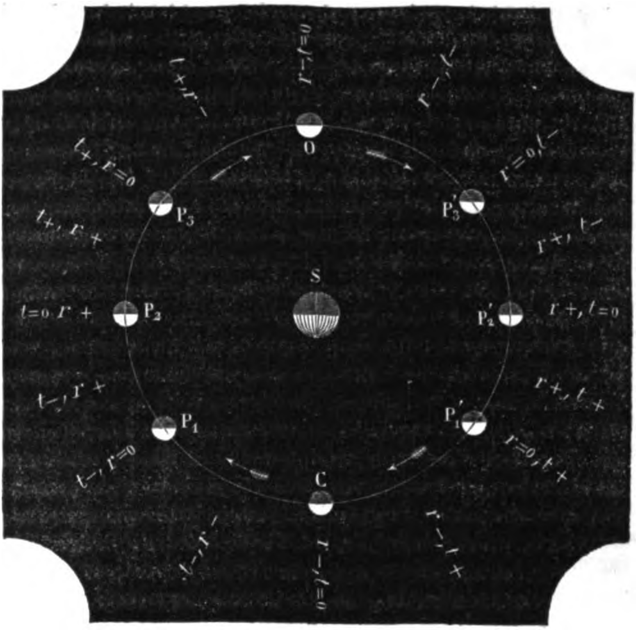


Fig. 837.

quadrant until the moon arrives at opposition O, where the disturbing force again becomes radial, and being at right angles to the tangent, the tangential component vanishes. After passing O, the disturbing force forms an obtuse angle with the direction of the moon's motion, and its tangential component is therefore contrary in direction to the motion of the moon, and consequently negative.

It continues negative through the third quadrant until, the moon arriving at P_2, the direction of the disturbing force becomes radial and at right angles to the tangent, and the tangential component again vanishes. After passing this point the disturbing force forms an acute angle with the direction of the moon's motion, and its tangential component therefore being in the direction of such motion is positive, and continues positive throughout the fourth quadrant until it vanishes at conjunction.

Thus it appears, that the tangential component is negative in

the first and third quadrants, throughout which it has therefore a tendency to retard the moon's motion; and that it is positive in the second and fourth quadrants, throughout which therefore it has a tendency to accelerate the moon's motion.

Let us now trace the changes of direction and sign, which affect the radial component of the disturbing force.

It has been already shown that at conjunction the radial component is negative, and therefore directed from the earth. This circumstance arises from the fact that the sun's attraction on the moon at C, is greater than its attraction upon the earth at S, and the disturbing force being the excess of the attraction towards the sun at C, above the attraction towards the sun at S, is directed from S. As the moon moves from C to P_1 the direction of the disturbing force lies outside the tangent, and consequently its radial component is directed from S and is therefore negative. It gradually decreases from C to P_1, because the angle under the disturbing force and the tangent gradually decreases. This angle vanishes at P_1, where the disturbing force coincides with the tangent, and where therefore its radial component vanishes. Thus it appears that the radial component which is negative at C, continually decreases from C to P_1 where it is$=0$.

After passing P_1, the direction of the disturbing force falling within the tangent, its radial component is directed towards S, and is therefore positive; and the angle which the direction of the disturbing force makes with the tangent, increasing from P_1 to P_2, the radial component continually increases until at P_2 the direction of the disturbing force being at right angles to the tangent, the whole disturbing force becomes radial. After passing P_2, the angle under the direction of the disturbing force and the tangent again becomes acute, and the radial component being positive, decreases and continues to decrease until at P_3 the angle under the direction of the disturbing force and the tangent vanishes, and there accordingly the radial component also vanishes, the whole disturbing force being tangential.

After passing P_3, the direction of the disturbing force falling outside the tangent, the radial component is again directed from the centre, and is therefore negative; and the angle under the disturbing force and the tangent continually decreases until the moon arrives at opposition O, where it becomes a right angle,

and the whole disturbing force becoming radial, is directed from S, and is therefore negative. After passing opposition, the angle under the disturbing force and the tangent again becomes acute and continually diminishes, and therefore the radial component continually decreases, being still negative until at P'_3 the angle under the direction of the disturbing force and the tangent vanishes, the whole disturbing force becoming tangential, and the radial component being therefore $= 0$. After passing P'_3, the direction of the disturbing force again falls within the tangent, and from P'_3 to P'_2 it forms an acute angle with the tangent which continually increases until at P'_2 it becomes a right angle. The radial component therefore, from P'_3 to P'_2, is positive and continually increases until at P'_2 the whole disturbing force is radial. After passing P'_2, the direction of the disturbing force again forms an acute angle with the tangent, and the radial component is therefore still positive; and as this angle continually decreases, the radial component decreases until the moon arriving at P'_1, the disturbing force takes the direction of the tangent, and the radial component is $= 0$. After passing P'_1, the direction of the disturbing force lying outside the tangent, the radial component is directed from the earth, and is therefore negative, and the angle formed by the direction of the disturbing force with the tangent continually increasing, the negative radial component continually increases until the moon arrives at conjunction where the whole disturbing force is negative and radial.

It appears, therefore, that through the arcs $C\,P_1$ and $C\,P'_1$, extending to $54° \, 44' \, 7''$ on each side of conjunction, and $O\,P_3$, $O\,P'_3$ extending to the same distance on each side of opposition, the radial component of the disturbing force is negative, being greatest at conjunction and opposition, and gradually decreasing as the distance of the moon from those points increases until it vanishes at the points P_1, P'_1, P_3, and P'_3.

It appears further, that throughout the arcs of the synodic orbit included between P_1 and P_3, and between P'_1 and P'_3, which are bisected by the points of quadrature P_2 and P'_2, the radial component of the disturbing force is positive, being greatest at the points of quadrature P_2 and P'_2, and gradually diminishing from those points to the extreme points of the arcs $P_1\,P_3$, and $P'_1\,P'_3$, where it vanishes.

Since, therefore, a negative radial component acts in anta-

gonism with the central attraction of the earth, and a positive radial component coincides in direction with that attraction, it follows, that the radial component of the sun's disturbing force has the effect of diminishing the intensity of the earth's attraction throughout the arcs $P_1 C P'_1$ and $P_3 O P'_3$; and that, on the contrary, it has a tendency to increase the earth's attraction throughout the arcs $P_1 P_2 P_3$ and $P'_1 P'_2 P'_3$.

The successive changes of sign of the radial component r, and the tangential component t, of the sun's disturbing force, are indicated on *fig.* 837., in a manner that will be readily understood after what has been explained above.

3194. *Effects of the disturbing force on the moon's motion. The annual equation.*—It now remains to explain the manner in which the disturbing force of the sun affects the moon's motion and the form of its orbit.

We shall first explain those lunar inequalities which are independent of the elliptic orbit; and, in doing this, we shall regard the undisturbed orbit as a circle, having the earth at its centre. When these inequalities have been explained, we shall consider those which affect the elliptic orbit of the moon.

After what has been explained it will be apparent, that the total effect of the radial component of the disturbing force during a synodic revolution, will be to decrease the intensity of the earth's attraction upon the moon.

It has been already shown that the intensity of the negative radial component of the disturbing force at syzygies, is about twice that of the positive radial component at quadratures. It follows therefore, that at C and O the disturbing action of the moon diminishes the earth's attraction twice as much as that by which the positive disturbing action at P_2 and P'_2 increases it; and it will be apparent that nearly the same proportion will prevail between the intensities of the negative action of the disturbing force through the arcs $P_1 C P'_1$ and $P_3 O P'_3$ and its positive action through the arcs $P_1 P_2 P_3$ and $P'_1 P'_2 P'_3$. It appears, therefore, that the intensities of the positive radial components do not amount to more than one half those of the negative radial components of the disturbing force during an entire synodic period.

But besides this excess of intensity of the negative over the positive radial components, it is to be considered that while the negative radial components are in operation through four arcs

of 54° 44′ 7″ of the synodic revolution, the positive radial components are in operation only through the four arcs complementary to these, — that is, through four arcs whose magnitude is 35° 15′ 53″.

The total effect, therefore, of the radial components, during an entire synodic period, must be to diminish the earth's central attraction upon the moon, —*first*, because the intensity of the negative radial components is greater than that of the positive in the proportion of nearly 2 to 1; and, *secondly*, because the total length of the arcs, through which the former act, is greater than that of the arcs through which the latter act, in the proportion of 54 to 35 nearly. The total effect, therefore, of the radial component of the disturbing force is to diminish the earth's attraction upon the moon. Now it appears, from the general principles of central force, which have been so fully explained in former chapters of this volume, that the angular motion of a body, revolving at a given distance round a centre of attraction, will be more or less rapid according to the greater or less intensity of that attraction. Whatever, therefore, diminishes the central attraction of the earth upon the moon, must produce a corresponding diminution of the rate of the moon's mean motion round the earth. Since, therefore, the disturbing force of the sun diminishes the central attraction of the earth, it necessarily retards the motion of the moon round the earth, or, what is the same, it increases the length of its period.

But it remains to be seen how far this effect may be modified by the tangential component of the disturbing force. Now, from what has been explained, it appears that the tangential component is negative in the first and third, and positive in the second and fourth quadrants, and therefore it retards the moon's motion in the former, and accelerates it in the latter, and these effects in the two quadrants are very nearly equal; the consequence is, that the contrary effects of the tangential force, in accelerating and retarding the moon's motion, neutralize each other, and leave the effect of the radial component unimpaired.

It appears, therefore, that the mean apparent motion of the moon is thus rendered less by the sun's disturbing force, than it would be if that force did not act; and if that force be subject to any variation, it is obvious that the mean apparent motion of

the moon must be subject to a corresponding variation, increasing and decreasing with the increase and decrease of the disturbing force. But since the sun's disturbing force bears a certain proportion to the sun's whole attraction, it must increase and decrease with such attraction. Now, since the sun's distance from the earth varies, being least when the earth is in perihelion, and greatest when it is in aphelion, and since the sun's attraction on the earth increases in the same proportion as the square of the earth's distance from it decreases, it follows, that the sun's attraction on the earth and moon, and therefore its disturbing force, is greatest when the earth is in perihelion, and least when it is in aphelion, and that it continually decreases while the earth passes from aphelion to perihelion.

Now, since, from what has been just explained, the mean apparent motion of the moon is diminished by every increase of the sun's disturbing force, it will follow that this mean apparent motion will be least when the earth is in perihelion, and greatest when it is in aphelion; and that it will gradually increase while the earth is passing from perihelion to aphelion, and gradually decrease while the earth is passing from aphelion to perihelion.

If we suppose an imaginary moon to revolve round the earth uniformly in the same time as the real moon does variably, the apparent motion of this imaginary moon would be the mean apparent motion of the moon, and its place would be the mean place of the moon.

The difference between the places of this imaginary moon and the true moon, produced by the inequality of the moon's mean motion which has been just explained, is called the *annual equation*, because this difference must continually increase for one half-year, and diminish for another half-year, according as the true motion exceeds or falls short of the mean motion.

This inequality or variation of the moon's mean apparent motion was discovered by observation at a very early period in the progress of astronomical science, and long before its physical cause was disclosed by the discovery of gravitation. Its discovery by observation was due to Tycho Brahè, about the year 1590. The greatest value of this annual equation, or, what is the same, the greatest difference between the mean and

true places of the moon, so far as they are affected by this cause, is about 10′.

3195. *Acceleration of the moon's mean motion.* — It might very naturally be expected that a mean between the greatest and least values of the moon's mean motion, as above explained, would be equal to the moon's mean motion when the earth is at its mean distance from the sun. Thus, if M' express the moon's mean apparent motion when the earth is in aphelion, and M'' its mean apparent motion when in perihelion, and M be the mean between these, we shall have

$$\text{M} = \tfrac{1}{2}(\text{M}' + \text{M}'').$$

Now, if m express the mean motion of the moon, subject to the disturbing action of the sun, when the earth is at its mean distance from the sun it might be expected that we should have $m = \text{M}$, and if this were the case, M would necessarily be invariable, because, as will hereafter appear, the earth's mean distance from the sun is subject to no secular variation, and consequently the disturbing force of the sun at this distance must be equally invariable. It is found, however, that M is not only not equal to m, but that it is not invariable.

It is found to be greater than m, and variable in a manner depending on the inequality of the earth's distances from the sun at the apsides; the more unequal these distances are, the greater is M found to be. But since the inequality of these distances depends on the excentricity of the earth's orbit, and since this excentricity is, as will appear hereafter, subject to a slow secular variation, it follows that M, or the moon's mean apparent motion, is subject to a corresponding secular variation. It will appear that, for thousands of years back, and for a corresponding period to come, the excentricity of the earth's orbit has been, and will be, subject to a slow decrease; the consequence of which is, that the moon's mean apparent motion, which increases as the inequality of the apsidal distances decrease, has been, and is still, subject to a slow secular increase, which, like the annual equation, was discovered as a fact by observation long before the physical cause was known, and was designated, in astronomical science, as the *acceleration of the moon's mean motion.*

Its cause, as just explained, was traced to the secular variation of the excentricity of the earth's orbit by Laplace, in 1787.

3196. *Effect upon the form of the moon's orbit.*—The combined effect of the two components of the disturbing force upon the form of the moon's orbit, the undisturbed orbit being supposed to be circular, is to elongate it in the direction of the line of quadratures, and to contract it in the direction of the line of syzygies so as to convert it into a sort of oval, the longer axis of which is at right angles to the line of direction of the sun. It consequently follows, that as the earth moves round the sun, this oval continually shifts the direction of its axis, the longer axis constantly turning so as to keep itself at right angles to the radius vector of the earth.

To explain this, it is only necessary to consider the general conditions which determine the curvature of the path of a body moving under the influence of a central attraction, and to compare them with the components of the disturbing force at different parts of the synodic revolution. The curvature of the path of such a body depends in general on the intensity of the central attraction and the velocity of the body. Whatever diminishes the central attraction or increases the velocity, must diminish the curvature of the path of the body, and *vice versâ*. Now it has been shown that, throughout two arcs of the lunar orbit extending to 54° 44′ 7″ at each side of the line of syzygies, the radial component of the disturbing force is negative, and therefore diminishes the central attraction, and consequently also diminishes the curvature. But it has also been shown that the tangential component is positive while the moon approaches the syzygies, and negative after it passes them. The motion of the moon is therefore accelerated by this component until it arrives at syzygies, and retarded after it passes these points. The velocity of the moon, therefore, being most augmented at syzygies by the tangential component, the curvature of its path is there diminished.

It appears, therefore, that throughout the arcs extending 54° 44′ 7″ on either side of the syzygies, both components of the sun's disturbing force have a tendency to diminish the curvature of the moon's path.

On the other hand, throughout the arcs which extend 35° 15′ 53″ on each side of the quadratures, the radial force, being positive, augments the central attraction of the earth,

and therefore increases the curvature of the orbit; and at the same time the tangential component of the disturbing force, being negative as the moon approaches the quadratures, diminishes the velocity, and consequently increases the curvature of the moon's path.

For these reasons, the curvature of the moon's orbit is greatest at the quadratures and least at the syzygies, which is equivalent to stating that the lunar orbit has an oval form arising from these causes, the longer axis of the oval being in the direction of the quadratures, and the lesser axis in the direction of syzygies.

3197. *Moon's variation.* — If the radial component be alone considered, it is easy to see that the moon, moving in such an orbit, would have a varying angular motion, which would be greatest at syzygies where the moon is nearest the earth, and least at quadratures where it is most distant from the earth. This will follow immediately from the principle of the equable description of areas, which is never affected by a radial disturbing force, since that principle rests on no other condition than that the revolving body be affected only by a force directed to a fixed centre. It is clear, from what has been proved in (2614.), that the angular velocity, varying inversely as the square of the moon's distance from the earth, will, in such an oval orbit as has just been described, be greatest where the distance is least, — that is, at syzygies ; and least where the distance is greatest, — that is, at quadratures. So far, therefore, as relates to the radial component of the disturbing force, the moon's apparent motion, as seen from the earth, which is, in fact, its angular motion round the earth, is greatest at syzygies, and least at quadratures.

But if we take into account, also, the effect of the tangential component, this variation of the apparent motion will be still greater. This component, according to what has been shown, continually accelerates the moon's motion in approaching the syzygies, and continually retards it in approaching the quadratures, so that, so far as depends on it, the moon's velocity will be greatest at syzygies, and least at quadratures.

Thus the two components of the sun's disturbing force combine to render the moon's apparent motion greatest at conjunction and opposition, and least at quadratures.

This inequality of the moon's apparent motion, which passes

through all its phases in the synodic period, is called the *variation*, and was discovered about the same time with the annual equation by Tycho Brahè.

If we suppose an imaginary moon to perform the synodic period with a uniform angular motion, while the motion of the true moon is subject to this alternate acceleration and retardation at syzygies and qaadratures, the distance between the two moons will be the *variation*, and its greatest amount will be about 32′.

3198. *Parallactic inequality.* — In this explanation, however, we have assumed that the disturbing forces are equal at conjunction and opposition. Now, it has been already shown that, at these points, they are slightly unequal,—the disturbing force at conjunction exceeding that at opposition, in the proportion of about 63 to 62. It follows, therefore, that the effect of the disturbing forces will not be exactly equal at the two syzygies; and a small inequality is thus, as it were, superposed upon the variation, or, so to speak, *a variation of the variation* is produced, which is called the *parallactic inequality*.

3199. *Inequalities depending on the elliptic form of the lunar orbit.* — In the preceding paragraphs we have omitted the consideration of the elliptic character of the moon's orbit. We shall now explain those inequalities which depend on the varying length of the moon's radius vector.

3200. *Equation of the centre.* — The first inequality of this class which we shall notice is one which appertains to the problem of two bodies, rather than that of three bodies, and which has been already noticed in relation to elliptic motion in general. The equable description of areas by the moon in its elliptic orbit, causes its angular motion at perihelion to be greater than at other points; and, as it moves from perihelion, this angular motion gradually diminishes, and continues to diminish, until it arrives at aphelion, where it is least. From aphelion to perihelion, on the contrary, the angular motion gradually and continually increases. If we suppose an imaginary moon to move from perihelion through aphelion back to perihelion, with a uniform angular velocity, the motion of this moon would be the mean motion of the moon, its place the mean place, and its anomaly, or its distance from perihelion, the mean anomaly; and the distance between this imaginary moon and the true moon is called the *equation of the centre*.

Starting from perihelion, the motion of the true moon being greater than that of the imaginary moon, the true moon is in advance of the imaginary moon, and it continues to gain upon the imaginary moon to a certain point, after which the mean moon begins, in its turn, to gain upon the true moon, and overtakes it at aphelion ; the distance between the two moons, mean and true, at any point, is the *equation of the centre.* From aphelion to perihelion, on the contrary, the mean moon precedes the true moon, and the distance between them increases to a certain point, after which the mean moon begins to overtake the true moon, and does overtake it on returning to perihelion.

The equation of the centre is, therefore, positive from perihelion to aphelion, and negative from aphelion to perihelion.

3201. *Method of investigating the variations of the elliptic elements of the lunar orbit.*—After what has been explained in the present and preceding chapters, there will be no difficulty in tracing the effects produced by the sun's disturbing force upon the magnitude, form, and position of the moon's elliptic orbit. The effects produced in general upon any elliptic orbit whatever, by positive and negative, radial and tangential disturbing forces (317. *et seq.*), and the successive changes of direction and intensity of the radial and tangential components of the sun's disturbing force acting on the moon, as explained above, being fully comprehended, it is only necessary to apply the general principles to the particular case of the moon in order to explain all the phenomena. For this purpose it will be necessary to consider successively the cases in which the moon's perigee assumes every variety of position with relation to the line of syzygies, and in each position to investigate the effects produced upon the elements of the instantaneous ellipse in the different positions which the moon assumes during an entire revolution in its orbit.

3202. *Moon's mean distance not subject to secular variation.*—It may be stated, generally, that the effects of the disturbing force of the sun upon the moon's mean distance or major axis of its orbit neutralise each other; the increase which it produces on that element in some synodic positions being exactly compensated by the decrease it produces in others.

In the first place, it must be observed that since the excentricity of the lunar orbit is very small, the radial component

produces no effect on the moon's orbital velocity, and, therefore, none upon the magnitude of its major axis.

The tangential component being negative in the first and third, and positive in the second and fourth, quadrants, diminishes the axis in the former and increases it in the latter. If it can be shown that, on the whole, the increase is equal to the decrease, it will follow that the magnitude of the mean

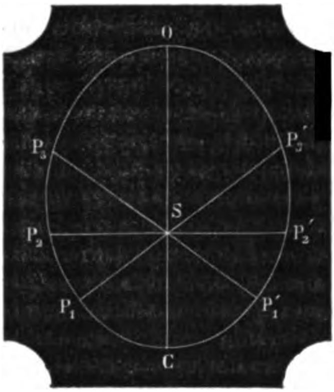


Fig. 838.

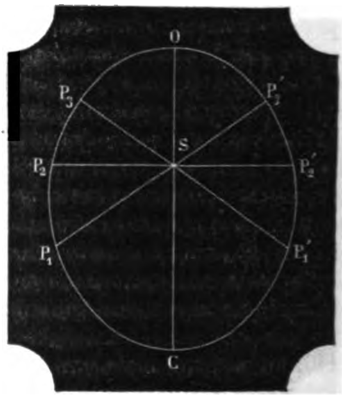


Fig. 839.

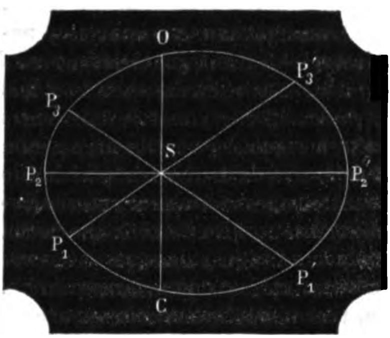


Fig. 840.

distance or major semi-axis suffers no ultimate change. For this purpose it will be necessary to examine the effects in different positions of the lunar orbit relatively to the syzygies and quadratures.

If the line of apsides be in syzygies, as represented in *fig.* 838., when perigee is in conjunction, and in *fig.* 839. when apogee is in conjunction, the elliptic orbit will be divided symmetrically by the four synodic quadrants; and, since the intensity of the disturbing force is the same at equal angular distances from the apsides, it follows that in each case the diminution of the mean distance produced by

the tangential component in the first and third quadrants is equal to the increase produced in the second and fourth quadrants.

If the apsides be in quadrature, as represented in *fig.* 840., the same will obviously be true.

In these cases, therefore, the major axis of the orbit suffers no ultimate change from the action of the disturbing force.

But if, as in *fig.* 841., the line of apsides $p\ a$ be inclined at an oblique angle to the line of syzygies C O, the elliptic orbit will not be symmetrically divided by the four synodic quadrants C P_2, P_2 O, O P'_2, and P'_2 C, and in that case the decrease and increase of the axis produced in the alternate quadrants will no longer be equal, and a complete compensation will not as before, be effected in a single synodic revolution. But if the orbit be taken in two positions in which the line of apsides $p\ a$ and $p'\ a'$ is equally inclined on different sides of the line of syzygies, the effect of the disturbing force on the mean distance in a complete synodic revolution in one position will be compensated by the equal and contrary effect produced in the other position. This will be apparent by considering that the intensities of the disturbing force at equal inclinations to the line of syzygies are proportional to the moon's distance from the earth. It follows from this that the effects of the disturbing force in the quadrants C p P_2 and C p' P'_2, and in the quadrants $P_2 a$ P'_2 and $P_2 a'$ P'_2 are equal, and in the same manner that the effects are equal in the other corresponding quadrants. It will therefore be apparent that, taking the orbit in the two positions, the increase and decrease which the major axis suffers in two complete synodic periods are equal, and that, therefore, the major axis suffers no ultimate change of magnitude.

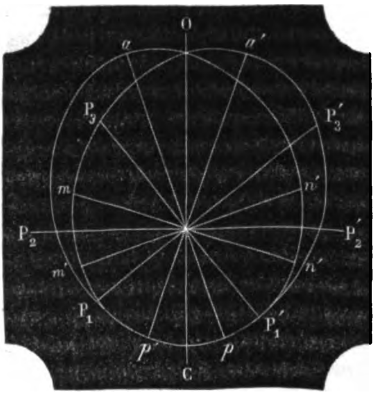


Fig. 841.

But since, in the revolution of the earth and moon round the sun, the line of apsides takes successively every inclination to the line of syzygies, it will necessarily assume, at regular inter-

vals, the equal inclinations at which the effects of the disturbing force upon the axis of the orbit are mutually compensatory, and it follows, therefore, that no ultimate decrease of magnitude of the axis takes place.

It remains, therefore, to investigate the effects of the disturbing force of the sun on the other elements of the lunar orbit; that is, upon the direction of the apsides, or, what is the same, on the longitude of perihelion and the excentricity.

As these effects will vary according to the varying position of the line of apsides, with relation to the line of syzygies, we shall consider successively the variation of each of the elements during a synodic revolution of the moon, when the apsides are in syzygies, in quadratures, and between these points.

First Case.

WHEN PERIGEE IS IN CONJUNCTION.

3203. *Motion of the apsides.*—Let the lines $P_1 P'_3$ and $P'_1 P_3$ be drawn, making angles of 54° 44′ 7″ with the line of syzygies C O, *fig.* 838. Let us consider, first, the effect of the radial, and secondly, the effect of the tangential component.

First. The radial component, being negative while the moon moves through the arcs P'_1 C P_1, and P_3 O P'_3 (3193.), a regressive motion will be imparted to the apsides in the former and a progressive motion in the latter (3193.).

The same component being positive while the moon moves through the arcs $P_1 P_3$ and $P'_3 P'_1$, a progressive motion will be imparted to the apsides in $P_1 P_2$ and $P'_2 P'_1$, and a regressive motion in $P_2 P_3$ and $P'_3 P'_2$.

To determine the effect produced upon the apsides during the whole synodic revolution, it will be necessary to take into account the varying intensity of the disturbing force in these several arcs.

Since, at equal inclinations to the line of syzygies, the intensity of the disturbing force is in the direct ratio of the moon's distance from the earth (3192.), it is evident that its effect through the arc P_3 O P'_3 will be greater than its effect through the arc P'_1 C P_1, and that its effect through the arcs $P_2 P_3$ and $P'_3 P'_2$ will also be greater, though in a much less degree, than its effect through the arcs $P_1 P_2$ and $P'_2 P'_1$.

But besides the greater intensity of the disturbing force throughout the arc $P_2 O P'_2$, its effect is augmented by the slower motion of the moon supplying a longer interval for its action.

It follows, therefore, that the progressive motion imparted to the apsides while the moon moves from P_3 to P'_3 is much greater than the regressive motion imparted while it moves from P'_1 to P_1; and that the regressive motion imparted while it moves through the arcs $P_2 P_3$ and $P'_3 P'_2$ very little exceeds the progressive motion imparted while it moves through the arcs $P_1 P_2$ and $P'_2 P'_1$.

The consequence is, that, in a complete synodic revolution, the progressive motion considerably exceeds the regressive; and therefore, on the whole, the apsides are moved forward.

Secondly. The tangential component being negative from C to P_2 and from O to P'_2, and positive from P_2 to O and from O to P'_2, it imparts a regressive motion to the apsides from P'_1 to P_1, from P_2 to P_3, and from P'_3 to P'_2, and a progressive motion from P_3 to P'_3, from P'_2 to P'_1, and from P_1 to P_2 (3193.)

It may be shown, as in the former case, that the progressive motion in a complete synodic revolution exceeds the regressive motion, and, therefore, that on the whole the tangential component imparts a progressive motion to the apsides.

It follows, therefore, that when perigee is in conjunction, the disturbing force of the sun, acting during a complete synodic revolution of the moon, causes the line of apsides to move forward in the direction of the sun's motion, so that, at the end of a synodic revolution, the longitude of perigee will be greater than it was at its commencement, — the longitude of that point, however, having during such revolution alternately increased and decreased.

3204. *Effects on the excentricity.* — To ascertain the variation of the excentricity of the lunar orbit, produced by the sun's disturbing force in the same position of the apsides, we shall, as before, consider, first, the effect of the radial, and, secondly, that of the tangential component.

First. The radial component being negative from P'_1 to P_1 and from P_3 to P'_3, and positive from P_1 to P_3 and from P'_3 to P'_1, it follows, that it will cause the excentricity to increase from C to P_1 and from P_3 to O, and to decrease from

P_1 to P_3, while, in the other half of the synodic revolution, it will cause it to decrease from O to P'_3 and from P_1 to C, and to increase from P'_3 to P'_1 (3193.). Now it is evident that the effects of the disturbing force through the arcs C P_1, $P_1 P_3$, and P_3 O, are respectively equal to its effects through the arcs O P'_1, $P'_1 P'_3$, and P'_3 O, and therefore that the increments and decrements which the excentricity receives during a complete synodic revolution are equal, and that so far as depends on the radial component of the disturbing force, it suffers no ultimate variation.

Secondly. Since the tangential component is negative in the first and third and positive in the second and fourth quadrants, it will follow, from what has been proved (3156.), that this component will cause the excentricity to decrease throughout the first and second and to increase throughout the third and fourth quadrants.

But it will be evident, from the same reasoning as has been used in the former case, that the intensity of the disturbing force in the first and second quadrants, being respectively equal to its intensity in the fourth and third, the decrease of the excentricity in the first quadrant will be equal to its increase in the fourth, and its decrease in the second quadrant will be equal to its increase in the third; the consequence of which will be that, in a complete revolution, the excentricity will suffer no change from the operation of the tangential component of the disturbing force.

It follows, therefore, that when the moon's perigee is in conjunction, the excentricity of its orbit, at the end of each synodic revolution, will be the same as at the beginning, but that during such revolution it will alternately increase and decrease within certain narrow limits.

Second Case.

WHEN PERIGEE IS IN OPPOSITION.

3205. *Motion of the apsides.* — As before, let the lines $P_1 P'_3$ and $P'_1 P_3$ *fig.* 839. be drawn, making angles of 54° 44′ 7″ with the line of syzygies C O, and let us consider, first, as in the former case, the effect of the radial, and secondly, the effect of the tangential, component.

First. The radial component being negative, while the moon

moves from P'_1 to P_1 and from P_3 to P'_3, a progressive motion will be imparted to the apsides in the former, and a regressive motion in the latter (3193.) The same component being positive, while the moon moves from P_1 to P_3, and from P'_3 to P'_1, a regressive motion will be imparted to the apsides from P_1 to P_2 and from P'_2 to P'_1, and a progressive motion from P_2 to P_3 and from P'_3 to P'_2; and it will appear by the same reasoning as in the former case, that the total effect through the arcs in which the motion is progressive, will exceed considerably the total effect through the arcs in which the motion is regressive, and therefore that the effect of the radial component during a complete synodic revolution will be, to carry forward the line of apsides.

Secondly. The tangential component being negative in the first and third, and positive in the second and fourth quadrants, it will impart a progressive motion to the apsides from P'_2 to P_2, and a regressive motion from P_2 to P'_2 (3193.).

It will follow, as in the former case, that the progressive motion arising from this component in a complete synodic revolution exceeds the regressive motion. It follows therefore, precisely as in the former case, that when perigee is in opposition, the sun's disturbing force, during a complete synodic revolution, gives a progressive motion to the line of apsides, that line however, during such revolution, receiving an alternate motion of progression and regression within certain limits.

3206. *Effects on the excentricity.*—These effects may be explained by reasoning precisely similar to that used in the former case.

First. The radial component being negative from P'_1 to P_1 and from P_3 to P'_3, and positive from P_1 to P_3 and from P'_3 to P'_1, it will cause the excentricity to decrease from C to P_1 and from P_3 to O, and to increase from P_1 to P_3; while in the other half of the synodic revolution, it will cause it to increase from O to P'_3 and from P'_1 to C, and to decrease from P'_3 to P'_1 (3193.); and it is evident, as before, that the effects in each half orbit being compensatory, no ultimate variation is produced by this component upon the excentricity in a complete synodic revolution.

Secondly. The effects of the tangential component are, in like manner, shown to be compensatory in this case by reasoning so completely similar to the former that it need not be repeated.

Third Case.

WHEN THE APSIDES ARE IN QUADRATURE.

3207. *Motion of the apsides.*—Let the lines $P_1 P'_3$ and $P'_1 P_3$, *fig.* 840., as before, be drawn, making angles of 54° 44′ 7″ with the line of syzygies C O. We shall, as in the former case, consider first the effect of the radial, and secondly that of the tangential component.

First. The radial component being negative from P'_1 to P_1 and from P_3 to P'_3, and positive from P_1 to P_3 and from P'_3 to P'_1, it follows that a regressive motion will be imparted to the apsides while the moon moves from C to P_1, from P_3 to O, and from P'_3 to P'_1, and that a progressive motion will be imparted to it in the intermediate arcs, that is from P_1 to P_3, from O to P'_3, and from P'_1 to C.

But, from the principle already so often referred to, in virtue of which the intensity of the disturbing force at equal inclinations to the line of syzygies is proportional to the distance of the moon from the earth, it will be evident that the total effect of the radial component in imparting a regressive motion to the apsides from P'_3 to P'_1 will be much greater than its total effect in imparting a progressive motion from P_1 to P_3, while the difference of its effects in the other arcs will be comparatively small. It will follow, therefore, that after an entire revolution the effect of this component in imparting regression will greatly predominate over its effect in imparting progression, and that, on the whole, the apsides will be made to regress.

Secondly. The tangential component being, as before, negative in the first and third, and positive in the second and fourth quadrants, it will impart to the line of apsides a progressive motion through the arc C P_2 O, and a regressive motion through the arc O P'_2 C (3163).

It is evident, as before, that in this case the regressive effects predominate over the progressive, and that, therefore, the effect of this component throughout a complete synodic revolution is to impart to the apsides a regressive motion.

3208. *Effect on the excentricity.*—We shall, as before, first consider the effect of the radial, and secondly of the tangential component.

It will appear, from what has been already explained (3163.), that the radial component being positive from P_1 to P_3 and from P'_3 to P'_1, the excentricity will increase from P_1 to P_2 and from P'_2 to P'_1, and will decrease from P_2 to P_3 and from P'_3 to P'_2; and the radial component being positive from P_3 to P'_3 and from P'_1 to P_1, the excentricity will increase throughout the former and decrease throughout the latter arc.

If then the several arcs of the ellipse in which the excentricity increases be compared with those in which it decreases, they will be found to be perfectly equal and symmetrical, so that the intensity of the radial components which produce increase will be equal on the whole to those which produce decrease, consequently the effects of this component on the excentricity will be compensatory.

Secondly. Since the tangential component is negative in the first and third, and positive in the second and fourth quadrants, it will follow from what has been explained (3163.), that the excentricity will continually increase through the same ellipse P_2 O P'_2, so that here again the effects of the component are compensatory.

It follows, therefore, that when the line of apsides is in quadrature, the excentricity at the end of a complete synodic revolution is precisely what it was at the commencement, suffering, nevertheless, variations of increase and decrease during such revolution.

3209. *The motion of the apsides greater in syzygies than in quadrature.*—It has been already shown (3191.) that the intensity of the disturbing force directed from the earth when the moon is in syzygies, is twice as great as its intensity directed to the earth when in quadrature.

So far, therefore, as depends on the radial component, it may be inferred that the progressive motion imparted to the apsides when in syzyzies, will be greater than the regressive motion imparted to them in quadrature.

But if the effect of the tangential component on the line of apsides in quadrature be compared with its effect in syzygies, it will be found to be nearly the same. The effect, therefore, of the greater intensity of the radial component not being affected by that of the tangential, the progressive motion imparted by the disturbing force to the apsides in syzygies is found to be greater than the regressive motion imparted to them in quad-

rature; in fact it is found that in each synodic revolution of the moon, when the apsides are in syzygies, a progressive motion of 11° is imparted to them, while the regressive motion imparted to them in each synodic revolution when in quadrature is only 9°. If then two such synodic revolutions be compared, one in syzygies and the other in quadrature, the result will be a progression of the apsides amounting to $11° - 9° = 2°$.

Several circumstances attending the moon's motion combine in producing this difference in the effects of the apsides in the two positions. The synodic motion of the moon is slower at apogee than at perigee in the ratio of 9 to 13; that is to say, while the moon departs from the sun at perigee through 13°, it will depart at apogee through only 9°. The consequence of this is, that the slower synodic motion at apogee leaves a longer time for the operation of the sun's disturbing force upon the earth than at perigee. Thus, this force is not only more energetic at apogee than at perigee, since its intensity is *proportional* to the mean distance from the earth, but the more intense force acts for a longer time.

When perigee is in conjunction, the motion of the apsides being progressive, and at the rate of 11° in each synodic revolution, while the progressive motion of the sun in the same time is about 27°, it follows that, in each synodic revolution, the sun will depart from perigee through a distance of $27° - 11° = 16°$.

But when the sun is in quadrature, the regressive motion of the apsides in each revolution being 9°, while the progressive motion of the sun is, as before, 27°, the sun and perigee will depart from each other, in each synodic revolution, through $27° + 9° = 36°$.

It appears, therefore, that the separation of the sun and perigee, in each synodic revolution when perigee is in syzygies, is greater than when it is in quadrature, in the proportion of 36 to 16, or 9 to 4.

It is evident from this, therefore, that another cause operates in favour of the more continued action of the disturbing force in producing a progressive motion in syzygies than in producing a regressive motion in quadratures, inasmuch as, from what has been just explained, the sun separates itself from the position favourable to the action of the disturbing force more than twice as rapidly in quadratures than in syzygies.

Fourth Case.

WHEN THE APSIDES ARE OBLIQUE TO THE LINE OF SYZYGIES.

3210. *Motion of the apsides.* — It has been shown that when the apsides are in syzygy the disturbing force imparts to them a progressive motion at the rate of about 11° in each synodic revolution, and that when they are in quadrature it imparts to them a regressive motion at the rate of about 9° in each synodic revolution. It might therefore be expected, that if the line of apsides assume successively increasing inclinations with the line of syzygies, from 0° to 90°, the progressive motion of 11° imparted to the apsides at syzygies would gradually decrease, and at some intermediate inclination between 0° and 90° would become nothing, after which the motion imparted to the apsides becoming regressive, would gradually increase until it becomes 9° in each synodic revolution, when the line of apsides is in quadrature.

If the conditions which determine the effect of each component of the disturbing force upon the direction of the line of apsides, in relation to that of syzygies, be clearly fixed in the mind, the student will have no difficulty in seeing that this in fact will be the case. For this purpose, it is only necessary to draw the elliptic orbit with its major axis inclined to the line of syzygies at successively increasing angles, and to examine and compare carefully the different effects produced by each component of the disturbing force upon the moon at different elongations from the sun, in each position of the apsides in relation to syzygy. It will be found that when the line of apsides makes a very small angle with the line of syzygies, the effect of the disturbing force is very little less than at syzygies, and that, accordingly, a progressive motion is imparted to the apsides, very little less than 11°; and on the other hand, that when the line of apsides makes with that of syzygies an angle but little less than 90°, the regressive motion imparted to the apsides is little less than 9°.

It will not be necessary here to multiply the details of this analysis, by going through the particulars of all such cases; but it may be useful to illustrate the mode of investigating them, by showing the effects of the disturbing force on the line of apsides, when that line is inclined to that of syzygies at the angle of

54° 44′ 7″, at which the effects of the disturbing force in imparting progressive and regressive motion to the apsides are compensatory, or nearly so, and where, therefore, the apsides at the end of a synodic revolution have the same direction as at its commencement.

3211. ***When the moon's perigee is 54° 44′ 7″ before the point of conjunction.***—Let C O, *fig.* 842., be the line of syzygies, C being the point of conjunction when the moon's perigee *p* is 54° 44′ 7″ before it, and when, consequently, the apogee *a* is at the same angular distance from the point of opposition O. The line of apsides will then coincide with the line which in the preceding diagrams has been marked P_1 P'_3, and the line *m n* passing through S at right angles to the line of apsides, will lie in advance of the line P'_1 P_3, which is inclined to the line of syzygies on the other side of it at the angle 54° 44′ 7″.

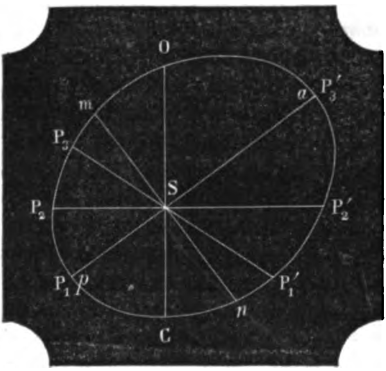


Fig. 842.

We shall consider, *first*, the effect of the radial, and *secondly*, of the tangential component.

First. From what has been already proved (3163.), it appears that a positive radial component will render the apsides regressive while the moon passes from *m* to *n*, and progressive while it passes from *n* to *m*, and, consequently, that a negative radial component will produce a contrary effect. By combining these with the conditions which determine the changes of sign of the radial component, explained in the present chapter, it may be easily inferred, that the apsides will be progressive while the moon moves from *p* to P_3, from *m* to P'_3, and from P'_1 to *n*, and that they will be regressive while the moon moves through the arcs P_3 *m*, *a* P'_1, and *n* P_1; and it will be easy to perceive that the aggregate length of these arcs is not only nearly equal, but that their distances from S are nearly the same, consequently the extent of the orbit through which the radial component renders the apsides progressive, is nearly equal to that through

which it renders it regressive. The effects of this component are therefore compensatory.

Secondly. It appears from what has been proved (3163.) that a positive tangential component renders the apsides progressive while the moon moves from *p* to *a*, and regressive while it moves from *a* to *p*, and that a negative tangential component has contrary effects. By combining these with what has been proved of the changes of sign of the tangential component in the present chapter, it will be easy to infer that this component will render the apsides progressive from P_2 to O, from *a* to P'_1, and from C to *p*, and regressive from *p* to P_2, from O to *a*, and from P'_2 to C; and by comparing these arcs as before, it will be obvious that they are nearly equal in length and at nearly equal distances from S, and that consequently the effects of this component of the disturbing force are also compensatory.

It follows therefore, generally, that in this position of the moon's perigee no motion is imparted to the line of apsides in a complete synodic revolution, but that alternate motions of progression and regression of equal total amount are imparted to it during such revolution.

3212. *When the moon's perigee is* 54° 44′ 7″ *behind the point of opposition.*—This case is represented in *fig.* 843., the moon's apogee being the same distance behind the point of conjunction C.

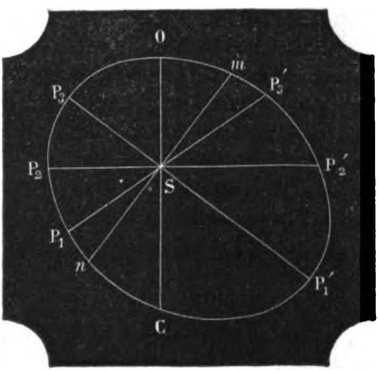


Fig. 843.

We shall, as in the former case, consider, *first*, the effect of the radial, and *secondly*, that of the tangential component.

First. Upon the same principles as were applied in the preceding case, it will follow that, by the effect of the radial component, the apsides will be progressive while the moon moves from *m* to P'_3, from P'_1 to *n*, and from P'_1 to P_3, and regressive while the moon moves from P_3 to *m*, from P'_3 to P'_1 and from *n* to P_1; and, as before, if these arcs be compared

both as to length and distance from S, it will be found that the effects of this component on the lines of apsides are compensatory.

Secondly. By the same principles as before, it will follow that, by the effect of the tangential component, the apsides will be progressive while the moon moves from P_3 to O, from P'_2 to P'_1, and from C to P_2, and regressive while the moon moves from *m* to P_2, from P'_1 to C, and from P_2 to P_3; and, as before, it will be apparent by comparing these arcs that these effects on the apsides are compensatory.

It follows therefore, as before, that in this case the disturbing force, in a complete synodic revolution, produces no change in the position of the line of apsides, and that this line oscillates with an alternate progressive and regressive motion during such revolution.

3213. *When perigee is* 54° 44′ 7″ *before the point of opposition.*—In this case, which is represented in *fig.* 844., the point of apogee will be at the same angular distance before the point of conjunction C.

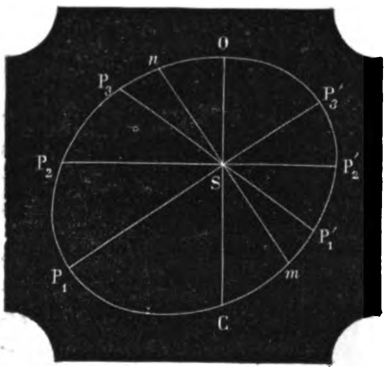


Fig. 844.

First. In the same manner as in the former case, it will be apparent that by the effects of the radial component the line of apsides will be progressive from P'_3 to P'_1, from *m* to P_1, and from P_2 to *n*, and regressive from *n* to P'_3, from P'_1 to *m*, and from P_1 to P_2; and, in the same manner as before, the effects through these arcs will be shown to be compensatory.

Secondly. In the same way it follows that by the effects of the tangential component the apsides will be rendered progressive while the moon moves from O to P'_3, from P'_2 to C, and from P_1 to P_2, and that they will be regressive while it moves from P_2 to O, from P'_3 to P'_2, and from C to P_1; and it will appear as before that the effects are compensatory.

It follows, therefore, in general, as in the former cases, that in this position of perigee the line of apsides suffers no

change of direction from the action of the disturbing force after a complete synodic revolution, but that, during such revolution as before, it oscillates on either side of its mean position.

3214. *When perigee is 54° 44′ 7″ behind conjunction.*—In this case, which is represented in *fig.* 845., the line of apogee is at the same angular distance behind the point of opposition.

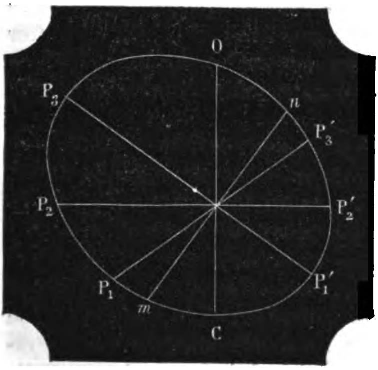


Fig. 845.

First. By the effects of the radial component the apsides will be rendered progressive while the moon moves from P_3 to n, from P'_3 to P'_1, and from m to P_1, and regressive while it moves from P_1 to P_3, from n to P'_3, and from P'_1 to m; and, as before, it may be shown that the effects in these cases respectively are compensatory.

Secondly. By the effects of the tangential force it appears, in like manner, that the apsides will be rendered progressive from O to P'_2, from P'_1 to C, and from P_2 to P_3, and regressive from P_3 to O, from P'_2 to P'_1, and from C to P_2; and that, in like manner, these effects are compensatory.

It follows, therefore, that in this position of the moon's perigee no effect is produced by the disturbing force upon the direction of the line of apsides after a complete synodic revolution, but that line, as before, during such revolution oscillates on the one side and on the other of its mean position.

3215. *Summary of the motions of the apsides.*—After what has been explained above, the motion of the line of apsides in all its positions with relation to the line of syzygies may be easily inferred. It must be remembered that the lines of syzygies and apsides being both affected with a mean progressive motion, that of syzygies, however, being much more rapid than that of apsides, it will follow that the line of syzygies after each successive synodic revolution will gain upon the line of apsides, advancing constantly before it.

Let us then imagine, first, that the moon's perigee is in conjunction.

The line of apsides will then, according to what has been proved, after one synodic revolution be affected by a progressive motion. After the next synodic revolution the line of syzygies will have advanced before that of apsides, and the progressive motion imparted to the latter will be less than before, and after each successive synodic revolution the line of syzygies advancing further and further in advance of the line of apsides, the progressive motion imparted to the latter will become less and less until the line of syzygies having advanced to the distance of 54° 44′ 7″ from the line of apsides the progressive motion ceases, and the line of apsides becomes stationary.

After this, the line of syzygies advancing to a still greater angular distance from the apsides, the motion imparted after each revolution to the latter becomes regressive, and its regressive amount increases every successive synodic revolution until the line of apsides comes into quadrature, when the regressive motion of the apsides produced in a complete synodic revolution becomes a maximum. After this when the line of syzygies has advanced more than 90° from the line of apsides the regressive motion imparted in each revolution becomes less and less until the line of apsides has approached within 54° 44′ 7″ of opposition, when the regressive motion vanishes and the line of apsides again becomes stationary; after this it becomes progressive, the amount of its progressive motion continually increases until the point of perigee comes into opposition, where it is a maximum. While perigee passes successively from opposition to conjunction the same variations of the motion of the apsides takes place, but in a contrary order.

Thus it appears that the arc of each synodic revolution through which the disturbing force produces a progressive effect is $54° 44′ 7″ \times 2 = 109° 28′ 14″$, while the arc through which it produces a regressive motion is only $35° 15′ 53″ \times 2 = 70° 31′ 46″$. The predominance of the progressive over the regressive effect results therefore from the greater range though which the former is produced, combined with the fact, that within this range the intensity of the disturbing force is equal to twice its intensity through the shorter arc, over which the motion is regressive.

3216. *Effects of the disturbing force upon the excentricity.*—

It remains now to examine the effects produced upon the excentricity of the moon's orbit by the disturbing force when the line of apsides has the same positions.

It has been already shown, that when the line of apsides is in syzygies and quadratures, the effects of the disturbing force upon the excentricity are compensatory, and that this element at these points undergoes no change of magnitude. If, therefore, it be subject to no variation, it is plain that at these points it must be either a maximum or a minimum, since it is neither on the increase or decrease.

3217. *When perigee is* 54° 44′ 7″ *before the point of conjunction.* — We shall consider as before, *first*, the effects of the radial, and *secondly*, those of the tangential component upon the excentricity.

First. It appears from what has been proved, that a positive radial component will cause the excentricity to diminish, while the moon moves from perigee to apogee, and to increase while it moves from apogee to perigee, and that a negative radial component will have a contrary effect. It will follow, therefore, in this case by combining this principle with the conditions determining the change of sign of the radial component explained in the present chapter, that it will cause the excentricity to increase while the moon moves from P_3 to P'_1, *fig.* 842., and to decrease while it moves from P'_1 to P_3. But it will be evident by comparing the lengths of these two arcs, and their distances from S, that the effect of the disturbing force will be much greater in the former than in the latter, and consequently, that the increase of the excentricity in the former must greatly exceed the decrease in the latter, and therefore, that the effect of the radial component in an entire synodic revolution will be to increase the excentricity.

Secondly. It appears from what has been explained that the effect of a positive tangential component will be to decrease the excentricity while the moon moves from *m* to *n*, and to increase it while it moves from *n* to *m*, and that a negative tangential component will have contrary effects. By combining this with the changes of sign explained in the present chapter, it will follow that the excentricity will be increased while the moon moves from P_2 to *m*, from O to P'_2, and from *n* to C, and that it will be decreased while it moves from C to P_2,

from m to O, and from P'_2 to n, and by comparing these arcs both with relation to their extent and their distance from S, it will be apparent that the increase of the excentricity must exceed the decrease, and that consequently the result of a whole synodic period will be to cause the excentricity to increase.

It appears, therefore, that both components of the disturbing force in this position of the line of apsides will cause the excentricity to increase during each synodic revolution, being subject nevertheless during such revolution to alternate increase and decrease.

3218. *When perigee is 54° 44′ 7″ behind opposition.*—*First.* By the effects of the radial component it may be shown, as before, that the excentricity will constantly increase while the moon moves from P_1 to P_3', *fig.* 843., and will continually decrease while it moves from P_3' to P_1, and it will be obvious, from considering the magnitude of these arcs and their distances from S, that the decrease of the excentricity will considerably exceed the increase; and that, therefore, this component of the disturbing force will, during an entire revolution, cause the excentricity to decrease.

Secondly. By the effects of the tangential force, it may be shown, as before, that the excentricity will be increased while the moon moves from P_2 to O, from m to P'_2, and from C to n, and that it will be decreased while the moon moves from n to P_2, from O to m, and from P'_2 to C, and by comparing these arcs it will be obvious that the total decrease will exceed the total increase.

It results therefore from this, that both components of the disturbing force in this position of perigee causes the excentricity to decrease in a complete synodic revolution.

3219. *When perigee is 54° 44′ 7″ before opposition.*—*First.* It may be shown as before, that the radial component will cause the excentricity continually to increase while the moon moves from P'_1 to P_3, *fig.* 844., and to decrease continually while it moves from P_3 to P'_1, and by comparing these arcs as before, it will be apparent that the increase greatly exceeds the decrease, and therefore, so far as relates to the radial component, the excentricity during each synodic revolution will suffer an increase.

Secondly.—By the effects of the tangential component, the excentricity will increase while the moon moves from C to P_2,

from n to O and from P'_2 to m, and will decrease while it moves from P_2 to n, from O to P'_2 and from m to C, and by comparing these arcs as before, it will be apparent that the increase will exceed the decrease.

It follows, therefore, that the effect of both components in this position of the apsides during an entire synodic revolution is to cause the excentricity to increase.

3220. *When perigee is* 54° 44′ 7″ *behind conjunction.*— *First.* In this case it may be shown that the radial component will cause the excentricity to decrease while the moon moves from P_1 to P'_3, *fig.* 845., and to increase while it moves from P'_3 to P_1, and by comparing these arcs, it will appear, as before, that the decrease will greatly exceed the increase, and that therefore on the whole in each synodic revolution the radial component will cause the excentricity to decrease.

Secondly. By the effect of the tangential force, the excentricity will increase while the moon moves from m to P_2, from O to n, and from P'_2 to C, and will decrease while it moves from P_2 to O, from n to P'_2 and from C to m, and by comparing these arcs as before, it will be apparent that the decrease will exceed the increase.

It follows, therefore, that the result of the whole disturbing force in this position of the apsides is to cause a decrease of the excentricity in each synodic revolution.

3221. *Extreme and mean values of excentricity.* It appears therefore generally, that when perigee is in conjunction, the excentricity does not vary, and that as the point of conjunction recedes from perigee, the excentricity decreases, and continues to decrease until the point of conjunction is 90° from perigee, when the excentricity again becomes stationary, its decrease ceasing. When the conjunction advances more than 90° from perigee, the excentricity again increases and continues to increase until the point of conjunction has moved 180° before perigee, when the increase ceases. After this, the excentricity again decreases, and continues to decrease until the point of conjunction gains another quadrant on perigee, when the increase ceases and the decrease commences, which is continued through another quadrant. It appears, therefore, that the excentricity is a maximum when the apsides are in quadrature, and a minimum when they are in conjunction, and that consequently it gradually increases while the apsides move from con-

junction to quadrature, and gradually decreases while they move from quadrature to conjunction.

If e' express the value of the excentricity while the apsides are in quadrature, and e'' their value when in conjunction, the mean value being e, it is found that

$$e' : e : e'' = 1{\cdot}50 : 1{\cdot}25 : 1{\cdot}00;$$

so that the extreme range of variation of the excentricity of the moon's orbit is as 3 to 2.

EFFECTS OF THE DISTURBING FORCE UPON THE LUNAR NODES AND INCLINATION.

From what has been proved in general (3158. *et seq.*), it will appear that when the moon is less distant than the earth from the sun, the orthogonal component of the disturbing force will have a tendency to draw it out of the plane in which it is moving towards the side on which the sun is placed, and that when the moon is more distant than the earth from the sun, the orthogonal component will have a tendency to draw it out of the place in which it moves to the side opposite to the sun.

But it has also been proved that the nodes will have a progressive or regressive motion according to the direction of the orthogonal force in the successive quadrants of its orbit between node and node.

It will, therefore, be necessary to consider, successively, the effects of the disturbing force in the various positions which the lines of syzygies and quadratures may assume with relation to the line of nodes.

FIRST CASE.

3222. *When the line of syzygies is in the line of nodes.* — In this case, it is evident that the orthogonal component of the disturbing force will be nothing, since the whole attraction of the sun is in the plane of the moon's orbit, and, consequently, no part of the attraction can act at right angles to that plane.

Second Case.

3223. *When the line of nodes is in quadrature.*— Let ACOA (*fig.* 846.) represent the moon's orbit, and ADA the

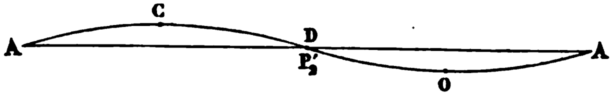


Fig. 846.

ecliptic seen in its own plane and projected into a straight line. Let A be the ascending and D the descending node, and let the points of syzygies be supposed to be at C, and O the points of the moon's orbit most distant from the ecliptic, while the points of quadrature P_2 and P_2' are at the nodes A and D.

It has been already shown, that when the moon is in quadratures the whole disturbing force is radial, and being directed to the sun and in the plane of the moon's orbit, it can have no component perpendicular to that plane, and therefore the disturbing force at these points can have no tendency to change the plane of the moon's orbit.

While the moon moves from A through C to D, its distance from the sun being less than that of the earth, and the sun being supposed to be below the plane of the orbit, the orthogonal component of the disturbing force will everywhere have a tendency to draw the moon nearer to the plane of the ecliptic A B; and, according to what has been proved (3161.), it follows that, through the entire semicircle A C D, the orthogonal component will impart a regressive motion to the line of nodes, and from A to C, it will cause the inclination to decrease, and from C to D to increase.

While the moon moves from D through O to A, being at a greater distance than the earth from the sun, the orthogonal component has a tendency to repel the moon further from the sun, which, in this case, will have the effect of drawing the moon from the semicircle D O A towards the plane of the ecliptic D A; and, consequently, according to what has been proved (3161.), its effect will be to impart a regressive motion to the line of nodes throughout the semicircle D O A, and to cause the inclination to decrease from D to O, and to increase from O to A.

Thus it appears that, in this position of the lines of quadrature and syzygies, a continual motion of regression is imparted to the nodes, except at the points A and D, where the effect is nothing. Commencing from A, the regression of the nodes continually increases with the increase of the orthogonal force, until the moon arrives at C, when the regression is a maximum; it then decreases and continues to decrease until the moon arrives at D, where it vanishes; it again increases from D to O, where it is again a maximum, and decreases from O to A, where it vanishes.

The change of inclination produced by the disturbing force being nothing at A, the inclination decreases continually from A to C, and increases from C to D. It is, therefore, a minimum at C. After passing D, it decreases from D to O, and, consequently, is a maximum at D. After passing O, it increases from O to A, and is, consequently, a minimum at O.

Thus it appears that the inclination is least at conjunction and opposition, and greatest at quadratures, — that is, in the present position of the lines of syzygies and quadrature, it is least when the moon's latitude is greatest, and greatest when the moon's latitude is nothing.

THIRD CASE.

3224. *When the line of syzygies is less than* 90° *before the line of nodes.*—This case is represented in fig. 847., where, as

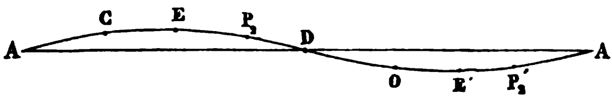


Fig. 847.

before, A is the ascending and D the descending node, C and O being the extremities of the line of syzygies, and P_2 and P'_2 those of the line of quadratures, the points E and E′ being those at which the moon's orbit is most distant from the ecliptic, and, therefore, the middle points of the semicircles A D and D A.

From what has been explained, it is evident that the orthogonal component at P_2 and P'_2 will be nothing, since at these points the disturbing force is radial; and it has been already shown that at the nodes A and D the orthogonal component is also nothing.

While the moon moves from P'_2 through A and C to P_2 the

disturbing force tends to draw it from the plane of its orbit towards the ecliptic, and while it moves from P_2 through D and O to P'_2 the disturbing force tends to draw it up from the plane of her orbit; therefore in moving from P'_2 to A the disturbing force draws it from the plane of the ecliptic, and while it moves from A to P_2 through C and E the disturbing force draws it towards the plane of the ecliptic.

While the moon moves from P_2 to D it draws the moon from the plane of the ecliptic, and while it moves from D through O and E′ to P_2 it draws it towards the plane of the ecliptic.

By combining these results with what has been proved in 3161., it will follow that while the moon moves from A to P_2 and from D to P'_2 a regressive motion, and while it moves from P_2 to D and from P'_2 to A a progressive motion, is imparted to the line of nodes. Now it will appear by inspection of the figure, that the arc A P_2 is greater than the arc P_2 D, and that the arc D P'_2 is greater than P'_2 A, and, consequently, it follows that the sum of the arcs through which a retrograde motion is imparted is much greater than the sum of the arcs through which a progressive motion is imparted, and consequently, after the synodic revolution has been completed, the line of nodes on the whole will have retrograded.

It follows, also, from what has been stated that from P'_2 to E and from P_2 to E′ the inclination will decrease, and from E to P_2 and from E′ to P'_2 it will increase. But since the arcs P'_2 E and P_2 E′ are respectively greater than E P_2 and E′ P'_2, the sum of the former will be greater than the sum of the latter, and consequently the arcs through which the inclination decreases being much greater than those through which it increases, there will be on the whole a decrease of the inclination after the synodic revolution has been completed.

Fourth Case.

3225. *When the line of syzygies is more than 90° before the line of nodes.*—In this case while the moon moves from P'_2 to P_2, *fig.* 848., the disturbing force has a tendency to draw it down

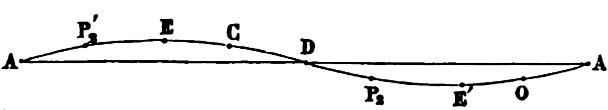


Fig. 848.

towards the ecliptic, and from P_2 to P'_2 it has an opposite effect. It follows, therefore, that while the moon moves from P'_2 to D the line of nodes regresses, and while it moves from D to P_2 it progresses. While it moves from P_2 to A it regresses, and while it moves from A to P'_2 it progresses.

By comparing the lengths of these arcs, as before, it will appear that those where regress is produced exceed those where progress is produced, and it follows, therefore, that in an entire synodic revolution the line of nodes regresses. It also follows that while the moon moves from P'_2 to E and from P_2 to E′ the inclination is decreased, and while it moves from E to P_2 and from E′ to P'_2 the inclination is increased, and by comparing the magnitudes of these arcs it will be evident that in the entire synodic revolution an increase of the inclination takes place.

The same conclusions would follow if the moon's orbit were supposed to be inclined to the ecliptic in the other direction.

Thus it appears in general that when the line of nodes is in syzygies no change takes place either in the position of that line or in the magnitude of the inclination. While it passes from syzygies to quadrature the line of nodes regresses and the inclination diminishes; when it is in the line of quadratures the line of nodes regresses, but the inclination is unchanged; and when it is between quadratures and syzygies the line of nodes still regresses and the inclination is increased. Thus when the sun has described in its apparent motion nearly one half a revolution of the ecliptic, there is, on the whole, a regression of the node and an alternate increase and decrease of the inclination; and during its motion through the other half of the ecliptic similar changes are produced in the same order. It follows, that the inclination is a maximum when the line of nodes is in syzygies, and a minimum when it is in quadratures.

3226. General summary of the lunar inequalities.— From all that has been stated in the present chapter it appears that the principal lunar inequalities are as follows;—

1°. The annual equation, which depends on the variation of the disturbing force due to the varying distance of the earth from the sun in its elliptic orbit.

2°. The variation, which depends on the difference of the disturbing force arising from the synodic place of the moon.

3°. THE ACCELERATION OF THE MOON'S MEAN MOTION, depending on the effect produced upon the disturbing force by the secular variation of the excentricity of the earth's orbit.

4°. THE PARALLACTIC INEQUALITY, depending on the difference between the disturbing forces of the sun in conjunction and opposition.

5°. THE EQUATION OF THE CENTRE, an inequality which, however, cannot properly be called a perturbation, inasmuch as it depends only on the elliptic form of the lunar orbit, which would subsist without any disturbing force.

6°. THE ALTERNATE PROGRESSION AND REGRESSION OF THE APSIDES, depending on the synodic place of the moon.

7°. THE MEAN PROGRESSION OF THE APSIDES, being the excess of the progression over the regression during a synodic period.

8°. THE VARIATION OF THE EXCENTRICITY, depending on the synodic place of the moon.

9°. THE ALTERNATE REGRESSION AND PROGRESSION OF THE NODES, arising from the effect of the orthogonal component of the disturbing force.

10°. THE MEAN REGRESSION OF THE NODES, arising from the excess of the regressive over the progressive motion during the synodic revolution.

11°. THE ALTERNATE INCREASE AND DECREASE OF THE INCLINATION.

The combined effects of the seventh and eighth of the preceding inequalities depending on the synodic position of perigee are called by the common name, *evection*. This is the greatest inequality to which the moon's place is subject, producing a variation in the moon's longitude, the extreme range of which is $2\frac{1}{2}°$. It was discovered by observation about the year A.D. 140, by Ptolemy.

The inequality arising from the alternate regression and progression of the nodes and the alternate increase and decrease of the inclination were discovered by Tycho, about the year 1590. This is the greatest of the inequalities which affect the moon's latitude. Its range, however, is limited to about 16′.

3227. *Other lesser inequalities.*—The preceding inequalities, numerous as they may seem, are nevertheless only the principal effects of lunar perturbations. There are many others which

depend on differences of intensity of the disturbing force, which have not here been taken into account; for example, the rate of the progression of the apsides, as well as the diminution of the lunar orbit, is affected by the difference of the intensities of the disturbing force at conjunction and opposition.

The variation of the intensity of this force due to the excentricity of the earth's orbit, affects also, to a sensible extent, the variation of the motion of the apsides, and the variation of the excentricity.

The parallactic inequality is also affected by the position of the moon in relation to the apsides of the earth's orbit.

There are also several small inequalities affecting the plane of the moon's orbit, depending on the excentricity of the orbits of the earth and moon.

In fact, the number of corrections, or equations as they are called, which are applied to the moon in the computation of its true place are not less than forty.

CHAP. XXII.

Theory of the Jovian System.

3228. *Analogy of the Jovian to the terrestrial system.*— What the moon is to the earth, each of the Jovian satellites is to Jupiter, and the sun stands in the same physical relation to both these systems.

It might, therefore, be expected that the same series of inequalities which arise from the disturbing force of the sun acting on the earth and moon, would be equally produced in the case of each of Jupiter's satellites, and that such disturbances do act and that like inequalities are produced cannot be doubted.

3229. *Why the same inequalities are not manifested.*— But when we come to calculate the quantities of these inequalities in the case of Jupiter, they are found to be so utterly insignificant in their numerical values, that they are altogether incapable of being appreciated by the nicest ob-

servation, except in the case of the fourth satellite, in whose motions inequalities of very minute amount, analogous to the moon's variation, evection, and annual equation are barely observable, the inequality corresponding to the annual equation in this case amounting to no more than 2′, and the other inequalities being much less.

The cause of this insignificant amount of the disturbing force of the sun will be easily understood.

The whole Jovian system subtends at the sun a visual angle less than one-half the apparent diameter of the sun as seen from the earth, and consequently, lines drawn from the sun to all points in that system will be practically parallel, and with the exception of the fourth satellite, as already mentioned, the variation of the distances of the different satellites from the sun is so utterly insignificant, compared with the whole distance, that the corresponding variation of the intensity of the sun's attraction upon the satellites and the central body is so minute as to produce no perceptible disturbing effect. In a word, the sun's attraction upon the Jovian system may be regarded as a force acting with equal intensities in parallel lines on all parts of the system, exactly as the force of gravitation would act upon any small group of heavy bodies placed near the surface of the earth.

3230. *Mutual perturbations of the satellites.*—In this secondary system, therefore, contrary to what might be expected, there is no analogy whatever to the lunar theory, and all the perturbations which are observable are those due to the mutual gravitation of the four satellites one upon another.

The investigation of these perturbations is greatly simplified by the following conditions which prevail in the system:

First. That the undisturbed orbits of all the satellites are very nearly circular, those of the first and second being exactly so;

Secondly. That they are very nearly in the common plane of the planet's equator; and

Thirdly. That the mean motions of the three inner satellites are commensurable in the remarkable manner already expressed (2762.)

3231. *Retrogression of the lines of conjunction of the first three satellites.*—As some of the most remarkable consequences of the mutual disturbing forces in this system depend upon the

relation between the mean motions of the three inner satellites just mentioned, we shall, in the first instance, explain the effect of this relation upon the successive positions assumed by their lines of conjunction.

Let the three inner satellites be expressed by s', s'', s''', and their periods by P, P'', P'''.

By the line of conjunction of any two satellites is to be understood that line which would be drawn through their places from the centre of Jupiter, where they have the same direction as seen from that centre.

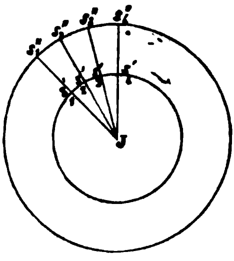


Fig. 849.

Thus, if J, *fig.* 849., be the centre of the planet, and s'_1 that of the first satellite, the second satellite will be in conjunction with it, if it be at s_1''. Now if the two satellites move each in its proper orbit, in the direction of the arrows from this position, the angular motion of the first will be $\frac{360°}{P'}$ and that of the second $\frac{390°}{P''}$ and since P' is less than P'', the latter angular motion will be more rapid than the former, and the first satellite will continually gain upon the second, and after the lapse of the interval called their synodic period, the first will overtake the second, and they will be again in conjunction.

The new direction of their line of conjunction, relatively to the former, will depend upon the relation which subsists between their periodic times, and consequently between their mean motions. Now it appears that the mean motions of the first and second are very nearly, though not exactly, in the proportion of 2 to 1. By reference to the tabular synopsis of the elements of the Jovian system, in (2999.), it will be seen that the proportion of their periodic times is as 1769 to 3551, that is, as 1000 to 2007. It follows, therefore, that the mean motion of the first satellite is a little more than twice as rapid as the mean motion of the second. If the mean motion of the first were exactly twice the mean motion of the second, the first would make two complete revolutions while the second

would make one; and therefore, the second having revolved once in its orbit, and returned to s''_1, the first would have revolved twice and would also have returned to s'_1, and in this case, their line of conjunction would always have the same fixed direction $J\ s'_1\ s''_1$. But since the periodic time of the first is a little less than half that of the second, the first will overtake the second before it has quite completed two revolutions, and the consequence will be, that their next line of conjunction $J\ s'_2\ s''_2$ will be behind the former; so that in a single synodic revolution, their line of conjunction will have retrograded through the angle $s''_2\ J\ s''_1$, and in the same manner, in another synodic period, it will have retrograded through an equal angle, and will assume the direction $J\ s'\ s''_3$, and in the same manner at every successive conjunction it will have retrograded through an equal angle, the mean motion of the satellite being supposed to remain the same.

3232. *Change of direction of line of conjunction in each synodic revolution.*—To determine this, let ϕ be the angle formed by the line of conjunction at the termination of each revolution with the direction it had at the commencement, such angle being measured from the latter position, in the direction of the motion of the satellites, so that, in fact, ϕ will express the increase of longitude which the line of conjunction may receive in the synodic period. Let T' express the synodic period of the satellites s' and s''. We shall then have (2589.)

$$\frac{1}{T'} = \frac{1}{P'} - \frac{1}{P''} \qquad T' = \frac{P''P'}{P''-P'};$$

and since s'' moves through $\frac{360^\circ}{P''}$ in the unit of time, and the advance of the line of conjunction in the time T' is equal to the angle through which s'' moves in that time, we shall have

$$\phi' = \frac{360^\circ}{P''} \times T = 360^\circ \times \frac{P'}{P''-P'}.$$

In like manner, if ϕ'' express the same angle for the line of conjunction of the satellites s'' and s''', we shall have

$$\phi'' = 360^\circ \times \frac{P''}{P'''-P''}.$$

3233. *Application to the three inner satellites.*—Now in the case of these three satellites we have by (2999.) Table V.

$$P' : P'' : P''' = 17691 : 35512 : 71546,$$

from which it appears that

$$\frac{P'}{P''-P'} = 0\cdot9927 \qquad \frac{P''}{P'''-P''} = 0\cdot9855\,;$$

and consequently

$$\phi' = 357^\circ\cdot37 \qquad \phi'' = 354^\circ\cdot78.$$

It appears, therefore, that in each synodic revolution of s′ and s″, their line of conjunction advances through 357°·37, and is, therefore, 2°·23 *behind* its first position; and that in the case of s″ and s‴, the corresponding line advances through 354°·78, and is, therefore, 5°·12 *behind* its first position.

3234. *Regression of the lines of conjunction of the three satellites equal.*—But the synodic periods T and T′, of s′, s″ and s″, s‴ are (2577)

$$T' = 3\cdot525 \qquad T'' = 7\cdot050\,;$$

and consequently the angles of regression of the two lines of conjunction in the same time are as

$$2\cdot23 \times 7\cdot050 : 5\cdot12 \times 3\cdot050 :: 157 : 156,$$

so that *the rate of regression of the two lines of conjunction is the same.*

3235. *Line of conjunction of the first and second in opposition to that of the second and third.*—It follows from this that the two lines of conjunction, thus regressing at the same rate, must always be inclined to each other at the same angle. Now it is found by observation, that this invariable angle is 180°, so that *the line of conjunction of the first and second satellites is always in immediate opposition to the line of conjunction of the second and third satellites, as seen from the planet.*

We shall now see the remarkable consequences of these relations in their effects upon the mutual perturbations of the satellites.

3236. *Effects of their mutual perturbations upon the forms of their orbits.*—The undisturbed orbits of the first and second satellites are sensibly circular, the excentricity of the orbit of

the third being extremely small (2999.), Table V. Their mutual disturbing forces render the orbits elliptical. The major axis of the ellipses are the same as the diameter of the undisturbed orbits, which are derived from the periodic times by the harmonic law. The forms of the orbits, or, what is the same, their excentricities, are invariable, but their lines of apsides are moveable.

3237. *Motion of the apsides equal to that of the lines of conjunction.*—The motions imparted to the apsides being exclusively the effects of the disturbing force, and that force being most effective where the satellites are in conjunction, and varying in its intensity and direction with the angular distance of the disturbed from the disturbing, as seen from the central body, it is evident that the position of the line of apsides must be always the same in relation to the line of conjunction, and, consequently, that the motion of the lines of apsides must be the same as that of the lines of conjunction, both as to rate and direction. The line of apsides of s' and s'', and that of s'' and s''', must, therefore, have a regressive motion exactly equal to that of the lines of conjunction; and since the motions of the latter are equal, the motions of the lines of apsides of the three orbits must likewise be equal.

3238. *The lines of apsides coincide with the lines of conjunction.*—In the exposition of the general theory of perturbations, it has been demonstrated that when the disturbed and disturbing bodies are in conjunction, the excentricity of the disturbed orbit varies, if the line of apsides be inclined to that of conjunction, and is only invariable when these lines coincide. Now in the present case, the excentricities of the disturbed orbits are subject to no variation; and it follows, consequently, that the lines of apsides of the disturbed orbits must always coincide with the lines of conjunction.

3239. *Positions of the perijoves and apojoves of the three orbits.*—But this being admitted, the apsides may be presented in either of two opposite directions. If we consider s' as disturbed by s'', either the perijove or apojove of s' (as the apsides of the satellites are called) may be in conjunction with s''. It results, however, from what has been proved, that if the perijove be in conjunction, the disturbing force of s'' will render the apsides of s' regressive, and if the apojove be in conjunction, it will render that motion progressive. But

since the motion of the apsides of s′ is regressive, the perijove must be in conjunction.

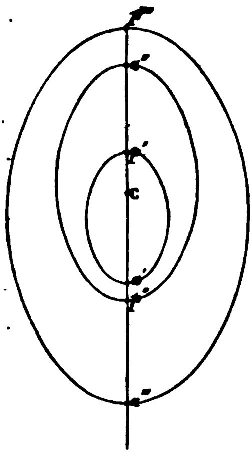


Fig. 850.

If we consider s″ disturbed by s′, we have the case of an exterior disturbed by an interior body, the latter being at a distance from the centre greater than half that of the exterior body. In this case, the motion imparted to the apsides of s″ would be progressive, if s″'s perijove were in conjunction, and regressive if its apojove were in that position. But since the motion of the apsides is actually regressive, the apojove of s″ must be in conjunction with s′.

In the same manner it may be shown, that in consequence of the disturbing forces mutually exerted by s″ and s‴, the former must be in perijove and the latter in apojove when in conjunction.

By combining these consequences with the relative positions of the lines of conjunction of s′ s″ and of s″ s‴ already indicated (3235.), it will be apparent that the perijove of s′ and the apojove of s″, when s′ and s″ are in conjunction, are in opposition with the perijove of s″ and the apojove of s‴, when s″ and s‴ are in conjunction, so that the relative position of the three orbits in this case is that which is represented in *fig.* 850., where p' is the perijove, and a' the apojove of s′, p'' and a'' those of s″, and p''' and a''' those of s‴.

3240. *Value of the excentricity.* — Since the motion of the apsides and the excentricity are exclusively due to the disturbing force, which in this case is given, the constant value of the excentricity will so depend on the motion of the apsides, that the latter being given, the former may be determined. Now the regressive motion of the apsides being exactly equal to that of the line of conjunction is known, and therefore the value of the excentricity can be determined.

The excentricity which thus arises exclusively from the agency of the disturbing forces, though less than the excentricities of the undisturbed orbits of the planets generally, is

nevertheless not inconsiderable, exceeding, for example, that of the orbit of Venus.

3241. *Remarkable precision in the fulfilment of these laws.*—These remarkable laws, of which there is no other example in the solar system, are fulfilled with such precision, that in the thousands of revolutions of the satellites which have taken place since their discovery, not the smallest deviation from them has ever been observed, except such as has arisen from the slight ellipticity of the undisturbed orbit of the third satellite. The greatest and most irregular perturbations of the planet or the satellites, provided they come on gradually, do not interrupt the play of these laws, nor change the relation of the motions resulting from them. The effect of a resisting medium will not affect them, though each of these causes would alter the motions of all the satellites, and though similar causes would wholly destroy the conclusions which mathematicians have drawn as to the stability of the solar system, with regard to the elements of the orbits of the planets.*

3242. *Effects of the excentricity of the undisturbed orbit of the third satellite.*—In the preceding paragraphs, the orbit of the third satellite s''' is considered as having no other excentricity except that which proceeds from the effects of perturbation. It has, however, a certain small original excentricity independent of these effects, the consequence of which is, that when its conjunction with s'' takes place near the perijove of its undisturbed orbit, the effect of its disturbing force on s'''s orbit is somewhat more considerable than in other synodic positions. The consequence of this is to produce a small variation both in the excentricity of s'''s orbit, and in the motion of its apsides, which variation will depend on the elongation of the line of conjunction from the apsides of s''''s undisturbed orbit.

The variation of the angular motion of s''', consequent on the small ellipticity of its orbit, also produces a corresponding variation in the rate of the regression of the line of conjunction.

Similar irregularities are incidental to the orbit of s''', proceeding from the same causes. The disturbing force excited by s'' on s''', being that of an interior upon an exterior body,

* See Airy on Gravitation.
The physical explanation of these laws was first given by Laplace, in 1784.

(3180.), has on the whole the effect of increasing the effective attraction of the central body; and this effect is chiefly due to the action when s″ and s‴ are at or near the line of conjunction; consequently, where that line is near the perijove of s‴'s undisturbed orbit, the disturbing force of s″ is most effective, and, as that line revolves, the angle under it and that of the apsides of s‴'s undisturbed orbit continually varying, the effect of s″'s disturbing force alternately increases and decreases. This is attended with an irregularity in the major axis, and consequently in the mean motion of s‴ which depends on its synodic position.

The disturbing force of an exterior exerted on an interior body tends, on the whole, to diminish the effective attraction of the central body (3177.). It follows, therefore, that s‴ exerts upon s″ a disturbing force which produces an irregularity depending on the synodic position of the perijove of s‴'s undisturbed orbit.

Each of the small inequalities noticed above, depending on the excentricity of s‴'s undisturbed orbit reacting on the other satellites s′ and s″, produce corresponding small inequalities, which if attempted to be introduced in the general theory of the system would render it extremely complicated. Their almost infinitely small amounts, however, render them comparatively unimportant.

3243. *Perturbations of the fourth satellite.*—The theory of the perturbations produced and sustained by the fourth satellite s⁗ has nothing in common with that of the three others, inasmuch as no such remarkable commensurability prevails between its mean motion and those of the others. As it sustains small inequalities from the action of the sun similar to the lunar disturbances, so it also produces and sustains a system of inequalities analogous to those of the planets. Thus this satellite s⁗ presents at once an example on a small scale of the application of the principles of both the lunar and the planetary theories.

To explain the theory of the fourth satellite we shall first suppose that the undisturbed orbit of s‴ is circular, while that of s⁗ has a sensible excentricity. We shall assume (what will be more fully explained hereafter) that a slow progressive motion is imparted to its apsides by the spheroidal shape of Jupiter, this motion being such that the apsides make a complete revolution in about 11,000 revolutions of s⁗.

Owing to the periods not being nearly commensurable, like those of the inner satellite, the line of conjunction of s''' and s'''' will, after a few hundred revolutions of the satellites, have assumed every possible direction. Now, since the mutual disturbing action of the two satellites is greatest when the perijove of s'''' is at or near conjunction, the question is what will be the form of orbit that will be impressed on s''' by the disturbing force, subject to the condition of its excentricity being invariable.

Now it is evident, from all that has been explained, that if the perijove of s''' be in conjunction, the disturbing force of s'''' will cause its apsides to regress, and the rate of this regression will be greater as the excentricity is smaller (3151.). It may therefore be such as to neutralise the progressive motion which is imparted to s''''s apsides by the spheroidal shape of the central planet, and, therefore, such as to render the actual progressive motion of s''''s apsides equal to that of s'''''s. But the motion of s'''''s apsides will be also affected, though in a very slight degree, by the action of s''' in the same position, and will receive from that action a small increase of its progressive motion.

When the increased progressive motion of the line of apsides of s'''' is equal to the diminished progressive motion of the line of apsides of s''', this state of the system will be permanent, and thus the progressive motion of the apsides of s'''' will be somewhat increased, and the orbit of s''' will have a compression corresponding in direction to the perijove, and an elongation in the same direction as the apojove of s''''.

In this reasoning we have assumed that the undisturbed orbit of the third satellite is circular, but similar effects will ensue if it have a small excentricity.

Let us next suppose that the undisturbed orbit of s'''' is circular, while that of s''' has a small excentricity. The disturbing force will be the greatest at the apojove of s''', and this will cause the line of apsides of s''' to progress; that is to say, it will increase the progressive motion already given to it by the spheroidal shape of the central planet. If then it be required to determine the form of the orbit of s'''' which will have at every revolution the same excentricity, and also have its line of apsides always corresponding with that of s''', and therefore progressing more rapidly than the spheroidal shape

of Jupiter alone would make it do, it is necessary to suppose that the perijove of s'''' is turned towards the apojove of s''', and by supposing the excentricity small enough, the disturbing force will impart to it a progressive motion as rapid as may be desired. Thus the effect of the excentricity of the orbit of s''' is, that its line of apsides will progress rather more rapidly, and that the orbit of s'''' will be compressed on the side nearest the apojove of s''', and elongated on the opposite side.

We have here assumed that the undisturbed orbit of s'''' is circular, but a similar distortion will be produced even if it have a small excentricity.

In effect the undisturbed orbits of both satellites are ellipses of small excentricity, and the preceding conclusions expressed with reference to undisturbed circular orbits will be equally applicable to them.

Besides the excentricity of the undisturbed orbit of s'''', it has also an excentricity impressed upon it by the disturbing force, opposite in kind to that of s'''s orbit; and besides the excentricity of the undisturbed orbit of s''', it has impressed upon it an excentricity of the same kind as that of s''''.

In the same manner, the orbits of s' and s'' have small excentricities impressed upon them, similar in their kind to those of s''' and s''''.

3244. *Complicated perturbations of this system.*—The inequalities which have been here briefly noticed as produced in the Jovian system, by the mutual perturbations of the satellites, and which are only the principal inequalities of this system, are so closely connected, and so completely entangled, that, though they admit of being, for popular purposes, explained under the point of view here presented, it would not be possible to reduce them in this way to computation; a mathematical process of the most abstruse kind, which would, at the same time, include the motions of all the four satellites, would alone be sufficient for this purpose.

Enough, however, has been done if, in what has been said above, a general idea may be obtained of the theory of these disturbances in the most curious and complicated system that has ever been reduced to calculation.*

* We are indebted for the substance of some parts of this chapter to the short but excellent tract on Gravitation by Professor Airy, to which we refer readers who may desire further details.

CHAP. XXIII.

THEORY OF PLANETARY PERTURBATIONS.

3245. *The theory simplified by those of the moon and the Jovian system.*—The investigation and solution of the more general and complicated cases of perturbation presented by the mutual action of the planets, will be greatly simplified and facilitated by the previous exposition of the theories of the moon and the Jovian system. The inequalities developed in each of these, are reproduced in very slightly modified forms, in the case of the planets. Thus the terrestrial disturbed by the major planets, present a class of perturbations similar to those of the moon disturbed by the sun. In both cases the disturbing is exterior to the disturbed body; in both, the mass of the disturbing is incomparably greater than that of the disturbed; in both, the distance of the disturbing from the central body, bears a large ratio to that of the disturbed body; and if in the lunar theory the mass of the disturbing body be much larger than in the case of the planets, its distances from the disturbed and central bodies, bearing also a much larger ratio to the distance of these bodies from each other, the intensity of its disturbing force is subdued and brought into closer analogy with the cases referred to.

The inequalities incidental to the three inner satellites of Jupiter, depending on the near commensurability of their periods, have also counterparts among the perturbations of the planets, some of the most remarkable of the planetary inequalities arising from the circumstance of the periods being very nearly in the ratio of whole numbers, as will presently appear.

In fine, other inequalities produced by the gravitation of planet on planet, are analogous to those found to prevail between the outer satellites of Jupiter.

3246. *Perturbations of the terrestrial by the major planets.*— If we suppose any one of the terrestrial to be disturbed by any one of the major planets, it will be easy to show that the points at which the disturbed planet and the sun are equidistant from the disturbing planet, and at which, therefore, the tangential component of the disturbing force vanishes, are in all cases very near the points of quadrature.

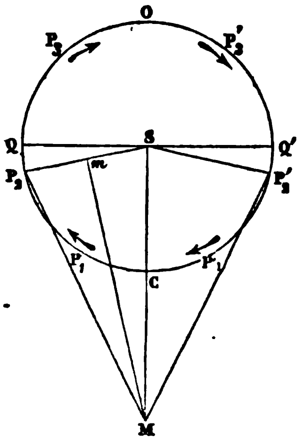


Fig. 851.

Let S, *fig.* 851., be the place of the sun, M the disturbing planet, QQ′ the places of the disturbed planet at quadrature, and P_2 P'_2 its places when at distances from M equal to SM. Let the arc QP_2, or the angle QSP_2, be expressed by a, SP_2 by r and SM by r', and draw Mm perpendicular to SP_2. It is evident, then, that the angle SMm $= a$, and consequently

$$\sin. a = \frac{Sm}{SM} = \frac{r}{2r'}.$$

Now, if the values of r and r' in each of the cases of the terrestrial and major planets be substituted, we shall find that the extreme values of a will be, for Mars disturbed by Jupiter, $a = 7° 21'$, and for Mercury disturbed by Neptune, $a = 0° 44'$, and for all other cases it will have intermediate values.

It follows, therefore, that in the cases here referred to, the points P_2 P'_2 are never so much as 8° removed from the points of quadrature.

By the application of the same process of investigation as that already adopted in the case of the lunar theory, it will be found that the points P_1, P_3, P'_3 and P'_1, will have positions very nearly the same as those assigned to them in the case of the moon.

It follows, therefore, that the several vanishing points of the components of the disturbing force, on the position of which the successive phases of the perturbations so mainly depend, are distributed around the synodic orbit of the terrestrial planets as disturbed by the major planets in a manner similar

in all respects, to the corresponding points in the lunar theory and periodical inequalities are accordingly developed in a like order and of a like character, differing only in their limiting magnitudes and the lengths of their periods.

Thus the disturbed orbit is less curved at C and O, and more so at P_2Q and P'_2Q', than elsewhere, so as to acquire an oval form, placed with relation to the line CO similarly to that of the moon, (3196.). Its curvature at C is more flattened than at O, (3198.). Inequalities affecting the place of the planet in approaching to and departing from syzygies, result from this, similar to the moon's variation, parallactic inequality, and annual equation.

The line of apsides is affected with a motion alternately progressive and regressive, but on the whole progressive (3215.). The disturbed orbit is rendered a little more excentric, when this line is in quadrature than when it is in syzygies. The effect of the disturbing force on the whole is, as in the case of the moon, to diminish the effective central attraction, and therefore to enlarge in a slight degree the orbit; and this effect is, of course, somewhat greater when the disturbing planet is near perihelion, while the disturbed planet is near aphelion.

It must, nevertheless, be observed, that these and other like periodical inequalities arising from similar causes, are not only smaller incomparably in magnitude, taken within their extreme limits, and slower in their rate of development, than in the case of the lunar perturbations, but that their absolute limits are so extremely narrow, that it is only those which are due to the predominant mass and greater proximity of Jupiter, which are productive of effects great enough to be appreciable by common observations.

3247. *Cases in which the disturbing is in closer proximity with the disturbed planet.* — In such cases the same close analogy to the lunar inequalities does not prevail. Nevertheless, even when the disturbing planet, being exterior to the disturbed, lies in comparatively close proximity with it, several of the inequalities manifested in the lunar motions, may still be recognised in a modified form. The vanishing points of the components are somewhat differently distributed in relation to the lines of syzygy and quadrature. The points P_2, P'_2 of equidistance from the disturbing body recede from quadrature and approach the point C of conjunction, and the vanishing

points of the radial components P_1 P'_1 approach conjunction C, while P_2 P'_2 approach opposition O. The disturbing force, however, still has a tendency to diminish the curvature of the orbit near C and O, and to increase it near P_2 and P'_2. The general effect is, as before, to diminish the effective central attraction, and consequently to enlarge the orbit of the disturbed planet.

3248. *Case in which the disturbing is within the orbit of the disturbed planet.* — In these cases the general effect of the disturbing force may be traced without difficulty, by the method explained in (3180.). In these cases the general effect of the disturbing force is to augment the effective central attraction, and consequently to diminish the magnitude of the orbit of the disturbed planet.

3249. *Perturbation affected by the position of the apsides and nodes in relation to the line of conjunction.* — In the general investigation of the planetary perturbations it is necessary to observe, that the effect produced by the disturbing force in each synodic revolution will necessarily depend on the position of the line of syzygies in relation to the lines of nodes and apsides, and will vary with that position.

If the orbits of the disturbing and disturbed planets were both circles, and in a common plane, the effect produced by the disturbing force in each synodic revolution, and in each synodic position of the planets, would be absolutely the same, whatever be the direction of the line of syzygies; for in that case the

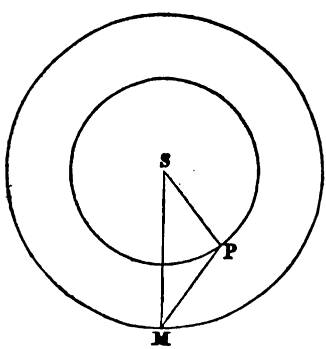


Fig. 852.

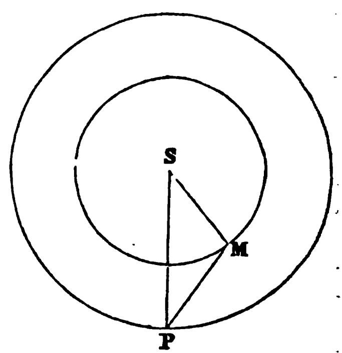


Fig. 853.

distances of M and P from S, *figs.* 852. 853., being always the

same, the distance M P between the disturbing and disturbed bodies, which corresponds to any angular distance M S P of one from the other as seen from the central body, would be always the same, and the angles at which M P is inclined to the lines M S and P S would be always the same. All the conditions, therefore, which can affect the intensity and direction of the disturbing force, would be absolutely identical; and it follows consequently that, no matter what may be the directions of the lines of syzygy and quadrature, the disturbing force during each synodic revolution would pass through precisely the same changes of intensity and direction, and consequently produce precisely the same effects upon the orbit of the disturbed planet.

If, however, the orbits, being still in a common plane, be either or both of them ellipses, the same identity of effects of the disturbing force during a synodic revolution will no longer prevail.

Let it first be supposed that the orbit of the disturbing planet M, *figs.* 854, 855., is circular; and that of the disturbed, elliptical. Let *p* be the point of perihelion, and *a* that of aphelion, *p'* and *a'* being the places of M corresponding to these points; and let *m*, *n* be the points at right angles to *p*, *m'* and *n'* being the corresponding positions of M.

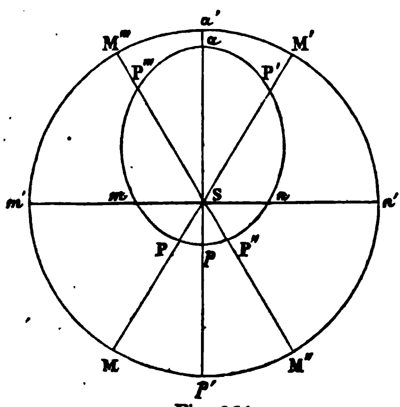


Fig. 854.

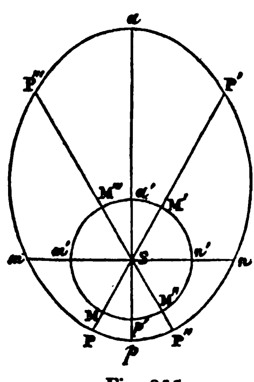


Fig. 855.

Upon comparing the varying distance of M, whether it be outside the orbit of P, as represented in *fig.* 854., or within it, as represented in *fig.* 855., it will be evident that the effects of the

disturbing force, during a synodic revolution, will be subject to a variation with the varying angle formed by the radius vector M S of the disturbing planet with the direction p S of the perihelion of the disturbed planet. Thus, when M, being outside P's orbit, is at a', it is evident that its distance from P, in any proposed synodic position, will be much less than its distance from P in the same synodic position when M is at p'; and consequently the effect of the disturbing force during a synodic revolution when M is at a', is much greater than when M is at p'; and the same may be said of any two opposite positions, such as M and M′, or M″ and M‴, which the disturbing planet can assume.

It is obvious, the like observations are applicable to the case represented in *fig.* 855., in which M is within the orbit of P.

But not only are the effects of the disturbing force different in their magnitude according as the radius vector of M takes different positions with relation to the line p S, but they are also different in their direction. The position of the vanishing points of the components of the disturbing force, and the distribution of the arcs through which they are alternately positive and negative round the orbit of the disturbed body, depend solely on the direction of the lines of syzygy and quadrature; but the effects which these components produce upon the different elements of the elliptic orbit of P, depend upon the position of these several arcs with relation to the line of apsides. In some positions the effect of the disturbing force in a complete synodic revolution, will be to augment, in others to diminish, one or other element; and in positions in which the same elements are augmented or diminished, they will be augmented or diminished in different degrees, according to the angle which the line of conjunction (that is, the line passing through P and M when seen in the same direction from S) forms with the line p S, connecting S with the perihelion of the disturbed orbit.

To simplify this explanation, we have here supposed that the orbit of the disturbing body is circular, while that of the disturbed is elliptical; but it will be apparent, that like observations, *mutatis mutandis*, will be applicable if, reversing this supposition, we suppose the orbit of P to be circular, and that of M elliptical.

In fine, if both orbits be supposed to be elliptical, a further

cause of variation will affect the disturbing force; for in that case, the distance between M and P, and the relative directions of the lines M P, M S, and P S, will vary, as well on account of the ellipticity of M's orbit as of that of P. The effects of this force on each of the elements will be subject to constant variation, depending on the angles which the line of conjunction forms with the lines drawn from S to the points of perihelion of the two orbits.

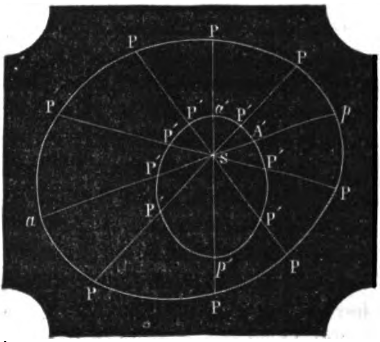


Fig. 856.

This will be very obvious by comparing the various positions which the line of conjunctions S P′ P of two such planets, mutually disturbing, may assume, with relation to the lines of apsides a S p and a' S p', as represented in *fig.* 856.

The orbits in the preceding illustration have been supposed to be in a common plane. If they be not so, but are inclined at any angle to each other, another cause of variation in the effects of the components of the disturbing force, during the synodic revolution, is introduced; and the entire effect of such force upon the elements, during each such revolution, will vary with the angle at which the line of conjunction is inclined to the line of nodes.

It will, therefore, be apparent that, in each position which the line of conjunction may assume with relation as well to the line of apsides as to the line of nodes, the disturbing force will, in each synodic revolution, produce a certain change, either by progression or regression, by increase or diminution in the elements severally of the disturbed orbit, the magnitude of which will depend on such position, so as to be always the same for the same position, but generally different for different positions; and when such positions are in extreme opposition, the effects of the elements severally are often also contrary in their character, so as mutually to destroy or compensate each other either wholly or partially, the progression or increase resulting from the effect of the disturbing force in one position

being compensated, wholly or partially, by an equal or nearly equal regression or decrease in the opposite position of the line of conjunction.

It follows from this that, if the motions of two planets were so related that the line of conjunction should always have the same position with relation to the lines of apsides and nodes, the effect of the disturbing force on each of the elements, in each synodic revolution, would be always the same; and the consequence would be that, after the lapse of a considerable number of such revolutions, the changes produced in each revolution accumulating, an alteration in the form and position of the disturbed orbit would be produced so great as completely to disturb the physical conditions of the planet and derange the harmony and order of the system.

But even though the place of the line of conjunction should not be rigorously the same after each successive synodic revolution, if nevertheless it be subject only to a small change of position, it is evident that the change in the character and magnitude of the effects of the disturbing force on the elements will be proportionally small; and that, therefore, such effects will continue to accumulate and to augment the variation of each of the elements of the disturbed orbit in the same direction, until by the long continuance of the slow change of position of the line of conjunction, that line at length shifts its direction so as to take up a position in which a contrary effect will be produced upon the elements. The variation of these latter will then change; what was previously increase will become decrease, and *vice versâ;* and this will continue until the line of conjunction, still slowly shifting its position, again resumes the direction favourable to the former change of the elements.

In this manner inequalities may be produced, of which the period may be of great length, but which, nevertheless, depending essentially on the direction of the line of conjunction, and therefore on the configuration of the disturbing and disturbed planets, are still *periodic*, and not *secular* variations. Since, however, the motions of the line of conjunction on which they depend, are, in all the cases of this class presented in the solar system, extremely slow, and consequently the periods of these inequalities are incomparably more protracted than those which arise from the varying synodic positions of the disturbed and disturbing planets, they have been denominated by astro-

nomers as the "*long inequalities;*" and the discovery of some of them by theory, before their detection by observation, has constituted one among the many triumphs of physico-mathematical science.

3250. *Method of determining the change of direction of the line of conjunction.* — From all that has been just explained it will be apparent, how much importance must attach to the problem to determine the change of position of the line of conjunction of any two planets after each synodic revolution.

Let P be the periodic time of the interior, and P′ that of the exterior; and let T express the synodic period or the interval between two successive conjunctions. It appears from what has been explained in (2577.), that

$$\frac{1}{T}=\frac{1}{P}-\frac{1}{P'};$$

and consequently,

$$T=\frac{P\times P'}{P'-P}.$$

In the time T, the interior planet describes 360°, besides overtaking the exterior planet, and therefore describes, in addition to 360°, the angle which the exterior planet describes in the time T; and since, at the beginning of the time T, the two planets are in conjunction, and again in conjunction at the end of that time, the angle formed by the direction of the line of conjunction at the end of the time T, with its direction at the beginning of the time T, measured in the direction of the planet's motion, will be the angle which the exterior planet describes in the time T. Let this angle be ϕ. Since the angle which the exterior planet describes in the unit of time is $\frac{360°}{P'}$ (2568.), the angle ϕ, which it describes in the time T, will be

$$\phi=\frac{360°}{P'}\times T = 360°\times\frac{P}{P'-P}.$$

This is, then, the angle, measured in the direction of the planet's motion, through which the line of conjunction advances in each synodic revolution.

3251. *Condition under which the direction of the line of conjunction is invariable.*—If the line of conjunction has always the same direction, it is evident that in a synodic revolution both planets must have made a complete number of revolutions, and consequently the angle ϕ must either be $360°$ or some exact multiple of $360°$. If $\phi = 360°$ we shall have $P = P' - P$ (2548), and, therefore, $P' = 2P$. In that case, while the exterior planet makes a single revolution, the interior makes two; so that, after each revolution of the exterior planet, the two planets come into conjunction always at the same point. In this case it is evident also, that the synodic time is equal to the periodic time of the exterior planet.

If $\phi = 2 \times 360°$, we shall have $P = 2P' - 2P$, and, therefore, $3P = 2P'$. In that case, while the exterior planet makes two complete revolutions, the interior makes exactly three, and the synodic period is equal to twice the periodic time of the exterior planet.

If $\phi = 3 \times 360°$, we shall have $P = 3P' - 3P$, and, therefore, $4P = 3P'$. In that case, therefore, the conjunctions are reproduced at the same point, after every three complete revolutions of the exterior planet.

In general, if $\phi = n \times 360°$, $P = n \times P' - n \times P$, and, therefore, $(n + 1) \times P = n \times P'$, and the conjunctions are reproduced constantly at the same point, after n revolutions of the exterior, and $n + 1$ revolutions of the interior planet.

The general condition on which the line of conjunctions shall have one invariable position, therefore, is that the periodic times of the two planets shall be such as can be exactly expressed by two whole numbers, of which the greater exceeds the less by 1, such as 1 and 2, 2 and 3, 3 and 4, &c.

3252. *To determine the condition under which the line of conjunctions shall have a limited number of invariable positions.*—Although the conjunctions may not be always reproduced at the same point, they may take place invariably at two, three, or more fixed points.

If $\phi = 180°$, they will take place invariably at two points which are diametrically opposed to each other. In that case, we shall have $2P = P' - P$ (3250.), and, therefore, $P' = 3P$ and $T = \frac{1}{2}P'$. To comprehend the motions of the planets in this case, let E and I (*fig.* 857.) be their positions at any proposed conjunction. The angular motion of I being three times that of E, while the

latter moves from E to E′ through a semicircumference, I moves through three semicircumferences, and, therefore, through the whole circumference II′I, and after that through the semicircumference II′, overtaking the exterior planet at I′, where, therefore, the next conjunction takes place. In like manner, the succeeding conjunction will take place at S I E, the next at S I′ E′, and so on, no conjunction being possible except in these two lines.

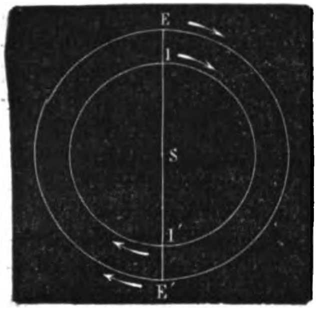


Fig. 857.

If $\phi = 120°$, or a third part of the circumference, the conjunctions will be reproduced continually in three fixed directions, dividing the circumference into three equal parts. In this case, $3P = P' - P$, and therefore $P' = 4P$, and $T = \frac{1}{3}P'$. To explain the motion in this case, let S I E (*fig.* 858.) be the position of the planets at any proposed conjunction. Let E move forward through 120° to E′. The angular motion of I being four times more rapid than that of E, it will move in the same time through $4 \times 120° = 360° + 120°$; that is, it will make a complete revolution and 120° more. It will, therefore, overtake the exterior planet at S I′ E′, 120° in advance of the last conjunction. In the same manner, it may be shown that the next conjunction will take place in the line S I″ E″, 120° in advance of S I′ E′. The following conjunction will, in the same way, take place in the line S I E, 120° in advance of S I″ E″. Thus each series of three conjunctions will take place in the lines S I E, S I′ E′, and S I″ E″, forming with each other angles of 120°; and no conjunctions can, under the proposed condition, take place in any other line.

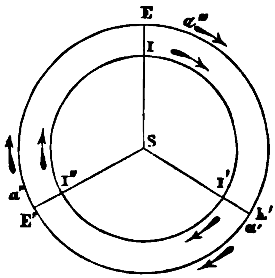


Fig. 858.

If $\phi = 90°$, it may be shown by precisely the same reasoning that $P' = 5P$ and $T = \frac{1}{4}P'$, and that the conjunctions will invariably take place in four fixed directions, at right angles to each other.

In general, in order that the conjunctions shall be reproduced in any proposed number n of fixed directions, it will be necessary that the period P' shall be exactly $n + 1$ times the period P. In that case the synodic time T will be n times P'; and the fixed directions in which the conjunctions will succeed each other, will divide the circumference into equal arcs or angles, the magnitude of which will be $\frac{360°}{n}$.

3253. *Effects of the disturbing force in cases of commensurable periods.*—It follows from what has been explained (3249.), that in such cases the effects of the disturbing force would accumulate indefinitely, without compensation or with imperfect compensation, through an indefinite succession of synodic revolutions. If, for example, $P' = 2P$, and therefore the conjunctions would always take place in the same line, the line of conjunctions being always inclined to the line of apsides at the same angle, the effect of the disturbing force on the several elements, in a synodic revolution would be always exactly the same, and would, therefore, accumulate indefinitely from revolution to revolution.

If $P' = 3P$, the lines of conjunction would have three, and only three, different positions in relation to the line of apsides; and although the effects of the disturbing force in a synodic revolution in these three positions would be different, and some of them would necessarily have contrary signs, and would produce, therefore, more or less compensation, such compensation would be imperfect; and after each series of three conjunctions, a residual inequality would remain, affecting each of the elements which, as before, would accumulate indefinitely during an indefinite succession of synodic revolutions.

In the same manner, if P' were any other exact multiple of P, the series of conjunctions which would take place in the directions of the fixed lines dividing the circumference into equal parts would still be imperfectly compensatory, and residual quantities would, as before, remain uneffaced, which would accumulate indefinitely.

In order that the conjunctions should take place always in certain fixed directions, it is not necessary that the periodic time of the exterior planet should be an exact multiple of that of the interior. The same will happen, if any exact multiple of one of the periods be exactly equal to an exact multiple of the

other, or, in other words, if the periods be commensurable. Thus, if $2\,P' = 5\,P$, it is evident that, counting from the epoch of any one conjunction, another will arrive in exactly the same place after every two complete revolutions of E, and every five of I. But, between these others will take place at fixed intermediate positions, for we should have

$$\frac{P}{P'-P} = \frac{2}{5-2} = \frac{2}{3};$$

and consequently,

$$\phi = 360° \times \tfrac{2}{3} = 240°.$$

In this case, the lines of conjunction would be distributed in the same manner as when $P' = 4\,P$, but the conjunctions would not follow in precisely the same manner. After the conjunction which takes place in the line S I E (*fig.* 858.), would succeed that which takes place in the line S I″ E″, and the third of the series would take place in the line S I′ E′; the fourth, or the first of the next series, taking place in the line S I E.

By pursuing this method of reasoning, it will be easily seen that, in all cases in which the periods of the two planets would be in the exact ratio of two whole numbers m and n, the conjunctions would invariably succeed each other in certain fixed lines. We should, in effect, have

$$\frac{P'}{P} = \frac{m}{n}, \quad \frac{P}{P'-P} = \frac{n}{m-n};$$

and consequently,

$$\phi = 360 \times \frac{n}{m-n};$$

and since m and n are whole numbers, this would give the magnitude of the angles into which the fixed directions of the lines of conjunction would divide 360°.

It is evident that, although complete compensation could not take place between the effects of the disturbing force upon the elements, so long as the line of conjunction is limited to fixed directions, their approach to compensation is closer and closer the more multiplied are the directions which the line of conjunction can assume.

3254. *Planets present no case of commensurable periods, but some nearly so.* — By reference to the table of periodic times

of the planets (2984.), it will be seen no case is presented in the solar system, in which the periods of two planets are exactly commensurable; but as several cases are found in which there is an approach, more or less close, to that condition, it will be convenient to investigate the effects which, in general, the disturbing force would produce in their approximate commensurability.

When the periods are nearly commensurable, the successive positions of the line of conjunction will necessarily be before or behind the positions which it would assume in the case of exact commensurability by a certain angular distance, which will be less or greater according as the periods depart less or more from exact commensurability.

Suppose, for example, that in the case of exact commensurability the positions of the lines of conjunction after each series of three synodic revolutions were S I E, S I′ E′, and S I″ E″, as represented in *fig.* 858., and suppose that the deviation of the periods from exact commensurability is such that the second conjunction, instead of taking place at E′, shall take place at a'. If a'' E″ = 2 a' E′, the third conjunction will take place at a''; and if E a''' = 3 E′ a', the next conjunction will take place at a''', and so on.

If, then, the distance a' E′ be very small, which it will be if the periods are very nearly commensurable, the lines of conjunction, though not rigorously in fixed directions will, for a considerable number of successive synodic revolutions, crowd about those fixed directions which they would have rigorously assumed if the commensurability had been exact, and during that interval, which, when the synodic time is of much length, will be of great duration, nearly the same inequality will be produced by the want of compensation in the effects of the disturbing force as if the directions of the line of conjunction were fixed.

But, however small the advance a' E′ of the line of conjunction in each synodic revolution may be, its continued accumulation through a long succession of synodic revolutions will carry that line at length round the whole circumference, causing it in slow but regular and inevitable succession to take all directions with relation to the lines of apsides of the two orbits. When it has made half a revolution, or revolved through 180°, it will have precisely the opposite position with relation to

these lines, and the disturbing force will produce contrary effects upon the elements of the disturbed orbit; and while the line of conjunction revolves through the other half revolution, the disturbing force, for like reasons, produces a series of effects on the elements which are the opposite to those it produced during the first half revolution.

3255. *Long inequalities.* Hence obviously arise a group of inequalities, affecting the elements severally of the disturbed orbit, the periods of which will correspond with the revolution of the line of conjunction.

What has been said of the varying position of the line of conjunction in relation to the line of apsides, will affect the inequalities of the major axis, the motions of the apsides, and the excentricity. The varying position of the same line with relation to the line of nodes, will in like manner affect the motion of that line and the variation of the inclination.

Having thus explained in general the principle which determines the successive phases of the LONG INEQUALITIES of the planets, we shall now briefly notice some of the most remarkable of these phenomena.

3256. *Long inequality of Jupiter and Saturn.* By (2984.) Table II. it will be seen that the periodic times of Saturn and Jupiter are

$$P' = 10759{\cdot}2198. \quad P = 4332{\cdot}5848,$$

and consequently

$$\frac{P'}{P} = 2{\cdot}48325 = \frac{5}{2} - 0{\cdot}01675,$$

from which it appears that five times the period of Jupiter, exceeds twice that of Saturn by a small fraction. Now, if $2\,P'$ were exactly equal to $5\,P$, the conjunctions would invariably take place in three lines at angles of 120° (3253.). But since

$$\phi = 360° \times \frac{P}{P' - P} = 242{\cdot}7°,$$

it follows that in the series of three successive conjunctions in which the line would assume the three directions at angles of 120°, if the periods were exactly commensurable, it advances beyond these directions successively by the angles, 2·7°,

$2{\cdot}7° \times 2 = 5{\cdot}4°$ and $2{\cdot}7 \times 3 = 8{\cdot}1°$, so that, after making a single revolution in three synodic periods, it advances $8{\cdot}1°$ beyond its first position; and as it will continue to advance at the same rate, in every three synodic periods it will make a complete revolution in

$$\frac{360}{8{\cdot}1} = 44{\cdot}44 \times \text{T}.$$

But the synodic period is

$$\text{T} = \frac{\text{P} \times \text{P}'}{\text{P}' - \text{P}} = 7253{\cdot}3 \text{ days};$$

and therefore the time of a complete revolution of the line of conjunction will be

$$7253{\cdot}3 \times 44{\cdot}44 = \overset{\text{days.}}{322666} = \overset{\text{years.}}{883{\cdot}2};$$

an interval which is equal to 29·96 revolutions of Saturn and to 74·405 revolutions of Jupiter.*

3257. *Period of this inequality about* 880 *years.*—It follows, therefore, from what has been explained, that the line of conjunction of these planets revolving from a given direction to one diametically opposite in about 440 years, and then completing its revolution, and returning to its original direction in the next 440 years, a series of inequalities will be produced upon the elements of the two orbits, which will go on increasing or decreasing for a period of 440 years, and will undergo the contrary variation, decreasing or increasing during the succeeding 440 years.

As already observed, however, it must not be assumed, in this or any like case, that the compensation produced by the contrary effects in the two intervals is necessarily complete, and that the increase effaces completely the decrease, or *vice versâ.* Such a perfect equilibrium between the effects of the perturbations rarely takes place.

3258. *Its effect upon the major axes and periods.*—One of the long inequalities resulting from this relation between the

* The Astronomer Royal, in his tract on Gravitation, gives 855 years for this interval, but observes that the numbers are not quite exact, the ratio of 29 to 72, which he takes as that of the periodic times, not being quite accurate.

mean motions of the planets, affects the major axes of their orbits, and consequently their periodic times. The major axis of one orbit increases, and that of the other decreases, continually for 440 years, and during the next 440 years the former decreases, and the latter increases. The consequence of this is that the mean motion of one planet continually increases, and that of the other continually decreases, during periods of 440 years. Although the changes produced upon the axes from this cause are so minute as to be scarcely appreciable, that of Saturn's orbit amounting when greatest to only the 1350th, and that of Jupiter's to the 8550th part of its length, the effects produced upon the motions of the planets are very considerable, the place of Saturn being affected to the extent of 48′, and that of Jupiter to 21′. The greatest inequality of any other planet does not affect its place to a greater extent than 3′; and those which are within Jupiter's orbit are much less affected, being never removed from their mean place by so much as half a minute.

3259. *Its effects upon the excentricities.*—It appears that, during the interval in which the line of conjunction moves through 120°, the excentricity of each of the two orbits increases, attains a maximum magnitude, and then decreases. The effect produced upon the planet's distance from the sun by the change of excentricity is much more considerable than the effect produced by the change in the magnitude of the major axis. In the case of Jupiter it amounts to the 1230th part of the entire distance, and in the case of Saturn to the 314th part.

3260. *Effect on the direction of the apsides.*—The effect upon the motion of the apsides is subject to a like period. A progressive motion is imparted to them for 440 years, and a regressive motion for the next 440 years. Between this motion of the apsides and the variation of the excentricity of each orbit, there is a necessary relation; the excentricity of each orbit having its mean value, when the progressive or regressive motion of the apsides has attained its limit; and when the excentricity is at its maximum or minimum, the apsides arrive at their mean places.

This long inequality of Jupiter and Saturn is a phenomenon of considerable historical celebrity and interest, owing to the apparent irregularity which it explained, having been observed long before its cause was discovered, and having given great

perplexity to astronomers. Its cause was demonstrated and the whole character and law of the phenomenon explained by Laplace, in 1785.

3261. *Long inequality of Venus.*—Next to that which has been just noticed, the most remarkable inequality of this class is the long inequality of Venus, arising from the near commensurability of the periods of that planet and the Earth. If P' and P express these periods, we shall have (2984.)

$$\frac{P'}{P} = \frac{365 \cdot 256}{224 \cdot 701} = 1 \cdot 6255 = \frac{13}{8} - 0 \cdot 0005.$$

Thus, 13 P exceeds 8 P' by 0·004 P, that is, by the 250th part of P.

To determine the value of ϕ, we have

$$\phi = 360° \times \frac{1}{0 \cdot 6255} = 575 \cdot 53° = 576° - 0 \cdot 47°.$$

But if 8 P' were exactly equal to 13 P, each successive conjunction would take place 576° in advance of the last; and since 576° = 3 × 180° + 36°, it follows that in this case the line of each successive conjunction would be 36° in advance of that point diametrically opposite to the last conjunction. By following this out it will be seen, that five successive conjunctions would take place in lines, dividing the whole circumference into five equal angles of 72°. But in consequence of 8 P' being a little greater than 13 P, the line of each successive conjunction will fall 0·47° behind the place it would occupy if the periods were exactly commensurable. By the continued accumulation of this deviation, the line of conjunction will take successively all positions round the circumference, shifting its direction through 0·47° in each synodic period. To determine the time in which it will make a complete revolution, it is only necessary to divide 72° by 0·47, and multiply the quotient by the synodic period. This gives

$$\frac{72 \times 584}{0 \cdot 47} = \overset{\text{Days.}}{89462} = \overset{\text{Years.}}{244 \cdot 9.}^{*}$$

This inequality, the discovery of which is due to the genius and research of the Astronomer Royal, notwithstanding the

* According to the Astronomer Royal, 239 years.

long interval of its accumulation, does not exceed, even at its maximum, a few seconds; and affords a striking example of the degree of precision to which our knowledge of the planetary motions has been carried by the application of the principles of the theory of gravitation.

3262. *Other long inequalities.*— There are several other inequalities of this class, incidental to the other planets, which need only be indicated here, their investigation and exposition being precisely similar to these already explained. Thus in the case of Mercury and the Earth

$$\frac{P'}{P}=\frac{365\cdot256}{87\cdot969}=4\cdot15=4+0\cdot15;$$

so that the one period is but a little more than four times the other. This produces an inequality whose period is about seven years.

In the case of Venus and Mercury we have

$$\frac{P'}{P}=\frac{224\cdot701}{87\cdot969}=2\cdot55=\frac{5}{2}+0\cdot05.$$

In the case of Mars and Venus

$$\frac{P'}{P}=\frac{686\cdot979}{224\cdot701}=3\cdot057=3+0\cdot057.$$

In the case of Uranus and Saturn

$$\frac{P'}{P}=\frac{30687}{10769}=2\cdot85=3-0\cdot15.$$

In the case of the Earth and Mars

$$\frac{P'}{P}=\frac{686\cdot979}{365\cdot256}=1\cdot88=2-0\cdot12.$$

In each of these cases long inequalities are produced, the periods of which may be determined by the method already explained.

3263. *Long inequalities of the nodes and inclination.*—The variation of the motion of the line of nodes and of the magnitude of the inclination, consequent upon the changes of position of the line of conjunction, in all cases of near commensurability of the periods, are so exactly similar to the changes

already explained, of the line of apsides and the excentricity, that it is only necessary here to observe that, *mutatis mutandis*, all that has been explained of the one is applicable to the other.

3264. *Secular inequalities.*—The inequalities noticed in the preceding paragraphs, have been exclusively those whose periods are determined by the variation of the relative positions of the disturbing and disturbed planets, or by what has been called their configuration; and which are denominated *periodical inequalities*, not because all other inequalities are not also periodical, but because the periods of the former are of much more limited length, and except in the case of the *long inequalities*, such as may be in general completed within the limits of astronomical records. The other class of inequalities are those which arise from the continual accumulation of the residual phenomena, which remain uncompensated after the disturbing and disturbed bodies have passed through all their phases of configuration, and recommence to pass through a like series of relative positions; these are the SECULAR INEQUALITIES.

3265. *Secular constancy of the major axes.*—No result of the researches of mathematicians in physical astronomy has excited so great and so just admiration, as the discovery of the fact that, although the major axes of the orbits of the planets are subject to small periodical variations, which cause their periodic times and mean motions to oscillate within narrow limits round certain mean values, yet that the mean values of these axes are, in the long run, rigorously invariable and subject to not the slightest variation from age to age, and cannot be subject to any, so long as the solar system is not interfered with by any agencies, save those which have play among the bodies, great and small, which compose it.

The importance of this theorem, and the interest with which its complete demonstration must be regarded, will be understood when it is considered that, upon the magnitude of the major axis of the orbit of a planet depends the apparent motion of the sun as seen from the planet, and the average supply of light and warmth received, in a given time from that luminary. Any continued and accumulated change in the major axis, such as would necessarily result from a secular inequality affecting it, would not only subvert the physical conditions to

which the organisation of the races which inhabit the planets are adapted, but would destroy the great landmarks of chronology, and deprive the heavenly bodies of some of their most important uses.

A wise provision in the physical structure and laws of the system has, however, rendered such derangements impossible, by making all the periodical perturbations to which the major axes of the planetary orbits are subject, rigorously compensatory.

First. The effects of the radial component of the disturbing force of one planet exerted upon another after they have passed through all possible configurations are rigorously compensatory.

If the disturbing planet be exterior, the radial component being alternately positive and negative, will on the whole produce a negative effect, the aggregate of its negative actions exceeding those of its positive. It will, therefore, on the whole diminish the average effective central attraction. In like manner, if the disturbing planet be interior to the disturbed, its results on the whole will be to augment the average effective central attraction. In the one case, the mean distance or major axis which corresponds to the periodic time by the harmonic law will be greater, and in the other case less, than it would be in the absence of the disturbing force; but in both cases, so long as the mean effective central attraction remains the same, the mean value of the major axis of the orbit will be invariable.

If we take an interval of time so great that each of the planets will have assumed, with relation to the other, every possible relative position, it will follow that the mean value of the radial component of the disturbing force corresponding to any proposed point, P, of the orbit of the disturbed planet, during such interval, will be found by taking a mean of the radial components of all the disturbing forces exerted by the disturbing planet in all the points M of its orbit upon the disturbed planet at the proposed point; for at one time or other in the assumed interval, provided it be sufficiently great, the disturbing planet must have been found at each of the points of its orbit, the disturbing planet being at the same moment at P. If, then, we imagine the radial components of the disturbing force exerted by the planet M, at each of the points of its

orbit upon the disturbed planet at the point P, and if we take the mean of all these components by dividing their sum by their number, the mean will be the mean value of the radial component of all the disturbing forces exerted by the disturbing planet upon the disturbed planet, when the latter was found at the point P during the assumed interval. Now, it is quite evident that this mean value must always be the same.

In the same manner, the mean value of the radial component for every other position of the disturbed planet may be found, and it will be apparent that the mean effect of the disturbing force estimated in the direction of the radius vector at each point of the orbit of the disturbed planet, is always the same, and consequently the effect produced by this component on the major axis is always the same. So far, therefore, as relates to this component of the disturbing force, the mean value of the major axis taken in an interval of time so great that the two planets will have assumed with relation to each other every possible position in it, is subject to no ultimate variation.

Secondly. The effects of the tangential components are most easily explained, by considering the whole attractive forces which the disturbing planet exerts upon the sun and upon the disturbed planet. It will be remembered, that the disturbing force exerted by M on P, is the resultant of the attractive force exerted by M on P and a force exerted on P, equal and opposite to the attractive force which M exerts on S. Now, if we take M successively at every point of its orbit, and find its attractive force on S, it will be apparent that the resultant of all these forces directed from S towards M, will be in equilibrium, and therefore, compensatory. It follows, therefore, that the forces equal and opposite to those which are assumed to act on P, must also be compensatory.

It remains, therefore, only to investigate the total effects of M's attraction on P, in all the positions which the two bodies can assume.

Let M be supposed to be at any given point of its orbit, and let P be taken at *every* point of *its* orbit. These are positions which are not successively assumed in the motions of the two bodies; but they are positions which at *some* times in the assumed interval they *must* have, if that interval be assumed of sufficient length; and as our present object is to obtain the aggregate effect of the forces during the entire interval, the

order in which they are exerted is immaterial. Now, if P be thus supposed to make a complete revolution while M maintains its position, it will follow, from a general principle of physics, that, when it returns to its primitive place, after completing a revolution, it will have exactly the same orbital velocity as when it started from that place. It follows, therefore, that the effects of the tangential component of M's attraction in accelerating it during its revolution must have been precisely equal to the effects of the same component in retarding it. But it has been shown (3154.) that every such acceleration produces an increase, and every such retardation a diminution, of the major axis of the orbit. It follows, therefore, that in such a revolution, the increments and decrements of the major axis would be equal, and a complete compensation would be effected.

Now, the same will be true for every position whatever which M can assume in its orbit, and it will therefore follow, that if, while M passes from point to point of its orbit, with an intermitting motion, P makes a complete revolution during the time it stops at each point, the result of the total action of all the disturbing forces developed during such a revolution of M on the major axis of P's orbit, would be absolutely compensatory, and consequently the major axis after such a revolution will have exactly the same magnitude it had at its commencement.

Now, it is true that such is not the way in which the two bodies M and P do actually move. The disturbing planet M does not stop at each point of its path while the disturbed planet P makes a revolution, and they consequently do not assume the various configurations in the order here assigned to them. The two planets move and continue to move simultaneously; but they do assume these various configurations, although they take place in a different order of succession. Since, however, that order does not affect the values of their aggregates, and since the sum of all the positive effects and the sum of all the negative effects will still be the same in whatever order they may take place, the same perfect compensation will be realised in the actual motions which has been here shown to take place in the supposed motions.

It will doubtless be objected to this reasoning, that we have supposed each planet to make its revolution in an orbit of fixed magnitude and form, without allowing for the displacement

which the disturbing force itself must inevitably produce during each revolution, which, though very small, is not quite inappreciable. This, however, has been taken into the account in some mathematical researches and it does not appear to affect the conclusion.

What is true in this reasoning of the effect of the disturbing force of any one planet upon another, will be equally true of all the planets, primary and secondary, on that other; and it may, therefore, be inferred, in general, that the major axes of the planetary orbits are not subject to any secular variation, and that in the course of ages the periods and mean motions, which by the harmonic law depend on the major axes, can never suffer any permanent change.

3266. *Secular variation of the apsides.*—It has been shown that the change of position of the apsides in a synodic revolution, depends on the position of the line of conjunction with relation to the perihelion of the disturbed orbit, but in an interval of time so long as to allow the line of conjunction to assume all possible positions, all possible effects will be produced upon the apsides. The magnitude of these effects will vary with the varying distance of the disturbing planet from the disturbed orbit. The greatest effect will obviously be produced at those parts of the disturbed which are nearest to the disturbing orbit; and in taking a mean of all the variations of the apsides during the entire interval these effects will predominate, so that it may be assumed that the final residual effect upon the apsides, will be identical in its character with the effect produced in those conjunctions which take place at the points where the two orbits are nearest to each other.

The position of these points will evidently depend upon the relative magnitude of the two major axes, their relative position, and the excentricities of the two orbits.

If that half of the disturbed orbit, in the middle of which aphelion is placed, be nearer than the other half to the orbit of the disturbing planet, the secular motion of the apsides will be progressive; if it be more remote, the motion will be regressive. If the position of the orbits be such that both halves are equidistant from the orbit of the disturbing planet, there will be no secular variation of the apsides.

The secular motion of the apsides will continue to have the same direction, until their change of relative position shall alter

the conditions and render them stationary, or reverse the direction of the motion.

3267. *Secular variation of the excentricity.*—The secular effect of the disturbing force upon the excentricity of the disturbed orbit is, like that of the apsides, similar in character to the effect produced at those points of the disturbed orbit, which are nearest to the disturbing orbit. The excentricity will accordingly, on the whole, either increase or decrease, or suffer no change, according to the position of the perihelion of the disturbed orbit at those points where the two orbits are in greatest proximity, and it will continue to be affected in the same manner, until the conditions are changed by the secular change of the apsides.

At the place where the two orbits are in closest proximity, both planets are in general moving either from aphelion to perihelion, or the contrary, so that one excentricity is increasing and the other decreasing.

The secular variation of the excentricities, if it continued to take place in the same way, either always increasing or always decreasing, would, after a period of time of great length, but still definite, so change as to derange in a serious degree the economy of the system, and to expose the planets to such vicissitudes of temperature as would be incompatible with their well-being, not to mention other causes of derangement which would attend such changes. It is found, however, by pursuing the inquiry, and tracing the changes in long periods incidental to the excentricity, that, slow and long continued as these are, they are still periodic, and ultimately compensatory. After varying in one way, either by continual increase or continual diminution, for many thousand years, each excentricity will at a certain epoch cease to vary, and will then begin to undergo a contrary variation, decreasing if it had before increased, and increasing if it had before decreased; and this will continue for periods of like length until it attain another limit at which it will becoming stationary, and so on without end.

3268. *Secular variation of the nodes.*—The same mode of explanation as was adopted in the case of the other elements, will serve to explain the secular variation of the nodes and inclinations. It has been shown that the components of the disturbing force, which are equal, parallel, and opposite to the planet's attraction on the sun, taken in every point of its orbit,

are exactly compensatory, and may therefore in this case be neglected. If, then, the forces exerted by the disturbing planet M, at all points of its orbit upon P, at any one point of its orbit be taken, it is easy to see that their resultant will be a force which will tend to draw P from the plane of its own orbit towards the plane of M's orbit. This will be apparent from the consideration that the total effect of M's attraction will be similar in character to its effect where most intense, that is, at the points where the two orbits are in closest proximity. It follows, therefore, that the nodes of P's orbit must secularly regress upon M's orbit.

It is necessary, nevertheless, to bear in mind that this secular regression on the orbit of the disturbing planet does not infer its regression on other places. A regression on one place may cause a progresssion of the nodes of the same orbit with another plane.

3269. *Secular variation of the inclination.*—If the orbits of both planets were absolutely circular, the periodical inequalities of the inclination would be exactly compensatory, and there would, therefore, be no secular variation of that element; for in such case at points equally distant from the point which is most remote from M, M exerts equal disturbing forces on the inclination, one tending to increase it, and the other to decrease it.

But if the orbits be elliptic, a certain point can be determined where the effect of the orthogonal component of the attracting force of M on P is greater than at any other point, and the total effect produced on the inclination by M taken in every part of its orbit, on P taken in every part of its orbit, in all positions of the line of conjunction with relation to the line of nodes, will be similar in character to this maximum effect; and such will be the character of the secular variation of the inclination.

The observations, however, which have been made (3267.), respecting the limits within which the secular variation of the excentricities are confined, are equally applicable to the inclinations. These also, though they may severally increase continually (subject to their periodical oscillations), for thousands of years, will necessarily decrease continually for periods of like duration, and the limits within which this secular oscillation is confined are in all cases extremely narrow.

3270. *Laplace's theorems of the relations between the excen-*

tricities and inclinations of the planetary orbits.—The researches of Laplace have led to the discovery of a beautiful mathematical relation which prevails between the excentricities and inclinations of the planetary orbits, which may be easily comprehended without any profound mathematical knowledge, although its demonstration does not admit of exposition on principles sufficiently elementary to allow of its introduction here.

THEOREM.

"If the numbers which express the square of the excentricity and the square root of the semi-axis major of each of the planetary orbits be multiplied together, and their product be multiplied by the number which expresses the mass of the planet, the sum of all such products for all the planets will always be the same, notwithstanding the secular variation of the excentricity."

This celebrated theorem may be conveniently and very concisely expressed algebraically as follows: Let a express the mean value of the semi-major axis. Let m express the mass of the planet, and e the excentricity; let all the axes and all the masses be expressed with relation to the same unit. If, then, the symbol Σ be used to express the result of the addition of the preceding products taken for the planets severally, we shall have

$$\Sigma (m \times \sqrt{a} \times e^2) = \text{C},$$

and it will be found that this sum C never has varied and never will vary.

In the same manner the relation of the inclination is expressed by the following

THEOREM.

"If the numbers which express the square of the tangent of the inclination of the orbit to a fixed plane, and the square root of the semi-axis major of each of the planetary orbits, be multiplied together, and their product be multiplied by the number

WHICH EXPRESSES THE MASS OF THE PLANET, THE SUM OF ALL SUCH PRODUCTS FOR ALL THE PLANETS WILL ALWAYS BE THE SAME, NOTWITHSTANDING THE SECULAR VARIATION OF THE INCLINATION."

If I express the inclination, this theorem may in like manner be expressed thus,

$$\Sigma(m \times \sqrt{a} \times \tan^2 \mathrm{I}) = \mathrm{C}',$$

and it will be found that C′ never has and never will vary.

It was shown in a paper by the author of this volume, read before the Astronomical Society, that if the present position of the plane of the ecliptic be taken as the fixed plane of reference, the values of these sums will be

$$\mathrm{C} = 0{\cdot}00000827941$$
$$\mathrm{C}' = 0{\cdot}00000314170.$$

3271. ***Conservative influence erroneously ascribed to these theorems.*** — These theorems have been represented by astronomical writers generally as affording a complete security against any undue secular increase of the excentricities or inclinations of the planetary orbits.*

It has been shown, however, in the memoir already referred to, by the author of this volume, that, except so far as relates to the major planets, these theorems have no such conservative influence, and that the excentricities and inclinations of the terrestrial planets, for anything contained in these formulæ, might increase to an extent which would utterly derange the economy of the system.

* See, for example, Herschel, Outlines of Astronomy, ed. 1849, p. 405. 432. ; Grant, History of Astronomy, chap. iv. p. 55.

CHAP. XXIV.

THEORY OF SPHEROIDAL PERTURBATIONS.

3272. *Attraction of planets would be central if their forms were exactly spherical.* — In the preceding investigations of the effects of the reciprocal attractions of the bodies composing the solar system, the attraction exerted by each of these upon the others, is considered to emanate from its centre in all directions around it, as luminous rays would from a radiant point. This would be strictly true if the gravitating bodies were all spherical; since it is a property of a sphere that the matter composing it, supposing it to be either uniformly dense, or to have a density varying according to some fixed law depending on the distance from the centre of the mass, exercises on all distant bodies exactly the same attraction as if its entire mass were concentratrated at its centre.

But if the attracting body be not spherical, this will not be true; and accordingly the attraction exerted by any such body, must be investigated with especial reference to its form.

3273. *Disturbing forces consequent on spheroidal forms.* — Now, although the planets generally, including the earth, are *very nearly*, they are not *exactly*, spherical, as has been already explained. The ellipticity of these spheroids, though too inconsiderable to produce any sensible effect upon their mutual attractions, or to require to be taken into account in any analysis of their perturbations, is nevertheless sufficient to produce very sensible effects on the mutual attractions of the spheroidal planets and their satellites, and even upon the phenomena resulting from the central attraction of the sun exerted upon them.

These effects are manifested in the motions of the planets themselves by periodical changes in the position of their axes of rotation, and in their satellites by similar changes in the elements of their orbits.

It is these effects that we denominate SPHEROIDAL INEQUALITIES, the name indicating their physical cause.

3274. *Effects which would be produced if a satellite were attached to the surface of the earth at the equator.*—If the earth were attended by a second satellite, revolving close to its surface and in the plane of its equator, its periodic time would be less than that of the moon, in a ratio which is easily ascertained by the harmonic law. Let m express the moon's period, r' its distance, p the period of such a satellite as is here supposed, and r its distance from the earth's centre. We should then have

$$\frac{p^2}{m^2} = \frac{r^3}{r'^3}.$$

But since the supposed satellite is close to the surface, its distance from the centre is the earth's semidiameter; and since the moon's distance is sixty semidiameters of the earth, we shall have

$$\frac{r^3}{r'^3} = \frac{1}{60^3};$$

and since $m = 655{\cdot}73$ hours (2470.), it follows that $p = 3{\cdot}613$ hours.

Such a satellite would be subject to the disturbing action of the sun, which would produce in its orbit inequalities similar in kind to, but different in magnitude from, those produced by the sun's disturbing force on the moon's orbit. Its nodes, that is, the equinoxial points (inasmuch as its orbit is by the supposition the plane of the equator), would receive a slow regressive motion; and its inclination, that is, the obliquity of the ecliptic, would be subject to a variation whose period would depend on that of the successive returns of the sun to the same equinoxial point.

This satellite would also be subject to the disturbing action of the moon, which would affect it in a manner nearly similar; since, in that case also, the disturbing body would be exterior to the disturbed. It would impart to the line of nodes of the supposed satellite, that is, to the intersection of the plane of its orbit with the plane of the earth's equator, a retrograde motion upon the former plane; and since that plane is inclined at a very small angle to the plane of the ecliptic, this would produce a like retrograde motion of the equinoxial points upon the ecliptic.

A variation of the inclination of the plane of the equator to that of the moon's orbit, and, therefore, to the plane of the ecliptic, would also be produced, the period of which would depend on the moon's motion.

But the moon's orbit would also be disturbed by the attraction of the supposed satellite. A regressive motion would be imparted to the line in which the plane of its orbit intersects that of the equator, and a periodical variation of inclination would likewise be produced, depending on the period of the supposed satellite.

Let us now imagine that the supposed satellite, instead of revolving in 3·613 hours, moves with a much slower motion, and revolves in 23 hours and 56 minutes, the time of the earth's rotation. The inequalities which it suffers and which it produces, will then be changed only in their magnitudes and periods, but will retain the same general character. But the supposed satellite now having the same motion precisely as the surface of the earth close to which it is placed, may be imagined to adhere to that surface, so as to form, in fact, a part of the earth, without in any way deranging the conclusions which have been deduced above.

3275. *Like effects would be produced by any number of such satellites, or what would be equivalent, by the spheroidal form.* — But the same observations would be equally applicable to any number of satellites similarly placed and similarly moving, which might, therefore, be imagined to be successively attached to the surface of the globe at and near the equator, until such a protuberance would be formed upon it, as would in effect convert it into the form of an oblate spheroid, such as the form of the earth is known to be.

It is, however, to be further considered, that the effects of the disturbing forces which thus act upon this protuberant matter, are necessarily modified by the inertia of the spherical mass within it, to which it is imagined to be attached. The protuberant mass which alone is acted on by the disturbing forces, cannot obey any action of these forces, without dragging with it this vast spherical mass to which it is united. The motions and changes of motion, therefore, which it receives, will be rendered slower in proportion to the mass with which such motions must be shared.

These observations are obviously applicable equally to any of

the other planets, which being attended by satellites have the spheroidal form. This, as has been already explained, is the case with Jupiter and Saturn, and would probably be found to prevail equally in the cases of the other major planets.

3276. *Precession of the equinoxes.*—Since, therefore, we may consider the spheroidal protuberance around the terrestrial equator as a satellite attached to the earth, it will follow that the general effect of the sun's disturbing force acting upon it, will be to impart to its nodes, that is, to the equinoxial points, a retrograde motion, which will be much slower than that which they would receive from the same cause, if this protuberant matter were not compelled to carry with it the mass of the earth contained within it.

The moon exercises a like disturbing force which produces a like regression of the nodes of the equator on the moon's orbit; and that orbit being inclined at a small angle to the ecliptic, this is attended with a like regression of the equinoxial points.

The mean annual regression of the equinoxial points upon the plane of the ecliptic arising from these causes, is 50·1″.

3277. *The sun returns to the equinoxial point before completing its revolution.* — Since the equinoxial points thus move backwards on the ecliptic, it follows that the sun, after it has in its annual course passed round the ecliptic, will arrive at either equinoxial point before it has made a complete revolution. The equinoxial point being 50·1″ behind the position it had when the sun started from it, the sun will return to it after having moved through 50·1″ less than a complete revolution. But since the mean hourly apparent motion of the sun is 147·8″ (2458.), it follows that the centre of the sun will return to the equinoxial point,

$$\frac{50{\cdot}1}{147{\cdot}8''} = 0{\cdot}33898 = \overset{\text{hrs.}}{}\overset{\text{min.}}{20}\ \overset{\text{sec.}}{20{\cdot}3},$$

before completing its revolution.

3278. *Equinoxial and sidereal year.* — Hence is explained the fact, which appears in (2984.), Table II., that while the sidereal year, or actual revolution of the earth round the sun, is

$$\overset{\text{days}}{365{\cdot}25637} = \overset{\text{days.}}{365}\ \overset{\text{h.}}{6}\ \overset{\text{m.}}{9}\ \overset{\text{sec.}}{10{\cdot}38},$$

the equinoxial revolution, or the time between two successive equinoxes of the same name, is

$$\begin{array}{cccccc} \text{days} & & \text{days} & \text{hrs.} & \text{min.} & \text{sec.} \\ 365{\cdot}242255 & = & 365 & 5 & 48 & 50{\cdot}4, \end{array}$$

the latter being less than the former by $20^{m}\ 20^{s}$.

The successive returns of the sun to the same equinoxial point must, therefore, always *precede* its return to the same point of the ecliptic, by $20^{m}\ 20^{s}$ of time, and by $50{\cdot}1''$ of space.

3279. *Period of the precession.*—To determine the period in which the equinoxial points moving backwards constantly at this mean rate would make a complete revolution of the ecliptic, it is only necessary to find how often $50{\cdot}1''$ must be repeated to make up 360°, or, what is the same, to divide the number of seconds in 360° by 50·1. This gives

$$\frac{1{,}296{,}000}{50{\cdot}1} = 25868 \text{ years.}$$

3280. *Its effect upon the longitudes of celestial objects.*—Although this motion, slow as it is, is easily detected from year to year by modern instruments, it was not until the sixteenth century that its precise rate was ascertained. Small as is its annual amount, its accumuluation, continued from year to year for a long period of time, causes a great displacement of all the objects in the heavens, in relation to the equinoxial points from which longitudes and right ascensions are measured. In 71·6 years, the equinoxes retrograde 1°, and therefore, in that time, the longitudes of all celestial objects of fixed position, such as the stars, have their longitudes augmented 1°. Since the formation of the earliest catalogues in which the positions of the fixed stars were registered, the retrogression of the equinoxial points has amounted to 30°, so that the present longitudes of all the objects consigned to these catalogues, is 30° greater than those which are there assigned to them.

3281. *Precession of equinoxes produces a rotation of the pole of the equator round that of the ecliptic.*—If two diameters of the celestial sphere be imagined to be drawn, one perpendicular to the plane of the equator and the other to that of the ecliptic, the angle included between them will obviously be equal to the angle under the equator and ecliptic; and since the extremities of these diameters are the poles of the equator and

ecliptic, it follows that the arc of the heavens included between these poles is equal to the obliquity of the ecliptic.

But since a plane passing through these diameters is at right angles both to the equator and ecliptic, the line of equinoxes or the intersection of the planes of the equator and ecliptic, will be at right angles to that plane. If, therefore, the equinoxial points revolve round the ecliptic in a retrograde direction, it follows that the plane passing through the diameters above mentioned, and through the poles of the two circles to which the line joining these points is at right angles, will revolve with a like motion, round that diameter of the sphere which is at right angles to the plane of the ecliptic, and which therefore terminates in its poles. But since the pole of the celestial equator is upon this circle at a distance from the pole of the ecliptic equal to the obliquity of the ecliptic, it follows that the pole of the equator will be carried round the pole of the ecliptic, in a lesser circle parallel to the plane of the ecliptic, with a retrograde motion exactly equal to that of the equinoxial points.

3282. *Distance of pole of equator from pole of ecliptic varies with the obliquity.*—And since the distance of the pole of the equator from that of the ecliptic must always be exactly equal to the obliquity of the ecliptic, it follows that every change which may take place from whatever cause, in the position of the plane of the equator, whether the change affect the angle at which it is inclined to the ecliptic, or the position of the equinoxial points, must be attended with a corresponding change, either in the apparent distance of the pole of the equator from that of the ecliptic, or in the rate or direction of the motion of the latter round the former.

3283. *Pole star varies from age to age.* — As the pole of the equator is carried with this slow motion round the pole of the ecliptic, its position for all popular, and even for some scientific, purposes is usually indicated by the nearest conspicuous star, for it rarely happens that any such star is found to coincide with its exact place. Such star is the pole star, for the time being; and it is clear from this motion of the pole, that the pole star must necessarily change from age to age.

The present polar star is a star of the second magnitude in the constellation called the "Lesser Bear," and its present distance from the exact position of the pole is 1° 24′.

The motion of the pole as above described, however, is such

that this distance is gradually diminishing, and will continue to diminish until it is reduced to about half a degree; after which it will increase, and after the lapse of a long period of time, the pole will depart from this star, and it will cease to bear the name, or serve the purposes, of a pole star.

3284. *Former and future pole stars.* — If upon any star-map a circle be traced round the pole of the ecliptic at a distance from it of 23·5°, such circle will pass through all positions which the pole of the equator will have in time to come, or has has had in time past; and it will then be easily seen which are the conspicuous stars in whose neighbourhood it will pass in after ages, and near which it has passed in past ages, and which will become in future, or have been in past times, the pole star of the age.

In 12,000 years from the present time, for example, it will be found that the pole will pass within a few degrees of the star of the first magnitude in the constellation of "Lyra," called *α Lyræ.*

In tracing back in the same manner the position of the pole among the stars, it is found that at an epoch 3970, or nearly 4000 years, before the present time, the pole was 55° 15′ behind its present position in longitude; and at this time the nearest bright star to it was the star γ, in the constellation of "Draco." The distance of this star, at that time, from the pole must have been 3° 44′ 25″.

3285. *Remarkable circumstance connected with the pyramids.* — In the researches which have been made in Egypt, a somewhat remarkable circumstance has been discovered, having relation to this subject.

Of the nine pyramids which still remain standing at Gizeh, six have openings presented to the north, leading to straight passages which descend at an inclination varying from 26° to 27°, the axes of the passages being in all cases in the plane of the meridian of the pyramid. Two pyramids, still standing at Abousseir, have similar openings leading to passages having similar directions.

Now, if we imagine an observer stationed at the bottom of any of these passages, and looking out along its axis as he would look through the tube of a telescope, his view will be directed to a point upon the northern meridian of the place of the pyramid at an altitude of between 26° and 27°, correspond-

ing with the slope of the passage. This is precisely the altitude at which the star γ Draconis must have passed the meridian below the pole, at the date of 3970 years before the present time, allowing for the difference of position of the pole according to the principle affecting the precession of the equinoxes explained above. Now, the date of the construction of the pyramids corresponds almost exactly with this epoch; and it cannot be doubted, that the peculiar direction given to these passages must have had reference to the position of γ Draconis, the pole star of that age.

3286. *Nutation.* — The regression, described above, of the equinoxial points upon the ecliptic, must be understood as their mean change of place produced by the disturbing forces of the sun and moon upon the protuberant matter of the equator in long periods of time. But this regression is not produced at a uniform rate. The disturbing forces vary in their action according to the general principles already explained, with the angles formed by lines drawn from the sun and moon to the centre of the earth with the plane of the equator. So far as relates to the sun, this variation in its effect goes through all its changes within a year. In the case of the moon, it will obviously vary from month to month and from year to year, with the change of position of the moon's nodes; and as these nodes have a regressive motion making a complete revolution in about nineteen years, the variation of the effect of the moon's disturbing force will pass through all its changes within that period. The regressive motion imparted to the equinoxial points, and also to the pole of the equator in moving round the pole of the ecliptic, as already described, by the sun and moon, is therefore subject to an alternate increase and decrease, whose period is a year for the sun, and nineteen years for the moon.

But these are not the only effects produced upon the position of the pole of the equator by the disturbing action of the moon and sun. According to what has been explained in general of the effects of the orthogonal component of the disturbing force, it will be easily understood that the protuberant matter of the equator being regarded as a satellite disturbed by the sun and moon, the inclination of the plane of the equator to the ecliptic will be subject to a variation proceeding from the disturbing force of the sun, whose period will be a year; and its inclina-

tion to the plane of the moon's orbit will be subject to a like variation, whose period is about nineteen years. These changes of the inclination of the plane of the equator to that of the ecliptic and the moon's orbit will be attended with a corresponding motion of the pole of the equator to and from the pole of the ecliptic.

This alternate approach and recess of the pole of the equator to and from the pole of the ecliptic, combined with the alternate increase and decrease of its regressive motion, is called the *Nutation;* that part of it due to the sun being called the *solar nutation;* and that due to the moon, the *lunar nutation.*

The solar nutation is an inequality of so small amount as altogether to escape observation, and therefore must be looked upon to have a merely theoretical existence.

It is otherwise, however, with the lunar nutation. By the alternate increase and decrease of the regressive motion of the pole, combined with its alternate approach and recess to and from the pole of the ecliptic, the pole is moved in such a manner that, if it were affected only by the disturbing force of the moon, it would describe an ellipse such as A B C D, *fig.* 859.; the major axis of which would be in the direction A E of the pole of the ecliptic, and would measure 18·5″, while the minor axis would be at right angles to this direction, and would measure 13·74″.

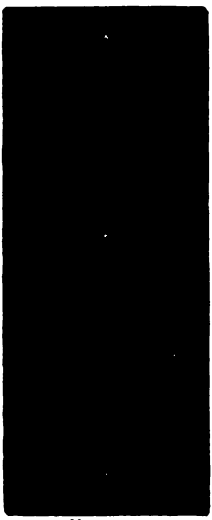


Fig. 859.

But while the pole of the equator describes this ellipse completing its revolution in nineteen years, it is carried by the common motion of precession, in a retrograde direction, as already described, at the rate of 50·1″ in each year, and will, therefore, in nineteen years be carried through 15·5′ in its motion round the pole of the equator. Now, by combining this motion with the elliptic motion already described, it will be easily seen that the pole of the equator would, in revolving round the pole of the ecliptic, alternately approaching to it and receding from it through

9·25″, describe an undulating line such as is represented in *fig.* 860., where P represents the pole of the ecliptic.

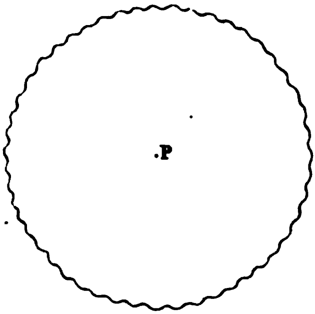


Fig. 860.

3287. *Equation of the equinoxes.* — Since the regression of the equinoxes does not take place at a uniform rate, but is subject to variations, alternately increasing and decreasing during every nineteen years, its true place will differ from its mean place. If we conceive an imaginary equinoxial point moving backward, with a uniform motion at the rate of 50° 1″, the place of such point would be the mean place of the equinoxial point. The true place would vary from this, preceding it when the disturbing force augments the rate at which the equinoxial point moves, and falling behind it when it decreases that rate.

The distance between the true and imaginary equinoxial points is called the *equation of the equinoxes.*

The mean place of the equinox for any proposed time is given by tables; and the equation of the equinoxes for the proposed time gives the quantity to be added to, or subtracted from, the mean place, to find the true place.

3288. *Proportion of the mean precession due to the disturbing forces of the moon and sun.*—If the entire amount of the mean precession in a given time be expressed by 7, the part due to the moon will be 5, and that due to the sun will be 2.

3289. *Like effects produced in the case of other planets.*—These disturbing effects produced upon the plane of the planet's equator, are not confined to the case of the earth. All the planets which have the spheroidal form, are subject to similar effects from the sun's attraction on their equatorial protuberance, the magnitude of these effects being, however, less as the distance from the sun is increased. In the case of the major planets, the sun's disturbing action on the planet's equator, proceeding from this cause, will be altogether insensible.

The disturbing forces of the satellites exerted upon the plane of the equator, in the cases of the major planets, however, must be considerable in magnitude, especially so far as relates to the

inner satellites, and very complicated in its character, the precession and nutation of each of the satellites separately being combined in affecting the actual position of the pole of the planet.

Since, however, these phenomena are necessarily local, and manifested only to observers on the planet, they offer merely speculative interest to the terrestrial astronomer.

3290. *Effects of spheroidal perturbation on the motions of the moon generally minute.*—The protuberant matter of the terrestrial spheroid disturbs the lunar orbit, in the same manner as would a satellite placed at the surface of the earth and in the plane of the equator. From what has been explained of the general effects when the disturbing body is within the disturbed orbit, it will follow that the terrestrial spheroid must impart a progressive motion to the moon's apsides, and a regressive motion to the nodes. These inequalities have, however, a theoretical existence only, being so minute, compared with the progression of the apsides and regression of the nodes due to the disturbing force of the sun, that they do not produce any observable change in these motions.

3291. *Spheroidal inequality of the inclination of the moon's orbit observable.*—A case, however, exists, in which the disturbing force of the terrestrial spheroid does produce sensible effects on the lunar orbit.

It has been already shown, that the moon's nodes move round the ecliptic with a retrograde motion, in about nineteen years. Twice in this period they must, therefore, coincide with the equinoxial points; and when they do so, the line of nodes must coincide with the inclination of the planes of the equator and ecliptic. In one of the two positions which they thus assume, the plane of the moon's orbit must lie between the planes of the ecliptic and equator as represented in *fig.* 861., where

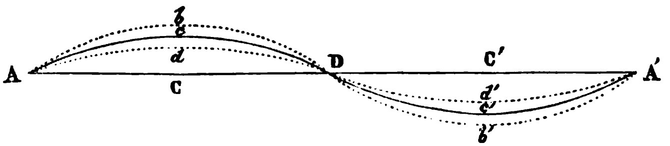


Fig. 861.

A C D C′ A′ represents the equator, A *c* D *c*′ A′ the ecliptic, and A *d* D *d*′ A′ the moon's orbit. In the other position the moon's

orbit will make a greater angle with the equator than the ecliptic does, and will lie above the ecliptic, as represented at A b D b' A'.

Now, if no disturbing force acted upon the plane of the moon's orbit, its obliquity to the plane of the ecliptic would remain invariable; and, therefore, in both positions here represented, the angles it would make with the ecliptic would be the same, that is to say, 5° 8′ 48″, which is the mean inclination of the moon's orbit.

But, if it be admitted that the spheroidal protuberance of the earth acts, as it has been shown to do, as a satellite would situate in the plane of the equator, this attraction would likewise tend to draw the moon towards that plane, and it would act more energetically when the plane of the moon's orbit makes a less, than when it makes a greater, angle with the equator. Thus the spheroidal perturbations of the earth will act more energetically in disturbing the plane of the moon's orbit, when it has the position A d D d' A', than when it has the position A b D b' A'. The consequence would be that the angle formed by A d D with A c D would be somewhat greater than the angle formed by A b D with A c D.

Another consequence, of still greater practical importance in producing an inequality, which would be still more easily observable, will follow from this. The greater energy of the disturbing force when the moon's orbit has the position A d D than when it has the position A b D, will impart to the moon a more rapid angular motion when it passes through the point of the orbit represented by A d D, than when it passes through that point of the orbit indicated by A b D; that is to say, the apparent motion of the moon will be more rapid than its mean motion, when the orbit has the former position, and less rapid when the orbit has the latter position. Now, a small variation of the moon's apparent motion is more easily observed than a small variation of the inclination of its orbit in the ecliptic.

These effects of the disturbing force of the spheroidal protuberance of the earth have accordingly been observed.

It has been found that the angle formed by A d D with A c D is greater, and the angle formed by A b D with A c D less, than the mean inclination of the moon's orbit to the ecliptic; and while the moon's orbit has the position A d D its apparent motion is greater, and while it has the position A b D its apparent

motion is less, than its mean motion. This difference is found to produce a variation of the moon's place in its orbit or of its longitude amounting to 8″, by which it gets in advance of its mean place when the orbit has the position A *d* D, and lags behind it when the orbit has the position A *b* D.

Since the interval between the epochs at which the moon's orbit has the two positions indicated in the figure, is one half the period of the regression of the nodes, that is about nine years and a half, it follows that the moon's inclination to the ecliptic gradually increases during that interval of nine years and a half, from its position A *d* D to its position A *b* D, and that, during the next nine years and a half, it gradually decreases from its position A *b* D to its position A *d* D; therefore, it follows that the moon's angular motion is continually greater than its mean value during nine years and a half, and continually less during the next nine years and a half.

Now, this inequality in its exact quantity depends altogether upon the oblateness of the terrestrial spheroid, or, what is the same, on the ratio of the equinoxial, to the polar diameter of the earth. If this ratio were greater than 301 to 300, its actual value, the inequality of the moon's motion consequent upon it would be greater, and accordingly its apparent place would be more than 8″ in advance of the true place in the one interval of nine years and a half, and less so in the succeeding nine years and a half; and, on the contrary, if the ratio of the equatorial to the polar diameter were less than 301 to 300, the variation of the moon's apparent motion would be less than 8″. Thus it appears, that this minute inequality of the moon's motion, developed in the protracted periods of nineteen years, supplies a measure of the spheroidal form of the earth, the result of which completely verifies the other methods already explained, of determining the ellipticity of the terrestrial spheroid.

3292. *Spheroidal inequalities of the Jovian system.* — The oblateness of Jupiter, still greater than that of the earth, produces inequalities of the motion of the satellites, similar to those produced by the spheroidal protuberance of the earth upon the moon. but greater in degree and more rapid in their development, in proportion to the greater ellipticity of the planet and the greater proportional proximity of the satellites. Thus, a progressive motion is imparted to the apsides, and a

regressive motion to the nodes of each of the satellites; and these motions are so much the more considerable the nearer the satellite is to the planet. The effect of this spheroidal perturbation on the inner satellites is so predominant, that the motion of these satellites is nearly the same as it would be if no other disturbing force whatever affected them.

It will be obvious that a very complicated system of inequalities must thus be produced upon the satellites of Jupiter, inasmuch as the disturbing effect of the spheroidal protuberance of the planet upon the elements of each of the orbits, is necessarily mixed up with the disturbing effects of the satellites upon each other.

3293. *Spheroidical inequalities of the Saturnian system.* — The disturbing forces produced by the spheroidal form of Saturn, are in all respects similar to those developed in the Jovian system. The effect of the ring is nearly the same as if the oblateness of the planet were augmented, — with this difference, however, that the ring, not being attached to the planet, is not clogged with the inertia of a great spherical mass within it, hanging upon it, as is the case with the spheroidal protuberance of the planet.

In consequence of this, the combined effects of the ring and the spheroidal protuberance of the planet, is to impart a very rapid progressive motion to the apsides, and a regressive motion to the nodes of the orbits of the several satellites. These effects must be obviously greater on the inner, than on the more remote satellites.

The theory of the perturbations of this system is rendered very simple, owing to the fact that the sun, in consequence of its remoteness and of the small visual angle which the whole system subtends at it, can produce no sensible perturbation. The satellites have, however, been so imperfectly observed that no data have been obtained, by which the results of the theoretical reasoning respecting the spheroidal perturbations can be verified with regard to any of the satellites, except the sixth. This has, however, been submitted to an elaborate series of observations by Professor Bessel, who has ascertained the rate of progression of its apsides, and the regression of its nodes.

By calculating what the progression and regression would be, which would arise from a given mass assigned to the ring, and

comparing the results of such calculation with the actual progression of the apsides and regression of the nodes, he found that the actual inequalities are such as would be produced, if the mass of the ring amounted to the 118th part of the entire mass of the planet. In this approximative calculation, however, the whole inequality incident to the nodes and apsides of the sixth satellite, is ascribed to the disturbing force of the ring. It is evident, therefore, that the value of the mass of the ring, thus obtained, must be more than its true value.

It appears, therefore, that while the entire mass of this annular appendage, stupendous as it is, is less than the 118th part of the mass of the planet, the mass of the moon amounts to so much as the 80th part of the mass of the earth. It follows, therefore, that relatively the entire mass of Saturn's ring is less than that of the moon, in the proportion of 2 to 3.

CHAP. XXV.

THE FIXED STARS. — STELLAR PARALLAX AND DISTANCE.

3294. *Creation not circumscribed by the solar system.* — The region of space, vast as it is, which is occupied by the solar system, forms but a small portion of that part of the material universe to which scientific inquiry and research have been extended. The inquisitive spirit of man has not rested content within such limits. Taking its stand at the extremities of the system, and throwing its searching glance toward the interminable realms of space which extend beyond them, it still asks—What lies there? Has the Infinite circumscribed the exercise of his creative power within these precincts—and has He left the unfathomable depths of space that stretch beyond them, a wide solitude? Has He whose dwelling is immensity, and whose presence is everywhere and eternal, remained inactive throughout regions compared with which the solar system shrinks into a point?

Even though scientific research should have left us without

definite information on these questions, the light which has been shed on the Divine character, as well by reason as by revelation, would have filled us with the assurance that there is no part of space however remote, which must not teem with evidences of exalted power, inexhaustible wisdom, and untiring goodness.

But science has not so deserted us. It has, on the contrary, supplied us with much interesting information respecting regions of the universe, the extent of which is so great that even the whole dimensions of the solar system supply no modulus sufficiently great to enable us to express their magnitude.

3295. *The solar system surrounded by a vast but limited void.*—We are furnished with a variety of evidence, establishing incontestably the fact, that around the solar system to a vast distance on every side there exists an unoccupied space; that the solar system stands alone in the midst of a vast solitude. It has been shown, that the mutual gravitation of bodies placed in the neighbourhood of each other, is betrayed by its effects upon their motions. If, therefore, there exist beyond the limits of the solar system, and within a distance not so great as to render the attraction of gravitation imperceptible, any mass of matter, such as another sun like our own, such a mass would undoubtedly exercise a disturbing force upon the various bodies of the system. It would cause each of them to move in a manner different from that in which it would have moved if no such body existed.

Thus it appears that, even though a mass of matter in our neighbourhood should escape direct observation, its presence would be inevitably betrayed by the effects which its gravitation would produce upon the planets. No such effects, however, are discoverable. The planets move as they would move if the solar system were independent of any external disturbing attraction. These motions are such, and such only, as can be accounted for by the attraction of the sun and the reciprocal attraction of the other bodies of the system. The inference from this is, that there does not exist any mass of matter in the neighbourhood of the solar system within any distance which permits such a mass to exercise upon it any discoverable disturbing influence; and that if any body analogous to our sun exists in the universe, it must be placed at a distance

so great, that the whole magnitude of our system will shrink into a point, compared with it.

3296. *The stars must be placed beyond the surrounding void.—Absence of sensible parallax.*—Yet when, on any clear night, we contemplate the firmament, and behold the countless multitude of objects that sparkle upon it, remembering what a comparatively small number are comprised among those of the solar system, and even of these how few are visible at any one time, we are naturally impelled to the inquiry, Where in the universe are these vast numbers of objects placed?

Very little reflection and reasoning, applied to the consideration of our own position and to the appearance of the heavens, will convince us that the objects that chiefly appear on the firmament, must be at almost immeasureable distances. The earth in its annual course round the sun moves in a circle, the diameter of which is about 200 millions of miles. We, who observe the heavens, are transported upon it round that vast circle. The station from which we observe the universe at one period of the year is, then, 200 millions of miles from the station from which we view it at another.

Now it is a fact, within the familiar experience of every one, that the relative position of objects will depend upon the point from which they are viewed. If we stand upon the bank of a river, along the margin of which a multitude of ships are stationed, and view the masts of the vessels, they will have among each other a certain relative arrangement. If we change our position, however, through the space of a few hundred yards, the relative position of these masts will not be the same as before. Two which before lay in line will now be seen separate; and two which before were separated are now brought into line. Two, one of which was to the right of the other, are now reversed; that which was to the right, is at the left, and *vice versâ;* nor are these changes produced by any change of position of the ships themselves, for they are moored in stationary positions. The changes of appearance are the result of *our own change of position;* and the greater that change of position is, the greater will be the relative change of these appearances. Let us suppose, however, that we are moved to a much greater distance from the shipping; any change in our position will produce much less effect upon the

relative position of the masts; perhaps it will require a very considerable change to produce a perceivable effect upon them. In fine, in proportion as our distance from the masts is increased, so in proportion will it require a greater change in our own position to produce the same apparent change in their position.

Thus it is with all visible objects. When a multitude of stationary objects are viewed at a distance, their relative position will depend upon the position of the observer; and if the station of the observer be changed, a change in the relative position of the objects must be expected; and if no perceptible change is produced, it must be inferred that the distance of the objects is incomparably greater than the change of position of the observer.

Let us now apply these reflections to the case of the earth and the stars. The stars are analogous to the masts of the ships, and the earth is the station on which the observer is placed. It might have been expected that the magnitude of the globe, being eight thousand miles in diameter, would produce a change of position of the observer sufficient to cause a change in the relative position of the stars, but we find that such is not the case. The stars, viewed from opposite sides of the globe, present exactly the same appearance; we must, therefore, infer that the diameter of the earth is absolutely nothing compared to their distance.

But the astronomer has still a much larger modulus to fall back upon. He reflects, as has been already observed, that he is enabled to view the stars from two stations separated from each other, not by 8000 miles, the diameter of the earth, but by 200 millions of miles, that of the earth's orbit. He, therefore, views the heavens on the 1st of January, and views them again on the 1st of July, the earth having in the meanwhile passed to the opposite side of its orbit, yet he finds, to his amazement, that the aspect is the same. He thinks that this cannot be—that so great a change of position in himself cannot fail to make some change in the apparent position of the stars;—that, although their general aspect is the same, yet when submitted to exact examination a change must assuredly be detected. He accordingly resorts to the use of instruments of observation capable of measuring the relative positions of the stars with the last conceivable precision, and

he is more than ever confounded by the fact that still no discoverable change of position is found.

For a long period of time this result seemed inexplicable, and accordingly it formed the greatest difficulty with astronomers, in admitting the annual motion of the earth. The alternative offered was this; it was necessary, either to fall back upon the Ptolemaic system, in which the earth was stationary, or to suppose that the immense change of position of the earth in the course of half a year, could produce no discoverable change of appearance in the stars; a fact which involves the inference that the diameter of the earth's orbit must be a mere point compared with the distance of the nearest stars. Such an idea appeared so inadmissible that for a long period of time many preferred to embrace the Ptolemaic hypothesis, beset as it was with difficulties and contradictions.

Improved means of instrumental observation and micrometrical measurement, united with the zeal and skill of observers, have at length surmounted these difficulties; and the parallax, small indeed but still capable of measurement, of several stars has been ascertained.

3297. *Annual parallax.—Parallactic ellipse.*—To render these results and the processes by which they have been attained intelligible, we must resume the explanation of the general effects of annual parallax, already briefly given (2442.).

The visual ray by which a star is seen, and which is its apparent direction, is carried by the annual motion of the earth round the surface of a cone, of which the earth's orbit (which we may here consider as a circle) is the base and of which the star is the apex. The line drawn from the centre of the earth's orbit to the star, which is its true or heliocentric direction, is the axis of this cone; and consequently, the parallax of the star is the angle under the latter line, and the visual ray by the motion of which the surface of the cone is formed.

The same optical effect would be produced by transferring the orbital motion of the earth to the star, the observer being supposed to be stationary and placed at the centre of the earth's orbit, and this supposition will render all the parallactic phenomena much more easily comprehended. Let the star, then, be imagined to move in a circle equal and parallel to the earth's orbit, the centre of the circle being the true place of the

star. The place of the star in this circle of parallax must always be diametrically opposite to the corresponding place of the earth in its orbit. The star so moving would suffer exactly the same apparent displacement as it would appear to suffer if it were, as it is, at rest in its true place, the earth moving in its proper orbit round the sun.

Let S, *fig.* 862., be the true place of the star, and let A B A′ B′ be its circle of parallax, the plane of which is parallel to that of the ecliptic, and the radius of which is equal to that of the earth's orbit. Let B B′ be the line in which the plane of this circle is intersected by a plane through the sun and star perpendicular to the ecliptic, or what is the same, by the plane of the circle of latitude passing through the star; and let A A′ be the diameter of the circle of parallax which is at right angles to that plane. When the longitude of the sun is the same as that of the star, the apparent place of the star will be at B′, that extremity of the diameter B B′ which is most remote from the sun; and when the longitude of the sun exceeds that of the star by 180°, it will be at B, the extremity of the same diameter which is nearest to the sun. When the longitude of the sun exceeds that of the star by 90°, the apparent place of the star will be at A′, the eastern extremity of the diameter A A′; and when it exceeds that of the star by 270°, it will be at A, the western extremity. It appears therefore, that while the sun makes a complete revolution of the ecliptic from the point at which it has the longitude of the star until its return to the same point, the star appears to move round the circle of parallax from B′ in the direction indicated by the arrows, taking successively the positions p and the angle B′ S p or the arc B′ p, being always equal to the difference of the longitudes of the sun and star measured from the star in the order of the signs.

Fig. 862.

But the plane of this circle A B′ A′ B being parallel to that of the ecliptic, will be inclined to the surface of the celestial sphere, at an angle equal to the complement of the latitude of the star; and by the common principles of projection it will, when projected optically on that surface, become an ellipse, of

which the major axis is the projection of the diameter A A′, and is parallel to the ecliptic, and the minor axis that of B B′, and which is perpendicular to the ecliptic, and therefore coincident with the circle of latitude of the star. The diameter A A′ being parallel to the surface on which it is projected, is not altered in apparent magnitude by projection; but the diameter B B′ is diminished by projection, in the ratio of the sine of the star's latitude to 1.

In the figure, this ellipse is represented above the circle of parallax.

It will be apparent that, when the star is projected on B or B′, its longitude is not affected by parallax, but its latitude is increased by S B at B, and diminished by S B′ at B′. In like manner, when the star is seen at A or A′, its latitude is not affected by parallax; but its longitude is increased by S A′ at A′, and diminished by S A at A.

When the star is seen at any of the intermediate points p of the parallactic ellipse, it is affected by parallax both in latitude and longitude, S n being the parallax in longitude, and $p\,n$ the parallax in latitude.

By the mere inspection of the figure it will be apparent, that the parallax in longitude S n is not affected by projection, being the same in the parallactic circle as in the ellipse; but that the parallax in latitude $p\,n$ is reduced in the ratio of the sine of the star's latitude to 1.

It is easy, and in some respects clearer, to express these relations in the symbols of arithmetic. Let ω be the excess of the sun's longitude above that of the star, let ϖ be the angle which the radius of the circle of parallax subtends at the earth, and let λ be the latitude of the star. The angle B′ S p in the parallactic circle will then be ω, and the parallax in longitude S n will be $\varpi \times \sin.\ \omega$. The parallax in latitude which is $p\,n$ in the parallactic ellipse will be $\varpi \times \cos.\ \omega \times \sin.\ \lambda$.

When the star is at B or B′, $\cos.\ \omega = 1$, and S B′ $= \varpi \times \sin.\ \lambda$, and when it is at A or A′, $\sin.\ \omega = 1$, and S A $= \varpi$.

3298. *Excentricity of parallactic ellipse depends on star's latitude.*—The excentricity of the parallactic ellipse increases as the star's latitude decreases. If the star be at the pole of the ecliptic, the plane of the circle of parallax being parallel to the surface of the celestial sphere, it is not altered by projection;

and the apparent motion of the star is in a circle, of which the centre is the true place of the star, and of which the radius is the star's parallax. When the latitude of the star is less than 90°, the obliquity of the plane of the circle of parallax to the visual direction gives an elliptical form to its projection, which becomes more and more elongated the nearer the star is to the ecliptic. When the star is in the ecliptic, the circle of parallax is viewed edgeways, and the parallactic ellipse is flattened into its major axis. The star in this position oscillates east and west of its true place to a distance equal to the parallax.

Fig. 863.

In *fig.* 863. the forms of the parallactic ellipse are represented as they vary in excentricity from a perfect circle at the pole of the ecliptic to a line equal to twice the parallax in the ecliptic itself.

3299. *Difficulty of determining the parallax, arises from its minute amount.*—It might be supposed, that where the character and laws of the phenomena are so clearly understood, the discovery of their existence could present no great difficulty. Nevertheless, nothing in the whole range of astronomical research has more baffled the efforts of observers than this question of the parallax. This has arisen altogether from the extreme minuteness of its magnitude. It is quite certain that the quantity we have designated by ϖ in the preceding paragraphs, does not amount to so much as $1''$ in the case of any of the numerous stars which have been as yet submitted to the course of observation which is necessary to discover the parallax. Now, since in the determination of the exact uranographical position of a star geocentrically and heliocentrically considered, there are a multitude of disturbing effects to be taken into account and eliminated, such as precession, nutation, aberration, refraction, and others, besides the proper motion of the star, which will be explained hereafter; and since besides the errors of observation, the quantities of these are subject to more or less uncertainty, it will astonish no one to be told that they may entail upon the final result of the calculation, an

error of 1″; and if they do, it is vain to expect to discover such a residual phenomenon as parallax, the entire amount of which is less than 1″.

3300. *How the distance is inferred from the parallax—parallactic unit.*—If in any case the parallax or the quantity which in the preceding paragraphs has been expressed by ϖ, and which is the semi-axis major of the parallactic ellipse, could be determined, the distance of the stars could be immediately inferred. For, if this value of ϖ be expressed in seconds or in decimals of a second, and if R express the semidiameter of the earth's orbit, and D the distance of the star, we shall have

$$D = R \times \frac{206265}{\varpi}.$$

If, therefore, $\varpi = 1''$ the distance of the star would be 206265 times the distance of the sun, and since it may be considered satisfactorily proved that no star which has ever yet been brought under observation has a parallax greater than this, it may be affirmed that the nearest star in the universe to the solar system is at a distance *at least* 206265 times greater than that of the sun.

Let us consider more attentively the import of this conclusion. The distance of the sun expressed in round numbers (which are sufficient for our present purpose) is 95 millions of miles. If this be multiplied by 206265, we shall obtain,—not indeed the distance of the nearest of the fixed stars,—but the *minor limit* of that distance, that is to say, a distance within which the star cannot lie. This limit expressed in miles is

$$D = 206265 \times 95{,}000000 = 19{,}595175{,}000000 \text{ miles},$$

or nearly *twenty billions of miles.*

3301. *Motion of light supplies a convenient unit for the stellar distance.*—In the contemplation of such numbers the imagination is lost, and no other clear conception remains, except of the mere arithmetical expression of the result of the computation. Astronomers themselves, accustomed as they are to deal with stupendous numbers, are compelled to seek for units of proportionate magnitude to bring the arithmetical expression of the quantities within moderate limits. The motion of light supplies one of the most convenient moduli for this purpose, and has, by common consent, been adopted as the

unit in all computations whose object is to gauge the universe. We have shown that light moves at the rate of 192000 miles per second. If, then, the distance D above computed be divided by 192000, the quotient will be the time, expressed in seconds, which light takes to move over that distance. But since even this will be an unwieldy number, it may be reduced to minutes, hours, days, or even to years.

In this manner we find that, if any star have a parallax of 1", it must be at such a distance from our system that light would take 3·235 years, or three years and eighty-five days, to come from it to the earth.

If the space through which light moves in a year be taken, therefore, as the unit of stellar distance, and ϖ be the parallax expressed in seconds or decimals of a second, we shall have

$$D = \frac{3 \cdot 235}{\varpi}.$$

3302. *Methods of ascertaining the parallax, and consequently the distance.* — It will easily be imagined that astronomers have diligently directed their observations to the discovery of some change of apparent position, however small, produced upon the stars by the earth's motion. As the stars most likely to be affected by the motion of the earth are those which are nearest to the system, and therefore probably which are brightest and largest, it has been to such chiefly that this kind of observation has been directed; and since it was certain that, if any observable effect be produced by the earth's motion at all, it must be extremely small, the nicest and most delicate means of observation were those alone from which the discovery could be expected.

One of the earlier expedients adopted for the solution of this problem was the erection of a telescope, of great length and power, in a position permanently fixed, attached, for example, to the side of a pier of solid masonry erected upon a foundation of rock. This instrument was screwed into such a position that particular stars, as they crossed the meridian, would necessarily pass within its field of view. Micrometric wires were, in the usual manner, placed in its eye-piece, so that the exact point at which the stars passed the meridian each night, could be observed and recorded with the greatest precision. The instrument being thus fixed and immovable, the transits of the

stars were noted each night, and the exact places where they passed the meridian recorded. This kind of observation was carried on through the year; and if the earth's change of position, by reason of its annual motion, should produce any effect upon the apparent position of the stars, it was anticipated that such effect would be discovered by these means. After, however, making all allowance for the usual causes which affect the apparent position of the stars, no change of position was discovered which could be assigned to the earth's motion.

3303. *Professor Henderson's discovery of the parallax of α Centauri.* — Notwithstanding the numerous difficulties which beset the solution of this problem, by means of observations made with the ordinary instruments Professor Henderson, during his residence as astronomer at the Royal Observatory at the Cape of Good Hope, succeeded in making a series of observations upon the star designated α in the constellation of the Centaur, which, being afterwards submitted by him to the proper reductions, gave a parallax of 1″. Subsequent observations made by his successor, Mr. Maclear, at the same observatory, partly with the same instrument, and partly with an improved and more efficient one of the same class, have fully confirmed this result, giving 0·9128, or $\frac{10}{11}$ths of a second as the parallax.

It is worthy of remark, that this conclusion of Messrs. Henderson and Maclear is confirmed in a remarkable manner, by the fact that like observations and computations applied to other stars in the vicinity of α Centauri, and therefore subject to like annual causes of apparent displacement, such as the mean annual variation of temperature, gave no similar result, showing thus that the displacement found in the case of α Centauri could only be ascribed to parallax.

Since the limits of error of this species of observation affecting the final result cannot exceed the tenth of a second, it may then be assumed as proved, that the parallax of α Centauri is 1″, and consequently that its distance from the solar system is such that light must take 3·235 years to move over it.

3304. *Differential method.*—In the practical application of the preceding and all similar methods of ascertaining the stellar parallax, it must not be imagined that every apparent deviation

from a fixed position that may be observed, is to be immediately placed to the account of parallax. There are a great number of other causes of apparent displacement, which must first be allowed for; and it is only after eliminating these, and discovering the quantity and direction of the residual displacement, that we are in a position to pronounce upon the existence and quantity of the parallax. But in all such calculations the various quantities to be thus taken into account and previously eliminated, are subject to errors, small in magnitude it is true, but still great enough on the whole to absorb the entire amount of a residual phenomenon so minute as the stellar parallax must in most cases be. All such methods of observation and calculation are therefore liable to be rendered abortive by the fact, that they are subject to sources of ultimate error, the amount of which may be greater than the quantity sought.

Independent of this class of errors, there are others which do not less impede the discovery of a quantity so exceedingly minute as the parallax, and which have a very different origin. All astronomical instruments are exposed to uncertain and variable changes of temperature, which cause the materials of which they are composed, to undergo equally uncertain and variable expansions and contractions. The piers of stone-work to which they are attached, nay, the very foundation on which these piers rest, is liable to these changes, which more especially affect the result of the comparison of observations made at intervals of six months, and therefore at opposite seasons of the year when the effects of difference of temperature are the most aggravated. "Hence," as Sir John Herschel observes, "arise slow oscillatory movements of exceedingly minute amount, which levels and plumb lines afford but very inaccurate means of detecting, and which *being also annual in their period* (after rejecting whatever is merely casual and momentary), mix themselves intimately with the matter of our inquiry," and give results which are especially liable to be mistaken for those of stellar parallax. Refraction itself, besides its casual and irregular changes, is subject to mean periodical variations which vary in different latitudes, and in different places in the same latitude, according to unascertained laws, but which, having periods dependent on the seasons, and therefore annual, must always be liable to be confounded with those of parallax.

It was, therefore, highly desirable to discover a method of

detecting the stellar parallax, which, while it would be free from the uncertainties attending the other sources of displacement which are eliminated subject to a certain limit of error, should also be independent of the sources of error of the latter class. This object was attained by an expedient which we shall now explain.

Let S, *fig.* 864., be a star which we will suppose to have sensible parallax; and let A B, A′ B′ be its parallactic ellipse. Let S′ be another star situate in the same parallel to the ecliptic, and therefore having the same latitude, and so near to it as to be included with S in the field of the telescope; and let S″ be another situate in the same circle of latitude, and therefore having the same longitude as S, and also so near to S as to be included in the field of view of the telescope.

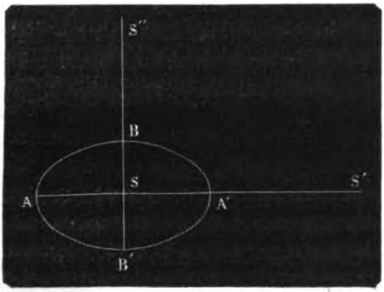


Fig. 864.

Let us suppose for the present, that the stars S′ and S″ have no sensible parallax, and therefore undergo no apparent change of position throughout the year. From what has been already explained, it will be evident that the apparent place of S will be A′, when the sun's longitude exceeds that of the star by 90°; it will be B, when it exceeds it by 180°; A, when it exceeds it by 270°; and B′, when the sun's longitude is the same as that of the star. The semi-axis major S A of the parallactic ellipse is the actual parallax, or the angle which the semidiameter of the earth's orbit subtends at the star. Let this be expressed by ϖ. S B the semi-axis minor will be found by multiplying this last by the sine of the star's latitude, as has been already explained. Let this latitude be expressed by λ; we shall therefore have

$$\mathrm{S\,A} = \varpi, \quad \mathrm{S\,B} = \varpi \times \sin \lambda.$$

Let $\mathrm{S'A} = \mathrm{D}$, $\mathrm{S'A'} = d$, and $\mathrm{S'\,S} = \Delta$. Since $\mathrm{S\,A} = \mathrm{S\,A'} = \varpi$, we shall have

$$\Delta = \tfrac{1}{2}\,(\mathrm{D}+d), \qquad \varpi = \tfrac{1}{2}\,(\mathrm{D}-d).$$

that is, the true distance between the stars S and S′ is half the sum

of their apparent distances at the times when the difference between their longitude and that of the sun is 270° and 90°, and the parallax of S is half the difference between the same apparent distances.

In like manner, let $S'' B' = D'$, $S'' B = d'$, and $S'' S = \Delta$, and we shall have

$$\Delta' = \tfrac{1}{2}(D' + d'), \qquad \varpi \times \sin.\lambda = \tfrac{1}{2}(D' - d');$$

that is, the true distance is half the sum of the apparent distances when the difference of longitude of the sun and star is 0° and 180°, and the parallax is half the difference of the same distances divided by the sine of the star's latitude.

It is evident, therefore, that under the supposition here made of the absence of all sensible parallax in the subsidiary stars S' and S'', the actual parallax of the star S would be found by measuring with the micrometer the distance between the stars S and S', at the epochs when the sun's longitude differs from that of the star by 90° and 270°.

In like manner, the parallax would be found by comparing the star S with the star S'' at the epoch when the difference between the longitude of the sun and that of the star is 0° and 180°.

It is evident, that the four observations here indicated, two upon each of the stars, will be made at intervals of three months, determined by the epochs at which the longitude of the sun exceeds that of the star by 0°, 90°, 180°, and 270°.

The precision with which micrometrical measurements can be effected, when applied to two or more objects which are simultaneously present in the field of view of the telescope, renders this method of observation susceptible of extraordinary exactness. It is also attended with the obvious advantage of being totally independent of all disturbing causes which, in this case, equally affect all the objects present simultaneously in the field of view; thus all the uncertainties attending the effects of refraction, aberration, precession, nutation, &c., may be here discarded, as not interfering in any way with the final result of the observations.

3305. *Position micrometer, its application to this problem.*— But these are not the only indications of the stellar parallax presented by this method of observation. An arrangement provided in the micrometers applied on the eye-piece of the as-

tronomical instrument, supplies the observer with the means, not only of measuring the apparent distance between two or more points which are present simultaneously in the field of view, but also the direction of the line joining these two points with relation to some fixed direction, such, for example, as a parallel or a perpendicular to the ecliptic. Thus, when the star S is seen at A, the direction of the line joining it with the star S′, is parallel to the ecliptic; but when it is seen at B, the line B S′ joining it with the star S′ is inclined to the ecliptic at the angle B S′ S.

The micrometric apparatus just indicated, which from its use is called the *position micrometer*, enables the observer to measure with the greatest precision the angle B S′ S, and in like manner to measure the equal angle B′ S′ S when the star S is seen at the lowest point B′ of the parallactic ellipse.

The same apparatus enables the observer to measure the angle A S″ S, which the line joining the stars S S′ when the sun's longitude exceeds that of the star by 90° makes with a perpendicular to the ecliptic, that is, the angle A S″ S, and in like manner he can measure the angle A′ S″ S.

Let the angle $\text{B S}' \text{S} = \phi$, and let the angle $\text{A S}'' \text{S} = \phi'$; we shall then have

$$\varpi \times \sin. \lambda = \tfrac{1}{2}(\text{D} + d) \times \tan. \phi,$$
$$\varpi = \tfrac{1}{2}(\text{D}' + d') \times \tan. \phi'.$$

If, therefore, the angles ϕ and ϕ' be ascertained, they supply further data by which the results of the combined observations may be verified.

If the subsidiary stars S′ and S″ be not in the exact position here assumed, but have latitudes and longitudes differing more or less from those of the star S, the question will be somewhat modified, but its investigation will present no difficulty.

3306. *Case of two stars having equal parallax.*—If the two stars seen at once in the field of view of the telescope have equal parallaxes, both being sensible, they will appear to describe similar parallactic ellipses, and will from time to time occupy similar positions in these ellipses, since their position will be determined by the difference of longitudes of the sun and the stars. It follows from this, that the lines drawn from the centres of the parallactic ellipses to any simultaneous positions of the stars in these ellipses, will be parallel and equal; and consequently the line joining the stars will always

be parallel to the major axis of the ellipse and always equal to the true distance between the stars. This will easily be comprehended by reference to *fig.* 865., where S and *s* are the true

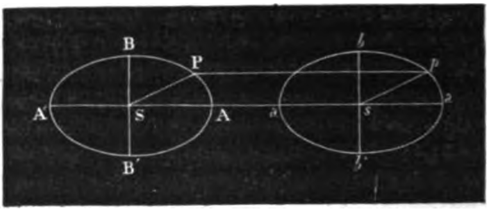


Fig. 865.

positions of the stars, and A B A′ B′, and *a b a′ b′*, the two parallactic ellipses, and P and *p* simultaneous positions of the stars in these ellipses. The semidiameters S P and S *p* being parallel, it is evident, that P *p* will be equal to S *s*, that is to say, the apparent distance between the stars will be equal to the true distance between them; and the line P *p*, joining the simultaneous positions of the stars will be constantly parallel to the line S *s*, that is, to the ecliptic. But if the star *s* be not, as here supposed, in the same parallel to the ecliptic with the star S, the same will nevertheless be true, the line joining the apparent places of the two stars being always parallel to the line joining their true places. It is evident, therefore, that if the two stars thus compared had exactly equal parallaxes, which they would have if they were at exactly equal distances from the solar system, this method of observation would not supply any means of determining the common value of their parallax.

3307. *Case in which they have unequal parallaxes.*—But if while the parallax of the subsidiary star is, on the one hand, not absolutely insensible, as first supposed, nor, on the other, equal to that of the principal star, but it is much less than that of the principal star, then the subsidiary star will appear to move in a parallactic ellipse proportionally smaller than that of the principal star.

Let A B A′ B′, *fig.* 866., be the parallactic ellipse of the principal star, and let *a b a′ b′* be the parallactic ellipse of the subsidiary star, which to simplify the explanation we will suppose, as before, to be in the same parallel to the ecliptic with the principal star. Since the two stars have the same latitude,

their parallactic ellipses will be similar, and their minor axes will consequently bear the same ratio to their major axes, as

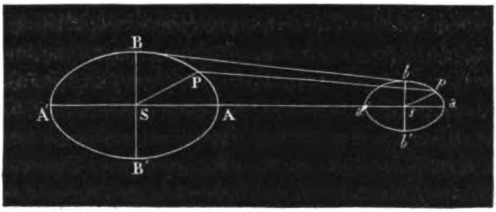


Fig. 866.

represented in the figure. The simultaneous places of the two stars in the parallactic ellipses, will also be such that the semidiameter of the ellipses S P and *s p* which pass through them, will always be parallel. But since, in this case, *s p* will always be less than S P, the line P *p*, which joins the simultaneous places of the stars, will not, as in the former case, be parallel to S *s*, nor equal to it. It will, on the contrary, be inclined to it at an angle which will vary with the position of the stars in the parallactic ellipses, and which will increase gradually from the points A *a* to the points B *b*, where its obliquity to the line S *s* is greatest.

Let A *a*, the apparent distance between the two stars when the sun's longitude exceeds that of the star by 90°, be expressed as before by D; and let A′ *a*′, their apparent distance when the sun's longitude exceeds that of the star by 270°, be expressed by *d*. Let the parallax S A of the principal star be expressed by ϖ, and let the parallax S *a* of the subsidiary star be expressed by ϖ'. The true distance, S *s*, between the stars, being expressed as before by Δ, we shall have

$$\Delta = \tfrac{1}{2}(D + d), \qquad \varpi - \varpi' = \tfrac{1}{2}(D - d).$$

The results are, therefore, absolutely the same as if the subsidiary star S had no sensible parallax, and the parallax of the principal star S were equal to the difference between its parallax and that of the subsidiary star. If the problem be pursued through its other details, it will be found that the changes of inclination of the line joining the apparent places of the two stars to a fixed line, such as the parallel to the ecliptic, will also be the same as they would be if the subsidiary star had no

sensible parallax, and the principal star had a parallax equal to the difference of the parallaxes.

3308. *Parallax of nine stars ascertained.*—Notwithstanding the great multitude of stars to which instruments of observation of unlooked-for perfection, in the hands of the most able and zealous observers, have been directed, the results of all such labours have hitherto been rather negative than positive. The means of observation have been so perfect, and their application so extensive, that it may be considered as proved by the absence of all measureable displacement consequent upon the orbital motion of the earth that, a very few individual stars excepted, the vast multitude of bodies which compose the universe and which are nightly seen glittering in the firmanent, are at distances from the solar system greater than that which would produce an apparent displacement amounting to the tenth of a second. This limit of distances is, therefore, ten parallactic units, or about two million times the space between the earth and sun.

Within this limit, or very little beyond it, nine stars have been found to be placed, the nearest of which is that already mentioned, of which Professor Henderson discovered the parallax. Those of the others are due to the observations of Messrs. Bessel, Struve, and Peters. In the following Table the parallaxes of these stars are given with their corresponding distances expressed in parallactic units, and also in the larger unit presented by the distance through which light moves in a year.

The parallax of the first seven of these stars may be considered as having been ascertained with tolerable certainty and precision. The very small amount of that of the last two is such as to render it more doubtful. What is certain, however, in relation to these is, that the actual amount of their parallax is less than the tenth of a second.

TABLE.

Nine Stars, with their ascertained Parallax and corresponding Distances.

Star.	Parallax.	Distance.		Observer.
		Sun's dist. = 1".	Ann. mot. of light = 1.	
α Centauri -	0·913"	225916	3·54325	Henderson.
61 Cygni - -	0·348	592712	9·29580	Bessel.
α Lyræ - -	0·261	790280	12·39400	Struve.
Sirius - -	0·230	896780	14·0650	Henderson.
1830 Groombridge	0·226	912660	14·3140	Peters.
ι Ursæ - -	0·133	1550800	24·3230	Peters.
Arcturus - -	0·127	1624100	25·4725	Peters.
Polaris - -	0·067	3078400	48·2838	Peters.
Capella - -	0·046	4484000	73·6400	Peters.

CHAP. XXVI.

MAGNITUDE AND LUSTRE OF THE STARS.

3309. *Orders of magnitude of the stars.* — Among the multitude of stars dispersed over the firmament, we find a great variety of splendour. Those which are the brightest and largest, and which are said to be of the *first magnitude*, are few; the next in order of brightness, which are called of the *second magnitude*, are more numerous; and as they decrease in brightness their number rapidly increases.

The number of stars of the first magnitude does not exceed twenty-four; the second, fifty; the third, two hundred; and so on, the number of the smallest visible without a telescope being from 12,000 to 15,000.

The stars which are capable of being seen by the naked eye

are usually resolved into seven orders of magnitude—the first being the brightest and largest, while those of the seventh magnitude are the smallest that the eye can distinctly see.

3310. *These varieties of magnitude caused chiefly by difference of distance.*—Are we to suppose, then, that this relative brightness which we perceive, really arises from any difference of intrinsic splendour between the objects themselves? or does it, as it may equally do, arise from their difference of distance? Are the stars of the seventh magnitude so much less bright and conspicuous than those of the first magnitude, because they are really smaller orbs placed at the same distance? or because, being intrinsically equal in splendour and magnitude, the distance of those of the seventh magnitude is so much greater than the distance of those of the first magnitude that they are diminished in their apparent brightness? We know that by the laws of optics the light received from a luminous object diminishes in a very rapid proportion as the distance increases. Thus at double the distance it will be four times less, at triple the distance it will be nine times less, at a hundred times the distance it will be ten thousand times less, and so on.

It is evident, then, that the great variety of lustre which prevails among the stars may be indifferently explained, either by supposing them objects of different intrinsic brightness and magnitude, placed at the same distance; or objects generally of the same order of magnitude, placed at a great diversity of distances.

Of these two suppositions, the latter is infinitely the more probable and natural; it has, therefore, been usually adopted: and we accordingly consider the stars to derive their variety of lustre almost entirely from their places in the universe being at various distances from us.

3311. *Stars as distant from each other generally as they are from the sun.*—Taking the stars generally to be of intrinsically equal brightness, various theories have been proposed as to the positions which would explain their appearance; and the most natural and probable is, that their distances from each other are generally equal, or nearly so, and correspond with the distance of our sun from the nearest of them. In this way the fact that a small number of stars only appear of the first magnitude, and that the number increases very rapidly as the magnitude diminishes, is easily rendered intelligible.

3312. *Why stars increase in number as they decrease in mag-*

nitude.—If we imagine a person standing in the midst of a wood, surrounded by trees on every side and at every distance, those which immediately surround him will be few in number, and by proximity will appear large. The trunks or stumps of those which occupy a circuit beyond the former, will be more numerous, the circuit being wider, and will appear smaller, because their distance is greater. Beyond these again, occupying a still wider circuit, will appear a proportionally augmented number, whose apparent magnitude will again be diminished by increased distance; and thus the trees which occupy wider and wider circuits at greater and greater distances will be more and more numerous, and will appear continually smaller. It is the same with the stars; we are placed in the midst of an immense cluster of suns, surrounding us on every side at inconceivable distances. Those few which are placed immediately about our system, appear bright and large, and we call them *stars of the first magnitude.* Those which lie in the circuit beyond, and occupy a wider range, are more numerous and less bright; and we call them stars of the second magnitude. And there is thus a progression increasing in number and distance and diminishing in brightness, until we attain a distance so great that the stars are barely visible to the naked eye. This is the limit of vision. It is the limit of the range of the eye in its natural condition; but an eye has been given us more potent still, and of infinitely wider range,—the eye of the mind. The telescope, a creature of the understanding, has conferred upon the bodily eye an infinitely augmented range, and, as we shall presently see, has enabled us to penetrate into realms of the universe, which, without its aid, would never have been known to us. But let us pause for the present and dwell for a moment upon that range of space which comes within the scope of natural vision.

3313. *What are the fixed stars?*—The extent of the stellar universe visible to the naked eye, and the arrangement of stars in it and their relative distances, have just been explained. But curiosity will be awakened to discover, not merely the position and arrangement of those bodies, but to ascertain what is their nature, and what parts they play on the great theatre of creation. Are they analagous to our planets? Are they inhabited globes, warmed and illuminated by neighbouring suns? Or, on the other hand, are they themselves suns, dispensing light and life to systems of surrounding worlds?

3314. *Telescopes do not magnify them like the planets.*— When a telescope is directed to a star, the effect produced is strikingly different from that which we find when it is applied to a planet. A planet, to the naked eye, with one or two exceptions, appears like a common star. The telescope, however, immediately presents it to us with a distinct circular disk similar to that which the moon offers to the naked eye, and in the case of some of the planets a powerful telescope will render them apparently even larger than the moon. But the effect is very different indeed when the same instrument is directed even to the brightest star. We find that instead of magnifying, it actually diminishes. There is an optical illusion produced when we behold a star, which makes it appear to us to be surrounded with a radiation which causes it to be represented when drawn on paper, by a dot with rays diverging on every side from it. The effect of the telescope is to cut off this radiation, and present to us the star as a mere *lucid point*, having no sensible magnitude; nor can any augmented telescopic power which has yet been resorted to, produce any other effect. Telescopic powers amounting to six thousand were occasionally used by Sir William Herschel, and he stated that with these the apparent magnitude of the stars seemed *less*, if possible, than with lower powers.

3315. *The absence of a disk proved by their occultation by the moon.*—We have other proofs of the fact that the stars have no sensible disks, among which may be mentioned the remarkable effect called the occultation of a star by the dark edge of the moon. When the moon is a crescent or in the quarters, as it moves over the firmament, its dark edge successively approaches to, or recedes from the stars. And from time to time it happens that it passes between the stars and the eye. If a star had a sensible disk in this case, the edge of the moon would gradually cover it, and the star, instead of being instantaneously extinguished, would gradually disappear. This is found not to be the case; the star preserves all its lustre until the moment it comes into contact with the dark edge of the moon's disk, and then it is instantly extinguished, without the slightest appearance of diminution of its brightness.

3316. *Meaning of the term magnitude as applied to stars.*— It may be asked then, if such be the case, if none of the stars, great or small, have any discoverable magnitude at all, with what meaning can we speak of stars of the first, second, or

other orders of magnitude? The term magnitude thus applied, was used before the invention of the telescope, when the stars, having been observed only with the naked eye, were really supposed to have different magnitudes. We must accept the term now to express, not the comparative magnitude, but the comparative brightness of the stars. Thus a star of the first magnitude, means of the greatest apparent brightness; a star of the second magnitude, means that which has the next degree of splendour, and so on. But what are we to infer from this singular fact, that no magnifying power, however great, will exhibit to us a star with any sensible magnitude? must we admit that the optical instrument loses its magnifying power when applied to the stars, while it retains it with every other visible object? Such a consequence would be eminently absurd. We are therefore driven to an inference regarding the magnitude of stars, as astonishing and almost as inconceivable as that which was forced upon us respecting their distances. We saw that the entire magnitude of the annual orbit of the earth, stupendous as it is, was nothing compared to the distance of one of those bodies, and consequently if that orbit were filled by a sun, whose magnitude would therefore be infinitely greater than that of ours, such a sun would not appear to an observer at the nearest star of greater magnitude than 1″; consequently would have no magnitude sensible to the eye, and would appear as a mere lucid point to an observer at the star! We are then prepared for the inference respecting the fixed stars which telescopic observations lead to. The telescope of Sir William Herschel, to which he applied a power of six thousand, did undoubtedly magnify the stars six thousand times, but even then their apparent magnitude was inappreciable. We are then to infer that the distance of these wonderful bodies is so enormous compared with their actual magnitude, that their apparent diameter, seen from our system, is above six thousand times less than any which the eye is capable of perceiving.

3317. *Why stars may be rendered imperceptible by their distance.*—It appears, therefore, that stars are rendered sensible to the eye, not by subtending a sensible angle, but by the light they emit. It has been already explained (1131.) that an illuminated or luminous object, such for example as the sun, has the same apparent brightness at all distances, and consequently, that the quantity of light which the eye of an observer receives from it being in the exact ratio of the apparent area of its

visual disk, is inversely as the square of its distance. It remains, however, to explain how it can be that, after it ceases to have a disk of sensible diameter, it does not cease to be visible. This arises from the fact that the luminous point constituting the image on the retina, is intrinsically as bright as when that image has a large and sensible magnitude. The eye is therefore sensible to the light, though not sensible to the magnitude of the image; and it continues to be sensible to the light, until by increase of distance the light which enters the pupil and is collected on the retina, though still as intense in its brilliancy as before, is so small in its *quantity*, that it is insufficient to produce sensation.

3318. *Classification of stars by magnitudes arbitrary and insufficient.*—The distribution of the stars visible to the naked eye into seven orders of magnitude, has been so long and so generally received, and is referred to so universally in the works of astronomers, ancient and modern, that it would be impossible altogether to supersede it, and if possible, such a change would be attended with great inconvenience. Nevertheless, this classification is open to many objections, and is, from its looseness and want of definiteness and precision, in singular discordance with the actual state of astronomical science. The stars which abound in such countless numbers on the firmament, are of infinite gradations, from that of Sirius, the most splendid object of this class, to the most faint stars which the sharpest and most practised eye can distinguish on the darkest and clearest night. To distribute such a series so imperceptibly decreasing in splendour, into seven orders of magnitude, must obviously be an arbitrary process, in which no two observers could possibly agree. There are no natural breaks of continuity by which the stars of the first magnitude could be separated from those of the second, the second from those of the third, and so on. Whatever be the stars assigned to any class, the brightest will be undistinguishable from the faintest of those of the next superior magnitude, and the faintest will be equally undistinguishable from the brightest of the next inferior magnitude.

The stars assigned to any order of magnitude, must in such a classification differ greatly one from another in brightness. Thus, of the 24 or 25 stars that are usually assigned to the first magnitude in the received classification, Sirius, the brightest, is about four times as bright as α Centauri, which may be

taken as the type of the average brightness of stars of this magnitude.

3319. *Importance of more exact astrometric expedients.*— When it is considered that the exact ratio of the apparent lustre of the stars, combined with their parallaxes when the latter are known, supplies the data by which the absolute splendour of these bodies may, as will presently appear, be calculated; and further, that they may be thus brought into immediate numerical comparison with the sun, which is itself only an individual of the same class of bodies, the importance of the expedients for the more exact estimation of their relative lustre, and a more precise basis of classification as to apparent magnitude, cannot fail to be felt and acknowledged. The importance of this is rendered still greater by the consideration that the parallax of a very small number of stars being found to have appreciable magnitude, the comparative lustre of these bodies taken in the mass, is the only ground upon which any estimate of their relative distances can be determined; and when the large number which are subject to observation is considered, and the improbability of their differing greatly in intrinsic magnitude taken collectively in classes, it must be admitted that their relative apparent brightness cannot fail to be a tolerably exact exponent of their comparative distances.

3320. *Astrometer contrived and applied by Sir J. Herschel.* —During his residence at the Cape, Sir J. Herschel contrived an apparatus for the more exact determination of the relative lustre of the stars, and applied it with great advantage to the determination of the relative brightness of a considerable number of these objects. This apparatus consisted of a rectangular glass prism, and a lens so mounted that two celestial objects might be seen in juxta-position, one directly, and the other by reflection and transmission through the prism and lens, the apparent brightness of the latter being capable of being varied at pleasure by the observer, so that, by proper adjustments, the two objects thus seen may be rendered sensibly equal in brightness. When this is accomplished, the arrangements of the apparatus are such, that by measuring the distance of the eye of the observer from the focus of the lens, a measure may be obtained by which the comparative lustre of any objects to which the apparatus may be successively directed may be determined.

To render this intelligible, let P, *fig.* 867. represent the rectangular prism, one of the faces of which is placed so as to

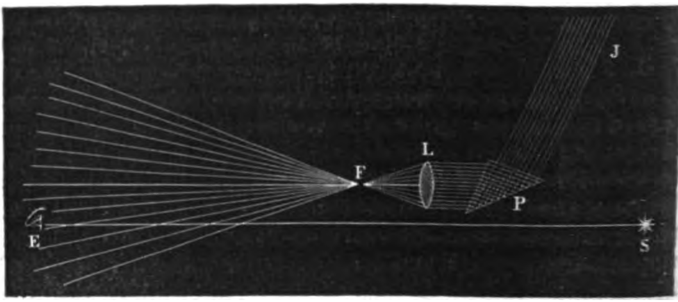


Fig. 867.

receive a pencil of rays passing from a distant object J perpendicularly upon it. These rays are totally reflected (1006.) by the back of the prism at P, and emerging from the other face of the prism, are received upon the lens L, and brought to a focus F, as if they came from the direction P F. The parallel pencil is thus converted into a divergent pencil, of which F is the focus, and the point F will appear to an eye, placed anywhere, as at E, within the limits of the divergent pencil as a star, the apparent brightness of which will be more or less according as the eye is nearer to or more distant from F. It results from the principles of optics, that the apparent brightness of the focal point F will be inversely as the square of the distance E F of the eye from this point. If, then, M express the apparent lustre of F when the eye is at the unit of distance from it, M divided by D^2 will express its apparent lustre when the eye is at the distance D.

Let us now suppose the apparatus so arranged in its position that while the eye, placed within the divergent pencil, sees the focus F, it may also see, in juxta-position with it, a star S, whose lustre is to be determined. Let the eye be moved to or from F until the lustre of the star becomes sensibly equal to that of F. If, then, the lustre of the star be expressed by S, we shall have

$$S = \frac{M}{D^2}.$$

Let the apparatus be then directed to another star, whose lustre s' is to be compared with the former, and let the same operation be repeated, the distance of the eye from F being so regulated as to render the apparent lustre of the point F equal to that of the second star. The distance of the eye from F being, in this case expressed by D', we shall then have

$$s' = \frac{M}{D'^2};$$

and consequently,

$$\frac{s}{s'} = \frac{D'^2}{D^2};$$

that is to say, the apparent lustres of the two stars are in the inverse numerical ratio of the squares of the distances of the eye from F, which would render the apparent lustre of F equal to those of the stars respectively.

In the series of observations made at the Cape by Sir J. Herschel, the moon was the object with which the stars were thus compared. The planet Jupiter would, perhaps, be more convenient; but any object which would retain an invariable brightness during the short interval necessary for the comparison of the stars under observation, would serve the purpose.

In this manner, Sir J. Herschel ascertained numerically the comparative brightness of a considerable number of stars under the fourth magnitude, and has given, in his "Cape Observations" a catalogue, exhibiting the relative magnitudes to two places of decimals.

3321. *Principle on which the successive orders of stellar magnitude should be based.* — Astronomers are not agreed as to the optical conditions by which the successive orders of stellar magnitudes should be fixed. It might appear, at first view, that a star of the second magnitude ought to have one half the brightness of one of the first magnitude, that a star of the third magnitude ought to have one third of the brightness, and so on.

But such a proportion would not be at all in accordance with the common classification of magnitudes.

The more generally received condition has been a succession of magnitudes, such as a star of a given intrinsic lustre would have if removed to a series of distances increasing in arithmetical progression. Thus, stars of the first magnitude would

be at the unit of stellar distance; those of the second magnitude would have a lustre due to twice this distance; those of the third magnitude, to three times this distance, and so on. Now, since the apparent lustre of an object is in the proportion of the inverse square of the distance, it would follow that, in this system, the succession of brightness would be as the numbers 1, $\frac{1}{4}$, $\frac{1}{9}$, $\frac{1}{16}$, and so on.

Meanwhile, whatever may be the principle adopted for this classification, the astrometric expedient contrived by Sir John Herschel, being sufficient for the numerical estimation of the relative brightnesses of different stars, it will be sufficient to determine a variety of interesting and important problems respecting the absolute lustre and magnitudes of those objects, not only compared with each other, but with the sun.

3322. *Comparative lustre of α Centauri with that of the full moon.* — By means of the instrument described above, Sir J. Herschel compared the full moon with certain fixed stars, and ascertained, by a mean of eleven observations, that its lustre bore to that of the star α Centauri, which he selected as the standard star of the first magnitude, the ratio of 27408 to 1; in other words, he showed that a cluster consisting of 27408 stars equal in brightness to that of α Centauri would give the same light as the full moon.

3323. *Comparison of the lustre of the full moon with that of the sun.* — Dr. Wollaston by certain photometric methods which are considered to have been susceptible of great precision, compared the light of the sun with that of the full moon, and found that the ratio was 801072 to 1; or in other words, that to obtain moon-light as intense in its lustre as sun-light, it would be necessary that 801072 full moons should be stationed in the firmament together.

3324. *Comparison of the sun's light with that of α Centauri.* — By the combination of these observations of Herschel and Wollaston, we are supplied with means of bringing into direct numerical comparison the sun and the star α Centauri. Since it appears that the light of α Centauri is 27408 times less than that of the full moon, while the light of the full moon is 801072 times less than that of the sun, it will evidently follow, that if we express by S the light of the sun, and by s that of α Centauri, we shall have

$$S = 27408 \times 801072 \times s = 21955{,}000000 \times s;$$

that is to say, the light of the sun is very nearly 22000,000000 times more intense than that of α Centauri.

To generalise these results, let m express the ratio of the light of the full moon to that of any star, as determined by Herschel's astrometer, we shall then have,

$$\text{S} = 801072 \times m \times s;$$

and consequently,

$$s = \frac{\text{S}}{801072 \times m}.$$

3325. *Comparison of the intrinsic splendour of the sun and a fixed star.* — Since all analogy and observation lead to the conclusion, that the stars, like the sun, are self-luminous bodies, although no telescopic power which we can command can exhibit them with a sensible disk, it cannot be doubted that they are, like the sun, spherical bodies. If, then, I express the intrinsic brightness, or what is the same, the absolute quantity of light emitted by a superficial unit of the visible surface of such a sphere, and if M express the superficial magnitude of the hemisphere presented to the eye, the total quantity of light emitted, or total intrinsic lustre, will be expressed by $\text{I} \times \text{M}$. But the apparent lustre will, according to the common optical law, decrease as the square of the distance of the observer increases, and consequently, if $\text{I} \times \text{M}$ express the lustre at the unit of distance, $\text{I} \times \frac{\text{M}}{\text{D}^2}$, will express it at the distance D, so that we shall have

$$\text{I} \times \text{M} = \text{L} \times \text{D}^2.$$

If the apparent lustre, and the distance of the star, therefore, be both known, the intrinsic lustre, which depends conjointly upon the magnitude of the luminous surface exposed to view and its intrinsic brightness, will be known.

3326. *Astrometer suggested by Dr. Lardner.* — To bring a fixed star into immediate comparison with the sun, and to obtain a measure of the visual magnitude of the star, supposing it to have an intrinsic lustre equal to that of the sun, would be easy if the distance could be ascertained to which it would be necessary to remove the sun, so that it shall present to the eye the same apparent lustre as the star, for in that case the visual magnitude of the sun, which could be calculated by means of its real magnitude and distance, would necessarily

be equal to the visual magnitude of the star. In this manner, a visual angle too small to be ascertained by direct instrumental measurement, would be determined by indirect means.

Let d = the real diameter of the sun, D = the distance to which it would be necessary to remove it from the observer, so that it might present to the eye the same appearance as a given star, and let ϕ = its visual diameter at that distance. We should then have,

$$\phi'' = 206265 \times \frac{d}{D},$$

and ϕ would then be the visual angle subtended by the star, if the star be supposed to have the same intrinsic lustre as the sun. But if the star be supposed to have a greater or less intrinsic lustre than the sun, then the visual magnitude of the star will be greater or less than ϕ.

Although the sun cannot be removed to increased distances, the same optical effect may be produced by the following expedient.

Let A B C D be a tube like that of a telescope, furnished with

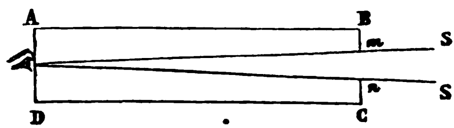


Fig. 868.

a diaphragm at B C, so constructed that by sliding pieces a circular aperture, having a diameter variable at pleasure within practical limits, may be made in its centre. Let a sliding tube having an eye-hole in a diaphragm at the end of it, like that in the eye-piece of a telescope, be attached to the other end A D of the tube, so that the distance of the eye-hole from the variable aperture $m\,n$ may be varied at pleasure within practical limits. It is evident, that the diameter of the aperture $m\,n$, and the distance from E to $m\,n$ being known, the visual angle subtended by $m\,n$ at E will be determined.

If the tube thus constructed and arranged be directed to the disk of the sun, a circular part of that disk having any desired visual diameter, can be made visible to an eye placed at E. This can always be accomplished within limits by the variation

of the diameter *m n* of the aperture, and the variation of the distance of E from *m n*.

But, by a well-understood principle of optics (1132.), the circular part of the sun's disk visible through the aperture, has exactly the same appearance both in apparent magnitude and brightness, as the sun itself would have if it were removed to such a distance from the observer, that it would subtend the same visual angle as that subtended by the aperture *m n* at the eye E.

If then, the apparatus be so adjusted, that the apparent lustre of the part of the sun seen through the aperture, shall be equal as exactly as can be determined by an observation of this kind, to the apparent brightness of any star, it will follow, that the visual angle subtended by the aperture seen from E, will be equal to the visual angle subtended by the star; and as the former can be calculated by knowing the real diameter of the aperture and its distance from E, the latter can be inferred.

In the practical application of this method, the difficulty arises from not being able to bring the luminous point seen in the tube, into immediate juxta-position with the star with which it is compared. The observer must rely upon his judgment and memory of the apparent brightness of the stars, to determine when that of the luminous point seen in the tube is equal to it.

3327. *Comparison of the sun and α Centauri.*—There will be no difficulty in the application of this principle to all stars whose parallax and relative distance with reference to a known standard, has been determined. Take, for example, the case of α Centauri. By Herschel's astrometric estimate, we have for this star,

$$\text{L} = \frac{\text{S}}{21955{,}000000},$$

S still expressing the light of the sun.

It appears by Henderson's observations, that the parallax of this star is 0·913″, which corresponds to a distance of 225916 semidiameters of the earth's orbit. We shall consequently have

$$\text{I} \times \text{M} = \frac{225916^2}{21955{,}000000} = 2{\cdot}324.$$

It follows, therefore, that this star, placed where the sun is, would have 2·324 times its splendour or illuminating power.

Now, this may arise either from the star having a greater superficial magnitude than the sun in that proportion, or if it have the same superficial magnitude, having greater intensity of light. But whichever may be the case, it is certain that its illuminating power would be greater than that of the sun, in the ratio of 2·324 to 1.

3328. *Comparison of the sun and Sirius.* — Sir John Herschel found by his astrometric observations that the lustre of the Dog-star is four times that of α Centauri. For the Dog-star we shall have, therefore,

$$\text{L} = \frac{\text{S}}{5488{,}750000};$$

and since it appears, by the observations of Peters, that the parallax of this star is 0·230″, its distance will be 896800 semidiameters of the earth's orbit. We shall consequently have, for Sirius,

$$\text{I} \times \text{M} = \frac{896800^2}{5488{,}750000} = 146{\cdot}53.^*$$

From which it appears that Sirius is a sun, whose lustre is such as, if placed in the centre of the solar system, would diffuse a light to the surrounding planets 146·53 times more intense than that afforded by the actual sun. If, therefore, the intensity of the lustre of the surface of this stupendous sphere be equal to that of the sun, it must have a diameter 12·11 times greater than that of our sun; and since the diameter of the latter is 882000 miles, that of Sirius would be

$$882000 \times 12{\cdot}11 = 10{,}676600 \text{ miles.}$$

3329. *Astrometric table of* 190 *principal stars.* — In the following table are collected the results of the observations of Sir J. Herschel, for the determination of the relative lustre of 190 principal stars. In addition to their astrometric magnitudes, as determined by Sir J. Herschel, we have computed from the data supplied by him, their relative brightness compared with that of the star α Centauri as a standard, and also their light in billionths of the light of the sun.

* Sir J. Herschel makes the proportion 63·02 which is certainly incorrect that being the ratio of the intrinsic brightness of Sirius to that of α Centauri and not that of Sirius to the sun. See Astronomy, p. 553., edit. 1849.

TABLE.

List of 190 stars, from the first to the third magnitude inclusive, with their magnitudes, according to the astrometric scale proposed by Sir John Herschel; and their apparent brightness compared with each other, and with the sun.

Stars.	Astrometric Magnitude. α Centauri = 1.	Star's Light. α Centauri = 1.	Star's Light. Billionths of Sun's Light.
I. First Mag.			
Sirius - -	0·49	4·165	189·7
Argus - -	Variab.	—	—
Canopus -	0·70	2·04	92·97
α Centauri -	1·00	1·00	45·55
Arcturus -	1·18	0·72	32·71
Rigel - -	1·23	0·66	30·11
Capella -	1·4	0·51	23·24
Lyra - -	1·4	0·51	23·24
Procyon -	1·4	0·51	23·24
α Orionis -	1·43	0·49	22·27
α Eridani -	1·50	0·44	20·24
Aldebaran -	1·5	0·44	20·24
β Centauri -	1·58	0·40	18·24
α Crucis -	1·6	0·39	17·79
Antares -	1·6	0·39	17·79
α Aquilæ -	1·69	0·35	15·95
Spica - -	1·79	0·31	14·21
II. 2 Mag.			
Fomalhaut -	1·95	0·26	11·98
β Crucis -	1·98	0·25	11·62
Pollux -	2·0	0·25	11·39
Regulus -	2·0	0·25	11·39
α Gruis - -	2·07	0·23	10·66
γ Crucis - -	2·14	0·22	9·95
ε Orionis -	2·25	0·20	8·99
ε Canis maj. -	2·27	0·194	8·84
λ Scorpii -	2·28	0·192	8·78
α Cygni - -	2·31	0·187	8·54
Castor - -	2·35	0·181	8·25
ε Ursæ maj. -	V. 2·36	0·179	8·18
α Ursæ maj. -	V. 2·37	0·178	8·11
ζ Orionis -	2·42	0·171	7·77
β Argus - -	2·44	0·168	7·65
α Persei - -	2·48	0·163	7·41
γ Argus - -	2·49	0·161	7·35
ε Argus - -	2·59	0·149	6·79
η Ursæ maj. -	V. 2·59	0·149	6·79
γ Orionis -	2·59	0·149	6·79
α Triang. aus.	2·64	0·143	6·54
ε Sagitt. -	2·67	0·140	6·39
β Tauri - -	2·69	0·138	6·29
Polaris -	2·69	0·138	6·29
θ Scorpii -	2·70	0·137	6·25
α Hydræ -	2·71	0·136	6·20
δ Canis maj. -	2·73	0·134	6·11
α Pavonis -	2·74	0·133	6·07
γ Leonis -	2·75	0·132	6·02
II. 2. Mag.			
β Gruis - -	2·77	0·130	5·94
α Arietis -	2·81	0·127	5·77
σ Sagitt. -	2·82	0·126	5·73
δ Argus - -	2·83	0·125	5·69
ζ Ursæ maj. -	2·84	0·124	5·52
β Androm. -	2·86	0·122	5·57
β Ceti - -	2·87	0·121	5·53
λ Argus - -	2·87	0·121	5·53
β Aurigæ -	2·89	0·120	5·45
γ Androm. -	2·91	0·118	5·38
III. 3 Mag.			
γ Cassiope -	2·93	0·116	5·32
α Androm. -	2·95	0·115	5·25
θ Centauri -	2·95	0·115	5·25
α Cassiope -	2·98	0·113	5·13
β Canis maj. -	2·99	0·112	5·09
κ Orionis -	3·00	0·111	5·06
γ Gemin. -	3·00	0·111	5·06
δ Orionis -	3·02	0·110	4·99
Algol - -	V. 3·03	0·109	4·96
ε Pegasi - -	3·03	0·109	4·96
γ Draconis -	3·03	0·109	4·96
β Leonis -	3·04	0·108	4·93
α Ophiuchi -	3·04	0·108	4·93
β Cassiope -	3·04	0·108	4·93
γ Cygni - -	3·04	0·108	4·93
α Pegasi - -	3·06	0·107	4·86
β Pegasi - -	3·06	0·107	4·86
γ Centauri -	3·09	0·105	4·77
α Coronæ -	3·10	0·104	4·74
γ Ursæ maj. -	3·12	0·103	4·68
ε Scorpii -	3·12	0·103	4·68
ζ Argus - -	3·13	0·102	4·65
β Ursæ - -	3·18	0·098	4·47
α Phœnicis -	3·19	0·098	4·47
ι Argus - -	3·21	0·097	4·42
ε Bootis - -	3·21	0·097	4·42
α Lupi - -	3·23	0·096	4·37
ε Centauri -	3·23	0·096	4·37
η Canis - -	3·26	0·094	4·29
β Aquarii -	3·26	0·094	4·29
δ Scorpii -	3·27	0·0935	4·26
ε Cygni - -	3·29	0·0924	4·21
η Ophiuchi -	3·30	0·0918	4·18
γ Corvi - -	3·31	0·0913	4·16
α Cephei -	3·31	0·0913	4·16
η Centauri -	3·32	0·0907	4·13

Stars.	Astrometric Magnitude	Star's Light.		Stars.	Astrometric Magnitude.	Star's Light.	
	α Centauri = 1.	α Centauri = 1.	Billionths of Star's Light.		α Centauri = 1.	α Centauri = 1.	Billionths of Star's Light.
III. 3 Mag.				**III. 3. Mag.**			
α Serpentis -	3·33	0·0902	4·11	β Aræ - -	3·72	0·0723	3·29
δ Leonis -	3·35	0·0891	4·06	α Toucani -	3·73	0·0719	3·27
κ Argus - -	3·35	0·0891	4·06	β Capric. -	3·73	0·0719	3·27
β Corvi - -	3·36	0·0886	4·03	ζ Argus - -	3·73	0·0719	3·27
β Scorpii -	3·37	0·0881	4·01	ζ Aquilæ -	3·73	0·0719	3·27
ζ Centauri -	3·37	0·0881	4·01	β Cygni - -	3·74	0·0715	3·26
ζ Ophiuchi -	3·38	0·0875	3·99	γ Persei -	3·75	0·0711	3·24
α Aquarii -	3·38	0·0875	3·99	μ Ursæ maj. -	3·76	0·0707	3·22
π Argus - -	3·39	0·0870	3·95	β Triang. bor.	3·76	0·0707	3·22
γ Aquilæ -	3·39	0·0870	3·95	π Scorp. -	3·76	0·0707	3·22
δ Cassiope -	3·40	0·0865	3·94	β Leporis -	3·76	0·0707	3·22
δ Centauri -	3·40	0·0865	3·94	γ Lupi - -	3·77	0·0704	3·21
α Leporis -	3·41	0·0860	3·92	δ Persei -	3·77	0·0704	3·21
δ Ophiuchi -	3·41	0·0860	3·92	φ Ursæ maj. -	3·77	0·0704	3·21
ξ Sagittarii -	3·42	0·0855	3·89	ε Aurig. -	V. 3·78	0·0700	3·19
η Bootis - -	3·42	0·0855	3·89	υ Scorp. -	3·78	0·0700	3·19
η Draconis -	3·43	0·0850	3·87	ι Orion. -	3·78	0·0700	3·19
π Ophiuchi -	3·46	0·0835	3·80	γ Lyncis -	3·80	0·0693	3·15
β Draconis -	3·47	0·0830	3·78	ξ Drac. -	3·81	0·0689	3·14
β Libræ - -	3·48	0·0826	3·76	α Aræ - -	3·81	0·0689	3·14
γ Virginis -	3·49	0·0821	3·74	σ Sag'tt. -	3·81	0·0689	3·14
μ Argus - -	3·49	0·0821	3·74	π Herc. -	3·82	0·0685	3·12
β Arietis -	3·50	0·0816	3·72	β Can. min. ?	3·82	0·0685	3·12
γ Pegasi - -	3·52	0·0807	3·68	ζ Tauri -	3·83	0·0682	3·10
δ Sagittarii -	3·52	0·0807	3·68	δ Drac. - -	3·83	0·0682	3·10
α Libræ - -	3·53	0·0803	3·65	μ Gemin. -	3·83	0·0682	3·10
λ Sagittarii -	3·54	0·0798	3·63	γ Boot. - -	3·84	0·0678	3·09
β Lupi - -	3·55	0·0793	3·61	ι Gemin. -	3·84	0·0678	3·09
ι Virginis ? -	3·55	0·0793	3·61	α Muscæ -	3·84	0·0678	3·09
α Columbæ -	3·56	0·0789	3·59	α Hydri. ? -	3·85	0·0675	3·07
θ Aurigæ -	3·58	0·0780	3·55	τ Scorp. -	3·85	0·0675	3·07
β Herculis -	3·59	0·0776	3·53	δ Herc. - -	3·85	0·0675	3·07
ι Centauri -	3·61	0·0767	3·49	δ Gemin. -	3·85	0·0675	3·07
δ Capricorni -	3·61	0·0767	3·49	φ Orion. -	3·86	0·0671	3·06
δ Corvi - -	3·63	0·0759	3·46	β Cephei -	3·86	0·0671	3·06
α Canum venat.	3·63	0·0759	3·46	θ Ursæ maj. -	3·86	0·0671	3·06
β Ophiuchi -	3·64	0·0755	3·44	ξ Hydræ -	3·86	0·0671	3·06
δ Cygni - -	3·65	0·0751	3·42	γ Hydræ -	3·87	0·0668	3·04
ι Persei - -	3·17	0·0742	3·38	β Triang. aus.	3·87	0·0668	3·04
η Tauri - -	3·67	0·0742	3·38	ι Ursæ maj. -	3·87	0·0668	3·04
β Rridani -	3·67	0·0742	3·38	η Aurig. -	3·87	0·0668	3·04
θ Argus - -	3·67	0·0742	3·38	γ Lyræ - -	3·88	0·0664	3·03
β Hydri - -	3·68	0·0738	3·36	η Gemin. -	3·89	0·0661	3·01
ζ Persei - -	3·68	0·0738	3·36	γ Ceph. - -	3·89	0·0661	3·01
ζ Herculis -	3·69	0·0734	3·34	κ Ursæ maj. -	3·90	0·0657	2·99
ι Corvi - -	3·69	0·0734	3·34	ε Cassiop. -	3·90	0·0657	2·99
ι Aurigæ -	3·70	0·0730	3·33	δ Aquil. -	3·91	0·0654	2·98
γ Ursæ min. -	3·71	0·0726	3·31	σ Scorp. - -	3·91	0·0654	2·98
η Pegasi -	3·72	0·0723	3·29	τ Argus - -	3·91	0·0654	2·98

3330. *Use of the telescope in stellar observations.* — Since no telescope, however great might be its power, has ever presented a fixed star with a sensible disk, it might be inferred that, for the purposes of stellar investigations, the importance of that instrument must be inferior to that which it may

claim in other applications. Nevertheless it is certain, that in no department of physical science has the telescope produced such wonderful results as in its application to the analysis of the starry heavens.

Two of the chief conditions necessary to distinct vision are, first, that the image on the retina shall have sufficient magnitude; or, what is equivalent to this, that the object or its image shall subtend at the eye a visual angle of sufficient magnitude (1116.); and, secondly, it must be sufficiently illuminated (1100.). When, by reason of their distance from the observer, visible objects fail to fulfil either or both of these conditions, the telescope is capable of restablishing them. It augments the visual angle by substituting for the distant object, which the observer cannot approach, an optical image of it close to his eye, which he can approach; and it augments the illumination by collecting, on each point of such image, as many rays as can enter the aperture of the object glass, instead of the more limited number which can enter the pupil of the naked eye; allowance, nevertheless, being made for the light lost by reflection from the surfaces of the lenses, and by the imperfect transparency of their material.

The increase of the visual angle is determined by the ratio of the focal length of the object glass to that of the eye glass (1212.), and the increase of illumination is determined by the ratio of the area of the aperture of the object glass to that of the pupil, which areas are proportional to the squares of the diameters of the object glass and the pupil. The illumination will, therefore, vary in the ratio of the square of the aperture of the telescope.

To explain the effect of the telescope applied to stellar observation, let the sun or any similar object be imagined to be transferred to a gradually increased distance from the observer. The effect will be the gradual decrease of its visual diameter, and a corresponding decrease of the image on the retina. The brightness or intensity of illumination of that image will remain always the same (1132.); and consequently, the total quantity of light which falls upon it, will be decreased in the exact ratio of its superficial magnitude, — that is, in the ratio of the square of its diameter. But this diameter is always proportional to the visual angle subtended at the eye by the object; and this angle decreases as the distance of the object

increases. It follows, therefore, that the total quantity of light incident on the retina, from the same or similar objects at different distances, decreases as the square of the distance increases.

Now, let the distance of the sun from the observer be imagined to be increased until the visual angle becomes so small that no sensible impression of the form or magnitude of the object is produced. Let this distance be expressed by D'. The appearance of the sun would then be that of a mere luminous point, without apparent magnitude or form. It would in fact, therefore, have the same appearance as that of a star or planet. Vision would depend on the mere excitation of the retina by the quantity of light acting upon it, and not on the form or magnitude of the picture produced upon it. The first of the above-mentioned conditions of distinct vision would fail to be fulfilled, but the second would be still fulfilled. Light without form or magnitude would, therefore, be the sensible impression on the observer.

If we now imagine the sun to continue to be transferred to greater and greater distances, the image on the retina will be proportionally diminished in magnitude; but as its magnitude has already ceased to be sensible because of its minuteness, this decrease of magnitude will necessarily also be insensible. But the total quantity of light falling upon the retina, will also be decreased, and this decrease will be in the ratio of the increase of the square of the distance. Now, since the apparent brightness of the luminous point to which the sun would be in this case reduced, must depend altogether on the total quantity of light falling on the retina, this brightness will be in the inverse ratio of the square of the distance.

Let L' be the total quantity of light falling on the retina, or the apparent brightness of the object at the distance D' at which it ceases to have a sensible disk, and let L be its apparent brightness, at any greater distance D. We shall then, according to what has just been explained, have

$$L : L' :: D'^2 : D^2;$$

and consequently,

$$L = L' \times \frac{D'^2}{D^2},$$

from which it appears again that L will decrease as D^2 increases.

By the continual increase of D, therefore, the apparent brightness of the luminous point to which the object has been reduced, would be continually diminished, and it would successively assume the appearance of stars of less and less magnitude, until at length the quantity of light falling on the retina would become so small that it would be insufficient to produce a sensible impression on the organ, and the object would cease to be seen. Let the distance at which this would take place be D''.

It appears, then, that in the gradations of the optical impression produced by such a continually receding object, there are two limiting distances, the lesser D' at which it ceases to have sensible magnitude but continues to be visible as a lucid point, and the greater D'' at which it ceases to be seen altogether; and that at intermediate distances D it appears as a lucid point of all degrees of brightness, less than that which it has at the distance D'.

If this reasoning be applied to different objects, it is evident that the distance D' will vary with the real diameter of the object, and will be exactly proportional to it. The distance D'' for objects having the same real diameter, will vary with their intrinsic lustre, or the relative quantities of light which they emit from their visible hemispheres, and will be greater in the ratio of the square root of the absolute quantity of light emitted.

If a telescope be directed to a star at any distance D greater than D'', its magnifying power will be incapable, however great it may be, of augmenting the visual angle to such an extent as to render it greater than it would be, if the star were at the distance D', at which the visual angle becomes so small as to be inappreciable by the eye. But in the same case, the power of the telescope to increase the quantity of light which enters the pupil, will produce effects which are not only very sensible, but which may be increased almost indefinitely, by augmenting the aperture of the telescope. In this way, although the magnifying power is altogether inefficacious so far as relates to the visual angle of the object, its power, so far as relates to the increase of light or increase of apparent brightness of the object, becomes of the greatest importance. Thus it is evident, that a telescope of a certain aperture directed to a star of the sixth magnitude, the light of which, according to the estimate

of Sir J. Herschel, is about the 100th part of the light of such a star of the first magnitude as α Centauri, would render it equal in apparent brightness to the latter, and would, therefore, have the effect of bringing it so much nearer to the observer, as the distance of an average star of the first magnitude is less than an average star of the sixth magnitude. But since the apparent brightness decreases as the square of the distance increases, it follows that a star of the sixth magnitude, being 100 times less bright than a star of the first magnitude, will be 10 times more distant. The telescope, therefore, in this case, would have the effect of bringing the star 10 times nearer to the observer.

By knowing the relation of the aperture of the telescope, whether it be a refractor or reflector, to the magnitude of the pupil, and the proportion of light lost in being transmitted to the eye by the lenses or specula of the instrument, it is easy to calculate the ratio in which it will increase the apparent brightness of a star, and this ratio being known, it will be easy to ascertain how much more distant such a star is than one which to the naked eye would have the same apparent brightness.

Let m express the ratio in which the telescope increases the apparent brightness L of a star, and let L′ be the brightness of the same star seen through the telescope. We should then have

$$L' = m \times L.$$

Now, let d be the distance of the star, and let d' be the distance at which, seen with the naked eye, it would have the brightness L′. We shall then have

$$L' = L \times \frac{D^2}{D'^2};$$

and therefore,

$$m = \frac{D^2}{D'^2} \qquad \sqrt{m} = \frac{D}{D'}.$$

The star is, therefore, brought nearer to the observer in the ratio of $\sqrt{m}$ to 1.

3331. *Space-penetrating power.*—This number $\sqrt{m}$ which expresses the power of the telescope to bring a star nearer to the observer, or what is the same, to enable the observer to see

distant stars with the same degree of distinctness or brightness as if they were at less distances, is called the SPACE-PENETRATING POWER.

Thus, if the light of a star of the sixth magnitude be 100 times less than that of a star of the first magnitude, a telescope which would augment the light 100 times, would exhibit it with the same apparent brightness as a star of the first magnitude; and for such a telescope we should have $m = 100$, and therefore $\sqrt{m} = 10$, so that the star of the sixth magnitude would be ten times more distant than the stars of the first magnitude.

Thus, for example, the reflecting telescope used by Sir William Herschel, in some of his principal stellar researches, had an aperture of eighteen inches, and twenty feet focal length with a magnifying power of 180. The space-penetrating power of this instrument was found to be seventy-five, the meaning of which is, that when directed to a star of any given brightness, it would augment its brightness so as to make it appear the same as it would be if at seventy-five times less distance, or what is the same, that a star which to the naked eye would appear of the same brightness as that star does when seen in the telescope would require to be removed to seventy-five times the actual distance, so that when seen through the telescope it would have the brightness it has when seen with the naked eye. Thus a star of the sixth magnitude, if removed to seventy-five times the actual distance, would appear in such an instrument still as a star of the sixth magnitude would to the naked eye, and if we assume with Sir John Herschel, that a star of the sixth magnitude has a hundred times less light than α Centauri, and is therefore at ten times a greater distance, it will follow that α Centauri would require to be removed to seven hundred and fifty times its actual distance, so that when viewed through such telescope it would be seen as a star of the sixth magnitude is to the naked eye.

If, then, it be assumed, as it may fairly be, that among the innumerable stars which are beyond the range of unaided vision, and brought into view by the telescope, a large proportion must have the same magnitude and intrinsic brightness, as the average stars of the first magnitude, it will follow that these must be at distances 750 times greater than the distance of an average star of the first magnitude, such as α Centauri. But

it has been already shown (3308.) that the distance of α Centauri is such that light would require 3·54325 years to come from it to the earth. It would, therefore, follow that the distance of the telescopic stars just referred to, must be such that light would take to come from them to the earth

$$3{\cdot}54325 \times 750 = 2657{\cdot}4375 \text{ years.}$$

If it be desired to ascertain the distance of such stars, taking the earth's distance from the sun as the unit, we shall have

$$225916 \times 750 = 169{,}437000.$$

It appears, therefore, that the distsnce of such a star would be about one hundred and seventy million times the distance of the sun, and since the distance of the sun expressed in round numbers is one hundred millions of miles, it will follow that the distance of such a star is seventeen thousand billions of miles.

We arrive, therefore, at the somewhat astonishing conclusion that the distance of these objects, the existence of which the telescope alone has disclosed to us, must be such that light, moving at the rate of 192000 miles per second, takes upwards of 2600 years to come from them to us, and consequently that the objects we now see are not those which now exist, but those which did exist 2600 years ago; and it is within the scope of physical possibility that they may have changed their conditions of existence, and consequently of appearance, or even have ceased to exist altogether, more than 2000 years ago, although we actually see them at this moment.

This incidentally shows that the actual perception of a visible object is no conclusive evidence of its present existence. It is only a proof of its existence at some anterior period.

3332. *Telescopic stars.*—It appears, therefore, that there are numerous orders of stars, which by reason of their remoteness are invisible to the naked eye, but which are rendered visible by the telescope; and these stars are, like those visible to the naked eye, of an infinite variety of degrees of magnitude and brightness, and have accordingly been classed by astronomers according to an order of magnitudes in numerical continuation of that which has been somewhat indefinitely or arbitrarily adopted for the visible stars. Thus, supposing that the last order of stars visible without telescopic aid is the seventh, the

first order disclosed by the telescope will be the eighth, and from these the telescopic stars, decreasing in magnitude, have been denominated the ninth, tenth, eleventh, &c. to the sixteenth or seventeenth magnitude, the last being the smallest stars which are capable of being rendered distinctly visible by the most powerful telescope.

3333. *Stellar nomenclature.*—Besides the classification of stars according to their estimated degrees of magnitude or brightness, they are also designated according to their distribution over the imaginary surface of the celestial sphere. Whether the apparent grouping of these objects depends on any physical relation existing between the members composing each group, or is the result of the fortuitous relation of the visual lines directed to them, the principal collections of the more conspicuous stars thus placed in near apparent vicinity, have been recognised from the most remote antiquity, and such groups have been commonly denominated CONSTELLATIONS.

Although in certain cases, it is probable that some physical relation may exist between the more close neighbours in these constellations, it is certain that the apparent juxta-position and relative arrangement of the component stars generally is altogether fortuitous. Imagination has, however, connected them together, and invested such constellations with the forms of mythological figures, animals, such as bears, dogs, lions, goats, serpents, and so on, from which they severally take their names. Unreasonable as such a system must be allowed to be, it is not without its use as a means of reference and an artificial aid to the memory. That a better system of signs and symbols might have been devised for these purposes, may be admitted; but when it is considered that the names and forms of the most conspicuous constellations have had their origin in remote antiquity — that they were handed down from the Chaldeans to the Egyptians, from the Egyptians to the Greeks, and from these to the moderns — that they are referred to in the works of every past astronomer, and registered in the memory of every living observer — that they are associated with the productions of art, and supply illustrations to the orator and the poet — it will be readily admitted that, even though a general change of the stellar nomenclature and symbols were practicable, it would neither be advantageous nor advisable.

As an example of a constellation, the group of seven conspi-

cuous stars, arranged nearly in the form of a note of interrogation, visible in the northern part of the firmament, and in these latitudes always above the horizon, may be referred to. This constellation is called Ursa major (the great bear). The seven stars are only the more conspicuous of those which compose the constellation, the entire number being eighty-seven, most of them, however, being telescopic; of the seven chief stars one only is of the first magnitude, three are of the second, and three of the third.

The seven principal stars of this constellation being all less than forty degrees from the north pole, will be always above the horizon in latitudes greater than forty degrees. Hence it is that this constellation is so familiarly known. They may serve as standards or *moduli* by which the astronomical amateur may estimate the orders of magnitudes of the stars generally. It is in the quarter of the heavens opposite to that in which the sun is in the month of March, and is therefore visible at midnight near the meridian above the pole at that season. In the month of September it is visible at midnight below the pole.

The stars which compose a constellation are designated usually by the letters of the Greek alphabet, the first letters being generally assigned to the most conspicuous. The order of the letters, however, does not always follow strictly the order of magnitudes. When the stars are not designated by letters, they are distinguished by numbers, and this is mostly the case with the smaller stars.

It is usual to express the constellations by their Latin names, and to designate the individual stars by the letter or number and the constellation, as *α Lyræ, β Ursæ majoris*, 61 *Ophiuchi*, 24 *Comæ*, &c.

In the cases of some of the more conspicuous stars, such as have been objects of observation in remote ages, they are also frequently distinguished by proper names. Thus, *α Canis major* is more commonly called *Sirius*, and sometimes the *Dog-star*, and is known as the most resplendent of the fixed stars. In like manner *α Piscis* is always called *Fomalhaut*, *α* and *β Gemini* are called *Castor* and *Pollux*, *β Orionis* is known as *Rigel*, *α Tauri* as Aldebaran, *α Virginis* as *Spica*, *α Bootis* as *Arcturus*, and so on.

The practical usefulness of the imaginary figures which give

names to the constellations, will thus be understood. If we desire to express the position of the star η *Ursæ majoris*, for example, we say that it is at the *tip of the tail* of the Great Bear. We indicate, in like manner, the place of three remarkable stars, by saying that they form the belt of Orion, and another Rigel by saying that it is on his foot. The star Sirius is on the nose of Canis major, and the bright star β on his right thigh.

3334. *Use of pointers.*—Those who desire to obtain an acquaintance with the stars, will find much advantage in practising the method of *pointers*, by which the position of conspicuous stars with which the observer is well acquainted is used to ascertain the places of others which are less known and less easily identified. This method consists in assigning two conspicuous stars so placed, that a straight line imagined to be drawn between them, and continued if necessary in the same direction, will pass through or near the star whose position it is desired to ascertain.

The most useful example of the application of this method, is the case of the pole star, which is α *Ursæ minoris*, a star of the third magnitude. Let the observer direct his eye to the two conspicuous stars, α and β *Ursæ majoris*, and supposing a straight line drawn from β to α, let him carry his eye along that line beyond α to a distance about six times the space between α and β, he will arrive at the Pole Star.

3335. *Use of star maps.*—To comprehend the preceding paragraphs, and profit by the instructions given in them, it will be necessary for the student to have in his hands a set of star maps. The GUIDE TO THE STARS* will be found to be one of the most convenient works for this purpose. In the maps there given, will be found indications of the most useful applications of the method of pointing.

3336. *Use of the celestial globe.*—A celestial globe may be defined to be a working model of the heavens. It is mounted like a common terrestrial globe. The visible hemisphere is bounded by the horizontal circle in which the globe rests. The brass circle at right angles to this, is the celestial meridian. The constellations with outlines of the imaginary figures from which they take their names, are delineated upon it.

* Twelve Planispheres, forming a Guide to the Stars for every Night in the Year, with an Introduction. — *Taylor and Walton*, London.

The globe will serve, not merely as an instrument of instruction, but will prove a ready and convenient aid to the amateur in astronomy, superseding the necessity of many calculations which are often discouraging and repulsive, however simple and easy they may be to those who are accustomed to such inquiries. Most of the almanacs contain tables of the principal astronomical phenomena, of the places of the sun and moon, and of the principal planets as well as the times when the most conspicuous stars are on the meridian after sunset. These data, together with a judicious use of the globe and a tolerable telescope, will enable any person to extend his acquaintance with astronomy, and even to become a useful contributor to the common stock of information which is now so fast increasing by the zeal and ability of private observers in so many quarters of the globe.

To prepare the globe for use, let small marks (bits of paper gummed on will answer the purpose) be placed upon it, to indicate the positions of the sun, moon, and planets, at the time of observing the heavens. The place of the sun on the ecliptic is usually marked on the globe itself. If not, its right ascension (that is, its distance from the vernal equinoxial point, measured on the celestial equator), and its declination (that is, its distance north or south of the equator), are given in the almanac, for every day. The moon's right ascension and declination are likewise given.

3337. *To find the place of an object on the globe when its right ascension and declination are known.*—Find the point on the equator where the given right ascension is marked. Turn the globe on its axis till this point be brought under the meridian. Then count off an arc of the meridian (north or south of the equator, according as the declination is given) of a length equal to the given declination, and the point of the globe immediately under the point of the meridian thus found, will be the place of the object. By this rule, the position on the globe of any object of which the right ascension and declination are known, may be immediately found, and a corresponding mark put upon it.

To adjust the globe so as to use it as a guide to the position of objects on the heavens, and as a means of identifying the stars and learning their names, let the lower clamping-screw of the meridian be loosened, and let the north pole of the globe be

elevated by moving the brass meridian until the arc of this meridian between the pole and the horizon be equal to the latitude of the place of observation. Let the clamping-screw be then tightened, so as to maintain the meridian in this position. Let the globe be then so placed that the brass meridian shall be directed due north and south, the pole being turned to the north. This being done, the globe will correspond with the heavens so far as relates to the poles, the meridian, and the points of the horizon.

To ascertain the aspect of the firmament at any hour of the night, it is now only necessary to turn the globe upon its axis until the mark indicating the place of the sun shall be under the horizon in the same position as the sun itself actually is at the hour in question. To effect this, let the globe be turned until the mark indicating the position of the sun is brought under the meridian. Observe the hour marked on the point of the equator which is then under the meridian. Add to this hour the hour at which the observation is about to be taken, and turn the globe until the point of the equator on which is marked the hour resulting from this addition is brought under the meridian. The position of the globe will then correspond with that of the firmament. Every object on the one will correspond in its position with its representative mark or symbol on the other. If we imagine a line drawn from the centre of the globe through the mark upon its surface indicating any star, such a line, if continued outside the surface toward the heavens, would be directed to the star itself.

For example, suppose that when the mark of the sun is brought under the meridian, the hour 5h. 40m. is found to be on the equator at the meridian, and it is required to find the aspect of the heavens at half-past ten o'clock in the evening.

	H.	M.
To -	5	40
Add -	10	30
	16	10

Let the globe be turned until 16h. 10m. is brought under the meridian, and the aspect given by it will be that of the heavens,

CHAP. XXVII.

PERIODIC, TEMPORARY, AND MULTIPLE STARS. — PROPER MOTION OF STARS. — MOTION OF THE SOLAR SYSTEM.

3338. *Telescopic observations on individual stars.*—Besides bringing within the range of observation objects placed beyond the sphere which limits the play of natural vision, the telescope has greatly multiplied the number of objects visible within that sphere, by enabling us to see many rendered invisible by their minuteness, or confounded with others by their apparent proximity. Among the stars also which are visible to the naked eye, there are many, respecting which the telescope has disclosed circumstances of the highest physical interest, by which they have become more closely allied to our system, and by which it is demonstrated that the same material laws which coerce the planets, and give stability, uniformity, and harmony to their motions, are also in operation in the most remote regions of the universe. We shall first notice some of the most remarkable discoveries respecting individual stars, and shall afterwards explain those which indicate the arrangement, dimensions, and form of the collective mass of stars which compose the visible firmament, and the results of those researches which the telescope has enabled astronomers to make in regions of space still more remote.

I. PERIODIC STARS.

3339. *Stars of variable lustre.*—The stars in general, as they are stationary in their apparent positions, are equally invariable in their apparent magnitudes and brightness. To this, however, there are several remarkable exceptions. Stars have been observed, sufficiently numerous to be regarded as a distinct class, which exhibit periodical changes of appearance. Some undergo gradual and alternate increase and diminution of magnitude, varying between determinate limits, and presenting these variations in equal intervals of time. Some are observed to attain a certain maximum magnitude, from which

they gradually and regularly decline until they altogether disappear. After remaining for a certain time invisible, they re-appear and gradually increase till they attain their maximum splendour, and this succession of changes is regularly and periodically repeated. Such objects are called *periodic stars.*

3340. *Remarkable stars of this class in the constellations of Cetus and Perseus.*—The most remarkable of this class is the star called *Omikron*, in the neck of the Whale, which was first observed by David Fabricius, on the 13th August, 1596. This star retains its greatest brightness for about fourteen days, being then equal to a large star of the second magnitude. It then decreases continually for three months until it becomes invisible. It remains invisible for five months, when it re-appears, and increases gradually for three months until it recovers its maximum splendour. This is the general succession of its phases. Its entire period is about 332 days. This period is not always the same, and the gradations of brightness through which it passes are said to be subject to variation. Hevelius states that, in the interval between 1672 and 1676, it did not appear at all.

Some recent observations and researches of M. Argelander, render it probable that the period of this star is subject to a variation which is itself periodical, the period being alternately augmented and diminished to the extent of 25 days. The variations of the maximum lustre are also probably periodical.

The star called *Algol*, in the head of *Medusa*, in the constellation of *Perseus*, affords a striking example of the rapidity with which these periodical changes sometimes succeed each other. This star generally appears as one of the second magnitude; but an interval of seven hours occurs at the expiration of every sixty-two, during the first three hours and a half of which it gradually diminishes in brightness till it is reduced to a star of the fourth magnitude, and during the remainder of the interval it again gradually increases until it recovers its original magnitude. Thus, if we suppose it to have attained its maximum splendour at midnight on the first day of the month, its changes would be as follows:—

D.	H.	M.		D.	H.	M.	
0	0	0	to	2	14	0	It appears of second magnitude.
2	14	0	to	2	17	24	It decreases gradually to fourth magnitude.
2	17	24	to	2	20	48	It increases gradually to second magnitude.

D.	H.	M.		D.	H.	M.	
2	20	48	to	5	10	48	It appears of second magnitude.
5	10	48	to	5	14	12	It decreases to fourth magnitude.
5	14	12	to	5	17	36	It increases to second magnitude.
	&c.			&c.			&c.

This star presents an interesting example of its class, as it is constantly visible, and its period is so short that its succession of phases may be frequently and conveniently observed. It is situate near the foot of the constellation *Andromeda*, and lies a few degrees north-east of three stars of the fourth magnitude which form a triangle.

Goodricke, who discovered the periodic phenomena of *Algol* in 1782, explained these appearances by the supposition that some opaque body revolves round it, being thus periodically interposed between the earth and the star, so as to intercept a large portion of its light.

The more recent observations on this star indicate a decrease of its period, which proceeds with accelerated rapidity. Sir J. Herschel thinks that this decrease will attain a limit, and will be followed by an increase, so that the variation of the period will prove itself to be periodic.

The stars δ in *Cepheus* and β in *Lyra* are remarkable for the regular periodicity of their lustre. The former passes from its least to its greatest lustre in thirty-eight hours, and from its greatest to its least in ninety-one hours. The changes of lustre of the latter, according to the recent observations of M. Argelander, are very complicated and curious. Its entire period is 12 days 21 hrs. 53 min. 10 sec., and in that time it first increases in lustre, then decreases, then increases again, and then decreases, so that it has two maxima and two minima. At the two maxima its lustre is that of a star of the 3·4 magnitude, and at one of the minima its lustre is that of a star of the 4·3, and at the other that of a star of the 4·5 magnitude.

In this case also the period of the star is found to be periodically variable.

3341. *Table of the periodic stars.*—In the following Table the stars periodically variable, discovered up to 1848, are given, with their periods and extremes of lustre. This Table has been collected from various astronomical records by Sir J. Herschel.

No.	Star.	Period.	Change of Mag. from	Change of Mag. to	Discovered by
		d. dec.	from	to	
1	β Persei (Algol)	2·8673	2	4	Goodricke, 1782.
2	λ Tauri	4 ±	4	5 4	Baxendell, 1848.
3	δ Cephei	5·3664	3·4	5	Goodricke, 1784.
4	η Aquilæ	7·1763	3·4	4·5	Pigott, 1784.
5	* Cancri R. A. (1800.) = 8h 32·5m N. P. D. 70° 15′	9·015	7·8	10	Hind, 1848.
6	ζ Geminorum	10·2	4·3	4·5	Schmidt, 1847.
7	β Lyræ	12·9119	3·4	4·5	Goodricke, 1784.
8	α Herculis	63 ±	3	4	Herschel, 1796.
9	59 B. Scuti R A. (1801.) = 18h 37m N. P. D. = 95° 57′	71·200	5	0	Pigott, 1795.
10	ι Aurigæ	250 ±	3	4	Heis, 1846.
11	ο Ceti (Mira)	331·63	2	0	Fabricius, 1596.
12	* Serpentis R. A. (1828.) = 15h 46m 45s; P. D. 74° 20′ 30″	335 ±	7 ?	0	Harding, 1826.
13	χ Cygni	396·875	6	11	Kirch, 1687.
14	υ Hydræ (B. A. C. 4501.)	494 ±	4	10	Maraldi, 1704.
15	* Cephei (B. A. C. 7582.)	5 or 6 years	3	6	Herschel, 1782.
16	34 Cyni (B. A. C. 6990.)	18 years ±	6	0	Janson, 1600.
17	* Leonis (B. A. C. 3345)	Many years	6	0	Koch, 1782.
18	κ Sagittarii	Ditto	3	6	Halley, 1676.
19	ψ Leonis	Ditto	6	0	Montanari, 1667.
20	η Cygni	Ditto	4·5	5·6	Herschel Jun., 1842 ?
21	* Virginis R. A. (1840) = 12h 3m N. P. D. 82° 8′	145 days	6·7	0	Harding, 1814.
22	* Coronæ Bor. (B. A. C. 5236.)	10½ months	6	0	Pigott, 1795.
23	7 Arietis (B. A. C. 581.)	5 years ?	6	8	Piazzi, 1798.
24	η Argûs	Irregular	1	4	Burchell, 1827.
25	α Orionis	Irregular	1	1·2	Herschel Jun., 1836.
26	α Ursæ majoris	Some years	1·2	2	Ditto, 1846.
27	η Ursæ majoris	Ditto	1·2	2	Ditto, 1846.
28	β Ursæ minoris	2 or 3 years ?	2	2·3	Struve, 1838.
29	α Cassiopeiæ	225 days ?	2	2·3	Herschel Jun., 1838.
30	α Hydræ	29 or 30 days ?	2·3	3	Ditto, 1837.
31	* R. A. (1847.) = 22h 58m 57 9s N. P. D. = 80° 17′ 30″	Unknown	8 ?	0	Hind, 1848.
32	* R. A. (1848.) = 7h 33m 55·2s N. P. D. = 66° 11′ 56″	Ditto	9	0	Ditto, 1848.
33	* R. A. (1848.) = 7h 40m 10·3s N. P. D = 65° 53′ 29″	Ditto	9	0	Ditto, 1818.
34	Near * R. A. 22h 21m 0·4s (1848) N. P. D. 100° 42′ 40″	Ditto	7·8	0	Rümker.
35	* R A. (1848) 14h 44m 39·6s N. P. D. 101° 45′ 25″	Ditto	8	9·10	Schumacher.
36	δ Ursæ Majoris	Many years	2 ?	2·3	Matter of general remark.

N. B. In the above list the letters B. A. C. indicate the catalogue of the British Association, B. the catalogue of Bode. Numbers before the name of the constellation (as 34 Cygni) denote Flamsteed's stars. Since this Table was drawn up, four additional stars, variable from the 8th or 9th magnitude to 0, have been communicated to us by Mr. Hind, whose places are as follow: (1.) R. A. $1^h\ 38^m\ 24^s$; N. P. D. 81° 9′ 39″; (2.) $4^h\ 50^m\ 42^s$, 80° 6′ 36″ (1846); (3.) $8^h\ 43^m\ 8^s$, 86° 11′ (1800); (4.) $22^h\ 12^m\ 9^s$, 82° 59′ 24″ (1800). Mr. Hind remarks that about several variable stars some degree of haziness is perceptible at their minimum. Have they clouds revolving round them as planetary or cometary attendants? He also draws attention to the fact that the red colour predominates among variable stars generally. The double star, No. 2718 of Struve's Catalogue, R. A. $20^h\ 34^m$ P. D. 77° 54′, is stated by Sir John Herschel to be variable. Captain Smyth (Celestial Cycle, i. 274.) mentions also 3 Leonis and 18 Leonis as variable, the former from 6^m to 0, P = 78 days, the latter from 5^m to 10^m, P = $311^d\ 23^h$, but without citing any authority. Piazzi sets down 96 and 97 Virginis and 38 Herculis as variable stars.

In the case of many of the stars in the preceding Table, the variations of lustre are subject to considerable irregularities. Thus No. 13 was scarcely visible from 1698, for the interval of three years, even at the epochs when it ought to have had its greatest lustre. The extremes of lustre of No. 9 are also very variable and irregular. In general the variations of No. 22 are so inconsiderable as to be scarcely perceivable, but they become sometimes suddenly so great that the star wholly disappears. The variations of No. 25 were very conspicuous from 1836 to 1840, and again in 1849, being much less so in the intermediate time.

3342. *Hypotheses proposed to explain these phenomena.*— Several explanations have been proposed for these appearances.

1. Sir W. Herschel considered that the supposition of the existence of spots on the stars similar to the spots on the sun, combined with the rotation of the stars upon axes, similar to the rotation of the sun and planets, afforded so obvious and satisfactory an explanation of the phenomena, that no other need be sought.

2. Newton conjectured that the variation of brightness might be produced by comets falling into distant suns and causing temporary conflagrations. Waiving any other objection to this conjecture, it is put aside by its insufficiency to explain the periodicity of the phenomena.

3. Maupertius has suggested that some stars may have the form of thin flat disks, acquired either by extremely rapid rotation on an axis, or other physical cause. The ring of Saturn affords an example of this, within the limits of our own system, and the modern discoveries in nebular astronomy offer other examples of a like form. The axis of rotation of such a body might be subject to periodical change like the nutation of the earth's axis, so that the flat side of the luminous disk might be present more or less towards the earth at different times, and when the edge is so presented, it might be too thin to be visible. Such a succession of phenomena are actually exhibited in the case of the rings of Saturn, though proceeding from different causes.

4. Mr. Dunn* has conjectured that a dense atmosphere surrounding the stars, in different parts more or less pervious to

* Phil. Trans. Vol. LII.

light, may explain the phenomena. This conjecture, otherwise vague, indefinite, and improbable, totally fails to explain the periodicity of the phenomena.

5. It has been suggested that the periodical obscuration or total disappearance of the star, may arise from *transits* of the star by its attendant planets. The transits of Venus and Mercury are the basis of this conjecture.

The transits of none of the planets of the solar system, seen from the stars, could render the sun a periodic star. The magnitudes, even of the largest of them, are altogether insufficient for such an effect. To this objection it has been answered that planets of vastly greater comparative magnitude may revolve round other suns. But if the magnitude of a planet were sufficient to produce by its transit these considerable obscurations, it must be very little inferior to the magnitude of the sun itself, or at all events, it must bear a very considerable proportion to the magnitude of the sun; in which case it may be objected that the predominance of attraction necessary to maintain the sun in the centre of its system could not be secured. To this objection it is answered, that although the planet may have a great comparative *magnitude*, it may have a very small comparative *density*, and the gravitating attraction depending on the actual mass of matter, the predominance of the solar mass may be rendered consistent with the great relative magnitude of the planet by supposing the density of the one vastly greater than that of the other. The density of the sun is much greater than the density of Saturn.

6. It has been suggested that there may be systems in which the central body is a planet attended by a lesser sun revolving round it as the moon revolves round the earth, and in that case the periodical obscuration of the sun may be produced by its passage once in each revolution behind the central planet.

Such are the various conjectures which have been proposed to explain the periodic stars; and as they are merely conjectures, scarcely deserving the name of hypotheses or theories, we shall leave them to be taken for what they are worth.

II. TEMPORARY STARS.

Phenomena in most respects similar to those just described, but exhibiting no recurrence, repetition, or periodicity, have

been observed in many stars. Thus, stars have from time to time appeared in various parts of the firmament, have shone with extraordinary splendour for a limited time, and have then disappeared and have never again been observed.

3343. *Temporary stars seen in ancient times.*—The first star of this class which has been recorded, is one observed by *Hipparchus*, 125 B. C., the disappearance of which is said to have led that astronomer to make his celebrated catalogue of the fixed stars; a work which has proved in modern times of great value and interest. In the 389th year of our era, a star blazed forth near *α Aquilæ*, which shone for three weeks, appearing as splendid as the planet Venus, after which it disappeared and has never since been seen. In the years 945, 1264, and 1572, brilliant stars appeared between the constellations of *Cepheus* and *Cassiopeia*. The accounts of the positions of these objects are obscure and uncertain, but the intervals between the epochs of their appearances being nearly equal, it has been conjectured that they were successive returns of the same periodic star, the period of which is about 300 years, or possibly half that interval.

The appearance of the star of 1572 was very remarkable, and having been witnessed by the most eminent astronomers of that day, the account of it may be considered to be well entitled to confidence. *Tycho Brahe*, happening to be on his return on the evening of the 11th November from his laboratory to his dwelling-house, found a crowd of peasants gazing at a star which he was sure did not exist half an hour before. This was the temporary star of 1572, which was then as bright as the Dog-star, and continued to increase in splendour until it surpassed Jupiter when that planet is most brilliant, and finally it attained such a lustre, that it was visible at mid-day. It began to diminish in December, and altogether disappeared in March, 1574.

On the 10th October, 1604, a splendid star suddenly burst out in the constellation of *Serpentarius*, which was as bright as that of 1572. It continued visible till October, 1605, when it vanished.

3344. *Temporary star observed by Mr. Hind.*—A star of the fifth magnitude, easily visible to the naked eye, was seen by Mr. Hind in the constellation of Ophiuchus, on the night of the 28th April, 1848. From the perfect acquaintance of that ob-

server with the region of the firmament in which he saw it, he was quite certain that, previous to the 5th April, no star brighter than those of the ninth magnitude had been there, nor is there any star in the catalogues at all corresponding to that which he saw there on the 28th. This star continued to be seen until the advance of the season and its low altitude rendered it impossible to be observed. It, however, constantly diminished in lustre until it disappeared, and has not since been seen.

3345. *Missing stars.*—To the class of temporary stars may be referred the cases of numerous stars which have disappeared from the firmament. On a careful examination of the heavens, and a comparison of the objects observed with former catalogues, and of catalogues ancient and modern with each other, many stars formerly known are now ascertained to be missing; and although, as Sir John Herschel observes, there is no doubt that in many instances these apparent losses have proceeded from mistaken entries, yet it is equally certain that in numerous cases there can have been no mistake in the observation or the entry, and that the star has really existed at a former epoch, and as certainly has since disappeared.

When we consider the vast length of many of the periods of astronomical phenomena, it is far from being improbable that these phenomena which seem to be occasional, accidental, and springing from the operation of no regular physical causes, such as those indicated by the class of variable stars first considered, may after all be periodic stars of the same kind, whose appearances and disappearances are brought about by similar causes. All that can be certainly known respecting them is, that they have appeared or disappeared once in that brief period of time within which astronomical observations have been made and recorded. If they be periodic stars, the length of whose period exceeds that interval, their changes could only have been once exhibited to us, and after ages have rolled away, and time has converted the future into the past, astronomers may witness the next occurrence of their phases, and discover that to be regular, harmonious, and periodic, which appears to us accidental, occasional, and anomalous.

III. DOUBLE STARS.

When the stars are examined individually by telescopes of a certain power, it is found that many which to the naked eye

appear to be single stars are in reality two stars placed so close together that they appear as one. These are called *double stars.*

3346. *Researches of Sir W. and Sir J. Herschel.* — A very limited number of these objects had been discovered before the telescope had received the vast accession of power which was given to it by the labour and genius of Sir William Herschel. That astronomer observed and catalogued 500 double stars; and subsequent observers, among whom his son, Sir John Herschel, holds the foremost place, have augmented the number to 6000.

3347. *Stars optically double.* — The close apparent juxtaposition of two stars on the firmament is a phenomenon which might be easily explained, and which could create no surprise. Such an appearance would be produced by the accidental circumstance of the lines of direction of the two stars as seen from the earth, forming a very small angle, in which case, although the two stars might in reality be as far removed from each other as any stars in the heavens, they would nevertheless *appear* close together. The *fig.* 869. will render this

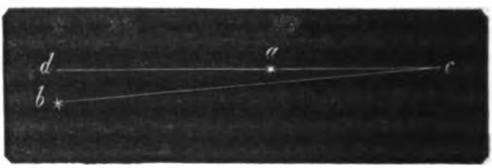


Fig. 869.

easily understood. Let *a* and *b* be the two stars seen from *c*. The star *a* will be seen relatively to *b*, as if it were at *d*, and the two objects will seem to be in close juxta-position; and if the angle under the lines *c a* and *c b* be less than the sum of the apparent semidiameters of the stars, they would actually appear to touch.

3348. *This supposition not generally admissible.* — If such objects were few in number, this mode of explaining them might be admitted; and such may, in fact, be the cause of the phenomenon in some instances. The chances against such proximity of the lines of direction are however so great as to be utterly incompatible with the vast number of double stars that have been

discovered, even were there not, as there is, other conclusive proof that this proximity and companionship is neither accidental nor merely apparent, but that the connection is real, and that the objects are united by a physical bond analogous to that which attaches the planets to the sun.

But apart from the proofs of real proximity which exist respecting many of the double stars, and which will presently be explained, it has been shown that the probability against mere optical juxta-position such as that described above is almost infinite. Professor Struve has shown that, taking the number of stars whose existence has been ascertained by observation down to the 7th magnitude inclusive, and supposing them to be scattered fortuitously over the entire firmament, the chances against any two of them having a position so close to each other as 4″ would be 9570 to 1. But when this calculation was made, considerably more than 100 cases of such duple juxta-position were ascertained to exist. The same astronomer also calculated that the chances against a third star falling within 32″ of the first two would be 173524 to one; yet the firmament presents at least four such triple combinations.

Among the most striking examples of double stars may be mentioned the bright star *Castor*, which, when sufficiently magnified, is proved to consist of two stars between the third and fourth magnitudes, within five seconds of each other. There are many, however, which are separated by intervals less than one second; such as ϵ *Arietis*, *Atlas Pleiadum*, γ *Coronæ*, η and ζ *Herculis*, and τ and λ *Ophiuchi*.

3349. *Argument against mere optical double stars derived from their proper motion.*—Another argument against the supposition of mere fortuitous optical juxta-position, unattended by any physical connection, is derived from a circumstance which will be fully explained hereafter. Certain stars have been ascertained to have a *proper motion*, that is a motion exclusively belonging to each individual star, in which the stars around it do not participate. Now, some of the double stars have such a motion. If one individual of the pair were affected by a proper motion, in which the other does not participate, their separation at some subsequent epoch would become inevitable, since one would necessarily move away from the other. Now, no such separation has in any instance been

witnessed. It follows, therefore, that the proper motion of one equally affects the other, and consequently, that their juxtaposition is real and not merely optical.

3350. *Struve's classification of double stars.*—The systematic observation of double stars, and their reduction to a catalogue with individual descriptions, commenced by Sir W. Herschel, has been continued with great activity and success by Sir J. Herschel, Sir J. South, and Professor Struve, so that the number of these objects now known, as to character and position, amounts to several thousand, the individuals of each pair being less than 32″ asunder. They have been classed by Professor Struve according to their distances asunder, the first class being separated by a distance not exceeding 1″, the second between 1″ and 2″, the third between 2″ and 4″, the fourth between 4″ and 8″, the fifth between 8″ and 12″, the sixth between 12″ and 16″, the seventh between 16″ and 24″, and the eighth between 24″ and 32″.

3351. *Selection of double stars.*—The double stars in the following Table have been selected by Sir J. Herschel from Struve's catalogue, as remarkable examples of each class well adapted for observations by amateurs, who may be disposed to try by them the efficiency of telescopes. (*See next page.*)

3352. *Coloured double stars.*—One of the characters observed among the double stars is the frequent occurrence of stars of different colours found together. Sometimes these colours are complementary (1059.); and when this occurs, it is possible that the fainter of the two may be a white star, which appears to have the colour complementary to that of the more brilliant, in consequence of a well-understood law of vision, by which the retina being highly excited by light of a particular colour is rendered insensible to less intense light of the same colour, so that the complement of the whole light of the fainter star finds the retina more sensible than that part which is identical in colour with the brighter star, and the impression of the complementary colour accordingly prevails. In many cases, however, the difference of colour of the two stars is real.

When the colours are complementary, the more brilliant star is generally of a bright red or orange colour, the smaller appearing blueish or greenish. The double stars ι Cancri and γ Andromachæ are examples of this. According to Sir J. Herschel, insulated stars of a red colour, some almost blood-

0″ to 1″.	1″ to 2″.	2″ to 4″.	4″ to 8″.	8″ to 12″.	12″ to 16″.	16″ to 24″.	24″ to 32″.
γ Coronæ Bor.	γ Circini.	α Piscium.	α Crucis.	β Orionis.	α Centauri.	α Can. Ven.	δ Herculis.
γ Centauri.	δ Cygni.	β Hydræ.	α Herculis.	γ Arietis.	β Cephei.	ε Normæ.	η Lyræ.
γ Lupi.	ε Chamæleontis.	γ Ceti.	α Gemin.	γ Delphini.	β Scorpii.	ζ Piscium.	ι Cancri.
ε Arietis.	ζ Bootis.	γ Leontis.	δ Gemin.	ζ Antliæ.	γ Volantis.	θ Serpentis.	κ Herculis.
ζ Herculis.	ι Cassiopeiæ.	γ Cor. Aus.	ζ Cor. Bor.	η Cassiopeiæ.	η Lupi.	κ Cor. Aus.	κ Cephi.
η Coronæ.	ι 2 Cancri.	γ Virginis.	θ Phœnicis.	θ Eridani.	ζ Ursa maj.	κ Tauri.	ψ Draconis.
η Herculis.	ξ Ursæ maj.	δ Serpentis.	κ Cephei.	ι Orionis.	κ Bootis.	24 Comæ.	κ Cygni.
λ Cassiopeiæ.	π Aquilæ.	ε Bootis.	λ Orionis.	f Eridani.	8 Monocerotis.	41 Draconis.	23 Orionis.
λ Ophiuchi.	σ Coro. Bor.	ε Draconis.	μ Cygni.	2 Can. Ven.	61 Cygni.	61 Ophiuchi.	
π Lupi.	2 Camelopar.	ε Hydra.	ξ Bootis.				
η Ophiuchi.	32 Orionis.	ζ Aquarii.	ξ Cephei.				
φ Draconis.	52 Orionis.	ζ Orionis.	π Bootis.				
φ Ursæ maj.		ι Leonis.	ρ Capricor.				
χ Aquilæ.		ι Trianguli.	ν Argus.				
ω Leonis.		κ Leporis.	ω Aurigæ.				
Atlas Pleiad.		μ Draconis.	μ Eridani.				
4 Aquarii.		μ Canis.	70 Ophiuchi.				
42 Comæ.		ρ Herculis.	12 Eridani.				
52 Arietis.		σ Cassiopeiæ.	32 Eridani.				
66 Piscium.		44 Bootis.	44 Herculis.				

red, occur in many parts of the heavens; but no example has been met with of a decidedly green or blue star unassociated with a much brighter companion.

3353. *Triple and other multiple stars.*—When telescopes of the greatest efficiency are directed upon some stars, which to more ordinary instruments appear only double, they prove to consist of three or more stars. In some cases one of the two companions only is double, so that the entire combination is triple. In others, both are double, the whole being, therefore, a quadruple star. An example of this latter class is presented by the star ε Lyræ. Sometimes the third star is much smaller than the principal ones, for example, in the cases of ζ Cancri, ξ Scorpii, 11 Monoceros, and 12 Lyncis. In others, as in θ Orionis, the four component stars are all conspicuous.

3354. *Attempts to discover the stellar parallax by double stars.*—When the attention of astronomers was first attracted to double stars, it was thought they would afford a most promising means of determining the annual parallax, and thereby discovering the distance of the stars. If we suppose the two individuals composing a double star, being situate very nearly in the same direction as seen from the earth, to be at very different distances, it might be expected that their apparent relative position would vary at different seasons of the year, by reason of the change of position of the earth.

Fig. 870.

Let A and B, *fig.* 870., represent the two individuals composing a double star. Let C and D represent two positions of the earth in its annual orbit, separated by an interval of half a year, and placed therefore on opposite sides of the sun S. When viewed from C, the star B will be to the left of the star A; and when viewed from D, it will be to the right of it. During the intermediate six months the relative change of position would gradually be effected, and the one star would thus appear either to revolve annually round the other, or would oscillate semi-annually from side to side of the other. The extent of its play compared with the diameter C D of the earth's orbit, would supply the data necessary to determine the

proportion which the distance of the stars would bear to that diameter.

The great problem of the stellar parallax seemed thus to be reduced to the measurement of the small interval between the individuals of double stars; and it happened fortunately, that the micrometers used in astronomical instruments were capable of measuring these minute angles with much greater relative accuracy than could be attained in the observations on greater angular distances. To these advantages were added the absence of all possible errors arising from refraction, errors incidental to the graduation of instruments, from uncertainty of levels and plumb-lines, from all estimations of aberration and precession; in a word, from all effects which, equally affecting both the individual stars observed, could not interfere with the results of the observations, whatever they might be.

3355. *Observations of Sir W. Herschel.*—These considerations raised great hopes among astronomers, that the means were in their hands to resolve finally the great problem of the stellar parallax, and Sir William Herschel accordingly engaged, with all his characteristic ardour and sagacity, in an extensive series of observations on the numerous double stars, for the original discovery of which science was already so deeply indebted to his labours. He had not, however, proceeded far in his researches, when phenomena unfolded themselves before him, indicating a discovery of a much higher order and interest than that of the parallax which he sought. He found that the relative position of the individuals of many of the double stars which he examined were subject to a change, but that the period of this change had no relation to the period of the earth's motion. It is evident that whatever appearances can proceed from the earth's annual motion, must be not only periodic and regular, but must pass annually through the same series of phases, always showing the same phase on each return of the same epoch of the sidereal year. In the changes of position which Sir William Herschel observed in the double stars, no such series of phases presented themselves. Periods, it is true, were soon developed; but these periods were regulated by intervals which neither agreed with each other nor with the earth's annual motion.

3356. *His discovery of binary stars.*—Some other explanation of the phenomena must, therefore, be sought for; and the

illustrious observer soon arrived at the conclusion, that these apparent changes of position were due to real motions in the stars themselves; that these stars, in fact, moved in proper orbits in the same manner as the planets moved around the sun. The slowness of the succession of changes which were observed, rendered it necessary to watch their progress for a long period of time before their motions could be certainly or accurately known; and accordingly, although these researches were commenced in 1778, it was not until the year 1803 that the observer had collected data sufficient to justify any positive conclusion respecting their orbital motions. In that and the following year, Sir William Herschel announced to the Royal Society, in two memorable papers read before that body, that there exist sidereal systems consisting of two stars revolving about each other in regular orbits, and constituting what he called *binary stars*, to distinguish them from double stars, generally so called, in which no such periodic change of position is discoverable. Both the individuals of a binary star are at the same distance from the eye in the same sense in which the planet Uranus and its attendant satellites are said to be at the same distance.

More recent observation has fully confirmed these remarkable discoveries. In 1841, Mädler published a catalogue of upwards of 100 stars of this class, and every year augments their number. These stars require the best telescopes for their observation, being generally so close as to render the use of very high magnifying powers indispensable.

3357. *Extension of the law of gravitation to the stars.*—The moment the revolution of one star round another was ascertained, the idea of the possible extension of the great principle of gravitation to these remote regions of the universe naturally suggested itself. Newton has proved in his *Principia*, that if a body revolve in an ellipse by an attractive force directed to the focus, that force will vary according to the law which characterises gravitation. Thus an elliptical orbit became a *test* of the presence and sway of the law of gravitation. If, then, it could be ascertained that the orbits of the double stars were ellipses, we should at once arrive at the fact that the law of which the discovery conferred such celebrity on the name of Newton, is not confined to the solar system, but prevails throughout the universe.

3358. *Orbit of star around star elliptic.*—The first distinct system of calculation by which the true elliptic elements of the orbit of a binary star were ascertained, was supplied in 1830, by M. Savary, who showed that the motion of one of the most remarkable of these stars (ξ *Ursæ majoris*), indicated an elliptic orbit described in 58¼ years. Professor Encké, by another process, arrived at the fact that the star 60 *Ophiuchi* moved in an ellipse with a period of 74 years. Several other orbits were ascertained and computed by Sir John Herschel, MM. Mädler, Hind, Smyth, and others.

The following Table is given by Sir J. Herschel, as containing the principal results of observation in this part of stellar astronomy up to 1850.

Star's Name.	Apparent semi-axis.	Excentricity.	Position of Node.	Perihelion from Node on Orbit.	Inclination.	Period in Years.	Perihelion Passage.	By whom computed.
1. Herculis - -	1·189″	0·44454	39° 26′	262° 4′	50° 53′	31·468	1829·50	Mädler.
2. η Coronæ B. - -	1·088	0·33760	24 18	261 21	71 8	43·246	1815·23	Ditto.
3. ζ Cancri - -	1·292	0·23486	1 28	266 0	63 17	58·910	1853·37	Ditto.
4. a. ξ Ursæ majoris -	3·857	0·41640	95 22	131 38	50 40	58·262	1817·25	Savary.
4. b. Ditto. - -	3·278	0·37770	97 47	134 22	56 6	60·720	1816·73	Herschel, junior.
4. c. Ditto. - -	2·417	0·41350	98 52	130 48	54 56	61·464	1816·44	Mädler.
5. ω Leonis - -	0·857	0·64338	135 11	185 27	46 33	82·533	1849·76	Ditto.
6. a. p. Ophiuchi - -	4·328	0·43007	147 12	125 22	46 25	73·862	1806·88	Encké.
6. b. Ditto - -	4·392	0·46670	137 2	145 46	48 5	80·340	1807·06	Herschel, junior.
6. c. Ditto - -	4·192	0·44880	126 55	142 53	64 51	92·870	1819·73	Mädler.
7. Σ 3062 - - -	1·255	0·44958	15 5	137 27	35 31	94·765	1837·41	Ditto.
8. ξ Bootis - - -	12·560	0·59374	359 59	100 59	80 5	117·140	1779·88	Herschel, junior.
9. δ Cygni - - -	1·811	0·60667	24 54	243 24	46 23	178·700	1862·87	Hind.
10. γ Virginis - -	3·580	0·87952	5 33	318 45	23 36	182·120	1836·43	Herschel, junior.
11. a. Castor - - -	8·086	0·75820	58 6	97 29	70 3	252·660	1855·83	Ditto.
11. b. Ditto - - -	7·008	0·79725	23 5	87 37	70 58	232·124	1913·90	Mädler.
11. c. Ditto - - -	6·300	0·24050	11 24	356 22	43 14	632·270	1699·26	Hind.
12. a. σ Coronæ B. - -	3·918	0·69978	25 7	64 38	29 29	608·450	1826·60	Mädler.
12. b. Ditto - - -	5·194	0·72560	21 3	69 24	25 39	736·880	1826·48	Hind.
13. μ 2 Bootis - -	3·218	0·84010	117 21	103 17	46 57	649·720	1852·50	Ditto.
14. α Centauri - -	15·500	0·95000	86 7	291 22	47 56	77·000	1851·50	Jacob.

The elements Nos. 1, 2, 3, 4 c, 5, 6 c, 7, 11 b, 12 a, are extracted from M. Mädler's synoptic view of the history of double stars, in vol. ix. of the Dorpat Observations: 4 a, from the Connoiss. des Temps, 1830: 4 b, 6 b, and 11 a, from vol. v. Trans. Astron. Soc. Lond.: 6 a, from Berlin Ephemeris, 1832: No. 8. from Trans. Astron. Soc. vol. vi.: No. 9, 11 c, 12 b, and 13 from Notices of the Astronomical Society, vol. vii. p. 22., and viii. p. 159., and No. 10 from Sir John Herschel's "Results of Astronomical Observations, &c., at the Cape of Good Hope," p. 297. The Σ prefixed to No. 7 denotes the number of the star in M. Struve's Dorpat Catalogue (Catalogus Novus Stellarum Duplicium, &c., Dorpat. 1827), which contains the places for 1826 of 3112 of these objects.

The "position of the node" in col. 4. expresses the angle of position of the line of intersection of the plane of the orbit, with the plane of

the heavens on which it is seen projected. The "inclination" in col. 6. is the inclination of these two planes to one another. Col. 5. shows the angle actually included *in the plane of the orbit*, between the line of nodes (defined as above) and the line of apsides. The elements assigned in this table to ω Leonis, ξ Bootis, and Castor must be considered as very doubtful, and the same may perhaps be said of those ascribed to μ 2 Bootis, which rest on too small an arc of the orbit, and that too imperfectly observed, to afford a secure basis of calculation.

3359. *Remarkable case of γ Virginis.*—The most remarkable of these, according to Sir John Herschel, is γ *Virginis;* not only on account of the length of its period, but by reason also of the great diminution of apparent distance and rapid increase of angular motion about each other, of the individuals composing it. It is a bright star of the fourth magnitude, and its component stars are almost exactly equal. It has been known to consist of two stars since the beginning of the eighteenth century, their distance being then between six and seven seconds; so that any tolerably good telescope would resolve it. Since that time they have been constantly approaching, and are at present hardly more than a single second asunder; so that no telescope that is not of very superior quality, is competent to show them otherwise than as a single star somewhat lengthened in one direction. It fortunately happens that Bradley, in 1718, noticed and recorded, in the margin of one of his observation-books, the apparent direction of their line of junction as being parallel to that of two

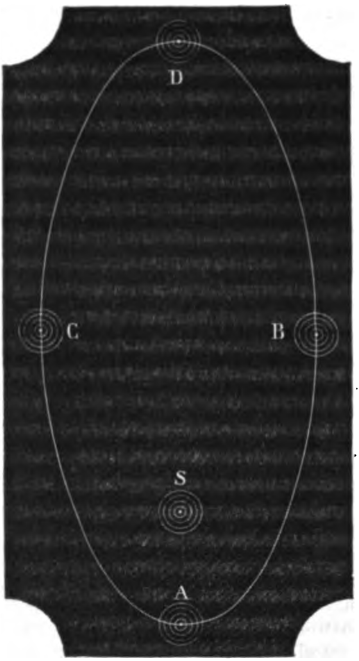


Fig. 871.

remarkable stars α and δ of the same constellation, as seen by the naked eye. They are entered also as distinct stars in Mayer's catalogue; and this affords also another means of recovering their relative situation at the date of his observations, which were made about the year 1756. Without particularising individual measurements, which will be found in their proper repositories, it will suffice to remark, that their whole series is represented by an ellipse.

3360. *Singular phenomena produced by one solar system thus revolving round another.* — To understand the curious effects which must attend the case of a lesser sun with its attendant planets revolving round a greater, let the larger sun, *fig.* 871., with its planets be represented at S, in the focus of an ellipse, in which the lesser sun accompanied by *its* planets moves. At A this latter sun is in its perihelion, and nearest to the greater sun S. Moving in its periodical course to B, it is at its mean distance from the sun S. At D it is at aphelion, or its most distant point, and finally returns through C to its perihelion A. The sun S, because of its vast distance from the system A, would appear to the inhabitants of the planets of the system A much smaller than their proper sun; but, on the other hand, this effect of distance would be to a certain extent compensated by its greatly superior magnitude; for analogy justifies the inference that the sun S is greater than the sun A in a proportion equal to that of the magnitude of our sun to one of the planets. The inhabitants of the planets of the system A will then behold the spectacle of *two suns* in their firmament. The annual motion of one of these suns will be determined by the motion of the planet itself in its orbit, but that of the other and more distant sun will be determined by the period of the lesser sun around the greater in the orbit A B D C. The rotation of the planets on their axes will produce two days of equal length, but not commencing or ending simultaneously. There will be in general *two sunrises* and *two sunsets!* When a planet is situate in the part of its orbit between the two suns, there will be no night. The two suns will then be placed exactly as our sun and moon are placed when the moon is full. When the one sun sets, the other will rise; and when the one rises, the other will set. There will be, therefore, continual day. On the other hand, when a planet is at such a part of its orbit

that both suns lie in nearly the same direction as seen from it, both suns will rise and both will set together. There will then be the ordinary alternation of day and night as on the earth, but the day will have more than the usual splendour, being enlightened by two suns.

In all intermediate seasons the two suns will rise and set at different times. During a part of the day both will be seen at once in the heavens, occupying different places, and reaching the meridian at different times. There will be *two noons*. In the morning for some time, more or less, according to the season of the year, one sun only will be apparent, and in like manner, in the evening, the sun which first rose will be the first to set, leaving the dominion of the heavens to its splendid companion.

The diurnal and annual phenomena incidental to the planets attending the central sun s will not be materially different, except that to them the two suns will have extremely different magnitudes, and will afford proportionally different degrees of light. The lesser sun will appear much smaller, both on account of its really inferior magnitude and its vastly greater distance. The two days, therefore, when they occur, will be of very different splendour, one being probably as much brighter than the other as the light of noonday is to that of full moonlight, or to that of the morning or evening twilight.

But these singular vicissitudes of light will become still more striking, when the two suns diffuse light of different colours. Let us examine the very common case of the combination of a *crimson* with a *blue* sun. In general, they will rise at different times. When the blue rises, it will for a time preside alone in the heavens, diffusing a blue morning. Its crimson companion, however, soon appearing, the lights of both being blended, a white day will follow. As evening approaches, and the two orbs descend toward the western horizon, the blue sun will first set, leaving the crimson one alone in the heavens. Thus a ruddy evening closes this curious succession of varying lights. As the year rolls on, these changes will be varied in every conceivable manner. At those seasons when the suns are on opposite sides of a planet, crimson and blue days will alternate, without any intervening night; and at the intermediate epochs all the various intervals of rising and setting of the two suns will be exhibited.

3361. *Magnitudes of the stellar orbits.* — It is evident that in any case in which the parallax of a binary star, and consequently its distance from our system, has been or may be discovered, the magnitude of the orbit of one described round the other, can be determined with a precision and certainty proportional to those with which the parallax is known. For, in that case, the linear value of 1″ at the star, will be found by dividing the earth's distance from the sun by the parallax expressed in seconds.

The binary stars 61 *Cygni* and *α Centauri* supply examples of the application of this principle. The parallax of these stars has been ascertained (2605.). That of 61 Cygni is 0·348, and the semi-axis of the elliptic orbit of one star round the other is 15·5″. The semidiameter of the earth's orbit being D, therefore, the linear value of 1″ at the star is $\frac{\text{D}}{0 \cdot 348}$, and the semi-axis a of the stellar orbit is

$$a = \text{D} \times \frac{15 \cdot 5}{0 \cdot 348} = 44 \cdot 54 \ \text{D}.$$

It appears, therefore, that the semi-axis of the orbit is greater than that of Neptune's orbit in the ratio of 3 to 2.

The angle subtended by the semi-axis of the elliptic orbit of α Centauri is not so certainly known, but is taken to be about 12″. The parallax of this star being 0·913″, we should then have

$$a = \text{D} \times \frac{12}{0 \cdot 913} = 13 \cdot 14 \ \text{D}.$$

The semi-axis of the stellar orbit would, therefore, be about one half greater than the orbit of Saturn.

3362. *Masses of binary stars determined by their parallax and period.* — Since by (2634.) the relation of the semi-axis of the orbits and the periodic times determines the relative masses of the central bodies, we are enabled to compare the mass of the central star of a binary system with that of the sun, in all cases in which the real semi-axis of the orbit and the periodic time are known. Thus, let M′ be the mass of the central star, a' the semi-axis of the orbit, and P′ the periodic time, and let M be the mass of the sun, a the semi-axis

of the earth's orbit, and P the earth's period, and we shall have

$$M' : M :: \frac{a'^3}{P'^2} : \frac{a^3}{P^2}.$$

The periods of the binary stars of known parallax have not been certainly determined; but if it be assumed, as it may probably be, that the period of 61 Cygni is about 500 times that of the earth, we shall have

$$M' : M :: \frac{44{\cdot}54^3}{500^2} : 1;$$

and, therefore,

$$M' = 0{\cdot}3533\ M;$$

so that the mass of the central star would be a little more than one third of that of the sun.

IV. PROPER MOTION OF THE STARS.

3363. In common parlance the stars are said to be *fixed*. They have received this epithet to distinguish them from the planets, the sun, and the moon, all of which constantly undergo changes of apparent position on the surface of the heavens. The stars, on the contrary, so far as the powers of the eye unaided by art can discover, never change their relative position in the firmament, which seems to be carried round us by the diurnal motion of the sphere, just as if the stars were attached to it, and merely shared in its apparent motion.

But the stars, though subject to no motion perceptible to the naked eye, are not absolutely fixed. When the place of a star on the heavens is exactly observed by means of good astronomical instruments, it is found to be subject to a change from month to month and from year to year, small indeed, but still easily observed and certainly ascertained.

3364. *The sun not a fixed centre.* — It has been demonstrated by Laplace, that a system of bodies, such as the solar system, placed in space and submitted to no other continued force except the reciprocal attractions of the bodies which compose it, must either have its common centre of gravity stationary or in a state of uniform rectilinear motion.

3365. *Effect of the sun's supposed motion on the apparent places of the stars.*— The chances against the conditions which would render the sun stationary, compared with those which would give it a motion in *some* direction with *some* velocity, are so numerous that we may pronounce it to be morally certain that our system is in motion in some determinate direction through the universe. Now, if we suppose the sun attended by the planets to be thus moved through space in any direction, an observer placed on the earth would see the effects of such a motion, as a spectator in a steamboat moving on a river would perceive his progressive motion on the stream by an apparent motion of the banks in a contrary direction. The observer on the earth would, therefore, detect such a motion of the solar system through space by the apparent motion in the contrary direction with which the stars would be affected.

Such a motion of the solar system would affect different stars differently. All would, it is true, appear to be affected by a contrary motion, but all would not be equally affected. The nearest would appear to have the most perceptible motion, the more remote would be affected in a less degree, and some might, from their extreme distance, be so slightly affected as not to exhibit any apparent change of place, even when examined with the most delicate instruments. To whatever degree each star might be affected, all the changes of position would, however, apparently take place in the same direction.

The apparent effects would also be exhibited in another manner. The stars in that region of the universe toward which the motion of the system is directed, would appear to recede from each other. The spaces which separate them would seem to be gradually augmented, while, on the contrary, the stars in the opposite quarter would seem to be crowded more closely toge-

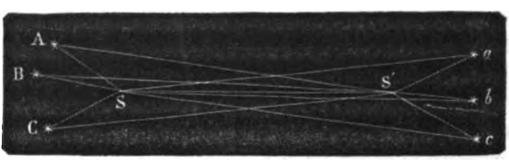


Fig. 872.

ther, the distances between star and star being gradually diminished. This will be more clearly comprehended by *fig.* 872.

Let the line SS′ represent the direction of the motion of the system, and let S and S′ represent its positions at any two epochs. At S, the stars ABC would be separated by intervals measured by the angles ASB, and BSC, while at S′ they would appear separated by the lesser angles AS′B, and BS′C. Seen from S′, the stars ABC would seem to be closer together than they were when seen from S. For like reasons the stars *abc*, towards which the system is here supposed to move, would seem to be closer together when seen from S, than when seen from S′. Thus, in the quarter of the heavens towards which the system is moving, the stars might be expected to separate gradually, while in the opposite quarter they would become more condensed. In all the intermediate parts of the heavens they would be affected by a motion contrary to that of the solar system. Such in general would be the effects of a progressive motion of our system.

3366. *Motion of the sun inferred from the proper motion of the stars.*— Although no general effect of this kind has been manifested in any conspicuous manner among the fixed stars, many of these objects have been found, in long periods of time, to have shifted their position in a very sensible degree. Thus, for example, the three stars, Sirius, Arcturus, and Aldebaran, have undergone, since the time of Hipparchus (130 B.C.), a change of position southwards, amounting to considerably more than half a degree. The double star 61 Cygni has, in half a century, moved through nearly 4·5′, the two stars composing it being carried along in parallel lines with a common velocity. The stars ϵ Indi and μ Cassiopeiæ move at the rate of 7·74″ and 3·74″ annually.

Various attempts have been made to render these and other like changes of apparent position of the fixed stars compatible with some assumed motion of the sun. Sir W. Herschel, in 1783, reasoning upon the proper motions which had then been observed, arrived at the conclusion, that such appearances might be explained by supposing that the sun has a motion directed to a point near the star λ Herculis. About the same time, Prevost came to a like conclusion, assigning, however, the direction of the supposed motion to a point differing by 27° from that indicated by Sir W. Herschel.

Since that epoch, the proper motions of the stars have been more extensively and accurately observed, and calculations of the motion of the sun which they indicate, have been made by

several astronomers. The following points have been assigned as the direction of the solar motion in 1790:—

R. A.	N. P. D.	
260° 34′	63° 43′	Sir W. Herschel.
256° 25′	51° 23′	Argelander.
255° 10′	51° 26′	Ditto.
261° 11′	59° 2′	Ditto.
252° 53′	75° 34′	Luhndahl.
261° 22′	62° 24′	Otto Struve.

The first estimate of Argelander was made from the proper motions of 21 stars, each of which has an annual motion greater than 1″; the second from 50 stars having annual proper motions between 1″ and 0·5″, and the third from those of 319 stars having motions between 0·5″ and 0·1″. The estimate of M. Luhndahl is based on the motions of 147 stars, and that of M. Struve on 392 stars.

The mean of all these estimates is a point whose right ascension is 259° 9′, and north polar distance 5° 23′, which it will be seen differs very little from the point originally assigned by Sir W. Herschel.

All the preceding calculations being based on observations made on stars in the northern hemisphere, it was obviously desirable that similar estimates should be made from the observed proper motions of southern stars. Mr. Galloway undertook and executed these calculations; and found that the southern stars gave the direction of the solar motion for 1790, to be towards a point whose right ascension is 260° 1′, and north polar distance 55° 37′.

No doubt, therefore, can remain that the proper motion of the stars is produced by a real motion of the solar system, and that the direction of this motion in 1790 was towards a point of space which seen from the then position of the system had the right ascension of about 260°, and the north polar distance of about 55°.

3367. *Velocity of the solar motion.*—It follows from these calculations, that the average displacement of the stars requires that the motion of the sun should be such as that if its direction were at right angles to a visual ray, drawn from a star of the first magnitude of average distance, its apparent annual motion would be 0·3392″; and taking the average

parallax of such a star at 0·209″, if D express the semi-axis of the earth's orbit, the annual motion of the sun would be

$$\frac{3392}{2090} \times D = 1{\cdot}623\ D.$$

It follows, therefore, that the annual motion of the sun would be

$$1{\cdot}623 \times 95{,}000000 = 154{,}200000 \text{ miles};$$

and the daily motion

$$\frac{154{,}200000}{365\frac{1}{4}} = 422000 \text{ miles};$$

a velocity equal to something more than the fourth of the earth's orbital motion.

3368. *The probable centre of solar motion.* — The motion of the sun, which has been computed in what precedes, is that which it had at a particular epoch. No account is taken of the possible or probable changes of direction of such motion. To suppose that the solar system should move continuously in one and the same direction, would be equivalent to the supposition that no body or collection of bodies in the universe would exercise any attraction upon it. It is obviously more consistent with probability and analogy, that the motion of the system is *orbital*, that is to say, that it revolves round some remote centre of attraction, and that the direction of its motion must continually change, although such change, owing to the great magnitude of its orbit, and the relative slowness of its motion, be so very slow as to be quite imperceptible within even the longest interval over which astronomical records extend.

Attempts have, nevertheless, been made to determine the centre of the solar motion; and Dr. Mädler has thrown out a surmise that it lies at a point in or near the small constellation of the Pleiades.

This and like speculations must, however, be regarded as conjectural for the present.

CHAP. XXVIII.

THE FORM AND DIMENSIONS OF THE MASS OF STARS WHICH COMPOSE THE VISIBLE FIRMAMENT.

3369. *Distribution of stars on the firmament.* — The aspect of the firmament might, at first, impress the mind of an observer with the idea that the numerous stars scattered over it are destitute of any law or regularity of arrangement, and that their distribution is like the fortuitous position which objects casually flung upon such a surface might be imagined to assume. If, however, the different regions of the heavens be more carefully examined and compared, this first impression will be corrected, and it will, on the contrary, be found that the distribution of the stars over the surface of the celestial sphere follows a distinct and well-defined law; that their density, or the number of them which is found in a given space of the heavens, varies regularly, increasing continually in certain directions and decreasing in others.

Sir W. Herschel submitted the heavens, or at least that part of them which is observable in these latitudes, to a rigorous telescopic survey, counting the number of individual stars visible in the field of view of a telescope of given aperture, focal length, and magnifying power, when directed to different parts of the firmament. The result of this survey proved that, around two points of the celestial sphere diametrically opposed to each other, the stars are more thinly scattered than elsewhere; that departing from these points in any direction, the number of stars included in the field of view of the same telescope increases first slowly, but at greater distances more rapidly; that this increase continues until the telescope receives a direction at right angles to the diameter which joins the two opposite points where the distribution is most sparse; and that in this direction the stars are so closely crowded together that it becomes, in some cases, impracticable to count them.

3370. *Galactic circle and poles.*—The two opposite points of the celestial sphere, around which the stars are observed to be most sparse, have been called the GALACTIC POLES; and the great circle at right angles to the diameter joining these points, has been denominated the GALACTIC CIRCLE.

This circle intersects the celestial equator at two points, situate 10° east of the equinoxial points, and is inclined to the equator at an angle of 63°, and, therefore, to the ecliptic at an angle of 40°.

In referring to and explaining the distribution of the stars over the celestial sphere, it will be convenient to refer them to this circle and its poles, as, for other purposes, they have been referred to the equator and its poles. We shall, therefore, express the distance of different points of the firmament from the galactic circle, in either hemisphere, by the terms north or south GALACTIC LATITUDE.

3371. *Variation of the stellar density in relation to this circle.*—The elaborate series of stellar observations in the northern hemisphere made during a great part of his life, by Sir W. Herschel, and subsequently extended and continued in the southern hemisphere by Sir J. Herschel, has supplied data by which the law of the distribution of the stars, according to their galactic latitude, has been ascertained at least with a near approximation.

The great celestial survey executed by these eminent observers, was conducted upon the principle explained above. The telescope used for the purpose had 18 inches aperture, 20 feet focal length, and a magnifying power of 180. It was directed indiscriminately to every point of the celestial sphere visible in the latitude of the places of observation.

It was by means of a vast number of distinct observations thus made, that the position of the galactic poles was ascertained. The density of the stars, measured by the number included in each "gauge" (as the field of view was called), was nearly the same for the same galactic latitude, and increased in proceeding from the galactic pole, very slowly at first, but with great rapidity when the galactic latitude was much diminished.

3372. *Struve's analysis of Herschel's observations.*—An analysis of the observations of Sir W. Herschel, in the northern hemisphere, was made by Professor Struve, with the view

of determining the mean density of the stars in successive zones of galactic latitude; and a like analysis has been made of the observations of Sir J. Herschel, in the southern hemisphere.

If we imagine the celestial sphere resolved into a succession of zones, each measuring 15° in breadth, and bounded by parallels to the galactic circle, the average number of stars included within a circle, whose diameter is 15′, and whose magnitude, therefore, would be about the fourth part of that of the disk of the sun or moon, will be that which is given in the second column of the following Table.

Galactic Latitude.	Average number of Stars in a circle 15′ diameter.
N 90° — 75°	4·32
„ 75° — 60°	5·42
„ 60° — 45°	8·21
„ 45° — 30°	13·61
„ 30° — 15°	24·09
„ 15° — 0°	53·43
S 0° — 15°	59·06
„ 15° — 30°	26·29
„ 30° — 45°	13·49
„ 45° — 60°	9·08
„ 60° — 75°	6·62
„ 75° — 90°	6·05

It appears, therefore, that the variation of the density of the visible stars in proceeding from the galactic plane, either north or south, is subject almost exactly to the same law of decrease, the density, however, at each latitude being somewhat greater in the southern than in the northern hemisphere.

3373. *The milky way.* — The regions of the heavens, which extend to a certain distance on one side and the other of the galactic plane, are generally so densely covered with small stars, as to present to the naked eye the appearance, not of stars crowded together, but of whitish nebulous light. This appearance extends over a vast extent of the celestial sphere, deviating in some places from the exact direction of the galactic circle, bifurcating and diverging into two branches at a certain point which afterwards reunite, and at other places

throwing out off-shoots. This appearance was denominated the *Via Lactea*, or the galaxy,* by the ancients, and it has retained that name.

The course of the milky way may be so much more easily and clearly followed by means of a map of the stars, or a celestial globe, upon which it is delineated, that it will be needless here to describe it.

3374. *It consists of innumerable stars crowded together.*— When this nebulous whiteness is submitted to telescopic examination with instruments of adequate power, it proves to be a mass of countless numbers of stars, so small as to be individually undistinguishable, and so crowded together as to give to the place they occupy, the whitish appearance from which the milky way takes its name.

Some idea may be formed of the enormous number of stars which are crowded together in those parts of the heavens, by the actual numbers so distinctly visible as to admit of being counted or estimated, which are stated by Sir W. Herschel to have been seen in spaces of given extent. He states, for example, that in those parts of the milky way in which the stars were most thinly scattered, he sometimes saw eighty stars in each field. In an hour, fifteen degrees of the firmament were carried before his telescope, showing successively sixty distinct fields. Allowing eighty stars for each of these fields, there were thus exhibited, in a single hour, without moving the telescope, four thousand eight hundred distinct stars! But by moving the instrument at the same time in the vertical direction, he found that in a space of the firmament, not more than fifteen degrees long, by four broad, he saw fifty thousand stars, large enough to be individually visible and distinctly counted! The surprising character of this result will be more adequately appreciated, if it is remembered that this number of stars thus seen in the space of the heavens, not more than thirty diameters of the moon's disk in length and eight in breadth, is fifty times greater than all the stars taken together, which the naked eye can perceive at any one time in the heavens, on the most serene and unclouded night!

On presenting the telescope to the richer portion of the *via lactea*, Herschel found, as might be expected, much greater

* From the Greek word, γάλα, γάλακτος, milk.

numbers of stars. In a single field he was able to count 588 stars; and for fifteen minutes, the firmament being moved before his telescope by the diurnal motion, no diminution of number was apparent, so that he estimated that in that space of time, 116000 stars must have passed in review before him; the number seen at any one time being greater than can be seen by the naked eye, on the entire firmament, except on the clearest nights.

3375. *The probable form of the stratum of stars in which the sun is placed.* — It may be considered as established by a body of analogical evidence, having all the force of demonstration, that the fixed stars are self-luminous bodies, similar to our sun; and that although they may differ more or less from our sun and from each other in magnitude and intrinsic lustre, they have a certain average magnitude; and that, therefore, in the main, the great differences which are apparent in their brightness, is to be ascribed to difference of distance. Assuming, then, that they are separated from each other by distances analogous to their distances from the sun, itself a star, the general phenomena which have been described above, involving the rapid increase of stellar density in approaching the galactic plane, combined with the observed form of the milky way, which, following the galactic plane in its general course departs nevertheless from it at some points, bifurcates resolving itself into two diverging branches at others, and at others throws out irregular off-shoots, conducted Sir W. Herschel to the conclusion, that the stars of our firmament, including those which the telescope renders visible, as well as those visible to the naked eye, instead of being scattered indifferently in all directions around the solar system through the depths of the universe, form a stratum of definite form and dimensions, of which the thickness bears a very small proportion to the length and breadth, and that the sun and solar system is placed within this stratum, very near its point of bifurcation, relatively to its breadth near its middle point, and relatively to its thickness (as would appear from the more recent observations) nearer to its northern than to its southern surface.

Let A C H D, *fig.* 873. represent a rough outline of a section of such a stratum, made by a plane passing through or near its centre. Let A B represent the intersection of this with the plane of the galactic circle, so that, *z* being the place of the solar system, *z* C

will be the direction of the north, and *z* D that of the south galactic pole. Let *z* H represent the two branches which bifurcate

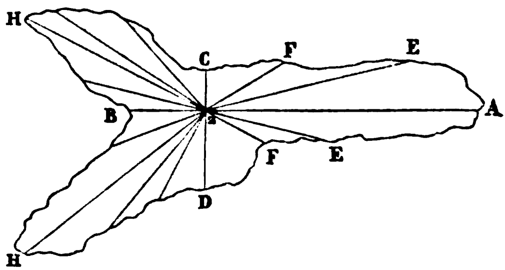


Fig. 873.

from the chief stratum at B. Now, if we imagine visual lines to be drawn from *z* in all directions, it will be apparent that those *z* C and *z* D, which are directed to the galactic poles, pass through a thinner bed of stars than any of the others; and since *z* is supposed to be nearer to the northern than to the southern side of the stratum, *z* C will pass through a less thickness of stars than *z* D. As the visual lines are inclined at greater and greater angles to *z* A, their length rapidly decreases, as is evident by comparing *z* A, *z* E, and *z* F, which explains the fact that while the stars are as thick as powder in the direction *z* A, they become less so in the direction *z* E, and still less in the direction *z* F, until at the poles in the directions *z* C and *z* D, they become least dense.

On the other side, *z* B being less than *z* A, a part of the galactic circle is found at which the stars are more thinly scattered; but in two directions, *z* H intermediate between *z* B and the galactic poles, they again become nearly as dense as in the direction *z* A.

This illustration must, however, be taken in a very general sense. No attempt is made to represent the various off-shoots and variations of length, breadth, and depth of the stratum measured from the position of the solar system within it, which have been indicated by the telescopic *soundings* of Sir W. Herschel and his illustrious son, whose wondrous labours have effected what promises in time, by the persevering researches of their successors, to become a complete analysis of this most marvellous mass of systems. Meanwhile it may be considered

as demonstrated that it consists of myriads of stars clustered together:

> "A broad and ample road, whose dust is gold,
> And pavement stars, as stars to us appear;
> Seen in the galaxy that Milky Way,
> Like to a circling zone powder'd with stars." — MILTON.

The appearance which this mass of stars would present if viewed from a position directly above its general plane, and at a sufficient distance to allow its entire outline to be discerned, was represented by Sir William Herschel as resembling the starry stratum sketched in Plate XXVI.

He considered that it was probable that the *thickness* of this *bed of stars* was equal to about eighty times the distance of the nearest of the fixed stars from our system; and supposing our sun to be near the middle of this thickness, it would follow that the stars on its surface in a direction perpendicular to its general plane would be at the fortieth order of distance from us. The stars placed in the more remote edges of its *length* and *breadth* he estimated to be in some places at the nine-hundredth order of distance from us, so that its extreme length may be said to be in round numbers about 2000 times the distance of the nearest fixed stars from our system. Such a space light would take 20000 years to move over, moving all that time at the rate of nearly 200000 miles between every two ticks of a common clock!

CHAP. XXIX.

STELLAR CLUSTERS AND NEBULÆ.

3376. *The stars which form the firmament a stellar cluster.— Analogy suggests the probable existence of others.*—It appears, then, that our sun is an individual star, forming only a single unit in a cluster or mass of many millions of other similar stars; that this cluster has limited dimensions, has ascertainable length, breadth, and thickness, and in short, forms what may be expressed by a *universe of solar systems.* The mind, still unsatisfied, is as urgent as before in its questions

regarding the *remainder of immensity!* However vast the dimensions of this mass of suns be, they are nevertheless finite. However stupendous be the space included within them, it is still *nothing* compared to the immensity which lies outside! Is that immensity a vast solitude? Are its unexplored realms dark and silent? Has Omnipotence circumscribed its agency, and has Infinite Beneficence left those unfathomed regions destitute of evidence of His power?

That the infinitude of space should exist without a purpose, unoccupied by any works of creation, is plainly incompatible with all our notions of the character and attributes of the Author of the universe, whether derived from the voice of revelation or from the light of nature. We should therefore infer, even in the absence of direct evidence, that *some* works of creation are dispersed through those spaces which lie beyond the limits of that vast stellar cluster in which our system is placed. Nay, we should be led by the most obvious analogies to conjecture that *other stellar clusters* like our own, are dispersed through immensity, separated probably by distances as much greater than those which intervene between star and star, as the latter are greater than those which separate the bodies of the solar system. But if such distant clusters existed, it may be objected, that they must be visible to us; that although diminished, perhaps, to mere spots on the firmament, they would still be rendered apparent, were it only as confused whitish patches, by the telescope; that as the stars of the milky way assume to the naked eye the appearance of mere whitish nebulosity, so the far more distant stars of other clusters, which cannot be perceived at all by the naked eye, would, to telescopes of adequate power, present the same whitish nebulous appearance; and that we might look forward without despair to such augmentation of the powers of the telescope as may even enable us to perceive them to be actual clusters of stars.

3377. *Such clusters of stars innumerable.* — Such anticipations have accordingly been realised. In various parts of the firmament objects are seen which, to the naked eye, appear like stars seen through a mist, and sometimes as nebulous specks, which might be, and not infrequently are, mistaken for comets. With ordinary telescopes these objects are visible in very considerable numbers, and were observed nearly a century ago.

In the *Connoissance des Temps*, for 1784, *Messier*, then so celebrated for his observations on comets, published a catalogue of 103 objects of this class, of many of which he gave drawings, with which all observers who search for comets ought to be familiar, to avoid being misled by their resemblance to them. The improved powers of the telescope speedily disclosed to astronomers the nature of these objects, which, when examined by sufficient magnifying powers, prove to be masses of stars clustered together in a manner identical with that cluster in which our sun is placed. They appear as they do, mere specks of whitish light, because of their enormous distance.

3378. *Distribution of clusters and nebulæ on the firmament.* —These objects are not dispersed fortuitously and indifferently on all parts of the heavens. They are wholly absent from some regions, in some rarely found, and crowded in amazing profusion in others. Their disposition, however, is not like that of the stars in general, determined by a great circle of the sphere and its poles. It was supposed that they showed a tendency to crowd towards a zone at right angles to the galactic circle, but a careful comparison of their position does not confirm this. According to Sirs W. and J. Herschel, the nebulæ prevail most around the following parts of the celestial sphere :—

1 The North Galactic Pole.
2 Leo major.
3 Leo minor.
4 Ursa major.
5 Canes Venatici.
6 Coma Berenici.
7 Bootes (precedingly).
8 Virgo (head, wings and shoulder).

The parts of the heavens, on the other hand, where they are found in the smallest numbers, are :—

1 Aries.
2 Taurus.
3 Orion (head and shoulders).
4 Auriga.
5 Perseus.
6 Camelopardus.
7 Draco.
8 Hercules.
9 Serpentarius (northern part).
10 Serpens (tail).
11 Aquila (tail).
12 Lyra.

In the southern hemisphere their distribution is more uniform.

3379. *Constitution of the clusters and nebulæ.*—What those objects are, and of what they severally consist, admits of no reasonable doubt. So far as relates to the stellar clusters, their

constituent parts are visible. They are, as their name imports, masses of stars collected together at certain points in the regions of space which stretch beyond the limits of our own cluster, and are by distance so reduced in their visual magnitude that an entire cluster will appear to the naked eye, if it be visible at all, as a single star, and when seen with the telescope will be included within the limit of a single field of view.

Different clusters exhibit their component stars seen with the same magnifying power more or less distinctly. This may be explained either by difference of distance, or by the supposition that they may consist of stars of different real magnitudes, and crowded more or less closely together. The former supposition is, however, by far the more natural and probable.

The appearance of the stars composing some of the clusters is quite gorgeous. Sir J. Herschel says, that the cluster which surrounds κ Crucis in the southern hemisphere, occupies the 48th part of a square degree, or about the tenth part of the superficial magnitude of the moon's disk, and consists of about 110 stars from the 7th magnitude downwards, eight of the more conspicuous stars being coloured with various tints of red, green, and blue, so as to give to the whole the appearance of a rich piece of jewellery.

Cluster compared with cluster show all gradations of smallness and closeness of the component stars, until they assume the appearance of patches of starry powder. These varieties are most obviously ascribable to varying distances.

Then follow those patches of starry light which are seen in so many regions of the heavens, and which have been denominated nebulæ. That these are still clusters, of which the component stars are indistinguishable by reason of their remoteness, there are the strongest evidence and most striking analogies to prove. Every augmentation of power and improvement of efficiency the telescope receives, augments the number of nebulæ which are converted by that instrument into clusters. Nebulæ which were irresolvable before the time of Sir W. Herschel, yielded in large numbers to the powers of the instruments which that observer brought to bear upon them. The labours of Sir J. Herschel, the colossal telescopes constructed by Lord Rosse, and the erection of observatories in multiplied numbers in climates and under skies more favourable to observation, have all tended to augment the number of nebulæ

which have been resolved, and it may be expected that this progress will continue, the resolution of these objects into stellar clusters being co-extensive with the improved powers of the telescope and the increased number and zeal of observers.

3380. *Nebular hypothesis.*—A theory was put forward to explain these objects, based upon views not in accordance with what has just been related. It was assumed hypothetically that the nebulous matter was a sort of luminous fluid diffused through different parts of the universe; that by its aggregation on certain laws of attraction solid luminous masses in process of time were produced, and that these nebulæ grew into clusters.

It would not be compatible with the limits of this work, and the objects to which it is directed, to pursue this speculation through its consequences, to state the arguments by which it is supported and opposed; and it is the less necessary to do so, seeing that such an hypothesis is not needed to explain appearances which are so much more obviously and simply explicable by the admission of a gradation of distances.

3381. *Forms apparent and real of the clusters.* — The apparent forms of these objects are extremely various, and subject to most extraordinary and unexpected changes, according to the magnifying power under which they are viewed. This ought, however, to excite no surprise. The telescope is an expedient by which a well-defined and strongly illuminated optical image of a distant object is formed so close to the observer, that he is enabled to view it with microscopes of greater or less power, according to the perfection of its definition, and the intensity of its illumination. Now, it is known to all who are familiar with the use of the microscope, that the apparent form and structure of an object change in the most remarkable and unexpected way when viewed with different microscopic powers. The blood, for example, which viewed with the naked eye, or with low powers, is a uniformly red fluid, appears as a pellucid liquid, having small red disks floating in it, when seen with higher powers (46.). Like effects are manifested in the cases of the nebulæ, when submitted to examination with different and increasing magnifying powers, of which we shall presently show many striking examples.

The apparent forms of the stellar clusters are generally roundish or irregular patches. The stars which compose them are always much more densely crowded together, in going from

the edges of the cluster towards the centre, so that at the centre they exhibit a perfect blaze of light.

The apparent form is that of a section of the real form, made by a plane at right angles to the visual ray. If the mass had a motion of rotation, or any other motion by which it would change this plane, so as to exhibit to the eye successively different sections of it, its real form could be inferred as those of the planets have been. But there are no discoverable indications of any such motion in these objects. Their real forms, therefore, can only be conjectured from comparing their apparent forms with their structural appearance.

The clusters having round apparent forms, and of which the stars are rapidly more dense towards the centres, are inferred to be either globular or spheroidal masses of stars, the greater apparent density in passing from the edges to the centre being explained by the greater thickness of the mass, in the direction of the visual line. Clusters of irregular outline which show also a density increasing inwards, are also inferred, for like reasons, to be masses of stars, whose dimensions in the direction of the visual rays correspond with their dimensions in the direction at right angles to those rays.

3382. *Forms apparent and real of the nebulæ.*—These objects exhibit forms much more various than those presented by the clusters. Some are circular, with more or less precision of outline. Some are elliptical, the oval outline having degrees of excentricity infinitely various, from one which scarcely differs from a circle, to one which is compressed into a form not sensibly different from a straight line. In short, the minor axis of the ellipses bears all proportions to the major axis, until it becomes a very small fraction of the latter.

To infer the real from the apparent forms of these objects with any certainty, there are no sufficient data. But in the cases in which the brightness increases rapidly towards the centre, which it very generally does, it may be probably conjectured that their forms are globular or spheroidal, for the reasons already explained in relation to the clusters, and this becomes the more probable when it is considered, that these nebulæ are in fact clusters, the stars of which are reduced to a nebulous patch by distance.

Nevertheless, these nebulæ may be strata of stars, of which the thickness is small compared with their other dimensions;

and supposing their real outline to be circular, they will appear elliptical if the plane of the stratum be inclined to the visual line, and more or less excentrically elliptical, according as the angle of inclination is more or less acute. In cases in which the brightness does not increase in a striking degree from the edges inwards, this form is more probable than the globular or the spheroidal.

Nebulæ may be conveniently classed according to their apparent form and structure; but whatever arrangement may be adopted, these objects exhibit such varieties, assume such capricious and irregular forms, and undergo such strange and unexpected changes of appearance according to the increasing power of the telescope with which they are viewed, that it will always be found that great numbers of them will remain unavoidably unclassified.

3383. *Double nebulæ.*—Like individual stars, nebulæ are found to be combined in pairs too frequently to be compatible with the supposition that such combinations arise from the fortuitous results of the small obliquity of the visual rays, which causes mere optical juxta-position.

These double nebulæ are generally circular in their apparent, and therefore probably globular in their real form. In some cases they are resolvable clusters.

That such pairs of clusters are physically connected does not admit of a reasonable doubt, and it is highly probable that, like the binary stars, they move round each other, or round a common centre of attraction, although the apparent motion attending such revolution is rendered so slow by their immense distance that it can only be ascertained after the lapse of ages.

3384. *Planetary nebulæ.*—This class of objects derive their name from their close resemblance to planetary disks. They are in general either circular or very slightly oval. In some cases the disk is sharply defined, in others it is hazy and nebulous at the edges. In some the disk shows a uniform surface, and in some it has an appearance which Sir J. Herschel describes by the term *curdled.*

There is no reason to doubt that the constitution of these objects is the same as that of other nebulæ, and that they are in fact clusters of stars which by mutual proximity and vast distance, are reduced to the form of planetary disks.

These objects, which are not numerous, present some remarkable peculiarities of appearance and colour. It has been already observed that, although the companion of a red individual of a double star appears blue or green, it is not certain that this is its real colour, the optical effect of the strong red of its near neighbour being such as would render a white star apparently blue or green, and no example of any single blue or green star has ever been witnessed. The planetary nebulæ, however, present some very remarkable examples of these colours. Sir J. Herschel indicates a beautiful instance of this, in a planetary nebula situate in the southern constellation of the Cross. The apparent diameter is 12″, and the disk is nearly circular, with a well-defined outline, and a "fine and full blue colour verging somewhat upon green." Several other planetary nebulæ are of a like colour, but more faint.

The magnitudes of these stupendous masses of stars may be conjectured from their probable distances. One of the largest, and therefore probably the nearest of them, is situate near the star β Ursæ majoris (one of the pointers). Its apparent diameter is 2′ 40″. Now, if this were only at the distance of 61 Cygni, whose parallax is known (2603.), it would have a diameter equal to seven times that of the extreme limit of the solar system; but as it is certain that its distance must be many times greater, it may be conceived that its dimensions must be enormous.

3385. *Annular nebulæ.* — A very few of the nebulæ have been observed to be annular. Until lately there were only four. The telescopes of Lord Rosse have, however, added five to the number, by showing that certain nebulæ formerly supposed to be small round patches are really annular. It is extremely probable, that many others of the smaller class of round nebulæ will prove to be annular, when submitted to further examination with telescopes of adequate power and efficiency.

3386. *Spiral nebulæ.* — The discovery of this class of objects, the most extraordinary and unexpected which modern research has yet disclosed in stellar astronomy, is due to Lord Rosse. Their general form and character may be conceived by referring to those represented in Plate XXII. *figs.* 1 and 3, and Plate XXIII. *fig.* 1. These extraordinary forms are so entirely removed from all analogy with any of the phenomena presented either in the motions of the solar system,

or the comets, or those of any other objects to which observation has been directed, that all conjecture as to the physical condition of the masses of stars which could assume such forms would be vain. The number of instances as yet detected, in which this form prevails, is not great; but it is sufficient to prove that the phenomenon, whatever be its cause, is the result of the operation of some general law. It is pretty certain, that when the same powerful instruments which have rendered these forms visible in objects which had already been so long under the scrutiny of the most eminent observers of the last hundred years, including Sir W. and Sir J. Herschel, aided by the vast telescopic powers at their disposition, without raising even a suspicion of their real form and structure, have been applied to other nebulæ, other cases of the same phenomenon will be brought to light. In this point of view it is much to be regretted, that the telescopes of Lord Rosse cannot have the great advantage of being used under skies more favourable to stellar researches, since the discovery of such forms as these not only requires instruments of such power as Lord Rosse alone possesses at present, but also the most favourable atmospheric conditions.

3387. *Number of nebulæ.* — The number of these objects is countless. The catalogues of Sir J. Herschel contain above 4000, of which the places are assigned, and the magnitudes, forms, and apparent characters described. As observers are multiplied, and the telescope improved, and especially when the means of observation have been extended to places that are more favourable for such observations, it may be expected that the number observed will be indefinitely augmented.

3388. *Remarkable nebulæ.* — Having noticed thus briefly the characters and appearances of the principal classes of these objects, it will be useful to illustrate these general observations by reference to examples of nebulæ and clusters of each class, assigning the position of each by its right ascension and north polar distance, and supplying, wherever it can be done on satisfactory authority, a telescopic view of such object. In the selection of these examples, it will be one of our chief purposes to show the extraordinary differences of form and structure, which the same object presents when viewed with telescopes of different powers. The drawings of the same nebulæ, which have appeared in the Philosophical Transactions by Sir J.

Herschel and the Earl of Rosse, supply numerous and instructive examples of this.

Plate XX. *fig.* 6. R A 15h 29m 9s. N P D 33° 27′. Diameter, 9s R A.—Drawn by Sir J. Herschel, who describes it as a faint large round nebula, which, by attentive examination, may be seen to be composed of excessively minute stars, appearing like points rubbed out. It is, in fact, a globular cluster.

Plate XXI. *fig.* 2. R A 21h 24m 40s. N P D 91° 34′. Diameter, 6s R A.—Drawn by Sir J. Herschel, who describes it as a most superb cluster of stars of the 15th magnitude, compressed towards the centre to a perfect blaze. It resembles a mass of fine luminous sand. It is resolvable with a six-inch aperture. The stars just visible with a nine-inch aperture (reflector).

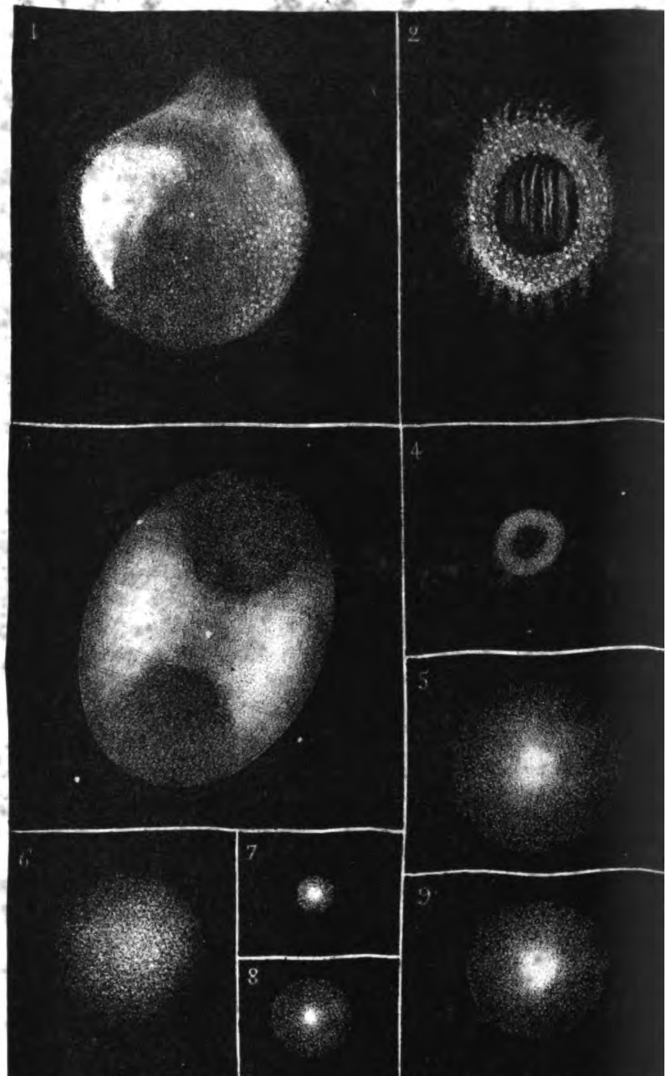
Plate XXI. *fig.* 1.—The same object as shown by the larger telescope of the Earl of Rosse. Lord Rosse thinks that no increased power is likely to alter materially its appearance. It would merely render the component stars brighter and less closely crowded.

Plate XXI. *fig.* 4. R A 5h 24m 16s. N P D, 68° 7′. Length 4′, breadth 3′, oval form.—A fine object. (Sir J. Herschel.)

Plate XXI. *fig.* 3.—The same object as shown by Lord Rosse's telescope. A considerable change of appearance is here produced by increased power, the oval resolvable nebula being changed into what the drawing represents. It is studded with stars mixed with a nebulosity, which a still higher power would evidently resolve into stars.

Plate XXII. *fig.* 2. R A 13h 32m 39s. N P D 41° 56′.—This is, in many respects, one of the most remarkable and interesting of its class, and has been submitted to elaborate examination by all the eminent observers. The distance of the centre of the small nebula from that of the large one, is given by Messier, as 4′ 35″, which may serve as a *modulus* for its other dimensions. It was described by Sir W. Herschel as a bright round nebula, surrounded by a halo or glory, and attended by a companion. Sir J. Herschel observed this object, and represented it as in the figure. He noticed the partial division of the ring as if it were split, as its most remarkable and interesting feature, and inferred that, supposing it to consist of stars, the appearance it would present to an observer, placed on a planet attached to one of them excentrically situate towards the north preceding quarter of the central mass, would be exactly similar to that of the milky way as seen from the earth, traversing in a manner precisely similar the firmament of large stars, into which the central cluster would be seen projected, and (owing to its greater distance) appearing like it to consist of stars much smaller than those in other parts of the heavens. "Can it be," asks Sir J. Herschel, "that we have here a brother system, bearing a real physical resemblance and strong analogy of structure to our own?" Sir J. Herschel further argues, that all idea of symmetry caused by rotation must be relinquished, considering that the elliptical form of the inner subdivided portion indicates with extreme probability, an elevation of that part above the plane of the

Nebulæ and Clusters
Telescopic views

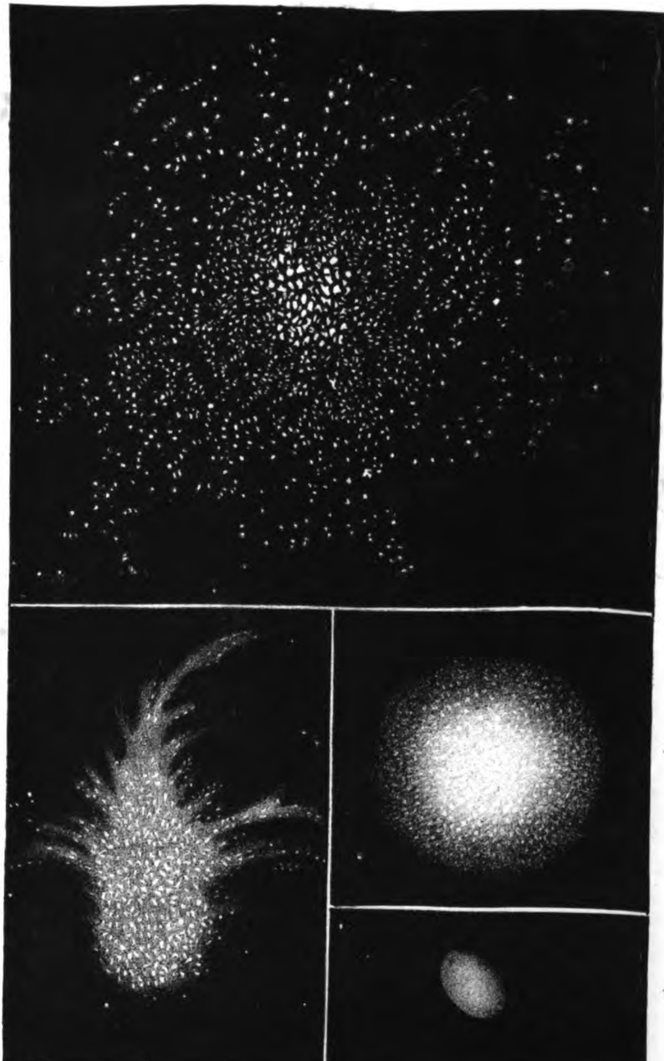


1. 2. By the Earl of Rosse.
3 to 9. By Sir J. Herschel.

Nebulæ and Clusters
Telescopic views

XVI

1



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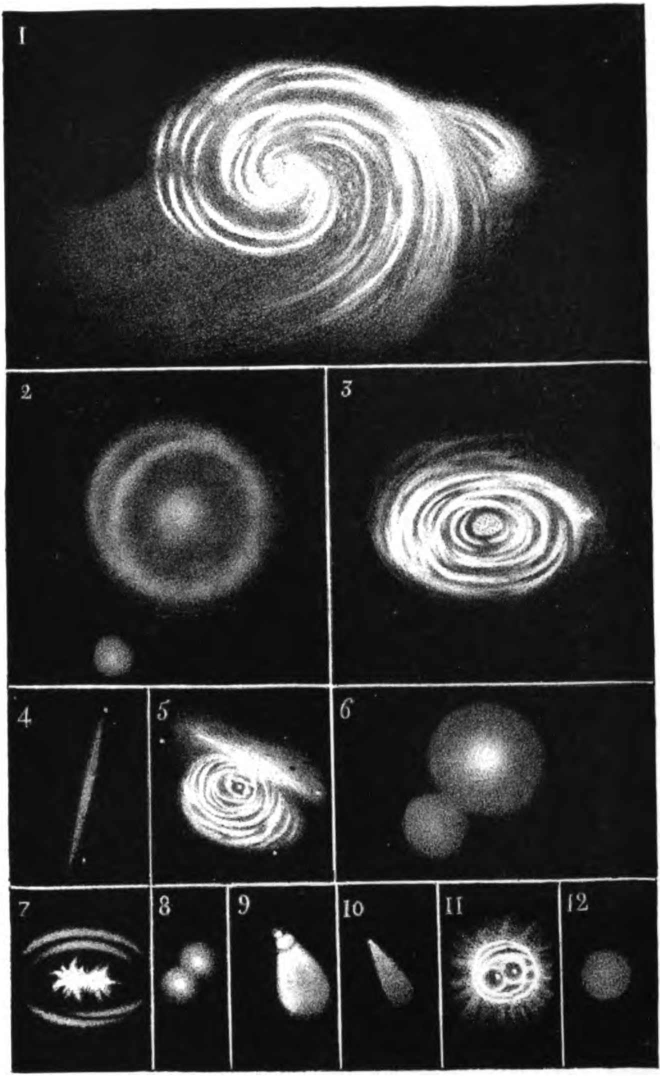
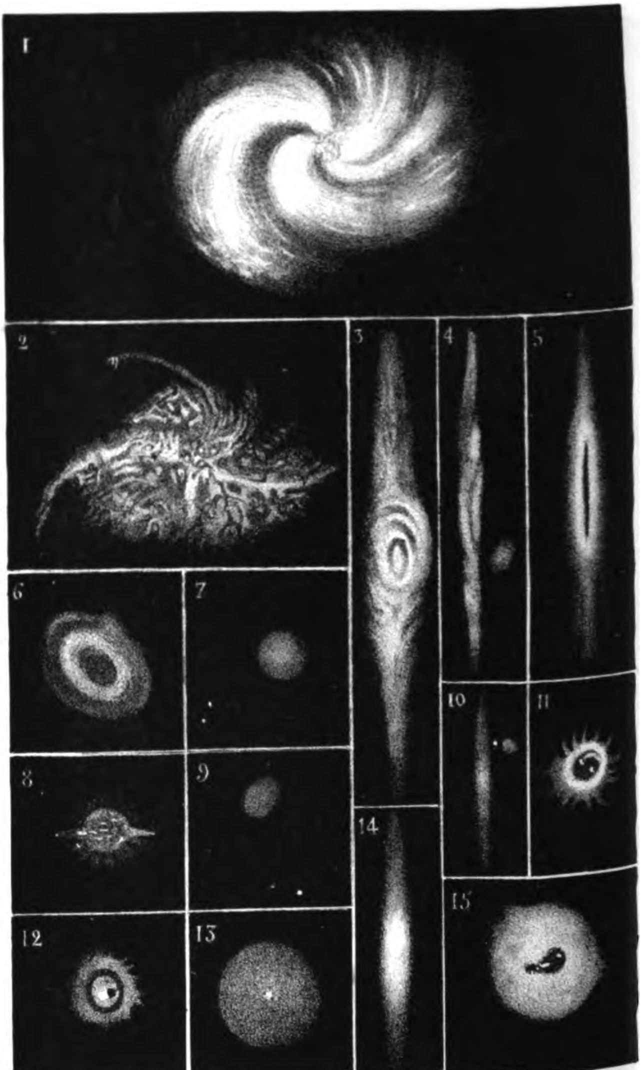
1. 3. by the Earl of Rosse
2. 4. by Sir J. Herschel.



1, 3, 5, 7, 9, 11, by the Earl of Rosse
2, 4, 6, 8, 10, 12, by Sir J Herschel

Nebulæ and Clusters
Telescopic views

VIII



1. 2. 3. 4. 5. 6. 8. 11. 12. 14. 15. by the Earl of Rosse
7. 9. 10. 13. by Sir J. Herschel.

rest; so that the real form must be that of a ring split through half its circumference, and having the split portions set asunder at an angle of 45°.

Plate XXII. *fig.* 1.—The same object as shown by Lord Rosse's telescope. This shows, in a striking manner, how entirely the appearances of these objects are liable to be varied by the increased magnifying power and greater efficiency of the telescope through which they are viewed. It is evident, that very little resemblance or analogy is discoverable between *fig.* 2. and *fig.* 1. Lord Rosse, however, says that if Sir John Herschel's be placed as it would be seen with a Newtonian telescope, the bright convolutions of the spiral shown in his own would be recognised in the appearance which Sir J. Herschel supposed to be that which would be produced by a split or divided ring. Lord Rosse further observes that, with each increase of optical power, the structure of this object becomes more complicated and more unlike anything which could be supposed to be the result of any form of dynamical law of which we find a counterpart in our system. The connection of the companion with the principal nebula, of which there is not the least doubt, and which is represented in the sketch, adds, in Lord Rosse's opinion, if possible, to the difficulty of forming any conceivable hypothesis. That such a system should exist without internal movement he considers in the last degree improbable. Our conception may be aided, by uniting with the idea of motion the effects of a resisting medium; but it is impossible to imagine such a system in any point of view, as a case of mere statical equilibrium. Measurements he therefore considers of the highest interest, but of great difficulty.

Plate XXIII. *fig.* 1.—This object is the 99th in Messier's catalogue. The spiral form of the nebula, represented in Plate XXII. *fig.* 1., was discovered by Lord Rosse, in the early part of 1845. In the spring of 1846, that represented in the present figure was discovered. The spiral form is here also presented, but of a different character. Lord Rosse conjectures, that the nebula No. 2370, and 3239 of Herschel's southern catalogue, are very probably objects of a similar character. As Herschel's telescope did not reveal any trace of the form of this nebula, it is not surprising that it did not disclose the spiral form presumed to belong to the others, and it is not, therefore, unreasonable to hope, according to his Lordship, that whenever the southern hemisphere shall be re-examined with instruments of greater power, these two remarkable nebulæ will yield some interesting results.

Lord Rosse has discovered other spiral nebulæ, but they are comparatively difficult to be seen, and the greatest powers of the instrument are required to bring out the details.

Plate XXII. *fig.* 6. R A $9^h\ 22^m\ 32^s$. N P D 67° 45′. Length, 3′.—This is described by Sir John Herschel as a very bright extended nebula, with an approach to a second nucleus, which, however, is very faint.

Plate XXII. *fig.* 3.—The same object as shown by Lord Rosse's telescope. This object was first observed with the great telescope, 24th March, 1846, when a tendency to an annular or spiral form was discovered. On the 9th March, 1848, in more favourable weather, the spiral form was distinctly seen in an oblique direction. The nebula was well resolved, particularly towards the centre, where it was very bright.

Plate XXII. *fig.* 4. R A 22^h 56^m 26^s. N P D 78° 36′. Length, 2′, breadth, 30″.—Described by Sir John Herschel as pretty bright and resolvable, and extended between two small stars, having two very small stars visible in it.

Plate XXII. *fig.* 5.—The same object as seen in Lord Rosse's telescope. It was frequently observed, both by Lord Rosse himself and several of his friends, and the drawing represents the form with great accuracy. It was doubtful whether the form was strictly spiral, or whether it were not more properly annular.

Plate XXIII. *fig.* 2. R A 1^h 24^m 15^s. N P D 60° 31′. Dimensions uncertain, but the diffused nebulæ estimated as extending through 15′.—This object has been the subject of observation by all the eminent observers. Sir John Herschel describes it as enormously large, growing very gradually brighter towards the middle, and having a star of the 12th magnitude, north, following the nucleus, and being characterised by irregularities of light, and even by feeble subordinate nuclei and many small stars. The drawing represents it as seen with the more powerful telescope of Lord Rosse. A tendency to a spiral form was distinctly seen on the 6th, 10th, and 16th September, 1849. The brightest of the spiral arms was that marked α, that marked δ was pretty bright, but short; β was distinct, and η only suspected; the branch γ was faint. The whole object was involved in a faint nebulosity, which probably extends past several knots which lie about it in different directions.

Plate XXII. *fig.* 8. R A 7^h 14^m 50^s. N P D 60° 11′.—This is described by Sir John Herschel as a curious bright double or an elongated bicentral nebula.

Plate XXII. *fig.* 7.—The same object as shown by Lord Rosse's telescope, on 22nd December, 1848. A bright star was visible between the nebulæ from which tails and curved filaments issued. The existence of an annulus surrounding the two nebulæ was suspected.

Plate XXIII. *fig.* 14. R A 11^h 10^m 2^s. N P D 75° 59′. Length 4′.—Described by Sir John Herschel, as large, elliptical in form, with a round nucleus, and growing gradually brighter towards the middle.

Plate XXIII. *fig.* 3.—The same object as shown by Lord Rosse's telescope, 31st March 1848. Described as a curious nebula, nucleus resolvable, having a spiral or annular arrangement about it. It was also observed with the same results on the 1st and 3rd April.

Plate XXIII. *fig.* 5. R A 15^h 1^m 47^s. N P D 33° 35′. Length 50″. Breadth 20″.—This nebula was not figured by Sir John Herschel; but is described by him as an object very bright, and growing much brighter towards the middle. The drawing *fig.* 5. represents the object as seen in Lord Rosse's telescope, in April 1848. It is described by Lord Rosse as a very bright resolvable nebula, but that none of the component stars could be distinctly seen even with a magnifying power of 1000. A perfectly straight longitudinal division appears in the direction of the major axis of the ellipse. Resolvability was strongly indicated towards the nucleus. According to Lord Rosse, the proportion of the major axis to the minor axis was 8 to 1; much greater than the estimate of Sir John Herschel.

Plate XXIII. *fig.* 10. RA 12^h 33^m 54^s. NPD 56° 30′.—Described by Sir John Herschel as a nebula of enormous length, extending across an entire field of 15′, the nucleus not being well defined. It was preceded by a star of the tenth magnitude, and that again by a small faint round nebula, the whole forming a fine and very curious combination.

Plate XXIII. *fig.* 4.—The same object as shown by Lord Rosse's telescope on 19th April 1849. The drawing is stated to be executed with great care, and to be very accurate. A most extraordinary object, masses of light appearing through it in knots.

Plate XXII. *fig.* 10. RA 6^h 29^m 53^s. NPD 81° 7′.—Described by Sir John Herschel as a star of the 12th magnitude, with a bright cometic branch issuing from it, 60″ in length, forming an angle of 60° with the meridian, passing through it. The star is described as ill defined, the apex of the nebula coming exactly up to it, but not passing it.

Plate XXII. *fig.* 9.—The same object as seen with Lord Rosse's telescope on 16th January 1850. Lord Rosse observed that the two comparatively dark spaces, one near the apex and the other near the base of the cone, are very remarkable.

Plate XXII. *fig.* 12. RA 11^h 4^m 49^s. NPD 34° 3′. Diameter 19″ time.—Described by Sir John Herschel as a large uniform nebulous disk, very bright and perfectly round, but sharply defined, and yet very suddenly fading away into darkness. A most extraordinary object.

Plate XXII. *fig.* 11.—The same object as shown by Lord Rosse's telescope. Two stars considerably apart, seen in the central part of the nebula. A dark penumbra around each spiral arrangement with stars as apparent centres of attraction. Stars sparkling in it and in the nebula resolvable. Lord Rosse saw two large and very dark spots in the middle, and remarked that all round its edge the sky appeared darker than usual.

Plate XXIII. *fig.* 11. RA 7^h 34^m 2^s. NPD 104° 20′ 25″. Diameter 3·75″ time.—Described by Sir John Herschel as a planetary nebula, of a faint equal light, and exactly round, having a very minute star a little north of the centre. Very velvety at the edges. In the telescope of Lord Rosse, however, it appears as an annular nebula as represented in the figure, with two stars within it.

Plate XXIII. *fig.* 7. RA 23^h 17^m 42^s. NPD 48° 24′ 24″. Diameter 12″—Figured by Sir John Herschel, who describes it as a fine planetary nebula. With a power of 240 it was beautifully defined, light, rather mottled, and the edges the least in the world unshaped. It is not nebulous, but looks as if it had a double outline, or like a star a little out of focus. It is perfectly circular.

Plate XXIII *fig.* 6.—The same object as shown in Lord Rosse's telescope, 16th—19th December 1848. A central dark spot surrounded by a bright annulus.

Plate XXIII. *fig.* 9. RA 20^h 54^m 5^s. NPD 102° 2′ 46″. Diameter 10″ to 12″ according to Herschel, but 25″ by 17″ according to Struve, who gives it a more oval form.—This figure is that given by Sir John Herschel, who describes it as a fine planetary nebula with equable light and blueish white colour.

Plate XXIII. *fig.* 8.—The same object as shown by Lord Rosse's telescope. Like a globe surrounded by a ring such as that of Saturn, the usual line being in the plane of the ring.

Plate XXIII. *fig.* 13. R A 7^h 19^m 8^s. N P D 68° 45′.—Described by Sir John Herschel as a star exactly in the centre of a bright circular atmosphere 25″ in diameter, the star being quite stellar, and not a mere nucleus, and is a most remarkable object.

Plate XXIII. *fig.* 12.—The same object as shown by Lord Rosse's telescope on 20th February 1849; described by him as a most astonishing object. It was examined in January 1850, with powers of 700 and 900, when both the dark and bright rings seemed unequal in breadth.

Plate XXIII. *fig.* 15. R A 5^h 27^m 7^s. N P D 96° 2′ 18″.—The star ι Orionis involved in a feeble nebula 3′ in diameter. (Sir J. Herschel.) The drawing shows this as seen with Lord Rosse's telescope.

Plate XX. *fig.* 3. A R 19^h 52^m 12^s. N P D 67° 44′.—Drawn by Sir John Herschel, who describes it as a nebula shaped like a dumb-bell, double-headed shot, or hour-glass, the elliptic outline being completed by a more feeble nebulous light. The axis of symmetry through the centres of the two chief masses inclined at 30° to the meridian. Diameter of elliptic light from 7′ to 8′. Not resolvable, but four stars are visible on it of the 12th, 13th, and 14th magnitude. The southern head is denser than the northern. This extraordinary object was also observed by Sir W. Herschel, who recognised the same peculiar form. Sir J. Herschel considers that the most remarkable circumstance attending it is the faint nebulosity which fills up the lateral concavities of its form, and in fact converts them into protuberances, so as to render the whole outline a regular ellipse, having for its shorter axis the common axis of the two bright masses. If it be regarded as a mass in rotation, it is around this shorter axis it must revolve. In that case, he considers that its real form must be that of an oblate spheroid; and as it does not follow that the brightest portions must be of necessity the densest, this supposition would not be incompatible with dynamical laws, at least supposing its parts to be capable of exerting pressure on each other. But if it consist of distinct stars this cannot be admitted, and we must then have recourse to other suppositions to account for the maintainance of its form. Sir John Herschel, it will be observed, failed to resolve this nebula.

Plate XXIV. *fig.* 1.—The same object as shown by the telescope of Lord Rosse, three feet aperture, twenty-seven feet focal length.

Plate XX. *fig.* 1.—The same object as shown with the great telescope of Lord Rosse, six feet aperture, fifty-three feet focal length.

The difference between these two representations and that given by Sir John Herschel of the same object, will illustrate in a very striking manner the observations already made on the effects of different magnifying and defining powers upon the appearance of the object under examination. These three figures could scarcely be conceived to be representations of the same object.

To explain the difference observable between the drawing Plate XXIV. *fig.* 1., made with the smaller telescope, and the drawing Plate XX. *fig.* 1., made with the larger instrument, Lord Rosse observes, that while the application

of a high magnifying power brings out minute stars not visible with lower powers, it completely extinguishes nebulosity which the lower powers render visible. The optical reason for this will be easily perceived; the circumstance was nevertheless overlooked when the observations were made from which the drawing Plate XXIV. *fig.* 1. was taken. Only one magnifying power, and that a very high one, was used on that occasion, the consequence of which was that, although the two knobs of the dumb-bell were more fully resolved, the nebulous matter filling the intermediate space, which Herschel considered to be the most remarkable feature of this nebula, was entirely extinguished in the optical image. If on that occasion a second eye-piece had been used of lower power, the intermediate nebulous matter would have been seen, as represented in the drawing, and the drawing would be as perfect as, and nearly identical with, that obtained with the greater telescope Plate XX. *fig.* 1., a lower power being used.

It will be observed that the general outline of this remarkable object which is so geometrically exact as seen with the inferior power used by Sir John Herschel, is totally effaced by the application of the higher powers used by Lord Rosse, and consequently Sir John Herschel's theoretical speculations based upon this particular form, must be regarded as losing much of their force, if not wholly inadmissible; and this is an example proving how unsafe it is to draw any theoretical inferences from apparent peculiarities of form or structure in these objects, which may be only the effect of the imperfect impressions we receive of them, and which, consequently, disappear when higher telescopic powers are applied. The case of the nebula represented in Plate XXII. *figs.* 1. and 2. presents another striking example of the force of these observations.

Plate XX. *fig.* 4. R A 8^h 47^m 13^s. N P D 57° 11.—This object, drawn by Sir J. Herschel, is the annular nebula between β and γ Lyræ. He estimates its diameter at 6·5″ R A. The annulus is oval, its longer axis being inclined at 57° to the meridian. The central vacuity is *not black*, but filled with a nebulous light. The edges are not sharply cut off, but ill defined; they exhibit a curdled and confused appearance, like that of stars out of focus. He considers it not well represented in the drawing.

Plate XX. *fig.* 2.—The same object as shown in the telescope of Lord Rosse. This drawing was made with the smaller telescope, three feet aperture, before the great telescope had been erected. The nebula was observed seven times in 1848, and once in 1849. With the large telescope, the central opening showed considerably more nebulosity than it appeared to have with the smaller instrument. It was also noticed, that several small stars were seen around it with the large instrument, which did not appear with the smaller one, from which it was inferred that the stars seen in the dark opening of the ring may possibly be merely accidental, and have no physical relation to the nebula. In the annulus near the extremity of the minor axis, several minute stars were visible.

Plate XX. *fig.* 5. R A 13^h 28^m 53^s. N P D 107° 0′ 50″. Diameter of faint nebula, 2′. Diameter of bright part, 10″ or 15″.—Described as a faint large nebula losing itself quite imperceptibly; a good type of its class. (Herschel.)

Plate XX. *fig.* 7. R A 17h 44m 42s. N P D 66° 52 41″. Perceptible disk 1″, or 1·5″ diameter. Surrounded by a very faint nebula.—A curious object. (Herschel.)

Plate XX. *fig.* 8. R A 19h 40m 19s. N P D 39° 54′.—A most curious object. A star of the 11th magnitude, surrounded by a very bright and perfectly round planetary nebula of uniform light. Diameter in R A 3·5″, perhaps a very little hazy at the edges. (Herschel.)

Plate XX. *fig.* 9. R A 10h 28m 7s. N P D 35° 36′ 32″.—A bright round nebula, forming almost a disk 15″ diameter, surrounded by a very feeble atmosphere. (Herschel.)

3389. *Large and irregular nebulæ.*—All the nebulæ described above, are objects generally of regular form and subtending small visual angles. There are others, however, of a very different character, which cannot be passed without some notice. These objects cover spaces on the firmament, many nearly as extensive as, and some much more extensive than, the moon's disk. Some of them have been resolved. Of those which are larger and more diffused, some exhibit irregularly shaped patches of nebulous light, affecting forms resembling those of clouds, in which tracts are seen in every stage of resolution, from nebulosity irresolvable by the largest and most powerful telescopes, to stars perfectly separated like parts of the milky way, and "clustering groups sufficiently insulated and condensed to come under the designation of irregular and, in some cases, pretty rich clusters. But, besides these, there are also nebulæ in abundance, both regular and irregular; globular clusters in every state of condensation, and objects of a nebulous character quite peculiar, which have no analogy in any other part of the heavens." *

3390. *Rich cluster in the Centaur.*—The star ω Centauri presents one of the most striking examples of the class of large diffused clusters. It is nearly round, and has an apparent diameter equal to two-thirds of that of the moon. This remarkable object was included in Mr. Dunlop's catalogue (Phil. Trans. 1828); but it is from the observations of Sir John Herschel, at the Cape, that the knowledge of its splendid character is derived. That astronomer pronounces it, beyond all comparison, the richest and largest object of the kind in the heavens. The stars composing it are literally innumerable; and as their collective light affects the eye hardly more than

* Herschel, Outlines of Astronomy, p. 613.

Nebulæ and Clusters
Telescopic views [illegible]

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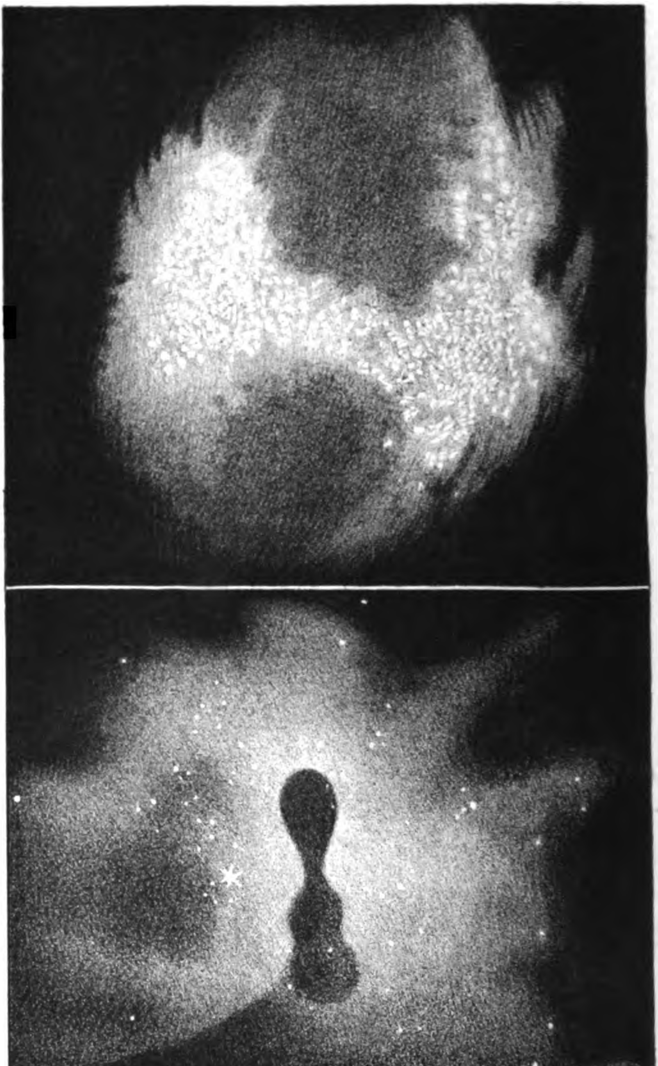
2



1. The Dumb-bell Nebula by the Earl of Rosse
2. Part of the great Nebula in Argo.

that of a star of the fifth magnitude, the minuteness of each of them may be imagined. The apparent magnitude of this object is such that, when it was concentric with the field of Sir J. Herschel's 20 ft. telescope, the straggling stars at the edges were beyond the limit of the field. In stating that the diameter is two-thirds of the moon's disk, it must be understood to apply to the diameter of the condensed cluster, and not to include the straggling stars at the edges. When the centre of the cluster was brought to the edge of the field, the outer stars extended fully half a radius beyond the middle of it.*

The appearance of this magnificent object resembles that shown in Plate XXI. *fig.* 1. only that the stars are much more densely crowded together, and the outline more circular, indicating a pretty exact globe as the real form of the mass.

3391. *The great nebula in Orion.* — The position of this extraordinary object is in the sword handle of the figure which forms the constellation of Orion. It consists of irregular cloud-shaped nebulous patches, extending over a surface about 40′ square; that is, one whose apparent breadth and height exceed the apparent diameter of the moon by about one-third, and whose superficial magnitude is, therefore, rather more than twice that of the moon's disk. Drawings of this nebula have been made by several observers, and engravings of them have been already published in various works.

In Plate XXIV. *fig.* 2. is given a representation of the central part of this object. The portion here represented measures 25′ in height, and 25′ in breadth; a height and breadth about one-sixth less than the diameter of the moon. An engraving upon a very large scale, of the entire extent of the nebula, with an indication of the various stars which serve as a sort of landmarks to it, may be seen by reference to Sir J. Herschel's "Cape Observations," accompanied by the interesting details of his observations upon it.

Sir J. Herschel describes the brightest portion of this nebula as resembling the head and yawning jaws of some monstrous animal, with a sort of proboscis running out from the snout. The stars scattered over it probably have no connection with it, and are doubtless placed much nearer to our system than the nebula, being visually projected upon it. Parts of this nebula,

* Cape Observations, p. 21.

when submitted to the powers of Lord Rosse's telescopes, show evident indications of resolvability.

3392. *The great nebula in Argo.* — This is an object of the same class, and presenting like appearances; it is diffused around the star η in the constellation here named, and formed a special subject of observation by Sir J. Herschel, during his residence at the Cape. An engraving of it on a large scale, giving all its details, may be seen in the "Cape Observations." The position of the centre of the nebula is, R A 10° 38′ 38″, N P D 148° 47′.

This object consists of diffused irregular nebulous patches, extending over a surface measuring nearly 7′ (time) in right ascension, and 68′ in declination; the entire area, therefore, being equal to a square space, whose side would measure one degree. It occupies, therefore, a space on the heavens about five times greater than the disk of the moon.

A part of the nebula immediately surrounding the central star, is represented in Plate XXV. The space here represented measures about one-fourth of the entire extent of the nebula, in declination, and one-third in right ascension, and about a twelfth of its entire magnitude.

No part of this remarkable object has shown the least tendency to resolvability. It is entirely compressed within the limits of that part of the milky way which traverses the southern firmament, the stars of which are seen projected upon it in thousands. Sir J. Herschel has actually counted 1200 of these stars projected upon a part of this nebula, measuring no more than 28′ in declination, and 32′ in right ascension, and he thinks that it is impossible to avoid the conclusion, that in looking at it we see through and beyond the milky way, far out into space through a starless region, disconnecting it altogether with our system.

3393. *Magellanic clouds.* These are two extensive nebulous patches also seen on the southern firmament, the greater called the *nubecula major*, being included between R A 4^h 40^m, and 6^h 0^m and N P D 156° and 162°, occupying a superficial area of 42 square degrees; and the other called the *nubecula minor*, being included between R A 0^h 21^m and 1^h 15^m and between N P D 162° and 165°, covering about 10 square degrees.

These nebulæ consist of patches of every character, some irresolvable, and others resolvable in all degrees, and mixed

Telescopic view of part of the great Nebula in Orion. XXV.

The Cluster in which the sun is placed.

XXVI.

with clusters; in fine, having all the characters already explained in the cases of the large diffused nebulæ described above. So great is the number of distinct nebulæ and clusters crowded together in these tracts of the firmament, that 278, besides 50 or 60 outliers, have been enumerated by Sir J. Herschel, within the area of the nebecula major alone.

CHAP. XXX.

NOTICES OF REMARKABLE ASTRONOMICAL INSTRUMENTS.

3394. *Classification of the instruments of observation.*—In the sixth Chapter of this book, an explanation of the principle of the construction, the form, and application of the most necessary instruments of an observatory, is given, such as was deemed sufficient to render intelligible the succeeding parts of the work. The instruments selected for that purpose were those which seemed best adapted to illustrate the theoretical principles afterwards developed, and to show, in the most simple manner, the mode in which the various data on which these principles rest were obtained. It was thought more conducive to the progress of the student to postpone the exposition of other instruments of observation until the importance and magnitude of the results attained by them could be more adequately appreciated, and the principles of their structure and mechanism more clearly comprehended.

We shall now therefore conclude this volume with a short notice of some of the most generally useful and some of the most remarkable instruments of astronomical observation, including the most important of those recently erected in different countries, and some which have been rendered memorable by the uses to which they have been subservient. If we omit some, more especially those in observatories erected and conducted at the charge of private individuals, it is from no undue sense of their value and importance, nor from any disinclination to award the praise due to the spirit which has prompted such devotion of time and fortune to the advancement of the noblest of the sciences, but from the necessity under which the objects

and purposes of this work place us to confine it within certain limits of bulk.

All astronomical observation is limited either to ascertain the magnitudes, forms, and appearance of celestial objects, or to determine the places they occupy at any given moment on the firmament.

To attain the former object, telescopes are constructed with the greatest practicable magnifying and illuminating powers, and so mounted as to enable the observer with all the requisite facility to present them to those parts of the heavens in which the objects of his observation are placed.

To attain the latter, it is necessary to provide an apparatus by which the direction of the visual line of the object of observation relatively to some fixed line and some fixed plane can be ascertained. The visual line being the straight line drawn from the eye of the observer to the object, at the moment of the observation, and having, therefore, no material tangible or permanent existence, by which it can be submitted to measurement, it is necessary to contrive some material line with which the visual line shall coincide. The telescope supplies an easy and exact means of accomplishing this. When it is directed so that the object or its centre, if it have a disk, is seen upon the intersection of the middle wires in the eye-piece, the visual direction of the object is the line drawn from the centre of the object-glass of the telescope to the intersection of the middle wires.

Now the telescope being attached to a graduated circle is so placed, that the line joining the centre of the object-glass with the intersection of the wires is parallel to a diameter of the circle. This diameter will therefore be the direction of the visual line. If the circle thus arranged be so mounted that a line drawn from the observer to the fixed point of reference, whatever that point be, shall be parallel also to a diameter of the circle, and if the circle be so mounted that, however its position may otherwise be changed, one of its diameters shall always pass through the fixed point of reference, the angular distance of the object of observation from the fixed point of reference will always be equal to the angle formed by the two diameters of the circle, one of which is parallel to the line joining the centre of the object-glass, with the intersection of the wires at the moment of the observation, and the other parallel

to the line drawn from the observer to the fixed point of reference.

But this is not yet enough to determine in a definite manner the position of the object on the heavens. A great many different objects may have the same angular distance from the fixed point of reference. If a plane be imagined to pass at right angles to a line drawn from the observer to the fixed point of reference, it will intersect the celestial sphere in a certain circle, every point of which will obviously be at the same angular distance from the point of reference. To render the position of the object of observation determinate, it is therefore necessary to know the position of the plane of the graduated circle, with relation to a circle whose plane is at right angles to that diameter of the celestial sphere which passes through the fixed point of reference.

The plane of the graduated circle may be fixed or moveable. If fixed, its position with relation to the fixed point of reference is ascertained once for all; after which, the position of the object of observation will be determined merely by its angular distance from the point of reference. If moveable, it is necessary to provide another graduated circle, the plane of which is perpendicular to the first, and upon which some fixed direction is marked. The position of the plane of the moveable circle, which carries the telescope, with relation to this latter fixed direction, is then ascertained by the arc of the second graduated circle, which is included between the moveable circle and such fixed direction.

All instruments of observation for determining the position of objects on the celestial sphere are constructed and mounted on one or other of these principles; and they differ one from another in respect to the point adopted as the fixed point of reference, and the plane at right angles to the diameter of the sphere passing through that point with relation to which the position of the circle, if it be moveable, is determined.

The fixed point of reference is, in all cases, either the zenith or the pole; and the plane of reference, consequently, either that of the horizon or the equator.

When the plane of the graduated circle carrying the telescope is fixed, it is almost invariably that of the meridian, but in certain instances the circle has been fixed in the vertical

plane at right angles to the meridian, that is to say, in the plane of the prime vertical.

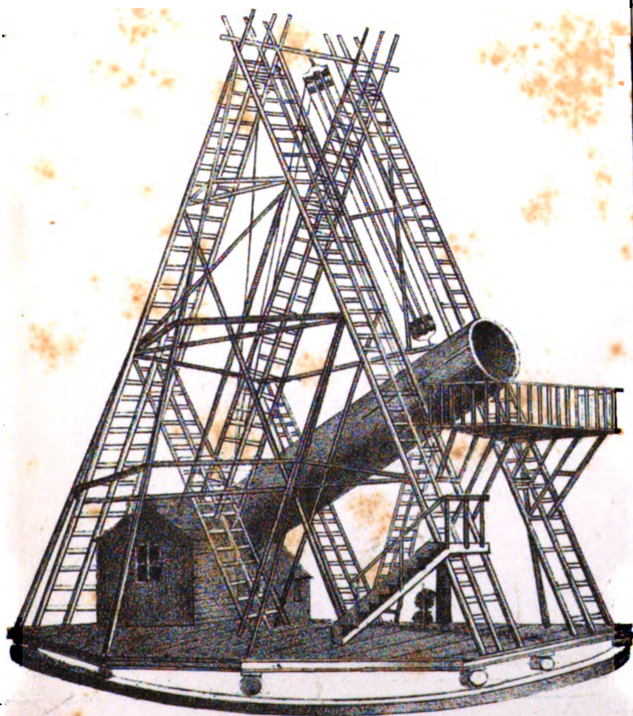
It will be obvious that the mural circle (2407.) and the transit (2397.) are meridional instruments.

When the fixed point of reference is the zenith, and the graduated circle is moveable, the immediate results of the observations made with the instrument are the zenith distance or altitude and the azimuth of the object observed. Such instruments are, consequently, denominated ALTITUDE and AZIMUTH CIRCLES, or ALTAZIMUTH INSTRUMENTS.

When the fixed point of reference is the pole and the graduated circle is moveable, the immediate results of the observations made with the instrument are the polar distance or declination and the right ascension of the object observed; and the motion of the instrument being parallel to the celestial equator, it is called an EQUATOREAL (2336.).

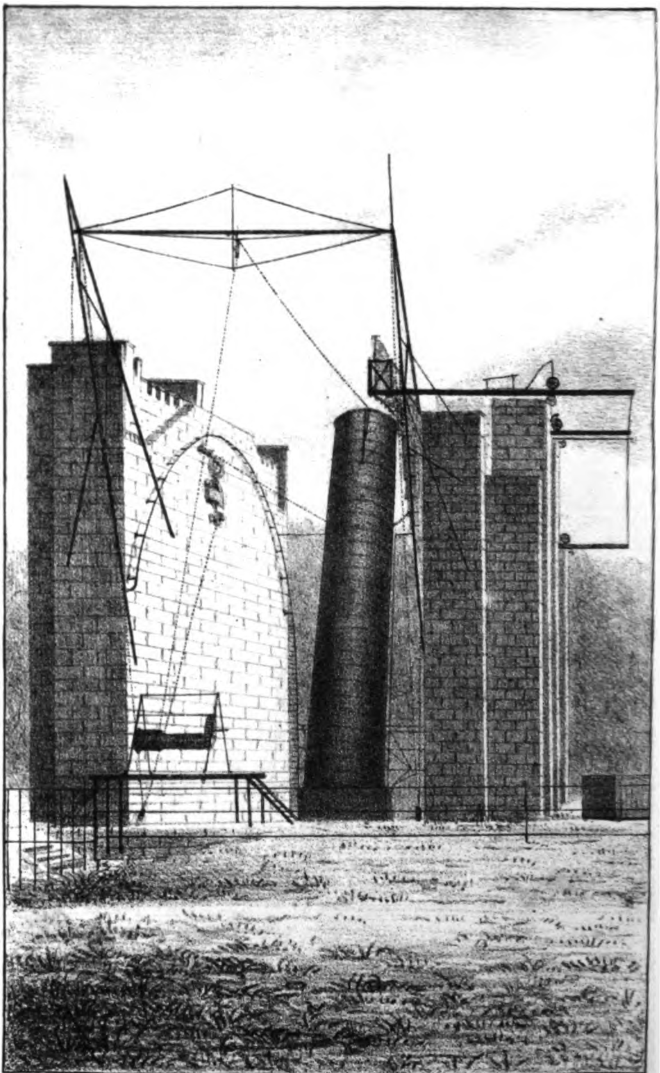
These general principles being understood, we shall give a few examples of each class of instrument, commencing with those which are adapted to the investigation of the magnitude, appearance, and structure of the celestial objects, without reference to their exact position in the firmament.

3395. *Sir W. Herschel's forty-feet reflector.* — This instrument, which is memorable as the first ever constructed upon a scale of such stupendous magnitude, and still more so for the vast discoveries made with it by its illustrious inventor and constructor, is represented in Plate XXVII. It will be seen in the drawing that the instrument is mounted on a platform which revolves in azimuth on a series of rollers. The telescope is placed between four ladders, which serve the double purpose of a framework for its support and a convenient means of approaching the superior end of the great tube. These ladders are united at the top by being bolted to a cross bar, to which the pulleys are attached. By one system of pulleys, the telescope is raised or lowered; and by another the gallery or balcony in which the observer stands is also raised or lowered, so as to enable him to look into the tube. These pulleys are each worked by a windlass established on the platform below. The framing is strengthened by another system of diagonal ladders, as well as various masts and braces which appear in the figure. The telescope is so mounted that it can be raised until its axis is vertical, so that an object in the zenith can be



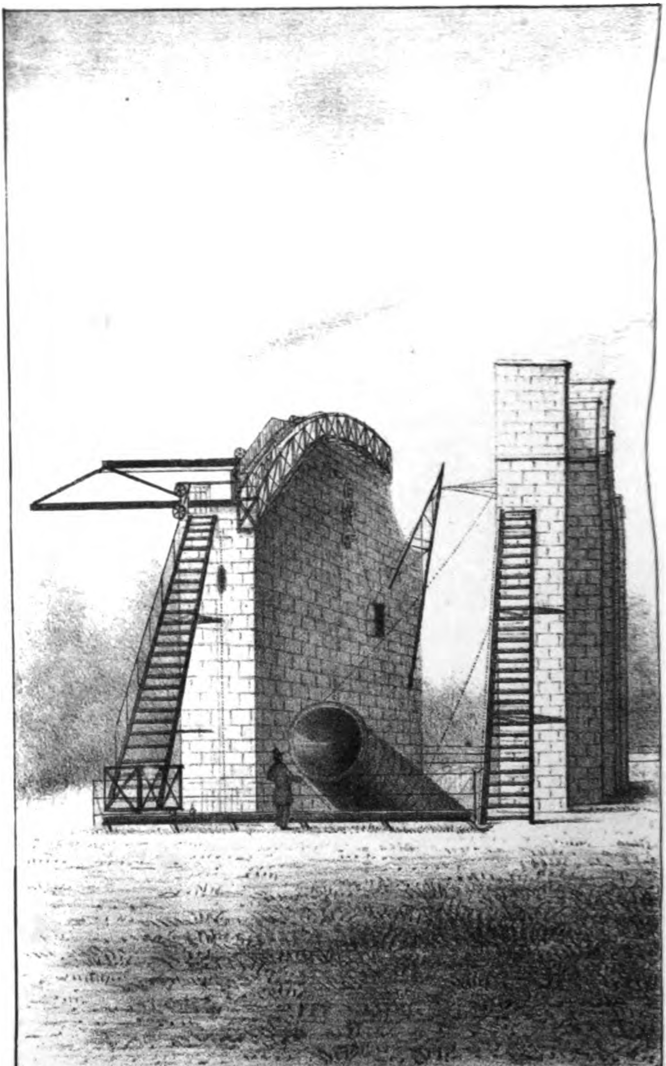
Focal length 40 f. Aperture 4 f.

The great Rosse Telescope
South Side.



I. Tuely del. & lith

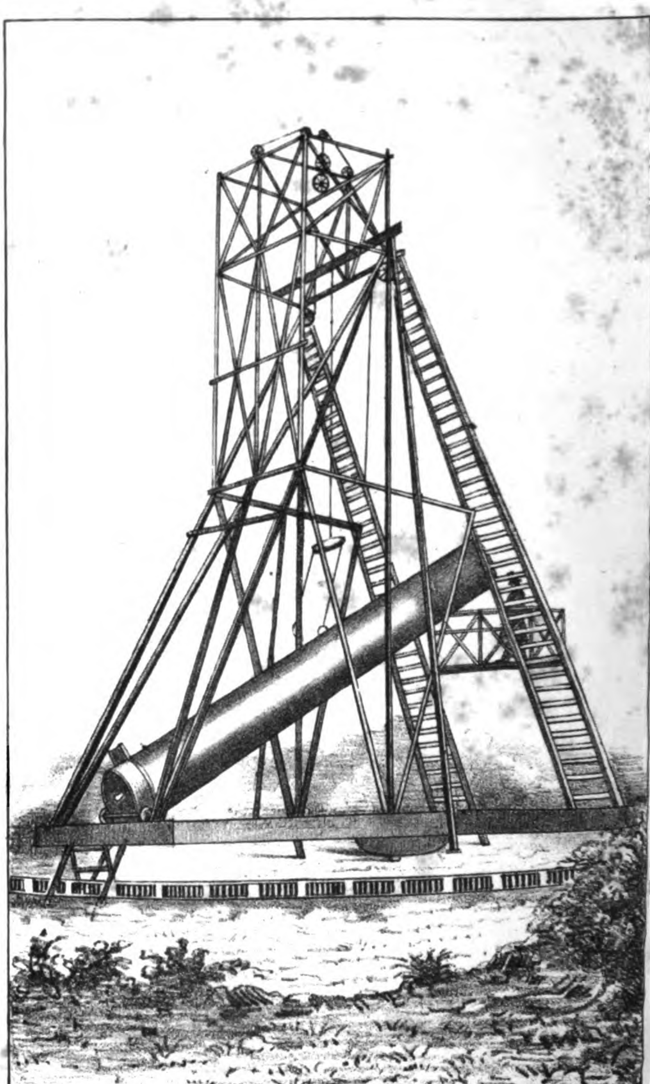
The great Rosse Telescope
North Side



L. Tenty del & lith.

Focal length 53 f. aperture 6 f.

The Lesser Rosse Telescope.



Im Lemercier, Paris

Focal length 27 f. Aperture 3 f.

observed with it. The observer's gallery rests in grooves upon the ladders, and slides up and down easily and smoothly by the operation of the pulley, so that when the telescope tube is elevated, even to the zenith, the observer can ascend and descend at pleasure by signals given to the man at the windlass. A small staircase is placed near the foot of one of the principal ladders, by which observers can mount into the gallery when it is let down to its lowest point.

The total length of the telescope tube is 39 ft. 4 in., and its clear diameter 4 ft. 10 in. It is constructed entirely of iron. The great speculum is placed in the lower end of the tube, the apparatus for adjusting it being protected by the wooden structure which appears in the figure. The diameter of the speculum is 4 ft., and the magnitude of its reflecting surface is consequently 12·566 square feet. In contains 1050 lb. of metal.

The axis of the speculum, when placed in the tube, is so inclined to the tube that its focus is at about two inches from the lower edge of the upper mouth of the tube, so that the observer, standing in the gallery with his back to the object, and looking over the edge of the tube towards the speculum, can direct an eye-piece conveniently mounted at that point upon the image of the object of observation formed by reflection in the focus.

Three persons are employed in conducting the observations: the observer, who stands in the gallery; his amanuensis, who may either be in the gallery or in the wooden house below, receiving the dictation of the observer by a speaking tube; and the person who works the windlass.

3396. *The lesser Rosse telescope.*—This instrument, with its mounting, is represented in Plate XXVIII. The arrangements are so similar to those of the Herschelian instrument described above, that they will be easily understood from the Plate without further description. The speculum is 3 feet aperture, and 7·068 square feet reflecting surface. The length of the telescope is 27 feet. It is erected upon the pleasure-grounds at Parsonstown Castle, the seat of its illustrious constructor. The weight of metal in the speculum is about 13 cwt.

3397. *The greater Rosse telescope.*—This stupendous instrument of celestial investigation, by far the largest and most powerful ever constructed, is represented in Plates XXIX. and XXX. from drawings made for this work under the superin-

tendence of his Lordship himself. Plate XXIX. presents a south, and Plate XXX. a north, view of the instrument.

The clear aperture is 6 ft., and consequently the magnitude of the reflecting surface is 28·274 square feet, being greater than that of Herschel's great telescope in the ratio of 7 to 3.

The instrument is at present used as a Newtonian telescope (1215.), that is to say, the rays proceeding along the axis of the great speculum are received at an angle of 45° upon a second small speculum, by which the focus is thrown towards the side of the tube where the eye-piece is directed upon them. Provision is, however, made to use the instrument also as an Herschelian telescope.

The great tube is supported at the lower end upon a massive universal joint of cast-iron, resting on a pier of stone-work buried in the ground, and is so counterpoised as to be moved with great ease in declination. In all such instruments, when it is required to direct them to an object, they are first brought to the desired direction by some expedient capable of moving them more rapidly, and they are afterwards brought exactly upon the object by a slower and more delicate motion. In this case, the quick motion is given by a windlass, worked upon the ground by an assistant at the command of the observer. The slow motion is imparted by a mechanism placed under the hand of the observer.

The extreme range of the telescope in right ascension, when directed to the equator, is 1 hour in time, or 15° in space; but when directed to higher declinations, its range is more extensive.

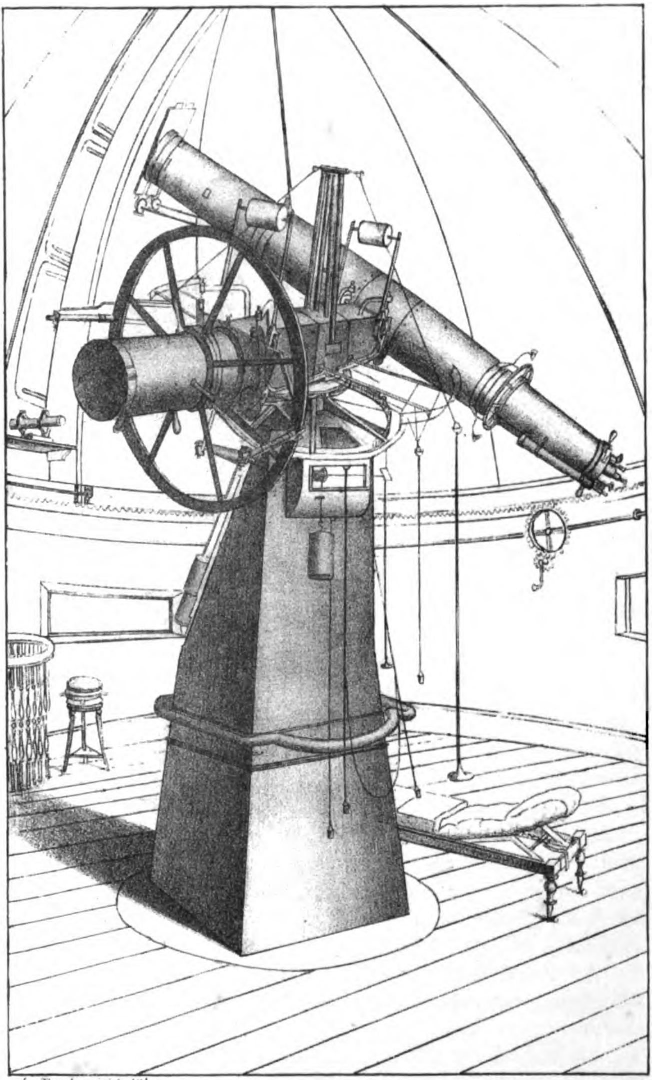
The tube is slung entirely by chains, and is perfectly steady, even in a gale of wind.

When presented to the south, the tube can be lowered until it is nearly horizontal; towards the north, it can only be depressed to the altitude of the pole. The apparatus of suspension is so arranged that the instrument may be worked as an equatoreal, and it is even intended to apply a clock-work mechanism to it.

The horizontal axis of the great universal joint, by which the lower end is supported, carries an index pointing to polar distance, and playing on a graduated arc of 6 feet radius. By this means, the telescope is easily set in polar distance.

The Oxford Heliometer.

XXXI.



L. Tank [illegible] lith.

The same object is also attained, and with greater precision, by a 20-inch circle attached to the instrument.

Two specula have been provided for the telescope, one of which contains 3½, and the other 4 tons of metal, the composition of which is 126 parts in weight of copper to 57½ of tin.

The great tube is of wood hooped with iron, and is 7 feet diameter, and 52 in length. The side-walls, 12 feet distant from the tube, are 72 feet in length, 48 feet in height on the outside, and 56 feet in the inside. These walls are built in the plane of the meridian.

The observer stands in one or other of four galleries, the three highest of which are drawn out from the western wall, while the fourth or lowest has for its base an elevating platform, along the surface of which a gallery is moved from wall to wall by a mechanism at the command of the observer.

3398. *The Oxford heliometer.* — This class of instrument, which derives its name from having been first applied to the measurement of the diameter of the sun, consists of a telescope equatoreally mounted, the object-glass of which is divided along a plane passing through its optic axis, each half of the lens being capable of being moved in its own plane, so that the axes of the two semi-lenses, being always parallel to each other and to the axis of the telescope, may be within certain limits separated from each other, more or less, at the pleasure of the observer.

From what has been explained in general of the structure of an equatoreal instrument (2336.), and from the drawing of this instrument given in Plate XXXI., the provisions for the direction of the telescope in right ascension and declination will be easily comprehended. The polar axis, round which the instrument turns in right ascension, is fixed upon the face of a block of Portland stone, and the graduated circle measuring right ascension is seen at the top and at right angles to the polar axis. This circle receives its motion in the usual way, from clockwork, which is attached to the stone pier, and which, with its impelling suspended weight, is seen in the drawing. Rods are provided by which the observer can, at pleasure, set the clock going, or stop it, and connect it with, or disengage it from, the equatoreal circle.

The circle for indicating polar distance or declination is placed upon the horizontal axis of the instrument, and also

appears in the drawing at the side opposite to that at which the telescope is attached.

The object-glass of this instrument, sometimes called the "divided object-glass micrometer," supplies a very accurate method of measuring angles which do not exceed a certain limit of magnitude.

It appears by the principles of optics, that when the image of a distant object is produced by a lens, *each point* of such image is formed by rays which proceed from *every point* of the lens. If, therefore, a part of the lens be covered by an opaque body or cut away, *each point* of the image will still be formed by the rays which proceed from *every point of the lens which is not covered or cut away.* The only difference which will be observed in the image will be, that it will be less strongly illuminated, being deprived of the rays which it received from the part of the lens covered or cut away, and that it will be less distinct in consequence of certain effects of diffraction which need not be noticed here.

It follows, therefore, that half a lens will produce at the focus an image of a distant object, and if two halves of the same lens be placed concentrically, they will form two images, the exact superposition of which will, in fact, constitute the image formed by the complete lens. But if the two halves be not concentrical, the two images will not be superposed, but will be separated by a space corresponding with, and proportional to, the distance between the centres of the two half lenses. Thus, if the lenses be directed to the sun, two images of the solar disc will be produced at the focus of the lenses, and these images may be shifted in their positions, the centres approaching to, or receding from, one another, according as the centres of the two half lenses approach to, or recede from, each other; and if the angular distance through which either image moves can be known, it is easy to see how, by this means, the apparent diameter of such an object as the sun can be measured. For this purpose, let the two half lenses be first placed concentrically, so that the two images shall be exactly superposed. Then let one of the two lenses be moved (the edges of the semi-lenses being always maintained in contact), until the image, formed by the

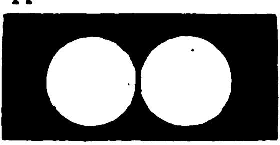


Fig. 874.

semi-lens, which is moved, shall be removed to such a position that the two images shall touch each other externally, as in *fig.* 874. In that case it is evident that the centre of the image formed by the semi-lens which has been moved, must have moved over a space equal to the diameter of the image of the disc, and if the angular value of such space be known, the apparent diameter of the sun will be known.

This was the application of the divided object-glass, from which the heliometer took its name. The instrument, however, has since been applied to so many other important purposes, that the name has ceased to express its uses.

The two semi-lenses forming the object-glass of the heliometer are set edge to edge in strong brass frames, which slide in grooves with a smooth and even motion. They are moved by fine screws which, by the intervention of cog-wheels, are turned by a pair of rods which pass along the tube of the telescope. The separation of the centres of the semi-lenses, and consequently the angular distance between the two images, is measured according to a known scale by the number of turns and parts of a turn of the screw which are necessary to produce the separation or to bring back the semi-lenses to a concentrical position, if they are separated.

It is obvious, that the same principle will be applicable to measure the apparent angular distance between any two objects, such as two stars, which are so near each other that they may be seen together in the field of view of the telescope. For this purpose, let the semi-lenses be first placed concentrically. The two stars S and S′ will then be seen in their proper positions in the field. Let the semi-lenses be then moved so that two images of each star will be visible. Let the motion be continued until the image of the star S by one semi-lens coincides with the image of the other star S′ by the other semi-lens. The angular distance corresponding to the separation of the lenses will then be the angular distance between the stars.

In this heliometer a very ingenious contrivance is introduced to enable the observer to read the scale by which the angular magnitude corresponding to the separation of the centres of the semi-lenses is indicated. This is accomplished by placing a scale behind the object-glass in the interior of the telescope tube, so that it can be read by means of a long microscope, the eye-glass of which is placed near the eye-piece of

the telescope. This interior scale is illuminated by a piece of platinum wire placed near it, which is rendered incandescent by a galvanic current transmitted upon it at pleasure by the observer. This current is produced by a Smee's battery placed in a room below that containing the heliometer.

A very splendid instrument of this class has been erected at the Pultowa observatory.

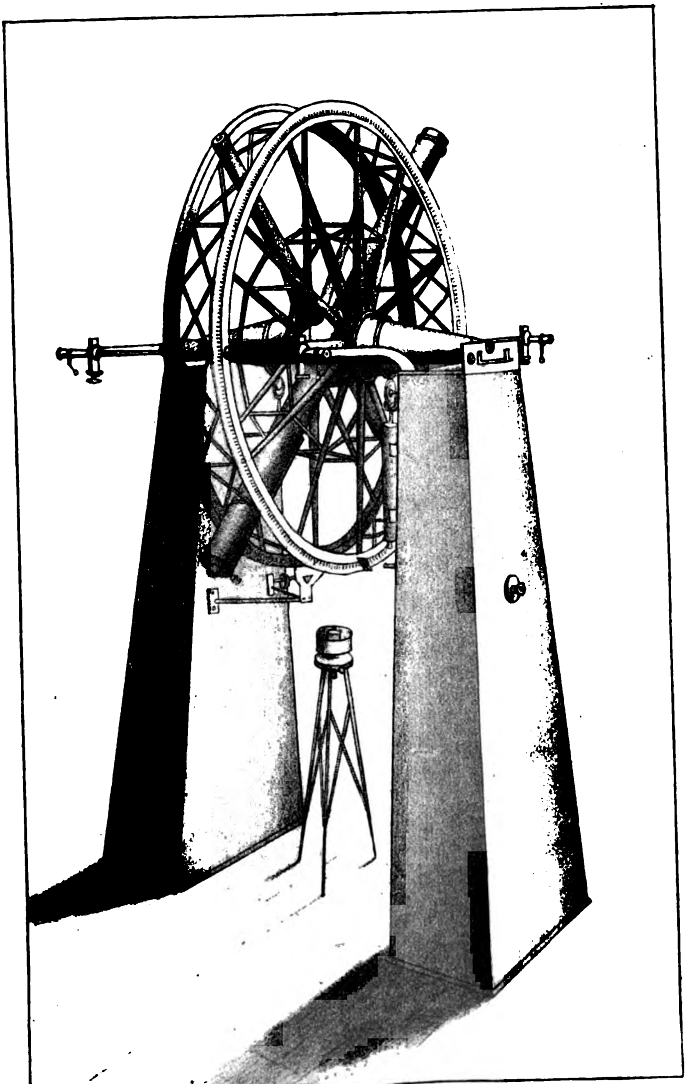
3399. *The transit circle, by Troughton.* — This instrument, which is represented in Plate XXXII., unites the functions of the mural circle and the transit instrument. The telescope is fixed between two parallel flat metallic circles or rings, the exterior face of each of which is graduated to 5′. These flat rings are connected with the horizontal axis, by two sets of radial hollow cones, so as to form two wheels, and they are connected with each other by various bars, crossing each other so as to form rhomboidal figures, as well as by a system of perpendicular rods, which appear in the figure. Each of these rings is 4 feet in diameter. The horizontal axis, which receives the spokes of each of the wheels, is cylindrical between the wheels, the parts projecting beyond them being strong cones, which rest in Ys fixed on two piers of solid stone-work, 5 feet 6 inches in height, and a little less than 3 feet apart. The length of the horizontal axis is 3 feet.

The faces of the stone piers coincide with the plane of the meridian. The Ys are provided with adjustments, one of which is capable of raising and lowering the Y, and the other of moving it horizontally through small spaces. When the instrument is placed on these supports, the line of collimation of the telescope will play nearly in the meridian, and it will be made to do so exactly, by means of the adjustments, according to the method explained in the case of the transit instrument (2398.), *et seq.*

The graduated faces of the two circles are surrounded by four or more microscopes, by which the observation is *read off* in the same manner and subject to the same conditions as have been already explained in the case of the mural circle (2408.), *et seq.*

3400. *The Greenwich transit circle.*—The great mural circle and the transit instrument have lately been superseded at the Royal Observatory, Greenwich, by an instrument upon the principle of that just described, but constructed upon a vast

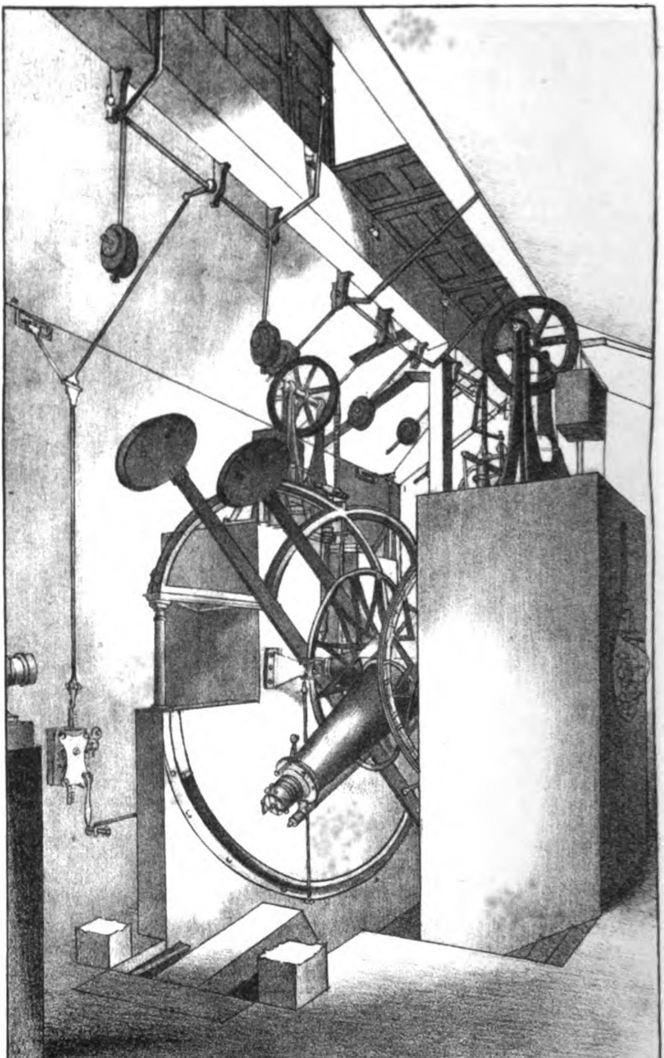
Troughton's Transit circle



L. Tanty del. & lith.

Greenwich Transit Circle

XXIII.



scale of magnitude, and combined with a variety of accessories by which its stability and the necessary precision of its indications are secured.

A perspective view of this instrument is presented in Plate XXXIII., made from original drawings taken by permission of the Astronomer Royal.

It was found, by the results of observations made with the great 10-feet transit instrument previously in use at Greenwich, that, although it was the best of its class, and had been constructed with the greatest degree of artistic skill, it was nevertheless so unstable as to produce errors in the determination of time, which it was possible, and therefore desirable, to remove by introducing improved principles of construction, which will be presently explained in relation to another instrument previously erected at the Observatory.

Like the transit circle of Troughton, already described, this instrument consists of a telescope fixed between two parallel circles, one of which is graduated, resting on horizontal supports, placed on two stone piers, so that the line of collimation moves in the plane of the meridian.

The telescope tube, which is nearly 12 feet long, consists of a hollow cube of metal, at the centre of which two cones are bolted by means of flanges. At the smaller end of one cone is the object-glass, and in that of the other the eye-piece. Each of these cones weighs 1·75 cwt., and the central cube with its pivots weighs 8 cwt. The whole length of the horizontal axis on which the instrument rests is 6 feet, the diameter of each of the bearings being 6 inches. The object-glass is 8 inches aperture, its optical power being sufficient for the observation of the faintest objects which are presented in the ordinary course of meridional observations.

In erecting the instrument experiments were made, with the view of determining the amount of drop produced by the weight of the cones of the telescope, when it was found that this quantity did not exceed the thousandth of an inch.

The parallel circles between which the telescope is fixed, are each 6 feet in diameter, and are firmly attached to cylindrical bands, one on each side of the central cube of the telescope. The clamping apparatus is applied to the eastern circle, and the western circle is graduated. The reading-off is effected by means of six microscopes, each 45 inches in length.

The graduation of the circle is such as to show approximately zenith distances; while a pointer fixed to a block projecting from the lower part of the pier, directed to another graduated band on the outer or eastern side of the circle, is used for setting the telescope, and gives approximately north polar distances. A small finder, with a large field of view, is attached to the side of the cone near the eye-piece, as well as sights for directing the telescope to stars by the naked eye.

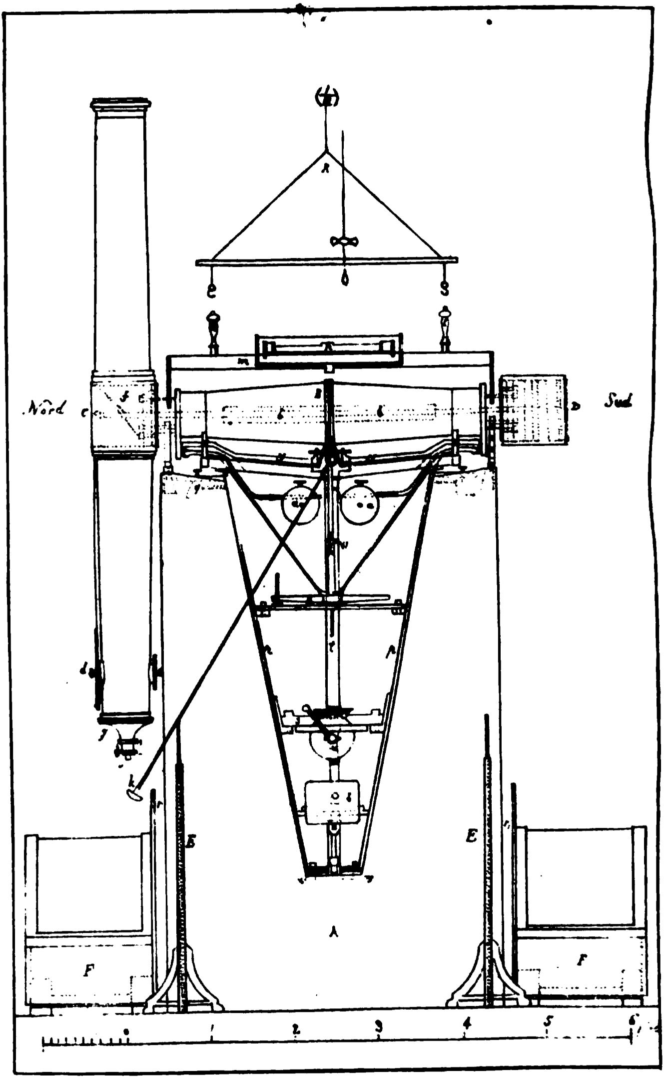
A large gas-light conveniently placed illuminates, by means of reflectors, the graduated arc of the circle at the points where the several microscropes are fixed, and also the field of the telescope.

A variety of other provisions and adjustments are attached to the instrument, which it would be impossible to render clearly intelligible without reference to the instrument itself, or very detailed and elaborate drawings of its several parts, which our limits do not permit us to introduce here.

3401. *The Pultowa prime vertical instrument.* — This instrument may be summarily described as a transit, whose line of collimation moves in the plane of the prime vertical, instead of that of the meridian. Nevertheless, its astronomical uses are essentially distinct from those of the transit instrument (2397.).

The first instrument made on this principle was erected, in the beginning of the last century, under the direction of the celebrated Roemer, whose name is rendered memorable by the discovery of the mobility of light (2959.). It was applied by that astronomer chiefly to observations on the sun near the equinoxes; but none of the purposes to which it has more recently subserved appear to have been contemplated, and the instrument was allowed to fall into disuse. Its revival, and the idea of its application to various important classes of observations in the higher departments of practical astronomy, and more especially to replace the zenith sector in observations having for their object the more exact determination of aberration and nutation, and for researches in stellar parallax, is due to Professor Bessel. Many of the improved details of construction exhibited in the Pultowa instrument are, however, due to Professor Struve, who, besides, has obtained such remarkable results by the system of observations which he has made with it.

Pultowa Prime Vertical Instrument.



The Pultowa prime vertical instrument was constructed, under the direction of Professor Struve, by Messrs. Repsold, of Hamburg. Two stone piers being erected in planes at right angles to the meridian, vertical chairs are fixed upon their summits in such a position that the line joining them is in the plane of the meridian. These chairs are the supports of the cylindrical extremities of the horizontal axis of the instrument, which is, therefore, also in the plane of the meridian. The extremities of this axis project beyond the chairs and the piers on each side, and the transit telescope is keyed on to one of them, while a counter-weight is keyed on to the other. The telescope, having its line of collimation adjusted at right angles to the horizontal axis, revolves with this axis outside the piers, in the same manner exactly as the transit telescope revolves between its piers; and as the line of collimation of the latter moves in the plane of the meridian, that of the transit telescope of the present instrument moves in the plane of the prime vertical.

Adjustments are provided in connection with the two chairs, one of which raises and lowers the axis, and the other moves it in azimuth, similar exactly to those described in the case of the transit instrument (2399.), *et seq.* By these means, and by proper levels, the axis is rendered truly horizontal, is brought exactly into the plane of the meridian, and the line of collimation is brought to coincide with the plane of the prime vertical by other expedients, similar in principle to those adopted in the case of the transit instrument.

The instrument, mounted on the piers, is represented in Plate XXXIV., as seen from the west, projected on the plane of the meridian, the telescope being on the north side, and placed so that the line of collimation is directed to the zenith. The telescope has 7 feet 7 inches focal length, with an object-glass having a clear aperture of 6·25 inches. The magnifying power commonly used is 270. In the eye-piece a system of seven parallel vertical micrometer wires is fixed, similarly to those of the transit instrument (2396.), and is similarly used with relation to the clock, as already described in the case of the latter instrument. A lamp is placed at a convenient distance from the centre of the telescope, the light of which, admitted by a plate of glass fixed in the side of the tube, is received upon a small

reflector at 45° within, and reflected along the tube, so as to illuminate the wires at night.

To enable the observer to direct the telescope to any required altitude, a small telescope, called a *finder*, is fixed to the outside of the great telescope, near the eye-piece, having attached to it a graduated circle, the plane of which is parallel to the prime vertical, and also a level. The line of collimation of the finder being parallel to that of the great telescope when the former is directed to any altitude by means of the level and graduated circle, the former will be similarly directed. This finder appears in the drawing outside the telescope, and a counterpoise to it is represented on the inside.

The process of reversion of the horizontal axis, which in the transit instrument is only used for the purpose of adjustment (2400.), constitutes, in the case of the prime vertical instrument, an essential part of every observation. It was, therefore, of the greatest importance that an easy, expeditious, and safe apparatus for reversion should be provided. This was contrived with great ingenuity by the makers, and attended with the most successful results, results to which M. Struve ascribes a great share of the advantage obtained by this instrument. A part of this apparatus, by which the horizontal axis, with the telescope, counterpoise, and their accessories, is elevated from the chairs, is represented in the drawing above the instrument. The two cords of suspension being attached by hooks to two points on the axis at equal distances from its centre, so as to maintain the equilibrium, the instrument is elevated by means of a windlass established on the floor below it and between the piers. When raised to the necessary height, it is turned through half a revolution in azimuth, so that the ends of the axis are brought directly over the chairs, into which they are then let down. So perfect is the performance of this apparatus, that, notwithstanding the magnitude and weight of the instrument, the whole process of reversion is completed in sixteen seconds; and the interval, from the moment the observer completes an observation with the telescope on the north side, to the moment he commences it on the south side, including the time of rising from the observing-couch, disengaging the clamps, withdrawing the key from the micrometer, reversing, directing the instrument on the south side to the object by means of the finder, closing the clamps, returning the key to the micro-

meter, and placing himself on the observing-couch, is only 80 seconds.

How essential to the practical use of the instrument this celerity is, will be understood when it is stated, that the same object which has been observed on one side must be also observed on the other *in the same transit.* The reversion, therefore, must be completed in less time than that which the object takes by the diurnal motion to pass over the space commanded by the field of the telescope in the two positions.

To comprehend the method of applying this instrument to the purposes of practical observation, it is necessary to remember that it is only applicable to objects moving in parallels of declination which intersect the prime vertical. Such objects must have northern declination (the instrument being supposed to be established in a place having north latitude), and a polar distance greater than that of the zenith of the observatory, that is to say, greater than its co-latitude. The parallels over which such objects are carried by the diurnal motion, all intersect the prime vertical at two points of equal altitude, one on the eastern, and the other on the western, quadrant of that circle. In passing from the east point of intersection to the west point, the object passes over the meridian, and it is evident that the moment of its meridional transit is precisely the middle of the interval between its two prime vertical transits. If, therefore, the exact times of the latter be observed, the time of the meridional transit can be deduced by a simple arithmetical process.

To prepare the instrument for observation, let the polar distance of the object about to be observed be taken from the Tables. Let it be expressed by ϖ; let the co-latitude of the observatory, or, what is the same, the polar distance of the zenith, be λ; and let the zenith distance which the object must have when it comes on the prime vertical, be z. These three arcs, ϖ, λ, and z, form a right-angled spherical triangle, the right-angle being included by λ and z. Let the angle at the pole, or the hour angle, be h. We shall then, by the elementary principles of trigonometry, have

$$\cos. z = \frac{\cos. \varpi}{\cos. \lambda} \ (1.); \qquad \cos. h = \frac{\tan. \lambda}{\tan. \varpi} \ (2.).$$

By the formula (1.), the zenith distance of the points at

which the object will cross the prime vertical is known. The observer, by means of the finder, and the circle and level attached to it, directs the great telescope to that zenith distance on the eastern quadrant of the prime vertical; and, although the exact position of the object is the thing sought, its approximate position is already known with that degree of precision which ensures its passage through the field of the instrument when it is set to the zenith distance given by the calculation made from the tabular polar distance.

When the telescope, being on the north side, is thus directed and clamped in its position, the observer awaits the transit, the time of which he already knows approximately by the formula (2.), which gives the hour angle from the meridian when the object is over the prime vertical. At the near approach of the transit he places himself on the observing-couch, and, seeing the object enter the field, notes the moments by the clock of its transits over the seven wires of the micrometer. Let these times be expressed by T_1, T_2, T_3, T_4, T_5, T_6, and T_7.

The moment the transit over the seventh wire has been observed he rises and performs all that is necessary for the reversion of the instrument, which being completed, he again places himself, and observes the transits over the seven wires on the south side; but in this case, owing to the change of position, the order of the transits is reversed. Let the times of transit be expressed by t_7, t_6, t_5, t_4, t_3, t_2, and t_1.

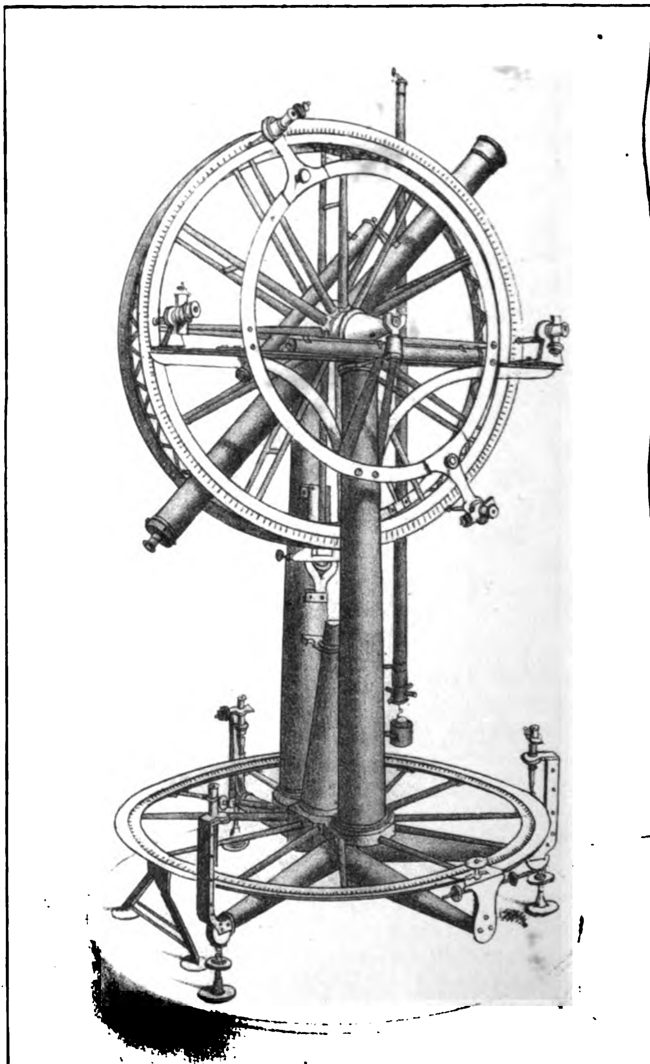
Now, it is evident that the true moment of the transit over the prime vertical will be found by taking a mean between the times of the transits over all the wires at both sides. If this time be expressed by T', we shall then have

$$T' = \frac{T_1 + T_2 + \ldots + T_7 + t_7 + t_6 + \ldots + t_1}{14}.$$

These observations being completed, the observer awaits the transit of the object over the western quadrant of the prime vertical, when he makes a similar series of observations on the transits, first with the telescope on the south side, in the position it had at the last observation, and then, after reversion, at the north side. The true moment of the transit is found, in this case, in the same manner as in the former.

By taking a mean of these two means, or, what would be

Troughton's Altitude and Azimuth Circle



L. Tinly, del. & lith

equivalent, a mean of the times of all the twenty-eight transits, the time of the meridional transit will be obtained.

The total length of the interval, necessary to observe the transits over the wires, north and south, in each quadrant of the prime vertical, is found to be about eleven minutes, less than 1¼ minute of which is employed in the reversion of the instrument and attendant arrangements.

The time which elapses between the observations on the eastern and western quadrants of the prime vertical, will necessarily vary with the polar distance of the object, and will be less in proportion as excess of that distance above the co-latitude is less. The observations which have been made with this instrument at Pultowa, have been chiefly confined to stars whose polar distance exceeds the co-latitude by less than 2°. In that case, the interval between the observations, east and west, would be less than three hours.

Professor Struve notices, in strong terms, the advantage which this instrument possesses over others in respect to the errors arising from the variation of the inclination of the line of collimation to the axis of rotation. In the prime vertical instrument, the deviation of the line of collimation from true perpendicularity to the axis of rotation, is assumed to be invariable only during the short interval of a single observation, whereas, in other instruments, its invariability is assumed for twelve hours, and in some cases for months, and even years. It has the further great advantage, that, by reversion in each quadrant, east and west, all optical imperfections which affect the precision of the image of the star are absolutely annihilated.*

3402. *Troughton's altitude and azimuth circle.*—The form of instrument best adapted to render the principle of altitude and azimuth instruments in general intelligible, is that which is represented in Plate XXXV. This circle was originally constructed by Troughton for the Royal Academy of St. Petersburg; but, at the time of the invasion of Russia by the French, the fear that Petersburg might be exposed to the same disasters as Moscow induced the Russian authorities to relinquish their claim on the instrument, and it passed into the hands of

* For a detailed account of the Pultowa prime vertical instrument, see Description de l'Observatoire Astronomique de Pultowa, par F. G. W. Struve. Also Astronom. Nachrichten, No. 468, *et seq.*

Dr. Pearson, at whose private observatory, at Kilworth, it was erected. Owing to the indisposition of Troughton, the graduation of the limbs was executed by Jones.

The drawing presents the circle so that all the important parts of it are visible. The instrument is supported on the capstone of a strong stone pedestal. Upon this, the fixed parts of the instrument, upon which it revolves in azimuth, are firmly established. They consist of a solid vertical cone, terminating below in an hexagonal solid mass of metal, each vertical face of which measures 3 inches square. From this proceed four radial cones, three of which, at angles of 120° with each other, are supported, on the stone pedestal, by feet supplied with adjusting screws, by means of which the instrument is levelled. The fourth radial cone, which forms an angle of 60° with two of the others, carries a clamp, which will be presently noticed.

The moveable part of the instrument terminates at the lower part in a hollow cone, which appears in the figure between the two vertical pillars, with which it is connected by a metallic collar. This hollow cone rests upon and conceals the solid cone already described, and, turning freely upon it, gives to the instrument its azimuth motion.

A horizontal circle, 3 feet in diameter, is connected by conical spokes, in the usual manner, with the lower ends of the two vertical pillars and the intermediate hollow cone.

To the three radial cones of the fixed base of the apparatus, at angles of 120°, are attached three pieces, which rise vertically outside the circle, and support three microscopes, by which the azimuthal angles are read off. This position for the microscopes gives some practical advantage, inasmuch as the observation is read at six different points of the limb, when the observation is repeated by turning the circle 180° in azimuth. To the fourth radial arm of the fixed base a clamp is attached, by which the position of the instrument is maintained in the position it has at the moment the observation is made. This clamp appears in the drawing with its tightening screw above it, and its tangent screw for giving the azimuth circle a slow motion to bring the instrument to its exact position. This tangent screw terminates in a square, which can be inserted in a square hole of corresponding magnitude, connected by an universal joint with a rod of convenient length, which the

observer holds at the moment of the observation, and by turning which the whole instrument is moved slowly in azimuth.

The vertical circle consists of two parallel limbs, having the telescope between them, constructed and connected in a manner exactly similar to the transit circle already described; and this circle is supported on the two vertical pillars in the same manner exactly, as already described in the case of the transit circle (3399).

One only of the two vertical circles is graduated, being divided as well as the horizontal circle to 5′. The clamp, with its tangent screw, is placed on the side not graduated. The focal length of the telescope is 44·4 inches, and its aperture $3\frac{1}{4}$ inches.

The observations of altitude are read off by four microscopes, the supports of which are shown in the figure. Two are supported by arms attached to the vertical pillar on the graduated side of the circle, and the other two to a hoop screwed on the pillar and to the arms which carry the two former.

The instrument is provided with a plumb-line and four levels, by which its proper position with relation to the vertical direction can be always verified.

3403. *The Greenwich alt-azimuth instrument.* — This, as the name imports, is an altitude and azimuth circle in principle similar to that just described, but in its construction and application different from any instrument of its class hitherto constructed. The purpose chiefly to which it is applied, and with the view to which it was conceived by the Astronomer Royal, is the improvement of the lunar theory by multiplying in a large ratio the observations which can be made from month to month on the moon, without in any degree impairing their precision. Such observations were always made with the mural circle and the transit, until this instrument was brought into operation, and they were consequently confined to the meridional transits of the moon. Now, these transits cannot be observed, even when the firmament is unclouded, for four days before and four days after the new moon, in consequence of the proximity of that body to the sun; an interval amounting to little less than one-third of the month. Besides this, it happens, in this climate, that, at the moment of the meridional transits at other parts of the month, the observation is frequently rendered

impracticable by a clouded sky. It was, therefore, highly desirable to contrive some means of making the observations in extra-meridional positions of the moon.

This could obviously be accomplished by means of an altitude and azimuth circle, such as that described above; but such an instrument, however perfect might be its construction, is not susceptible of the necessary precision. The Astronomer Royal, therefore, conceived the idea of an instrument on the same principle, which, while it would be capable of shifting its azimuth, would still be susceptible of as much precision in each vertical in which it might be placed, as the mural circle has in the meridian. He accordingly proposed to attain this object by adopting adequate engineering expedients to produce the necessary solidity and invariability of form. He adopted, as fundamental principles of construction, —

1. To produce as many parts as possible in a single casting;
2. To use no small screws for combining the parts;
3. To allow no power of adjustment anywhere.

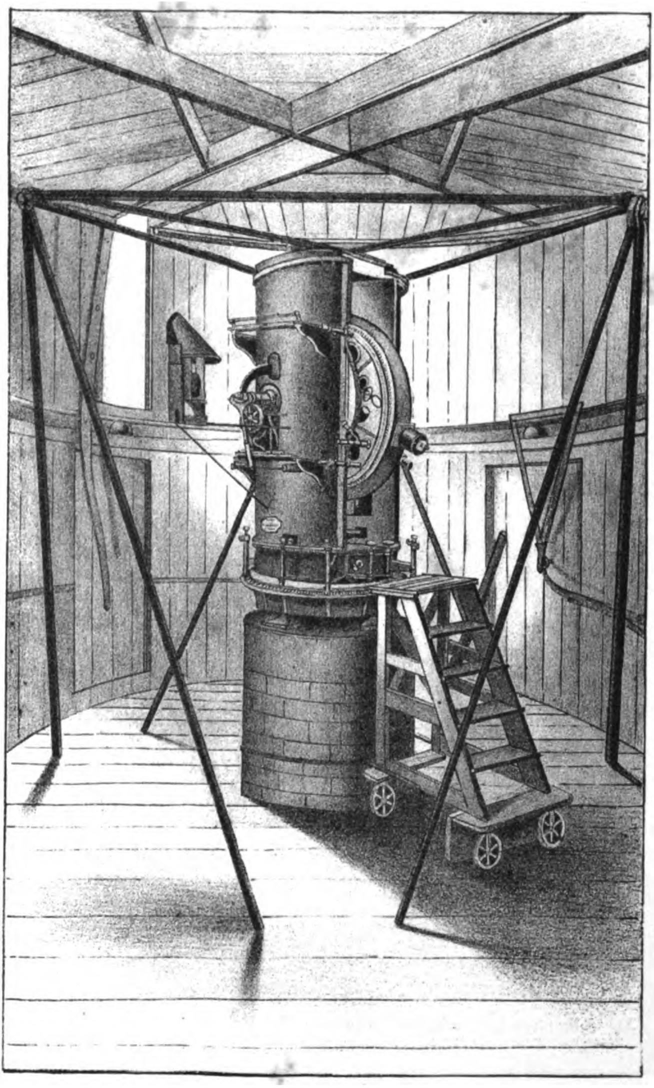
Following out these principles, the instrument represented in Plate XXXVI. was constructed, under his superintendence, by Messrs. Ransome and May, engineers, of Ipswich; the graduation being executed by Messrs. Troughton and Simms: and the first observation with it was made by Mr. Hugh Breen, jun., one of the assistants at the Observatory, on the 16th of May, 1847, which was found to be as good as any succeeding observation.

The instrument is mounted in a tower, raised to such a height as to command the horizon in all directions above the other buildings of the Observatory, except on the side of the south-east dome and the octagon room. The foundation of the instrument is a three-rayed pier of brickwork, carried up nearly to the level of the floor of the room appropriated to the instrument. Upon this pier is placed a cylindrical stone pillar, 3 feet in diameter, which appears in the drawing, and on which the instrument is placed. This pillar and the pier upon which it reposes are quite independent of, and unconnected with, the tower within which it is erected, and do not even touch the floor of the room through which they pass.

The fixed horizontal azimuth circle is solidly established upon this stone pillar. It is a circle 3 feet in diameter, the

Greenwich altazimuth instrument.

XXXVI



rim being connected with the centre in the usual way by spokes. The whole is constructed of hard gun-metal. In the upper surface of the rim a circular groove is left, which is filled with a band of silver, on which the divisions are engraved. This circle is divided into arcs of 5′ continuously from 0° to 360°. It is set with the zero towards the north and the numbering of the divisions runs from north to east, south and west. This azimuthal circle was cast in a single piece, and weighs 441 lbs.

Attached to this, and concentric with it, is another fixed horizontal circle, having teeth on the inside edge, in which the pinions work by which the azimuth motion is given to the instrument.

There are four microscopes placed at equal distances over the graduated arc, which are provided with micrometers, by which the observation in azimuth is read off. These microscopes are attached to the instrument so as to revolve with it. Their reflectors are illuminated by a lamp properly placed.

The lower pivot on which the instrument turns, is supported on a point in the stone pillar at the centre of the azimuthal circle. To support the upper pivot, an iron triangle is established on the three-rayed pier. On each side of this, is erected another iron triangle, whose plane is vertical, and whose sides unite in a vertex which forms one of the angles of a corresponding triangle above. This upper triangle supports three radial bars, which carry at their point of union the Y in which the upper pivot plays. The bars of the lateral triangles, which are apparent in the drawings, pass the holes in the floor without touching it.

The frame, revolving in azimuth and carrying the instrument with it, consists of a top and bottom connected by vertical cheeks, all of cast-iron. The supports of the four microscopes for reading off the azimuth on the lower circle are cast in the same piece with these vertical cheeks.

The vertical circle carrying the telescope is 3 feet in diameter, and, like the azimuth circle, is made of hard gun-metal. The aperture of the object-glass is $3\frac{3}{4}$ in. The top and bottom of the instrument each carries two levels, parallel to the plane of the vertical circle.

The dome over the instrument is cylindrical, with double

sides, between which the air passes freely. Its diameter is 10 feet.

The drawing represents the instrument as in use. The ladder revolves in azimuth, with the instrument, round the central pier,—to facilitate which motion, rollers are placed under it. Two boards are attached to the revolving frame, having their edges in a plane parallel to that of the vertical circle. The eye being directed along these to view the object, the instrument is placed very nearly in the proper azimuth, and the telescope is then accurately directed to the object by the ring-finder. These boards are omitted in the drawing.

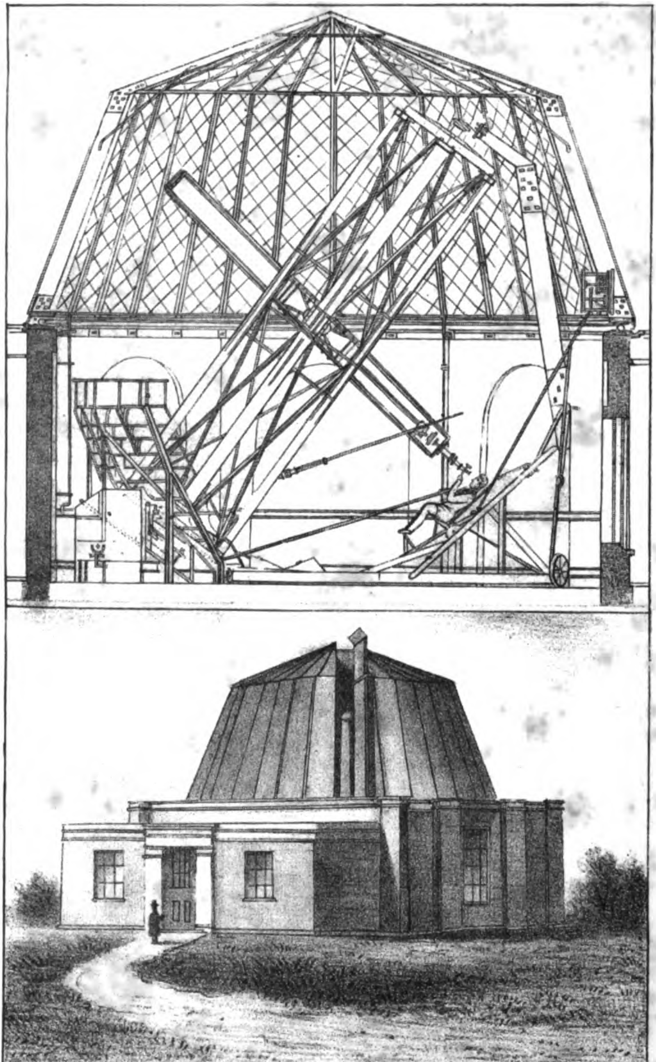
The drawing has been reduced from an engraving prefixed to the "Greenwich Observations for 1847," which, however, was originally published in the "Illustrated London News."

The results of the observations made with this instrument are stated to have fulfilled all the anticipations of the Astronomer Royal, as well as to the number of observations as to their excellence. At least twice the number have been made and of equal goodness with those formerly made with the meridional instruments. Some have been made even with a day of conjunction; and Mr. Main, the chief assistant at the Observatory, has expressed his conviction that in a few years observations with this instrument will remove all the deficiencies which still remain in the lunar theory.

3404. *The Northumberland telescope—Cambridge observatory.*—The late Duke of Northumberland, who filled during the latter part of his life the high and honourable office of Chancellor of the University of Cambridge, presented to that university this instrument, which, successively in the hands of the Astronomer Royal and Professor Challis, has contributed so effectually to the advancement of astronomical science.

The instrument, of which a perspective view is given in Plate XXXVII., together with a view of the building in which it is erected, consists of a refracting telescope of 19½ feet focal length and 11½ inches aperture equatoreally mounted. The polar axis, as appears in the drawing, consists of a system of framing composed of six strong deal poles, attached at the ends to two hexagonal frames of cast-iron, the centres of which support the upper and lower pivots on which the telescope revolves. These poles at the middle are braced by transverse iron bands, and by a system of diagonal rods of deal

Northumberland Equatoreal
Cambridge Observatory



abutting near the middle of the poles. These give stiffness to the entire framing of the polar axis and maintain the hexagonal frames square to it. Efficient means are provided to give elasticity to the supports of the pivots and smoothness to the equatoreal motion.

The tube of the telescope is made of well-seasoned deal, and attached to one side of it is a flat brass bar, 6 feet long, carrying a small graduated arc at right angles to it at one end, and turning at the other on a pin fixed in the telescope tube at a distance of 30 inches from the axis of revolution. This arc, which is called the declination sector, serves to measure small differences of declination, and is read by a micrometer microscope fixed to the telescope tube.

The hour circle, which measures the equatoreal motion, is 5½ feet diameter, and is so arranged that it can be clamped to the telescope, or disengaged from it, at pleasure. It has two indexes with verniers, one fixed to the support of the lower pivot, and the other to the hexagonal frame. By setting the latter to a certain angle, determined by an observation of a star of known right ascension, the telescope can be directed to any proposed right ascension by means of the other index. Observations of right ascension can be made to 1 second of time. The outer rim of the circle is cut into teeth, which are acted on by an endless screw connected at pleasure by a brass rod with a large clock, by which a motion can be given to the telescope corresponding with the diurnal motion of the heavens.

The hour circle is clamped to the frame of the axis by a tangent screw clamp fixed to the frame itself, by means of which, with the aid of a handle extending to the place of the observer, he can, when the endless screw is applied, give motion to the instrument through a limited space upon the hour circle. The rate of motion given to the hour circle by the clock is not affected by this movement. The hour circle, therefore, going according to sidereal time, small differences of right ascension can be measured by reading off the angles pointed to by the moveable index before and after the changes of position.

The dome which covers the instrument, and which, as well as the other details of its erection, was constructed under the direction of the Astronomer Royal, who was then the Cambridge astronomer, is supported so as to revolve on free balls between concave channels, holdfasts of peculiar construction

being provided to obviate the eventuality of the dome being dislodged or blown off by wind or any other unusual disturbance. The winch which acts on the machinery for turning the dome, is carried to the observer's chair, so that he can, while engaged in a long observation, turn the dome slowly without removing from his position.

The magnitude of the instrument, and the consequent extensive motion of the eye-piece, rendered it necessary to contrive adequate means by which the observer could be carried with the eye-piece by a common motion without any personal derangement which might disturb the observation. This is accomplished by means of an ingenious apparatus consisting of a frame, of which the upper edge is nearly a circular arc whose centre is the centre of the telescope, which frame traverses horizontally round a pin in the floor exactly below the centre of the telescope, the observer's chair sliding on the frame. The observer can, by means of a winch placed beside his chair, turn round the frame on which the chair is supported, and by means of a lever and ratchet wheel he can raise and lower the chair on the frame. He has also means of raising and depressing the back of the chair so as to give it the inclination he may at the moment find most convenient.

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Note. — This Index refers to the numbers of the paragraphs, and not t[o] comprises the entire contents of the three courses which compose th[e] may be convenient to the reader, who uses the work for the purpos[e] remember that the second course begins at paragraph 1304; and the th[ird] 2160.

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THE END.

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DELIVERED IN UNIVERSITY COLLEGE HOSPITAL.

BY RICHARD QUAIN, F.R.S.

Professor of Clinical Surgery in University College, and Surgeon to University College Hospital.

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